Homework Number 1, McCall Models

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[NOTE: If you dont know anything about computing yet, see me and you will do the Bewley economy instead to learn the basics.]

Consider the modified Ljungqvist and Sargent (1998) model in the slides. There is a finitely-lived continuum of workers with linear utility. To find jobs, workers exert search intensity s at utility cost $c(s)$. Given that effort, $\pi(s)$ is the probability of receiving an offer. They all draw offers from the stationary distribution $F(w)$. λ is probability of being laid off. α probability of dying each period. h is human capital, and take home pay is wh (linear in human capital).

Simplifying assumptions. Human capital lies on a grid $h \in \{\underline{h}, \underline{h} + \Delta, \ldots, \overline{h}\}$. If unemployed human capital falls by $\psi_U \times \Delta$ (ψ_U just determines how many grid slots it falls by). If fired, human capital falls by $\psi_F \times \Delta$ immediately (in the model, in periods of structural change, ψ_F increases (e.g. from 1 to 2), but we will ignore this for now). If employed, human capital increases by Δ per period. All unemployed workers receive b in perpetuity. Let $\hat{\beta} = (1 - \alpha)\beta$. The government levies taxes τ to pay for unemployment insurance (and yes, believe it or not, benefits are taxed in the US and elsewhere). The value function of unemployed is given by,

$$U(h) = \max_s (1 - \tau)b - c(s) + \hat{\beta}\Bigg[\pi(s)\int_w \max\{W(w, h'), U(h')\}dF(w)$$
$$(1 - \pi(s))U(h')\Bigg]$$

s.t. human capital is given by $h' = \max\{h - \psi_U\Delta, \underline{h}\}$ The value function of employed is

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given by,

$$W(w,h) = (1-\tau)wh + \hat{\beta}\Big[(1-\lambda)W(w,\min\{h+\Delta,\overline{h}\}) + \lambda U(\max\{h-\psi_F\Delta,\underline{h}\})\Big]$$

Calibration of Modified Ljungqvist & Sargent (1998):

i. Model period is 2 weeks, $\beta = .9985$

ii. Probability of dying is $\alpha = .0009$

iii. Probability of being laid off $\lambda = .009$

iv. Human capital h lies on grid of [1,2] with 201 evenly spaced nodes (8 years to the top)

v. The layoff loss is large: $\psi_F = 30$ ($\sim$ 15% H.C. loss)

vi. Unemployment entails continual losses, $\psi_U = 10$

vii. Newborns draw their human capital from a uniform distribution and begin life unemployed

viii. Search effort cost: $c(s) = .5s$

ix. Probability of contact: $\pi(s) = s^{.3}$

x. Wage offer distribution is normal with a mean of .5, a variance of .1, and that is truncated to the unit interval

xi. Let the wage grid have 41 evenly spaced points between [0,1]

xii. Let $b = .1$

xiii. Search effort lies on a grid [0,1] evenly spaced with 41 nodes.

Main Assignment: To complete this homework, you must,

1. Plot search effort as a function of h.

2. Plot the reservation wage as a function of h.

3. Solve the model economy for τ.

4. Report the equilibrium unemployment rate.

Suggested Solution method: How to solve the problem, heuristically (the attached sample code is a more solid, but relatively inefficient example),

i. Guess $\tau \in [0, 1]$.
   - It will work best to use bisection, set $\tau_H = 1$ and $\tau_L = 0$. The first guess will be $\tau_{guess} = (\tau_H + \tau_L)/2$, if the tax revenue is too high, set the new value for τ_H to τ_{guess}. Vice versa in the opposite case.

ii. Solve the employed and unemployed bellman equations (value function iteration is easy and clear).
   - Setup the grids
   - Guess zero for the initial value functions, and then iterate until the guess and the new value function are close enough(1e-3 is good enough for this assignment).

iii. Save the search policy function and reservation wage.

iv. Simulate a mass of N=10,000 workers for a large number of periods T=200, replacing the dead with newborns.
   - Draw several large NxT matrices of uniform random numbers.
   - Use these uniform random numbers to resolve any uncertainty, i.e. if the random number is below λ (which occurs with exactly λ probability) then the person is fired, etc.
   - Store the states of each worker, at each point in time in matrices.

v. Drop the first 100 periods.

vi. Calculate taxes among these workers over the remaining 100 periods, and check that the taxes cover the unemployment insurance.

vii. Update the guess of τ as per above.