Wireless Communication Signal Processing—Homework 2

Due on April 4 in class.

March 21, 2025

In the following questions, please provide a thorough explanation or outline the calculation process; otherwise, no points will be awarded

1. (20%) Assume there are Q users, each user's transmitted symbol is s and $\mathbb{E}\left\{\mathbf{s}\mathbf{s}^H\right\} = \mathbf{I}$, and $\mathbf{n}$ is independent and identically distributed (i.i.d.) additive Gaussian noise. The received signal chip sampled over the kth symbol is

$$\mathbf{x}(k) = \mathbf{H}\mathbf{A}\mathbf{s}(k) + \mathbf{n}(k),$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_Q]$: channel matrix,

$\mathbf{A} = \mathrm{diag}[\sigma_1, \sigma_2, \ldots, \sigma_Q]$: power control factor,

$\mathbf{s}(k) = [s_1(k), s_2(k), \ldots, s_Q(k)]^T$: transmit symbol.

Based on the Minimum Mean Square Error (MMSE) detector design criterion on p. 29 of Chapter 2, please derive the Wiener solution expressed in $\mathbf{R}_{xx}$ and $\mathbf{r}_{xs}$, which are signal correlation matrix and cross-correlation vector, respectively.

2. (30%) Consider the single-user RAKE, ZF, and MMSE detectors in CDMA systems. The simulation environment is assumed as follows:
   - It is assumed that user 1 is the desired signal with power σ_1^2, and the others are the MAI with the same transmit power σ_i^2.
   - The input SNR$_i$ and SIR$_i$ are defined as

$$\mathrm{SNR}_i = \sigma_1^2 / \sigma_n^2 \ ; \ \mathrm{SIR}_i = \sigma_1^2 / \sigma_i^2 .$$

   - Q active user signals are transmitted via independent Rayleigh fading paths with path delays assumed to be uniform over $[0, 5T_c]$ (i.e., $L = 5$).
   - All CDMA signals are BPSK modulated and spread by a random code of length 48 (a random sequence of ± 1).

- Perfect channel estimation is assumed.

Perform the following tasks and **COMMENT on your results**:

(a) Plot SINR_o (output SINR in dB) of the detectors as a function of SNR_i (from −20 to 20 dB with an increment of 2 dB) with $\text{SIR}_i = 0$ dB and $Q = 8$. (Using HW2_1_a_2025.m)

(b) Plot SINR_o (output SINR in dB) of the detectors as a function of Q (from 1 to 10) with $\text{SNR}_i = 0$ dB and $\text{SIR}_i = 0$ dB. (Using HW2_1_b_2025.m)

3. (50%) Consider the SIC and the PIC detector with the Rake receiver in CDMA systems, respectively. The simulation environment is assumed as follows:

- $Q = 3$ active user signals are transmitted via independent Rayleigh fading paths with path delays assumed to be uniform over $[0, 5T_c]$ (i.e., $L = 5$).
- All CDMA signals are BPSK modulated and spread by a random code of length 48 (a random sequence of ±1).
- Perfect channel estimation is assumed.
- The input SNR_i is defined as

$$\text{SNR}_i = \sigma_i^2 / \sigma_n^2$$

- The relative power levels of the three users are fixed and given by $\text{SNR}_1 = \text{X}$ dB, $\text{SNR}_2 = (\text{X}-6)$ dB, and $\text{SNR}_3 = (\text{X}-12)$ dB.
- The decision is performed according to

$$\text{Dec}(x) = \begin{cases} 1, & \text{if Real}(x) \geq 0 \\ -1, & \text{if Real}(x) < 0 \end{cases}$$

Perform the following tasks and **COMMENT on your results**:

(a) Perform SIC detection on the three users in the order of descending power levels, i.e., 1, 2, 3. Plot the average bit error rate of the three users as a function of input SNR_1 (from −10 to 20 dB with an increment of 5 dB). (Using HW2_2_SIC_2025.m)

(b) Perform SIC detection on the three users in the order of ascending power levels, i.e., 3, 2, 1. Plot the average bit error rate of the three users as a function of input SNR_1 (from -10 to 20 dB with an increment of 5 dB). (Using HW2_2_SIC_2025.m)

(c) Perform PIC detection on the three users. Plot the average bit error rate of the three users as a function of input SNR_1 (from -10 to 20 dB with an increment of 5 dB). (Using HW2_2_PIC_2025.m)

(d) Compare the results in (a)-(c). Which detection scheme is the best for such a scenario? Why?