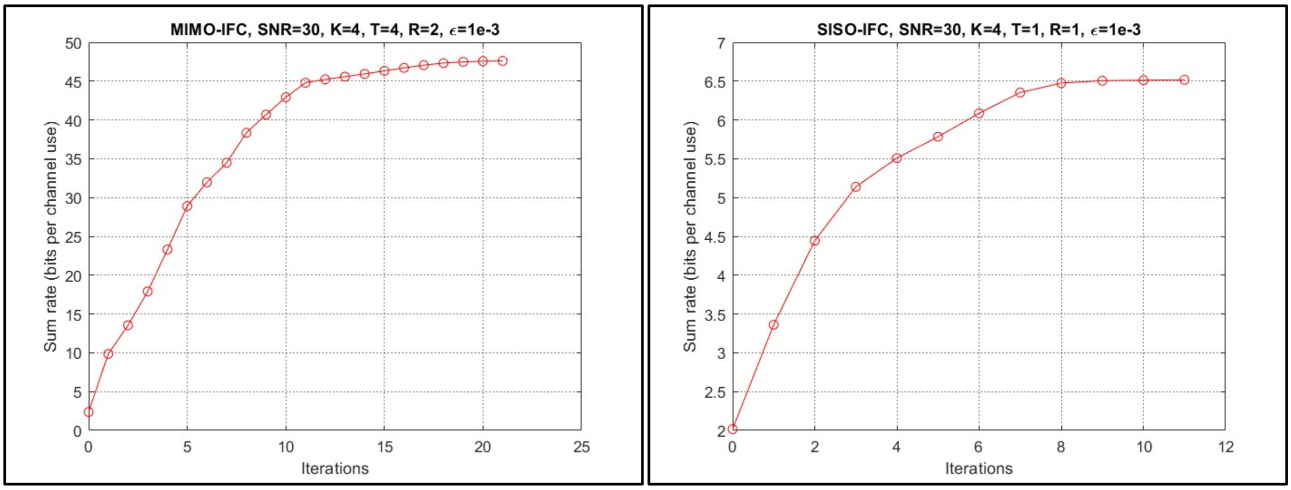
[1] Q. Shi, M. Razaviyayn, Z. -Q. Luo and C. He, ”An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel,” in IEEE Transactions on Signal Processing, vol. 59, no. 9, pp. 4331-4340, Sept. 2011.

(a) Plot the convergence properties, and discuss the simulation results.

a) Consider the MIMO case K = 4, T = 4, R = 2, I = 4, d = 4, SNR = 30 dB.

b) Consider the SISO case K = 4, T = 1, R = 1, I = 4, d = 4, SNR = 30 dB.



< discussion >

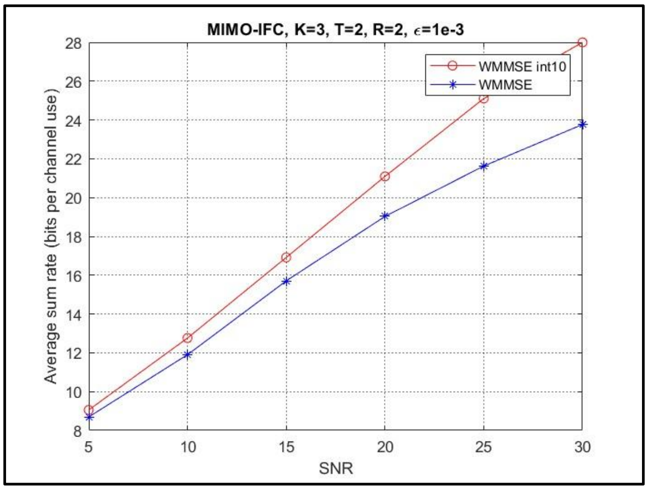
1. Since $d \le \min\{T, R\}$, the **SISO** case can transmit at most $d = 1$ data stream, which significantly **limits the achievable sum rate**.

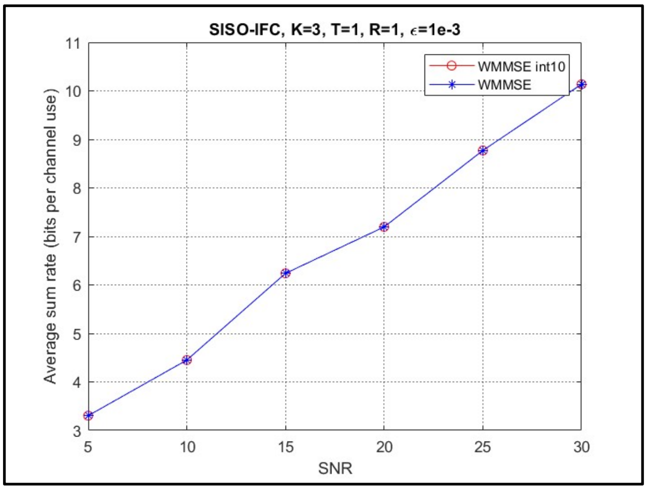
2. In the **SISO** case, the number of data streams per user is restricted to one, which eliminates interference among multiple streams of the same user. This results in a simpler mathematical structure and leads to **faster convergence**.

