Homework 6

Jing Leng (GSI: Jiahe)

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1

a

```
densitybeta=function(x){
  alpha=5;beta=2
  dbeta(x,alpha,beta)
}
curve(densitybeta,from=0,to=1,n=200,col='blue')

ARBeta1=function(n,alpha,beta){
  Vy=numeric(n);Vcpt=integer(n);

  for (j in 1:n) {
    cpt = 0
    u = runif(1)
    y = runif(1)
    cpt = cpt + 1
    while (u > y ^ (alpha - 1) * (1 - y) ^ (beta - 1)) {
      u = runif(1)
      y = runif(1)
      cpt = cpt + 1
    }
    Vy[j] = y
    Vcpt[j] = cpt
  }
  return(list(Vy,Vcpt))
}

ARBeta2=function(n,alpha,beta){
  a=min(alpha,beta)-1
  Vy=numeric(n);Vcpt=integer(n);

  for (j in 1:n) {
    u=runif(1); y=runif(1);cpt=1
    while (u > 4^a*y^(alpha-1)*(1-y)^(beta-1)) {
      u = runif(1)
      y = runif(1)
      cpt = cpt + 1
    }
    Vy[j] = y
    Vcpt[j] = cpt
  }

  return(list(Vy,Vcpt))
}
```

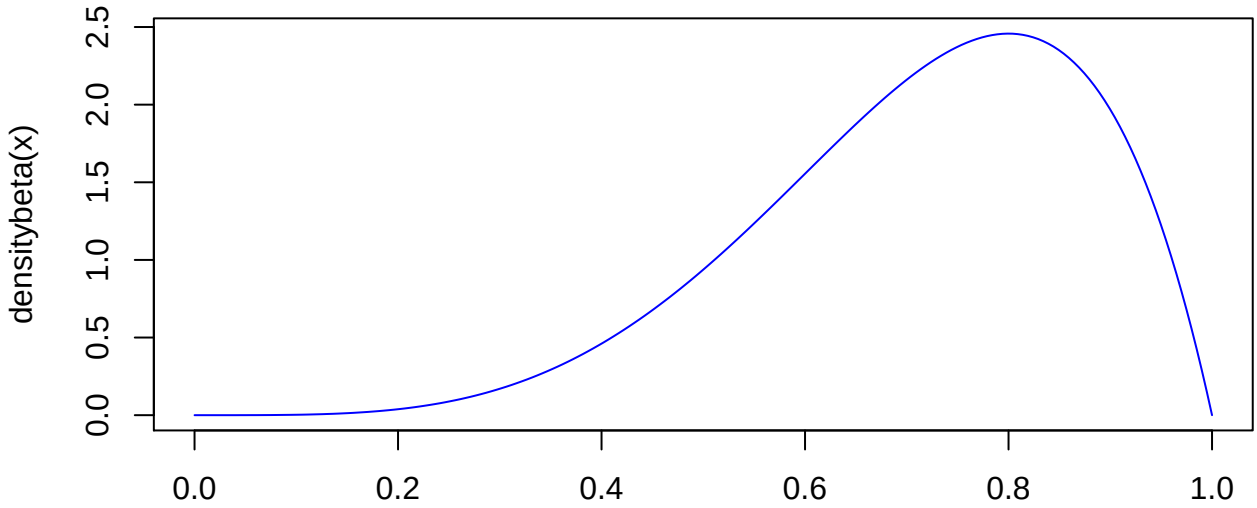
```
ARBeta3=function(n,alpha,beta){
  m=(alpha-1)/(alpha+beta-2)
  Vy=numeric(n);Vcpt=integer(n);

  for (j in 1:n) {
    u=runif(1); y=runif(1);cpt=1
    while (u > exp((alpha-1)*(log(y)-log(m)))) {
      u = runif(1)
      y = runif(1)
      cpt = cpt + 1
    }
    Vy[j] = y
    Vcpt[j] = cpt
  }
  return(list(Vy,Vcpt))
}

alpha=5;beta=2
n=3000;
system.time(Res1<-ARBeta1(n,alpha=5,beta=2));
system.time(Res2<-ARBeta2(n,alpha=5,beta=2));
system.time(Res3<-ARBeta3(n,alpha=5,beta=2));
c(mean(Res1[[2]]),mean(Res2[[2]]),mean(Res3[[2]]))

hist(Res1[[1]],prob=T,xlim=c(0,1),ylim=c(0,3),col='blue',nclass=200,main='Algo. I');
par(new=T)
curve(densitybeta,from=0,to=1,n=200,col='red', xlim=c(0,1),ylim=c(0,3))

curve(densitybeta,from=0,to=1,n=200,col='red',
      xlim=c(0,1),ylim=c(0,32))
M1=1/beta(alpha,beta)
abline(h=M1,col='green')
M2=(1/beta(alpha,beta))*((1/4)^(min(alpha-1,beta-1)))
abline(h=M2,col='blue')
m=(alpha-1)/(alpha+beta-2)
M3=(1/beta(alpha,beta))*m^(alpha-1)*(1-m)^(beta-1)
abline(h=M3,col='red')
legend(x='left',legend=c('Algo. I','Algo.II','Algo. III'),
       col=c('green','blue','red'),lty=c(1,1,1))
```

```
##    user  system elapsed
##   0.996   0.015   1.039
```

