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Macro, Money and Banking

Problem Set 3

1. Consider the economy in Amador and Bianchi (2022) without runs. In that economy, banks are subject to a borrowing constraints

$$b_{t+1} \leq \gamma_t p_{t+1} k_{t+1} \tag{1}$$

For a constant price p, the value of γ is given by the lowest solution to

$$\frac{\overline{z} + p(1 - \gamma R)}{\underline{z} + p} = (1 - \gamma)^{\beta} \qquad \text{for all} \quad t \geq 0. \tag{2}$$

Set values as follows: $R = 1.1,\ \beta = 0.9/R,\ \underline{z} = \beta/(1 - \beta),\ \bar{z} = 1.15\underline{z}$, and $\bar{K} = 1$.

a) Keeping all parameters constant, conduct a comparative static wirht respect to R. Specifically, plot the (partial equilibrium) value of γ and the borrowing limit γp as a function of a grid of values for $R \in [0.9, 1/\beta, 1/\beta]$ assuming $p = 1$

b) Find the general equilibrium value of f γ in a stationary equilibrium with repayment as a function of a grid of values for $R \in [1.001, 1/\beta]$. Recall that the asset price in a repayment stationary equilibrium is given by

$$p^R = \frac{\beta \overline{z}}{1 - \beta - (1 - \beta R)\gamma^R} \tag{3}$$

c Explain the intuition behind the differences in (a) and (b). In addition, discuss the intuition of the case with $\beta R = 1$.

d Redo [a]-[c] in an economy with runs. In that case, recall that

$$\beta \log\left(\frac{\overline{z} + p(1 - \gamma R)}{\overline{z} + p}\right) - \beta^2 \log\left(\frac{\overline{z} + p(1 - \gamma R)}{\overline{z} + p}\right) + \\ + \beta^2 \log(1 - \gamma) = \log\left(\frac{\overline{z} + p(1 - \gamma R)}{\underline{z} + p}\right) \tag{4}$$

and p^R is still given by (3).

How does the region where banks default due to runs change with R. Discuss the intuition?

2. Consider the economy in Amador and Bianchi (2022). Assume that at $t = 0$, banks face an idiosyncratic shock to the productivity under repayment $\bar{z}_0$. So in the absence of runs the value of repayment at $t = 0$ is given by

$$\hat{V}_0^R(n; \bar{z}_0) = A + \frac{1}{1-\beta} \log((\bar{z}_0 + p_0)K - Rb_0) + \frac{1}{1-\beta} \sum_{t=1}^{\infty} \beta^t \log(R^e_{t+1}), \tag{5}$$

where

$$R^e_{t+1} \equiv R^k_{t+1} + (R^k_{t+1} - R)\frac{\gamma_t p_{t+1}}{p_t - \gamma_t p_{t+1}}, \tag{6}$$

where $R^K_{t+1} \equiv \frac{\bar{z}+p_{t+1}}{p_t}$

Meanwhile, the value of default is:

$$V_0^D(k) = A + \frac{1}{1-\beta} \log((\underline{z} + p_0)k) + \frac{1}{1-\beta} \sum_{t=1}^{\infty} \beta^t \log\left(R^D_t\right), \tag{7}$$

where $R^D_{t+1} \equiv \frac{\underline{z}+p_{t+1}}{p_t}$

a) Show that the value of $\bar{z}_0$ that makes a bank indifferent between defaulting and repaying at $t = 0$ is given by

$$\hat{z}^f = (\underline{z} + p_0) \prod_{t=1}^{\infty} \left(\frac{R^D_t}{R^e_t}\right)^{\beta^t} - p_0(1 - R\frac{b_0}{k_0}) \tag{8}$$

a) Bonus. Assume that now runs can take place only at $t = 0$. Show that the value of $\bar{z}_0$ that makes a bank indifferent between repaying and defaulting is given by:

$$\hat{z}^{Run} = (\underline{z} + p_0) \left(\frac{R^D_1}{R^k_1}\right)^{\beta} \times \prod_{t=2}^{\infty} \left(\frac{R^D_t}{R^e_t}\right)^{\beta^t} - p_0(1 - R\frac{b_0}{k_0}) \tag{9}$$

Hint: the value of repaying for a bank facing a run at $t = 0$ is given by:

$$V_0^{Run}(n) = \max_{k' \geq 0, c > 0} \log(c) + \beta V_1^R\left((z + p_1)k'\right),$$

subject to

$$c = n - p_t k'.$$