Estimation and Detection (Fall - 2024)

*** Total points: 101**

*** Due 01/05/2025**

For all simulations, let the seed be 101, and the noise is WGN with variance σ^2 and zero mean, and please attach the code with your handwriting.

In the DC level in WGN problem, let $A \sim u[-A_0, A_0]$ and the noise $w[n] \sim N(0, \sigma^2)$. We would like to obtain Bayesian estimators using different risk functions.

1. (5 points)

   Derive the posterior probability $p(A|\mathrm{x})$.

   Now, let $A_0 = 2$ and $\sigma^2 = 0.1$.

2. (10 points)

   Let the number of observations x at each realization of A be $N = 10$. Randomly generate one realization of A. Plot $p(A|\mathrm{x})$, and $\hat{A}_{MMSE}$ (with bold dot in the figure) for 15 different realizations of x

3. (5 points)

   Now let $N = 100$. Repeat (2).

4. (12 points)

   Following (2), now let the realizations of A be $M = 1000$. Obtain the Bayesian MSE $B_{mse}(\hat{A}_{MMSE})$ using Monte Carlo simulations.

5. (5 points)

   Following (2), and let $N = 100$, repeat (4).

6. (32 points)

Now, the risk function is absolute error. Repeat (2)-(5). Your answer should be as detailed as possible to obtain full credit. Especially "HOW" you obtain $\hat{A}$ either in mathematical representation or via Matlab program.

7. (32 points)

   Now, the risk function is Hit-or-risk. Repeat (2)-(5). Your answer should be as detailed as possible to obtain full credit. Especially "HOW" you obtain $\hat{A}$ either in mathematical representation or via Matlab program.