University of Minnesota
ECON 8581
Homework 5
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Basic Block Recursivity

Proof of Block Recursivity

For this proof I follow the approach proposed by Menzio and Shi (2011). The case we are treating here is more straightforward since there is no on-the-job search. The intuition for the result is the following: allowing workers to self-select informs the firm about the **only** type of worker it will meet if it opens a vacancy in one specific submarket (w, θ), which makes the equilibrium independent of the distribution of agents across states. In random search environments if a firm post a vacancy, it can find **any** worker across the entire distribution of workers, then the equilibrium would not be block recursive.

Step 1. Define the value function for the worker $V : \{0,1\} \times [0,1] \times \Psi \to \mathbb{R}$.

$$V(e,w,b,\psi) = e\left(u(w) + \beta \mathbb{E}_{\psi'}\left[\Big(1-\delta(w,z')\Big)V(1,w,b,\psi') + \delta(w,z')V(0,w,w/2,\psi')\right]\right) \tag{1}$$

$$+(1-e)\left(\max_s u(b,s) + \beta \mathbb{E}_{\psi',b'}\left[\lambda(s)\max_{\hat{w}}\left\{\Big(p\Big(\theta(\hat{w},\psi')\Big)V(1,\hat{w},b,\psi')\Big) + \Big(1-p\Big(\theta(\hat{w},\psi')\Big)\Big)V(0,w,b',\psi')\right\}\right.\right.$$

$$\left.\left.\Big(1-\lambda(s)\Big)V(0,w,b',\psi')\right]\right)$$

Step 2. Let $w(\theta,\psi)$ denote the value offered to a worker in a submarket with tightness $\theta(w,\psi) = \theta > 0$. From free-entry:

$$\theta(w;\psi) = q^{-1}\left(\frac{\kappa}{J(w;\psi)}\right) \Longrightarrow J(w;\psi) = \frac{\kappa}{q(\theta)} \tag{2}$$

The above expression gives a closed-form expression for θ that does not depend on any of the distribution of agents μ and only depends on the aggregate state through z. From the firms' problem, solving for w:

$$w(\theta;\psi) \equiv w(\theta;z) = z - \frac{\kappa}{q(\theta)}\left[\frac{r - \mathbb{E}_{z'}[\delta(z',w)]}{1+r}\right] \tag{3}$$

Step 3. Lets rewrite (1) substituting w by $w(\theta,z)$ and $\theta(\hat{w};\psi)$ by θ:

$$T\Big(V(e,w,b,\psi)\Big) = e\Bigg(u(w) + \beta\mathbb{E}_{\psi'}\Big[\Big(1-\delta(w,z')\Big)V(1,w,b,\psi') + \delta(w,z')V(0,w,w/2,\psi')\Big]\Bigg) \tag{4}$$

$$+(1-e)\Bigg(\max_s u(b,s) + \beta\mathbb{E}_{\psi',b'}\Bigg[\lambda(s)\max_\theta\Big\{p(\theta)V(1,w(\theta,z),b,\psi') + \Big(1-p(\theta)\Big)V(0,w(\theta,z),b',\psi')\Big\}$$
$$\Big(1-\lambda(s)\Big)V(0,w,b',\psi')\Bigg]\Bigg)$$

By guessing that V does not depend on μ and it is a function of aggregate variables only through z we get that $T(V)$ also does not depend on μ. According to (2), the market tightness also does not depend on the distribution of agents. We thus conclude the model is *block recursive*. ■