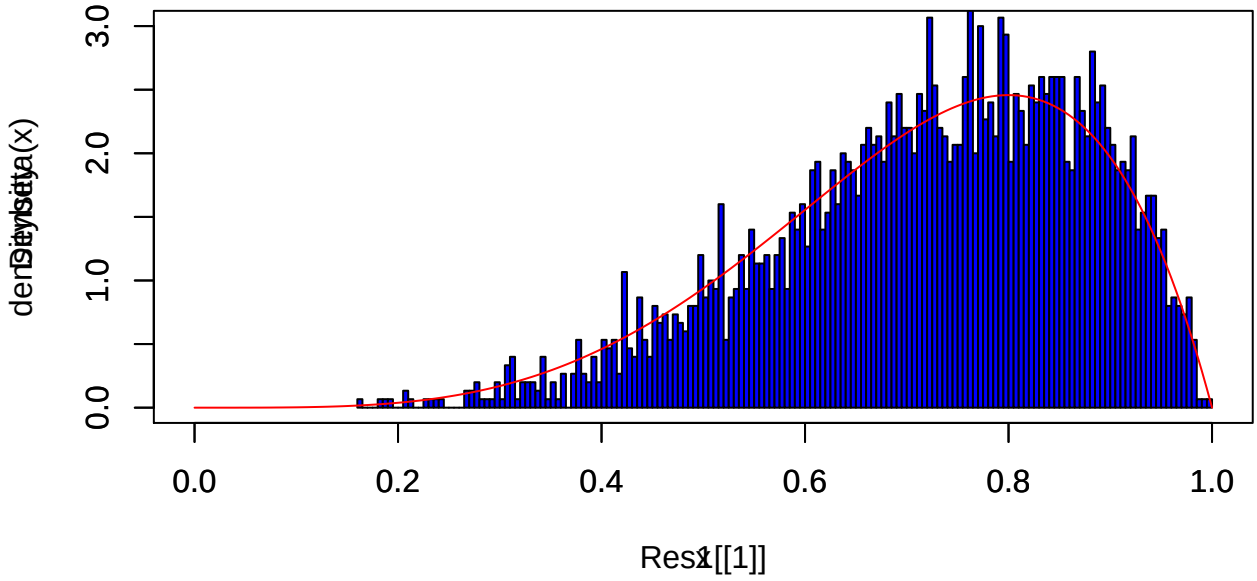
```
##    user  system elapsed
##   0.283   0.002   0.294
```