(b) **Plot the average sum-rate versus SNR, where the SNR ranges from 5 to 30 in increments of 5. Please plot ”WMMSE” and “WMMSE 10rand int”, each curve is averaged over 100 random channel realizations, and discuss the simulation results.**

a) Consider the MIMO case K = 3, T = 2, R = 2, I = 2, d = 4.

b) Consider the SISO case K = 3, T = 1, R = 1, I = 2, d = 4.





< discussion >

1. In the **MIMO** case, the **WMMSE 10rand int** method randomly initializes ten sets of precoders (V) and selects the best one as the starting point. This helps the algorithm **converge to a better local optimum** during the subsequent iterations.

   At **high SNR** levels, the impact of suboptimal beamforming becomes more significant, so the **performance gap** between WMMSE 10rand int and standard WMMSE **increases with SNR**. As a result, WMMSE 10rand int achieves a higher sum rate, especially in high-SNR scenarios.

2. In the **SISO** case, the **sum-rate curves** of WMMSE 10rand int and standard WMMSE **completely overlap**. This is because **both the transmitter and receiver have only one antenna**, which **makes beamforming impossible**. In this case, the precoder (V) is effectively just a complex scalar, meaning there is very limited freedom in the optimization. Therefore, different initializations lead to the same solution, resulting in no performance difference between WMMSE and WMMSE 10rand int.

(c) Explain why the weighted sum-rate maximization problem and the sum-MSE minimization problem is equivalent.

sum-MSE minimization problem:

$$\min_{\boldsymbol{W},\boldsymbol{U},\boldsymbol{V}} \sum_{k=1}^{K}\sum_{i=1}^{I_k} \alpha_{i_k}\left(Tr\left(\boldsymbol{W}_{\boldsymbol{i_k}}\boldsymbol{E}_{\boldsymbol{i_k}}\right) - logdet\left(\boldsymbol{W}_{\boldsymbol{i_k}}\right)\right) s.t. \sum_{i=1}^{I_k} Tr\left(\boldsymbol{V}_{\boldsymbol{i_k}}\boldsymbol{V}_{\boldsymbol{i_k}}^{H}\right) \le P_k, k = 1, 2, \ldots, K \quad (1)$$

substituting the optimal $\mathbf{U}_{\mathbf{i_k}}$ and $\mathbf{W}_{\mathbf{i_k}}$, for all $\mathrm{i_k} \in \mathcal{I}$, in (1), $\begin{cases} \mathbf{U}_{\mathbf{i_k}}^{\boldsymbol{opt}} = \mathbf{U}_{\mathbf{i_k}}^{\boldsymbol{mmse}} \\ \mathbf{W}_{\mathbf{i_k}}^{\boldsymbol{opt}} = \boldsymbol{E}_{\boldsymbol{i_k}}^{-\mathbf{1}} \end{cases}$

$$(1) \Rightarrow \max_{\{\boldsymbol{V}\}} \sum_{k=1}^{K}\sum_{i=1}^{I_k} \alpha_{i_k} logdet\left(\left(\mathbf{E}_{\mathbf{i_k}}^{\boldsymbol{mmse}}\right)^{-1}\right) s.t. \sum_{i=1}^{I_k} Tr\left(\boldsymbol{V}_{\boldsymbol{i_k}}\boldsymbol{V}_{\boldsymbol{i_k}}^{H}\right) \le P_k, \forall\, \mathrm{i_k} \in \mathcal{I} \quad (2)$$

defining $\boldsymbol{\gamma}_{\boldsymbol{i_k}} \triangleq \sum_{(l,j)\neq(i,k)} \boldsymbol{H}_{\boldsymbol{i_k j}}\boldsymbol{V}_{\boldsymbol{l_j}}\boldsymbol{V}_{\boldsymbol{l_j}}^{\boldsymbol{H}}\boldsymbol{H}_{\boldsymbol{i_k j}}^{\boldsymbol{H}} + \sigma_{i_k}^2\boldsymbol{I}$, we get

$$logdet\left(\left(\mathbf{E}_{\mathbf{i_k}}^{\boldsymbol{mmse}}\right)^{-1}\right) = logdet\left(\boldsymbol{I} + \boldsymbol{V}_{\boldsymbol{i_k}}^{\boldsymbol{H}}\boldsymbol{H}_{\boldsymbol{i_k k}}^{\boldsymbol{H}}\boldsymbol{\gamma}_{\boldsymbol{i_k}}^{-1}\boldsymbol{H}_{\boldsymbol{i_k k}}\boldsymbol{V}_{\boldsymbol{i_k}}\right)$$

