University of Minnesota
ECON 8581
Homework 3
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Replication of Shimer (2005)

We start by outlining the algorithm used to solve the model.
Algorithm:[1]

1. Put a grid on y size $n \times 1$.
   (a) Define a step size $\Delta > 0$, and let $y \in \{-n\Delta, -(n-1)\Delta, \ldots, 0, \ldots, (n-1)\Delta, n\Delta\}$
   (b) Compute the transition probabilities for y according to:
$$y' = \begin{cases} y + \Delta, & \text{w/ prob } \frac{1}{2}(1 - \frac{y}{n\Delta}) \\ y - \Delta, & \text{w/ prob } \frac{1}{2}(1 + \frac{y}{n\Delta}) \end{cases}$$
2. Let p^* and s^* be the long-run measures of productivity and separation, respectively. Since $p = z + e^y(p^* - z)$, and $s = e^y s^*$, the grids for productivity and separation shocks are automatically set.
3. For each y we compute $(p(y), s(y))$ and find the $\theta(y)$ that solves the wage equation:
$$\frac{r + \lambda + s(y)}{q(\theta(y))} + \beta\theta(y) = (1-\beta)\frac{p(y) - z}{c} + \lambda \mathbb{E}_y\left[\frac{1}{q(\theta(y'))}\right]$$
   But definition of $q(\theta(y)) = \frac{f(\theta(y))}{\theta(y)} = \frac{\mu\theta(y)^{1-\alpha}}{\theta(y)} = \mu\theta(y)^{-\alpha}$.
$$\frac{r + \lambda + s(y)}{\mu} \times \theta(y)^{\alpha} + \beta\theta(y) = (1-\beta)\frac{p(y) - z}{c} + \frac{\lambda}{\mu}\mathbb{E}_y\left[\theta(y')^{\alpha}\right] \tag{1}$$
   There is a very important thing to highlight in (1). Since the model is stated in continuous time, even though there are $2n$ values for y, the **only** values that y' can

[1] The MATLAB code used can be found here. I thank Sara Canilang and Johanna Torres Chain for the collaboration.

take are $y+\Delta$ and $y-\Delta$. In this case, the expectation in the above equations is simply:

$$\theta(y+\Delta)^{\alpha} \times \frac{1}{2}(1-y/(n\Delta)) + \theta(y-\Delta)^{\alpha} \times \frac{1}{2}(1+y/(n\Delta))$$

Or in terms of position on the grid:

$$\theta_{i+1}^{\alpha} \times \frac{1}{2}(1-y/(n\Delta)) + \theta_{i-1}^{\alpha} \times \frac{1}{2}(1+y/(n\Delta))$$

4. We then simulate the model for $T = 1212$ quarters, discard the first 1,000 observations and repeat the process 10,000 times.

Labor productivity shocks

I start with an attempt of replicating Table 3 in Shimer (2005). The numbers for standard deviations of each variable matched the ones found in the paper. [2]

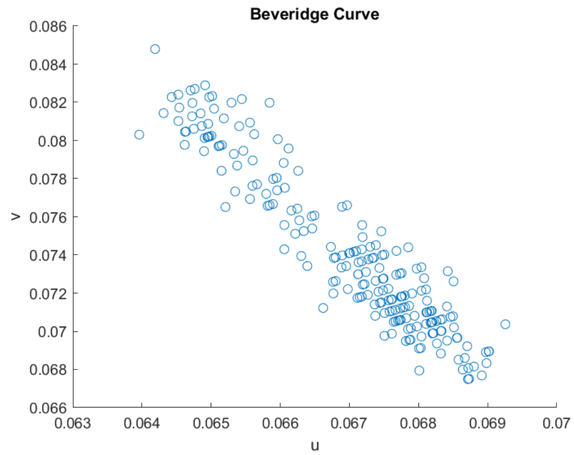
Table 1: Labor Productivity Shocks

	u	v	θ	f	p
Standard Deviation	0.009	0.028	0.035	0.010	0.020
	(0.001)	(0.004)	(0.005)	(0.001)	(0.003)
Autocorrelation	0.841	0.780	0.872	0.872	0.872
	(0.046)	(0.058)	(0.037)	(0.037)	(0.037)

	u	v	θ	f	p
u	1	-0.734	-0.8429	-0.8435	-0.8427
		(0.071)	(0.045)	(0.045)	(0.045)
v		1	0.984	0.984	0.983
			(0.004)	(0.004)	(0.004)
θ			1	1	0.999
				(0.000)	(0.000)
f				1	0.999
					(0.000)
p					1

After running the above algorithm I exhibit the Beveridge curve for one of our simulations.