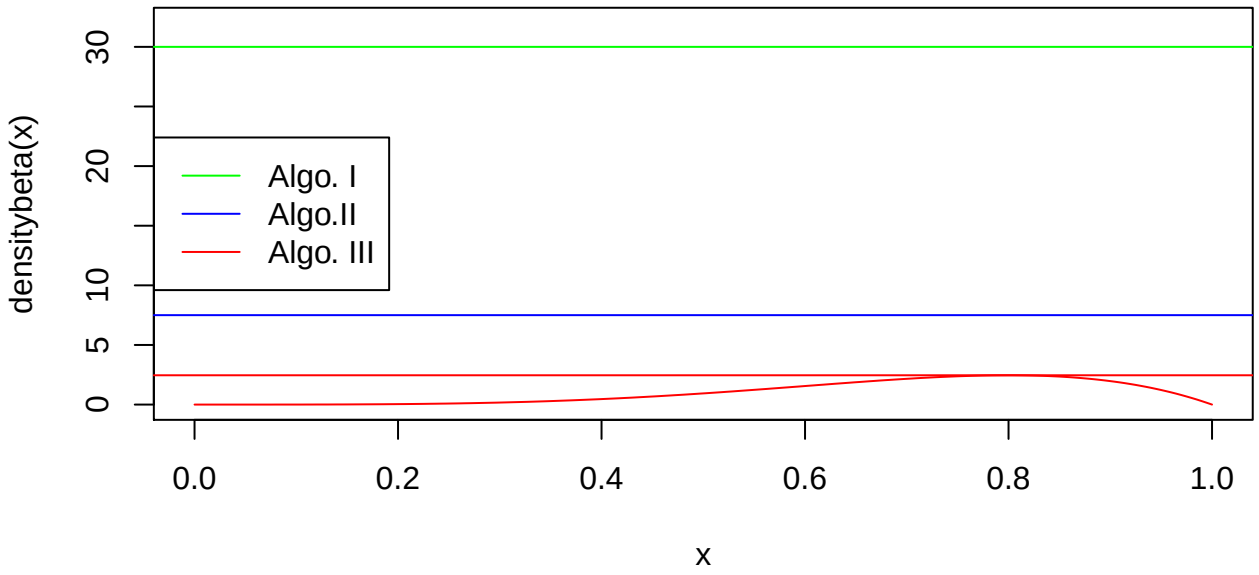
```
##    user  system elapsed
##   0.093   0.000   0.093
```

```
## [1] 30.107  7.309  2.850
```


