# Homework Number 5, Basic Block Recursivity

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## 1 A Simple Example with Search Effort and Expiring Benefits

Imagine there is a world in which the aggregate state $z \in Z \subseteq \mathbb{R}$ fluctuates over time and there is heterogeneity in unemployment benefits across agents $b \in \mathbb{R}_{++}$, wages $w \in \mathbb{R}_{++}$, and employment status $e \in \{W, U\}$ (W=Employed, U=unemployed). The distribution agents across states, $\mu(b, w, e) : \mathbb{R}_{++} \times \mathbb{R}_{++} \times \{W, U\} \rightarrow [0, 1]$, fluctuates over time with the aggregate state in a non-trivial way. Let $\psi = (z, \mu)$ summarize the aggregate state $z$, and the distribution of agents $\mu$ across states.

There is free entry of firms, and the vacancy posting cost $\kappa \in \mathbb{R}_{++}$ is fixed and does not depend on the aggregate distribution. Firms direct their search by posting vacancies in certain submarkets that are indexed by $w \in \mathbb{R}_{++}$. The posted wage $w$ is fixed once an employee is found. The submarket tightness is given by $\theta(w; \psi) = \frac{v(w;\psi)}{u(w;\psi)}$ where $v(w; \psi)$ is the number of vacancies posted in the $w$ submarket and $u(w; \psi)$ is the number of unemployed people in that submarket.[1] The constant returns to scale of the matching function will guarantee that the ratio of unemployment and vacancies is all that matters for determining job finding rates. Let the vacancy filling rate be given by $q(\theta(w; \psi)) = \frac{M(w;\psi)}{v(w;\psi)}$ and let the job finding rate be given by $p(\theta(w; \psi)) = \frac{M(w;\psi)}{u(w;\psi)}$. The value to a firm of posting a vacancy in submarket $w$ is given below:[2]

$$V(w; \psi) = -\kappa + q(\theta(w; \psi))J(w; \psi)$$

With free entry it must be the case that profits are competed away. Thus $V(w; \psi) = 0$ for any submarket that is visited with positive probability. Thus, the free entry condition is

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[1] Off equilibrium markets will have a tightness of 0

[2] Notice the timing- the firm is not subject to idiosyncratic risk

given below:

$$\kappa = q(\theta(w;\psi))J(w;\psi) \ \text{ if } \theta(w;\psi) > 0 \tag{1}$$

Given a sufficiently well behaved function $q$, it is possible to invert this equation to solve for the market tightness $\theta(w;\psi)$:

$$\theta(w;\psi) = q^{-1}\left(\frac{\kappa}{J(w;\psi)}\right) \ \text{ if } \theta(w;\psi) > 0 \tag{2}$$

The value of an ongoing match is given below. Notice that the expectation $\mathbb{E}_{\psi'}$ is over the aggregates which includes the distribution of people across states:

$$J(w;\psi) = z - w + \mathbb{E}_{\psi'}\Big[\frac{(1-\delta(z',w))}{(1+r)}J(w;\psi')\Big]$$

And I will assume that zero profit matches are destroyed with probability 1:[3]

$$\delta(z,w) = \begin{cases} \bar{\delta} & \text{if } z > w \\ 1 & \text{if } z < w \end{cases}$$

An employed household's problem is given below:

$$W(w;\psi) = u(w) + \beta\mathbb{E}_{\psi'}\big[(1-\delta(w,z'))W(w;\psi') + \delta(w,z')U(\frac{1}{2}w;\psi')\big]$$

Unemployed households choose search effort $s$ at some convex utility cost. With probability $\lambda(s)$ a household is able to look for a job, and unemployment benefits stochastically decline over time to capture the limited duration of benefits. The household is assumed to have access to a backyard production technology that produces $b$ units of final consumption good. Thus, the unemployed household's problem is given below:

$$\begin{aligned} U(b;\psi) = \max_s u(b,s) + \beta\mathbb{E}_{\psi',b'}\Bigg[\lambda(s)\max_{\hat{w}}\Big(p\big(\theta(\hat{w};\psi')\big)W(w;\psi') \\ + \big(1 - p\big(\theta(\hat{w};\psi')\big)\big)U(b';\psi')\Big) + (1-\lambda(s))U(b';\psi')\Bigg] \end{aligned}$$

# 2 Calibration

The period is monthly, and $\beta = .996$. The log of the labor productivity process follows an AR(1):

$$\log(z_t) = \rho\log(z_{t-1}) + \epsilon_t$$

[3]There is efficiency loss here since future surplus might be positive even though today's profits are zero.

