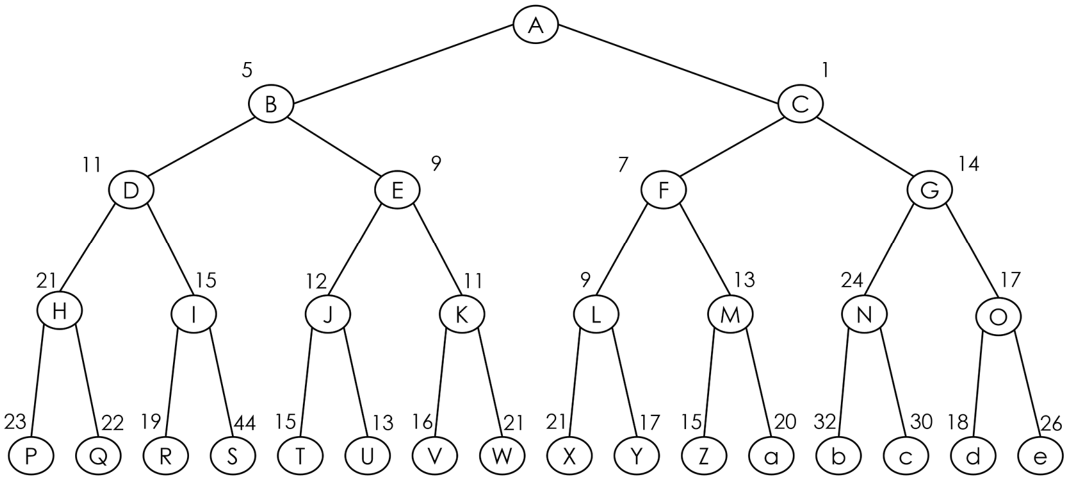
Wireless Communication Signal Processing – Final

Due on **June 6 at 23:59**.

June 4, 2025

1. **(10%)** Depict the search path in the following MIMO sphere decoder using the Schnorr-Euchner (SE) search method.



2. **(15%)** Consider a MISO system with four transmit antennas using a space-time block code (STBC) given as below:

$$\mathbf{X} = \begin{bmatrix} s_1^* & 0 & 0 & 0 & s_2^* & -s_3^* & s_4 \\ 0 & s_1^* & 0 & -s_2^* & 0 & -s_4^* & -s_3 \\ 0 & 0 & s_1^* & s_3^* & s_4^* & 0 & -s_2 \\ -s_4^* & s_3^* & s_2^* & 0 & 0 & 0 & s_1 \end{bmatrix},$$

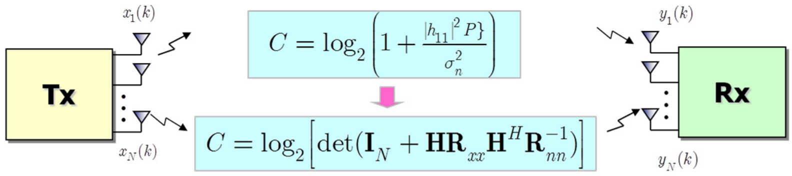
where the columns indicate different time slots. Assuming that the channel is time-invariant and noiseless during the transmission of the STBC, the received signal can be expressed as $\mathbf{y} = \mathbf{hX}$, where $\mathbf{y} = [y_1, \ldots, y_7]$ is a 1×7 vector such that y_i is the received signal at the ith timeslot, and $\mathbf{h} = [h_1, h_2, h_3, h_4]$ is a 1×4 vector with h_1, h_2, h_3, and h_4 being the channel gain for the first, second, third, and fourth antennas, respectively.

(a) **(4%)** Find the code rate R.

(b) **(4%)** Is this an orthogonal design code? Give your reason.

(c) **(7%)** After some manipulations, the received signal can be expressed as $\hat{\mathbf{y}} = \mathbf{Hs}$, where $\mathrm{s} = [s_1,\ s_2,\ s_3,\ s_4]^T$ is a 4×1 signal vector, $\mathbf{H}$ is a 7×4 matrix, and $\hat{\mathbf{y}}$ is a 7×1 vector. Find $\mathbf{H}$ and derive the expression of $\hat{\mathbf{y}}$.

3. **(15%)** Consider the MIMO channel capacity as described in course slides:



Assume that there are N transmit antennas and N receive antennas. Given that the total transmission power is P and the noise is i.i.d. with zero mean and variance σ_n^2. Derive capacity under the following conditions in terms of N, P, σ_n^2 and singular values of some matrix.

Note: Express your answers explicitly and define the parameters or variables used clearly. Derivation is required to get full score.

$\mathbf{H}$ denotes an i.i.d. Rayleigh fading channel matrix.

(a) **(5%)** Derive the channel capacity when CSI is unknown to the transmitter.

(b) **(5%)** Following (a), when SNR is high, show that the channel capacity can be approximated as

$$C \approx \log_2 \det\left(\mathbf{R}_{xx}\right) + \log_2 \det\left(\frac{1}{\sigma_n^2}\mathbf{H}\mathbf{H}^H\right)$$

and give the expression of $\mathbf{R}_{xx}$.

(d) **(5%)** Derive the channel capacity when CSI is known to the transmitter. Determine $\mathbf{R}_{xx}$ that maximizes the capacity.