densitybeta(x)
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
x

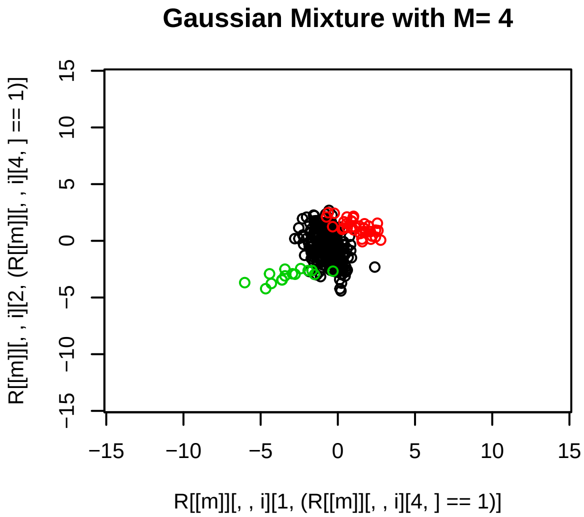
Algo. I

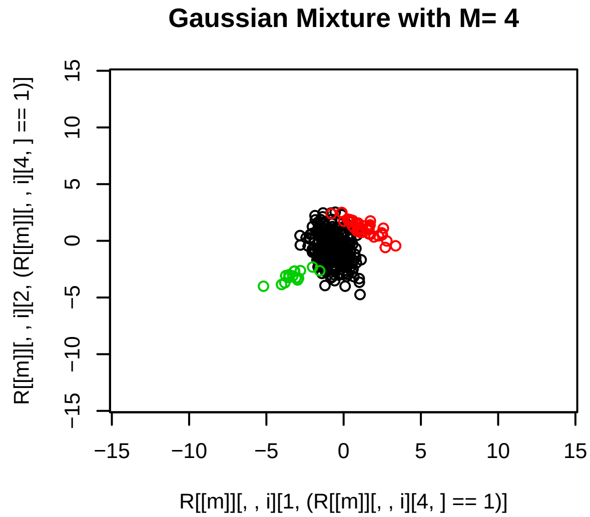

0.0
1.0
2.0
3.0
0.0
0.2
0.4
0.6
0.8
1.0
Res1[[1]]