Where $\epsilon_t \sim N(0, \sigma_e^2)$. Set $\rho = .98$ and $\sigma_e = .01$. Approximate this process using Tauchen's method (or Rouwenhurst), with the log of the process normalized to zero, using 15 grid points spread across $\pm 3$ standard deviations of the mean. Use the following function for the job finding rate $p(\cdot)$:

$$p(\theta) = \theta(1 + \theta^\gamma)^{-1/\gamma}$$

Let $\gamma = .6$ as in Menzio and Shi (2009) and set the vacancy posting cost $k = 1.89$. Let $\bar{\delta} = .012$. Use a utility function of the form below:

$$u(c, s) = \frac{c^{1-\sigma} - 1}{1 - \sigma} - \alpha s^\chi$$

Set $\sigma = 0$, so the utility function is linear, and the agent is not constrained. Set $\alpha = 1$ and $\chi = 2$ as in Krueger and Mueller (2008). Set the following function for job posting opportunities:

$$\lambda(s) = s^a$$

Set $a = 1/3$ as in Ljungqvist and Sargent (1998).

In terms of 'unemployment insurance', the replacement rate is 54%, and there is a 10% chance that the home production technology becomes useless which is meant to capture a 2 year expected unemployment benefit duration.

Set the wage grid to $w \in [.1, 1]$ with spacing of .025. Set the benefit grid to $b = .54 * w$. If benefits expire, workers receive the lowest possible UI $= .1 \times .54$ (for simplicity). Either put a grid on $s \in [.2, .4]$ with spacing of .02 or optimize it continuously.

# 3 Solution Algorithm

Put a grid on $w \in \{w_1, \ldots, w_{N_w}\}$ of length $N_w$ and a grid on $z \in \{z_1, \ldots, z_{N_z}\}$ of length $N_z$ (determined by Tauchen's method), and a grid for search effort $s \in \{s_1, \ldots, s_{N_s}\}$ .

Use Tauchen's method to obtain a transition matrix $P_z$ for $z$:

$$P_z = \begin{pmatrix} P_{1,1} & \ldots & P_{1,N_z} \\ \vdots & & \vdots \\ P_{N_z,1} & \ldots & P_{N_z,N_z} \end{pmatrix}$$

1. Guess $\mathbf{J}_0$ which is an $N_w$ by $N_z$ vector of zeros. **Always** organize $\mathbf{J}_0$ such that choice variables index the rows and the stochastic variables index the columns.

$$\mathbf{J}_0 = \begin{pmatrix} J_0(w_1, z_1) & \ldots & J_0(w_1, z_{N_z}) \\ \vdots & & \vdots \\ J_0(w_{N_w}, z_1) & \ldots & J_0(w_{N_w}, z_{N_z}) \end{pmatrix}$$

2. Iterate on $J$ until convergence. To do this efficiently, pull out an entire row of $\mathbf{J}_0$. Pull the jth row.

$$J_1(w_j,:) = T(J_0(w_j,:)) = [z_1,\ldots,z_{N_z}] - [w_j,\ldots,w_j] + \\ [\frac{(1-\delta([z_1,\ldots,z_{N_z}],[w_j,\ldots,w_j]))}{(1+r)} .* J(w_j,:)] * (P_z')$$

3. Solve for $\theta$ (which will be of dimension $N_w$ by $N_z$), setting $\theta = 0$ in the cases where $J < 0$
4. Guess $W_0$ and $U_0$
5. Iterate until convergence
6. Use policy functions to simulate $N = 40,000$ agents for $T = 200$ periods, burning the first $T_{burn} = 100$ period's worth of data.

**To complete the homework you must:**

1. Prove the model admits a block recursive solution.
2. Solve the model and plot:
   A. The search policy function as a function of UI.
   B. The wage policy function as a function of UI.
   C. Report the mean unemployment rate and the volatility of the unemployment rate relative to productivity.