Quantitative Results

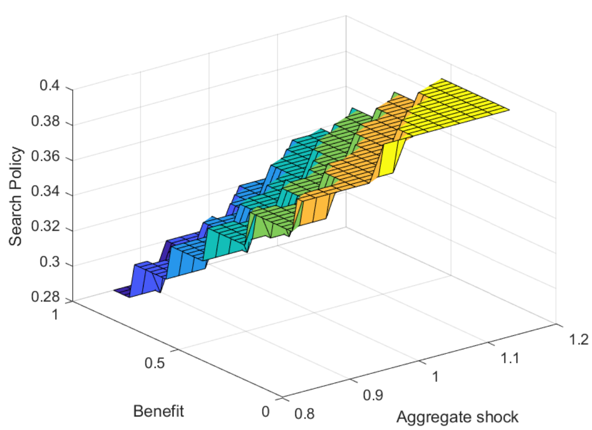
We follow the algorithm suggested and solved discretized the grid for possible effort values[1]. Following the hint we loop over the rows to find the wage in the grid that maximizes the value of searching. After computing the maximum value of searching we compute the RHS of the unemployed Bellman equation and maximize picking the values on the effort grid. The result for this sequence of operations provides us the update for the guess of the unemployed value function.

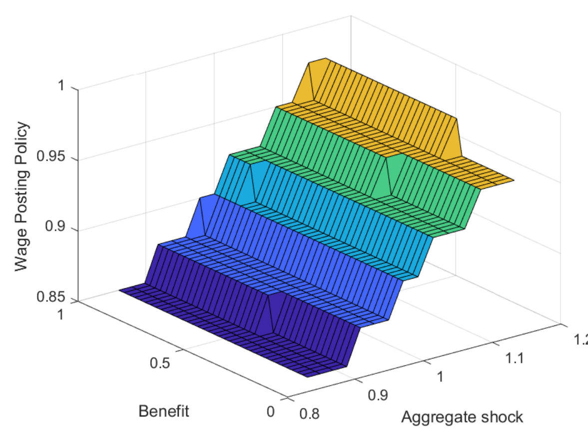
[1] The MATLAB codes used to solve/simulate the model can be found here

Table 1: Model Parameters

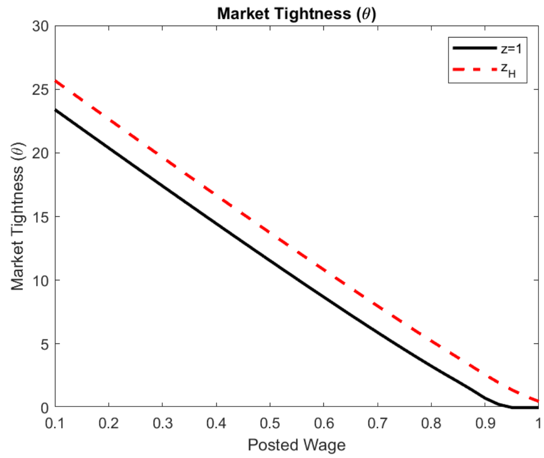
Parameter	Value
β	0.996
σ	0
κ	1.89
α	1
χ	2
a	1/3
γ	0.6
$\bar{\delta}$	0.012
σ_ϵ	0.01
ρ	0.98

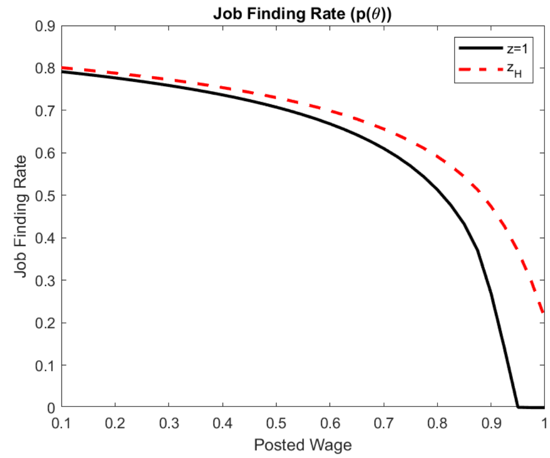
The results for the policy functions for wages posted and search policy as a function of the benefit and the aggregate shock are the following





I also exhibit results for market tightness and job-finding rate as a function of the wages posted.





Finally, we simulate the model for $T = 500$, burn 300 observations and compute the moments:

Table 2: Simulation Results

Moment	Value
Avg. Unemployment rate	0.702
σ_u/σ_z	10.72

References

G. Menzio and S. Shi. Efficient search on the job and the business cycle. *Journal of Political Economy*, 119(3):468–510, 2011.