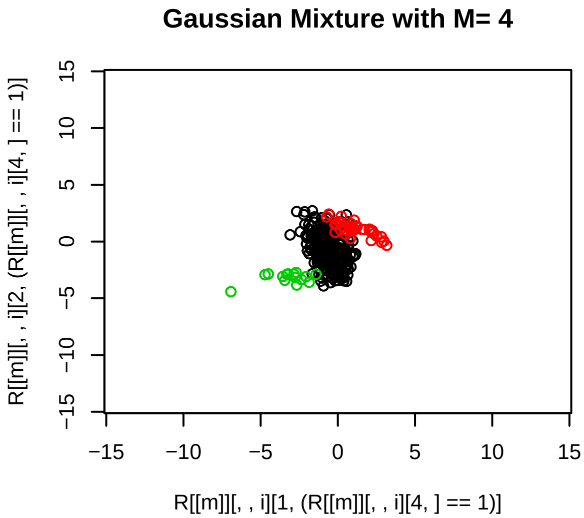


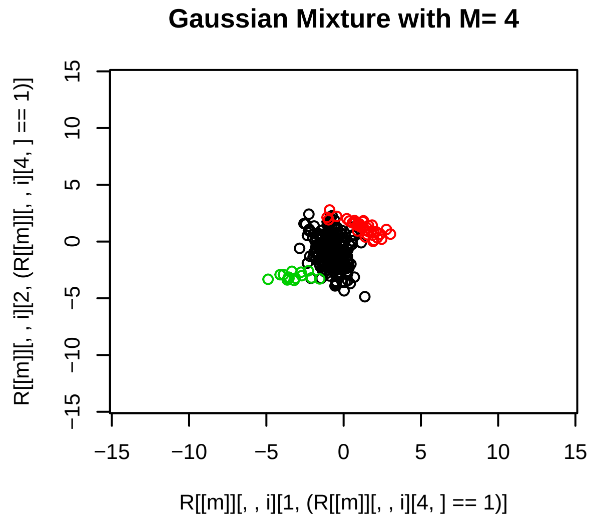
2

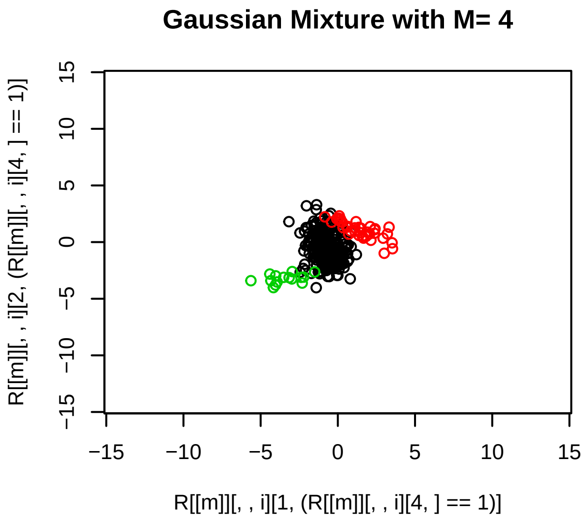
```
library(gtools)
mix = rdirichlet(1, c(1, 1, 1))
M = list(4, 10, 16)
mu = lapply(M, function(m) matrix(runif(3, -m, m), ncol = 3))
A = replicate(3, matrix(runif(9, -1, 1), nrow = 3), simplify = F)

N = 300
S = 6
d = 3
Z = sample(seq(3), size = N, replace = T, prob = mix)
R = sapply(1:length(M), function(m) replicate(S, sapply(Z, function(i) c(mu[[m]][,
    i] + t(A[[i]]) %*% rnorm(d, 0, 1), i), simplify = T), simplify = "array"),
    simplify = "array")
R = lapply(1:length(M), function(m) replicate(S, sapply(Z, function(i) c(mu[[m]][,
    i] + t(A[[i]]) %*% rnorm(d, 0, 1), i), simplify = T), simplify = "array"))

for (m in 1:length(M)) {
    # pdf(paste('plot', m, '.pdf', sep = ''))
    par(mfrow = c(3, 2))
    for (i in 1:S) {
        plot(R[[m]][, , i][1, (R[[m]][, , i][4, ] == 1)], R[[m]][, , i][2, (R[[m]][,
            , i][4, ] == 1)], xlim = c(-14, 14), ylim = c(-14, 14), col = 1,
            main = paste("Gaussian Mixture with M=", M[[m]]))
        points(R[[m]][, , i][1, (R[[m]][, , i][4, ] == 2)], R[[m]][, , i][2,
            (R[[m]][, , i][4, ] == 2)], col = 2)
        points(R[[m]][, , i][1, (R[[m]][, , i][4, ] == 3)], R[[m]][, , i][2,
            (R[[m]][, , i][4, ] == 3)], col = 3)
    }
}
```

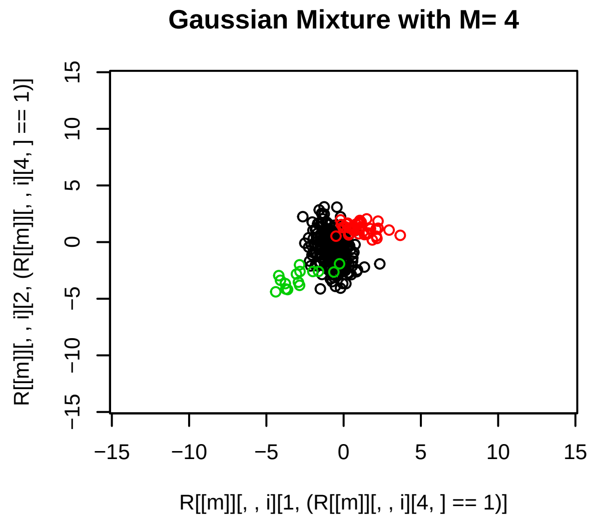

Gaussian Mixture with M= 4
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
−15 −10 −5 0 5 10 15
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]

