Estimation and Detection (Fall - 2024)

*** Total points: 100**

*** Due 12/02/2024**

For all simulations, let the seed be 101, and the noise is WGN with variance σ^2 and zero mean, and please attach the code with the handwriting part.

1. (100 points)

   In this homework, we'd like to evaluate the modified DC level in WGN problem in Ch7, which has noice variance A.

   (a) (5 points) Derive the MLE.

   (b) (15 points) Use the Monte Carlo mentioned in the class and shown below.

   $\widehat{\mathbb{E}\{\widehat{A}\}} = \frac{1}{M}\sum_{m=1}^{M}\widehat{A_m}$, $\widehat{\text{var}\{\widehat{A}\}} = \frac{1}{M}\sum_{m=1}^{M}(\widehat{A_m} - \widehat{\mathbb{E}\{\widehat{A}\}})^2$.

   Let $A = 1$ and $M = 10000$. Plot the Monte Carlo mean as a function of N for $N = 2 : 1 : 100$. Also, plot the Monte Carlo variance as a function of N for $N = 2 : 1 : 100$.

   (c) (10 points) Plot the histogram of the MLEs and the theoretical pdf in the same figure for $N = 5$, 20 and 100 like that in P.18 of the slides.

   (d) (15 points) We say that the MLE is consistently unbiased. We'd like to verify how large the value of N should be to validate this claim. "Conduct an experiment" and "explain with figures in details" the

minimum value of N should be to achieve an MSE of A less than 10^{-3}.

(e) (20 points) We say that this MLE asymptotically achieves the CRLB, and this was explained by using Taylor approximation. Using the same parameter settings, plot the histogram of the M values of u for $N = 5$, 20 and 100 separately. Please conduct an experiment and explain how accurate this Taylor approximation. (Hint: you may check the dynamic range of u for different values of N, and conclude an upper bound and lower bound of the approximate error via Monte Carlo simulation.)

(f) (20 points) Following (e), use this Taylor approximation and obtain the M MLEs for $N = 5$, 20 and 100 separately and plot the histograms. Does they look like Gaussian? Please have a discussion about what you have observed.

(g) (10 points) Following (e) and (f), similar to (d), we'd like to verify how large the value of N should be to validate the Taylor approximation. "Conduct an experiment" and "explain with figures in details" the minimum value of N should be to achieve an MSE of A less than 10^{-3} with Taylor approximation.