4. **(15%)** In this question, you are required to focus specifically on hybrid precoding methods designed for multi-user massive MIMO (MU-Massive MIMO) systems. Hybrid precoding is widely adopted to balance hardware feasibility and communication performance, particularly in high-frequency (e.g., mmWave) and massive antenna array settings. Please select journal or conference papers that address hybrid precoding for MU-Massive MIMO (not single-user MIMO, and not

purely digital/analog only) and write a brief technical survey. (clearly showcasing the reference sources)

5. **(25%)** Given a desired signal from $\theta_0 = 0^\circ$ and two interferers, design three receive beamformers based on the MVDR, MSINR, and MMSE beamforming methods, respectively. The received signal $\mathbf{x}(k)$ is generated according to the following formula:

$$\mathbf{x}(k) = \mathbf{a}(\theta_0)s_{\mathrm{p}}(k) + \mathbf{a}(\theta_1)s_1(k) + \mathbf{a}(\theta_2)s_2(k) + \mathbf{n}(k), k = 1,\ldots,10^4$$

where

$$\mathbf{a}(\theta) = \begin{bmatrix}1, & e^{j\pi\sin(\theta)}, & \ldots, & e^{j(N-1)\pi\sin(\theta)}\end{bmatrix}^T,$$

$s_{\mathrm{p}}(k)$ is the pilot signal known to the receiver, $\mathbf{n}(k)$ is AWGN, and the number of receive antenna $M = 16$. Please answer the following questions: (You should use the signal $\mathbf{x}$ and s_{p} given in the attached "signal.mat" file to answer the questions)

(a) **(5%)** Plot the power of the beampattern ($\left|\mathbf{a}^H(\theta)\mathbf{w}\right|^2, \theta = -90^\circ, \ldots 89^\circ$) of the beamformers, e.g., the figure in p. 23, Chapter 4. (y-axis: Power (dB); x-axis: Angle (degree))

(b) **(5%)** Plot the phase of the beampattern ($\angle\mathbf{a}^H(\theta)\mathbf{w}, \theta = -90^\circ, \ldots 89^\circ$) of the beamformers. (y-axis: Phase (radian); x-axis: Angle (degree))

(c) **(5%)** Observe the results you get in (a) and (b), where are the interferers? ($\theta_1 = ?, \theta_2 = ?$)

(d) **(5%)** Based on the results you get in (a) and (b), are the interferences coherent with the desired signal? Why?

(e) **(5%)** Compare the results of MVDR and MMSE beamformers in (a) and (b), which beamformer gives better SNR? Why?

6. **(30%)** Consider four MIMO detectors: MMSE detector, MMSE-OSIC detector, *K*-best sphere decoder, and Maximum-likelihood (ML) detector. Basic assumptions and some parameters are given as follows:

- Four antennas are employed at both Tx and Rx.
- The channel is assumed to be flat fading.
- Four data streams are transmitted and detected.
- QPSK modulation is adopted for each data stream.
- CSI is available at Tx, and precoding is adopted before transmission.
- A codebook of precoders is given (see the Matlab sample program).
- Select a precoder according to the maximum capacity criteria in p. 175 of Chapter 4.
- The selected precoder is known to Rx.
- K= 6 is adopted for the K-best sphere decoder.
- AWGN is assumed at Rx.
- The decision is performed as follows:

$$\hat{x}_{\mathrm{R}} = \begin{cases} 1, & \text{if Real}(x) \geq 0 \\ -1, & \text{if Real}(x) < 0 \end{cases}, \quad \hat{x}_{\mathrm{I}} = \begin{cases} 1, & \text{if Imag}(x) \geq 0 \\ -1, & \text{if Imag}(x) < 0 \end{cases}.$$

The channel matrix of each device is modeled as $\mathbf{H}_i = \mathbf{R}_{\mathrm{r}}^{1/2}\mathbf{H}_{w,i}\mathbf{R}_{\mathrm{t}}^{1/2}, i = 1,2$,

where $\mathbf{R}_{\mathrm{t}}$, $\mathbf{R}_{\mathrm{r}}$, and $\mathbf{H}_w$ are the correlation matrix at the Tx, the correlation matrix at the Rx, and a matrix of i.i.d. complex Gaussian random variables with zero mean and unit variance. Effective channel correlation can be adjusted by choosing a parameter ρ in $\mathbf{R}_{\mathrm{t}}$ and $\mathbf{R}_{\mathrm{r}}$ (see the Matlab sample program).

Conduct the following simulations under different channel conditions:

(a) **(7%)** Uncorrelated channel with $\rho = 0$.
(b) **(7%)** Medium-correlation channel with $\rho = 0.5$.
(c) **(7%)** Fully correlated channel with $\rho = 1$.

Now, instead of using codebook-based precoding, please implement the **singular value decomposition (SVD) precoding** on p.169 of Chapter 4

(d) **(9%)** Repeat the simulation of (a) with SVD precoding. Is there any difference between the performance of the four detectors? Why?

For each of (a)-(d), plot the average SER (i.e., SER averaged over different data streams) versus SNR for each of the four detectors and **COMMENT** on your results.

Note: the range of SNR is -10 dB to 20 dB with an increment of 5 dB.