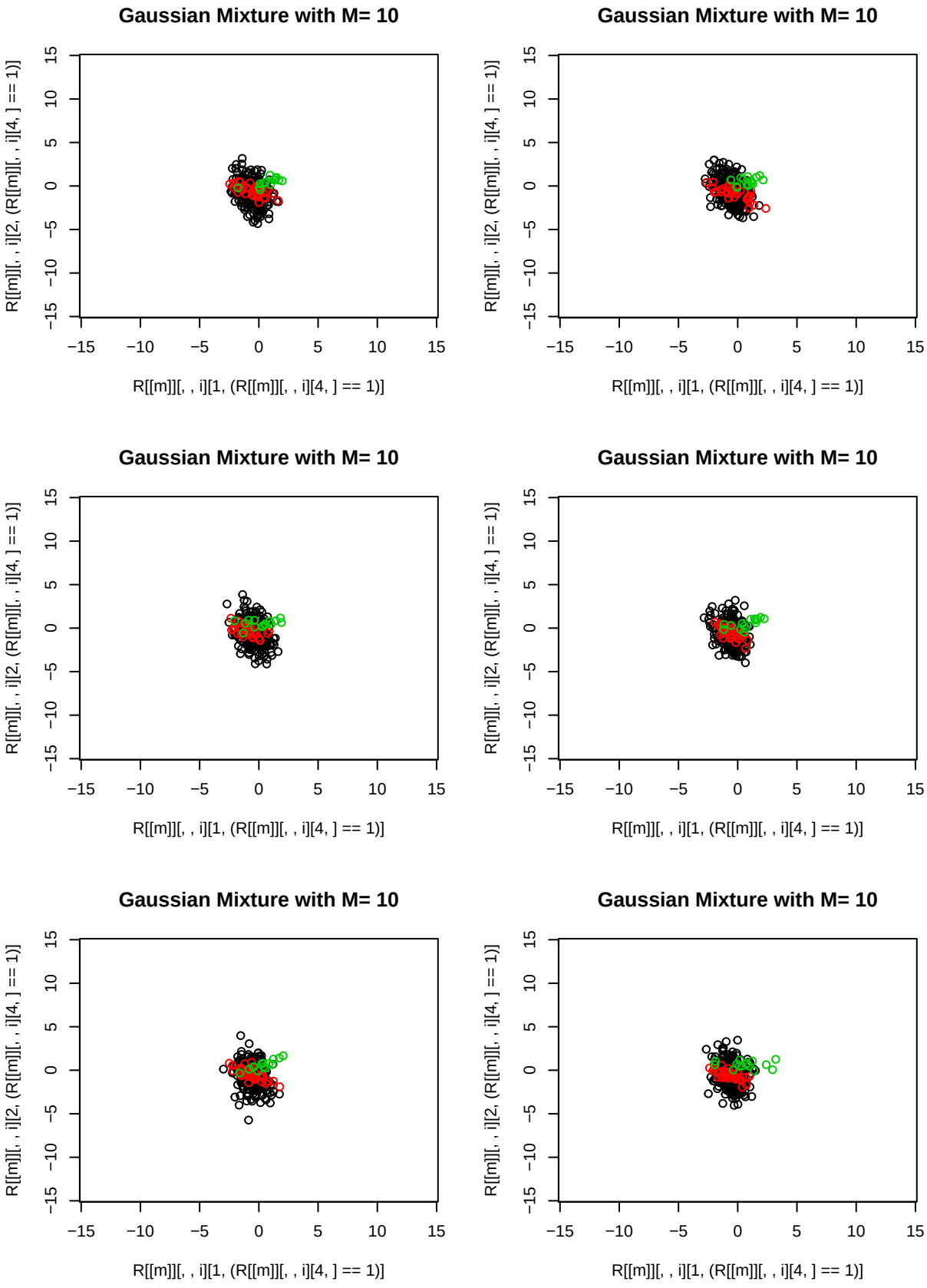

Gaussian Mixture with M= 4
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
−15 −10 −5 0 5 10 15
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]