$$= logdet\left(\boldsymbol{I} + \boldsymbol{H}_{\boldsymbol{i_k k}}\boldsymbol{V}_{\boldsymbol{i_k}}\boldsymbol{V}_{\boldsymbol{i_k}}^{\boldsymbol{H}}\boldsymbol{H}_{\boldsymbol{i_k k}}^{\boldsymbol{H}}\boldsymbol{\gamma}_{\boldsymbol{i_k}}^{-1}\right)$$

$$= logdet\left(\boldsymbol{I} + \boldsymbol{H}_{\boldsymbol{i_k k}}\boldsymbol{Q}_{\boldsymbol{i_k}}\boldsymbol{H}_{\boldsymbol{i_k k}}^{\boldsymbol{H}}\boldsymbol{\gamma}_{\boldsymbol{i_k}}^{-1}\right) \quad (3)$$

combining (2) and (3)

$$\Rightarrow \max_{\{\boldsymbol{V}\}} \sum_{k=1}^{K}\sum_{i=1}^{I_k} \alpha_{i_k} logdet\left(\boldsymbol{I} + \boldsymbol{H}_{\boldsymbol{i_k k}}\boldsymbol{Q}_{\boldsymbol{i_k}}\boldsymbol{H}_{\boldsymbol{i_k k}}^{\boldsymbol{H}}\boldsymbol{\gamma}_{\boldsymbol{i_k}}^{-1}\right) s.t. \sum_{i=1}^{I_k} Tr\left(\boldsymbol{V}_{\boldsymbol{i_k}}\boldsymbol{V}_{\boldsymbol{i_k}}^{H}\right) \le P_k, \forall\, \mathrm{i_k} \in \mathcal{I}$$

So, the sum-MSE minimization problem is equivalent to the weighted sum-rate maximization problem.

(d) Consider the general utility maximization problem in the paper. What is the conditions of $\mathbf{u_{ik}(.)}$, $\mathbf{c_{ik}(.)}$ and $\boldsymbol{\gamma_{ik}(.)}$. And according to the condition, please make an example of $\mathbf{u_{ik}(.)}$.

i. $\mathrm{u_{ik}}(.)$: strictly increasing function

$$\because \max_{\{\boldsymbol{V}\}} \sum_{k=1}^{K} \sum_{i=1}^{I_k} u_{i_k}(R_{i_k})\ s.t. \sum_{i=1}^{I_k} Tr\left(\boldsymbol{V}_{\boldsymbol{i_k}} \boldsymbol{V}_{\boldsymbol{i_k}}^H\right) \le P_k, k = 1, 2, \dots, K$$

$$\therefore \mathrm{u_{ik}}(.) \text{ must be a strictly increasing function.}$$

ii. $\mathrm{c_{ik}}(.)$: strictly concave function

$$\because \mathrm{c_{i_k}}\left(\boldsymbol{E}_{\boldsymbol{i_k}}\right) = -u_{i_k}(-logdet(\boldsymbol{E}_{\boldsymbol{i_k}}))$$

$$\therefore \mathrm{c_{i_k}}(.) \text{ must be a strictly concave function.}$$

iii. $\gamma_{\mathrm{ik}}(.)$: $\nabla \mathrm{c_{i_k}}(.)$ is invertible

$$\because \gamma_{\mathrm{ik}}(.) \text{ is the inverse mapping of the gradient map } \nabla \mathrm{c_{i_k}}\left(\boldsymbol{E}_{\boldsymbol{i_k}}\right)$$

$$\therefore \gamma_{\mathrm{ik}}(.) \text{ is well defined if } \nabla \mathrm{c_{i_k}}(.) \text{ is invertible}$$

$$\nabla \mathrm{c_{i_k}}(.) \text{ is invertible since } \mathrm{c_{i_k}}(.) \text{ is a strictly concave function for all } \mathrm{i_k}$$

iv. an example of $\mathrm{u_{ik}}(.) = \log(.)$

(e) Based on this paper, what are some promising avenues for future research?

i. Computational Complexity of WMMSE

WMMSE algorithms are computationally intensive, especially in large-scale MIMO systems. A promising research direction is to develop simplified or approximate versions of the WMMSE iterative algorithm that significantly reduce complexity while maintaining similar performance.

ii. Robustness in Time-Varying Channels

Extending WMMSE to fast time-varying or dynamic wireless environments is another important challenge. Future work can explore adaptive or real-time implementations that can track channel variations efficiently.

iii. Integration with AI Techniques

Combining WMMSE with AI-based methods, such as using deep learning models to predict optimal precoders and decoders, can help reduce convergence time and computation. This hybrid approach may offer both high performance and faster adaptation to system changes.