Wireless Communication Signal Processing—Homework 3

Due on Apr 25 in class

Apr 11, 2025

1. (30%) Use the demodulated signal in matrix form on page 64 of Chapter 3 to show that the received signal Y_k with insufficient CP is given by

$$Y_k = H_k X_k + \sum_{k'=0}^{N-1} \left[\mathbf{F} \ \mathbf{H}_{ISI} \mathbf{F}^{-1} \right]_{k+1,k'+1} X_{k'}^{(-)} - \sum_{k'=0}^{N-1} \left[\mathbf{F} \ \mathbf{H}_{ICI} \mathbf{F}^{-1} \right]_{k+1,k'+1} X_{k'} + N_k,$$

where $\mathbf{H}_{ISI}$ is a $N \times N$ upper triangular Toeplitz matrix with first row being

$[0,\ldots,0,h[L_h-1],\ldots,\ h[N_g+1]\ ]$, and $\mathbf{H}_{ICI} = \mathbf{H}_{ISI}\mathbf{J}^{Ng}$.

2. (70%) Consider an OFDM system with the following scenario:

(i) An OFDM signal is transmitted via 9 independent Rayleigh multipath fading paths with path delays being uniform over [0, ... , $9T_d$] (i.e., L_h = 10), where T_d is the sampling duration. The fading gains are assumed i.i.d. complex Gaussian random variables with zero-mean and unit variance, and treated as constant over a single packet due to the slow fading assumption (but vary from packet to packet).

(ii) The adopted system parameters are as follows:

- Number of subcarriers: $N = 256$
- OFDM SNR: $\mathrm{SNR}_i = \sigma_s^2 / \sigma_n^2$ with σ_s^2 and σ_n^2 denoting signal and AWGN power (note the SNR definition for OFDM)
- Data modulation: QPSK
- Length of CP: $N_g = 16$
- Channel is invariant during each packet
- 10 OFDM symbols per packet

Assuming perfect channel estimation, perform the following tasks and **COMMENT** on your results.

(a) Plot the SER curve for $\mathrm{SNR}_i = 0:5:30$ dB using 1000 packets.

(b) Repeat (a) with different CP lengths: $N_g = 16, 8, 4$ and 2.

(c) Repeat (a) with different frequency offsets: $f_{\text{offset}} T = 0, 0.1, 0.2, 0.3, 0.4, 0.5$ and 1.0.