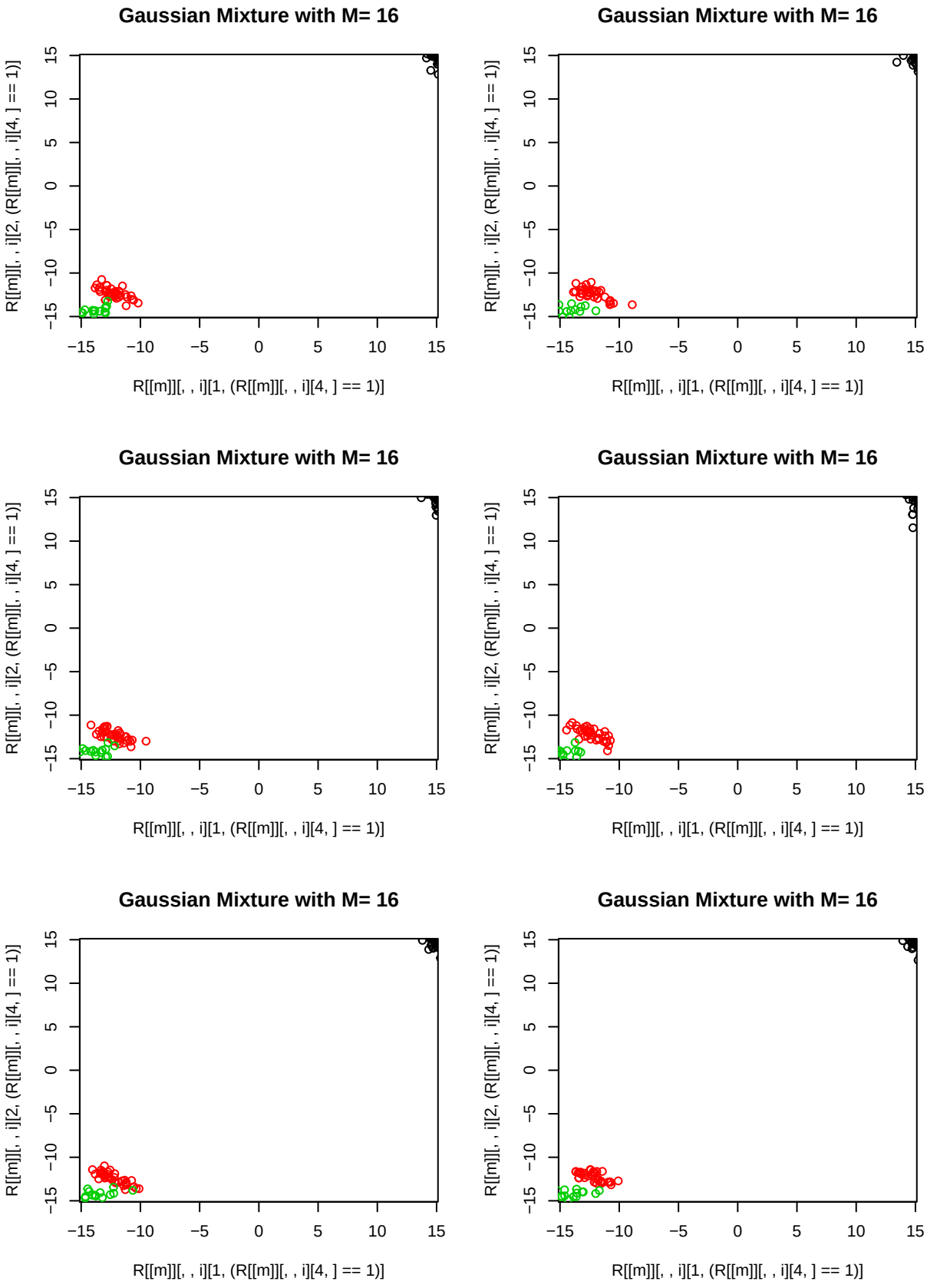

Gaussian Mixture with M= 4
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
−15 −10 −5 0 5 10 15
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]


Gaussian Mixture with M= 4
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
−15 −10 −5 0 5 10 15
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]


Gaussian Mixture with M= 4
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
−15 −10 −5 0 5 10 15
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]


Gaussian Mixture with M= 4
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
−15 −10 −5 0 5 10 15
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]


Gaussian Mixture with M= 10
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]
−15
−10
−5
0
5
10
15


Gaussian Mixture with M= 16
R[[m]][, , i][2, (R[[m]][, , i][4,] == 1)]
R[[m]][, , i][1, (R[[m]][, , i][4,] == 1)]
−15
−10
−5
0
5
10
15