[2] I could not exactly match correlations and autocorrelation for all the variables.



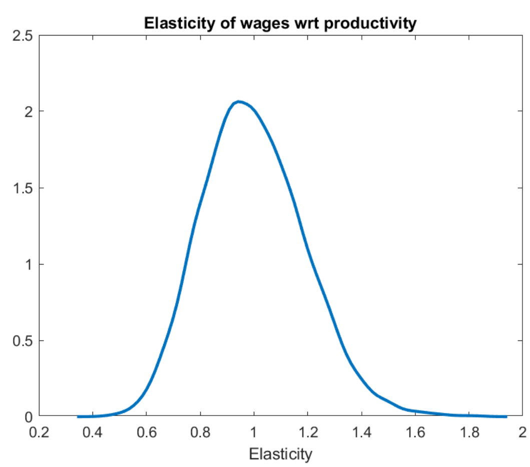
We also computed the simulated wages from the wage equation:

$$w(p) = (1-\beta)z + \beta(p + c\theta(p))$$

With wages in hand, we can compute the elasticity of wages with respect to productivity $\eta = d\log w / d\log p$ by running the regression for each simulated sample:

$$\log \text{wages}_t = \gamma + \eta \log p_t$$

We plot the distribution of elasticity estimates and highlight that it seems to have a mean of 1. This is an important result from the paper since it shows that the model has problems generating fluctuations in vacancies and unemployment. Large productivity shocks are totally converted into higher wages, which provides little incentive for firms to open new vacancies after a productivity shock.



Bargaining shocks

Next, we introduce shocks to the bargaining power parameter β. The idea is to make the worker's bargaining power counter-cyclical, so increases in labor productivity are associated with a higher share of the matching surplus to the firms, which should imply more vacancies after positive shocks. I chose the following functional form for bargaining power:

$$\beta(y) = \Phi(-p) + 0.5$$

Where Φ is the standard normal cdf. We consider a more extreme case where firms absorb all productivity shocks, that is, wages are constant across y: $w(y) = \overline{w}$. I set $\overline{w}$ and solve the following minimization problem:

$$(c, \mu) = \arg\min \|w(y) - \overline{w}\|_\infty$$

We could be way less restrictive by not assuming that all shocks are absorbed by the firms. In this case, we would choose (c, μ) such that the average job-finding rate and v-u rate remain unchanged.

Even though the code for this is already written, I could not find satisfying values for the recalibrated parameters so far.

Leisure shocks

Finally, we make the outside option of workers stochastic by setting $z(p) = p - \epsilon$.

Table 2: Leisure Shocks

	u	v	θ	f	p
Standard Deviation	0.000	0.000	0.000	0.000	0.0205
	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)
Autocorrelation	0.990	0.990	0.990	0.989	0.872
	(0.004)	(0.005)	(0.006)	(0.005)	(0.036)

Table 3: Leisure shocks - Correlation Matrix

	u	v	θ	f	p
u	1	0.9572	0.9615	0.9584	0.002
		(0.048)	(0.04)	(0.051)	(0.107)
v		1	0.948	0.941	0.002
			(0.057)	(0.062)	(0.112)
θ			1	0.9491	0.0027
				(0.063)	(0.1121)
f				1	0.0024
					(0.106)
p					1

As $\epsilon \to 0$ the wage equation reduces to

$$w(p) = p + \beta c \theta(p)$$

In this version, agents' outside option raises with positive productivity shocks in the exact same proportion. This induces a constant v-u ratio as we can see in the above tables and therefore, cannot replicate all the volatility in vacancies and unemployment observed in the data.

References

D. T. Mortensen and C. A. Pissarides. Job creation and job destruction in the theory of unemployment. *The review of economic studies*, 61(3):397–415, 1994.

R. Shimer. The cyclical behavior of equilibrium unemployment and vacancies. *American economic review*, 95(1):25–49, 2005.