Entrepreneurship and Misallocation in Production Network Economies*

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Abstract

This paper explores how sectoral linkages amplify or mitigate misallocation at intensive and extensive margins. Our analysis employs a multisector general equilibrium model with input-output connections, heterogeneous agents, and endogenous occupational choices. Distortions impact both the intensive use of production inputs and agents' career decisions, causing misallocation of entrepreneurs across diverse production sectors. We analytically demonstrate that input-output linkages amplify (diminish) misallocation losses when the most distorted sectors are upstream (downstream). Calibrating our model to the US economy, we quantify output losses due to sectoral corporate taxes, revealing that sectoral linkages magnify misallocation losses by over four times. We evaluate an entry subsidy program and find that it should ideally target sectors where "marginal" entrepreneurs face more significant profit losses due to distortions, even if these sectors are not the most distorted ones.

Keywords: Distortions, Firm Entry, Production Network, Aggregate Misallocation
JEL codes: E23, L26, O11, O41

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1. Introduction

It is well known that sectoral distortions reduce aggregate productivity (e.g., Hsieh and Klenow, 2009) and linkages between sectors can amplify the effects of such distortions (e.g., Jones, 2011; Bigio and La'o, 2020; Baqaee and Farhi, 2020b) on economic efficiency. Most of the papers in the literature on production networks and misallocation consider economic environments with a fixed number of firms and no endogenous entry. Existing distortions, however, not only affect the optimal scale of firms, but they also impact entry decisions. This paper investigates how sectoral distortions affect aggregate output in a framework with endogenous entrepreneurship and input-output linkages.

We first provide evidence of how sectoral shocks affect the economy through the production network. We empirically investigate the impact of financial shocks on sectoral employment and number of firms. We use the measure of External Finance Dependence developed by Rajan and Zingales (1998) to investigate the sectoral effects of an increase in aggregate credit in the United States. We explore the direct effect of the shock and how it propagates through the production network. We build measures of upstream and downstream exposure that capture how the shock spreads from sectors that respectively purchase and sell inputs to a specific sector. We find that both employment and number of firms in one sector expand when a positive shock hits sectors that buy its inputs. On the contrary the effect is significantly negative when the shock hits the suppliers. While the upstream effect can be easily explained with an expansion driven by higher demand, the downstream one can be rationalized with a reallocation of workers and entrepreneurs.

Based on this evidence, we build a multisector general equilibrium model in which sectoral output can be consumed or used by other sectors as input, similarly to Bigio and La'o (2020) and Carvalho and Tahbaz-Salehi (2019), among others. As in Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), distortions are represented by exogenous sectoral wedges between marginal revenues and costs. However, in our economy, returns to scale are decreasing, in the same spirit of Lucas Jr (1978). Individuals with heterogeneous managerial productivity can choose to be a worker or an entrepreneur in one of the production sectors. Therefore, the mass of workers and the mass of firms by sector and in the aggregate are endogenous objects.

We show analytically that aggregate output depends on the allocation of intermediate goods and workers at the intensive margins, and the allocation of agents between labor and entrepreneurship at the extensive margin. Extensive margin distortions can be described by two types of wedges: a "labor" wedge, which misallocates individuals between entrepreneurship and the labor force; and "entrepreneurship" wedges, which misallocate entrepreneurs among different sectors of production.

The "labor" wedge resembles the one described by Chari et al. (2007), which appears also in Bigio and La'o (2020) who investigate the effects of sectoral intensive margin distortions (in their case, financial shocks) on the macroeconomy. In their framework, labor supply is elastic, and the mass of firms is constant. In our model, labor supply of each worker is fixed, but the mass of workers and entrepreneurs are endogenous. We show that this wedge is represented by the ratio of the aggregate share of labor income in the distorted over the undistorted economies.

The "entrepreneurship" wedges affect instead the mass of entrepreneurs in each one of the sectors. Similarly to the "labor" wedge, they are given by the ratio of income shares of entrepreneurs in the distorted over the undistorted economies. However, since entrepreneurs earn heterogeneous profits, these wedges are based on the income of marginal entrepreneurs only.

The Hulten's Theorem (cf., Hulten, 1978) applies to our economy, as the first-order effects of sectoral productivity shocks on aggregate TFP are represented by the efficient-economy Domar weights. Shocks to sectoral distortions induce a reallocation of individuals between the labor force and entrepreneurship. In particular, a negative distortion shock - a rise in distortion - in sector i reduces the number of firms in this sector if it is characterized by low labor intensity. In addition, the mass of firms in other sectors is also reduced if those sectors are direct or indirect supplier to sector i and they have low labor intensity.

The second-order approximation of the aggregate output loss from distortions can be characterized by the variance of our "labor" and "entrepreneurship" wedges. We compare the output losses in a production network economy to the one suffered by an equivalent horizontal economy.[1] We show how input-output linkages diminish (amplify) the loss from distortions if they directly hit more downstream (upstream) sectors. Intuitively, sectors that are direct or indirect supplier of downstream sectors are negatively affected by a lower demand. This leads to a reallocation of resources which may rebalance the outflows of entrepreneurs from the originally distorted sector.

We complement our analysis with some quantitative exercises. We evaluate the cost of uneven sectoral tax rates in the United States, represented in our model by the exogenous wedges between marginal revenues and costs. We calibrate model parameters to match sectoral moments of the US economy. We compute the contribution of intensive and extensive margin misallocation on aggregate output loss. We show how the endogenous entry of firms is quantitatively important in amplifying distortions in network economies, especially in an augmented version of our model with fixed technological entry costs. Network linkages quadruple the loss from misallocation of entrepreneurs in the model without fixed cost, while the loss is twelve times larger in the model with fixed costs.

Finally, we use our calibrated model to analyze the effects of sectoral subsidies on output. We compare the effects of a subsidy program which targets one sector at the time but requiring the same total level of expenditure. This exercise resembles the one investigated by Liu (2019). The difference is that we consider subsidies in an environment with endogenous occupational choice. We show that the size of direct distortions are a good statistics to rank the return from subsidies in the equivalent horizontal economy, while they are not necessarily a good measure in production network economies. In the presence of input-output linkages, sectors

[1] The equivalent horizontal of a production network economy is defined as the economy with no input-output linkages but the same allocation of individuals across sectors at efficiency.

should be ranked by their "entrepreneurship" wedge, which represents the profit loss of the marginal entrepreneurs relative to the undistorted benchmark. Consequently, the knowledge of the production network structure is needed in designing this entry subsidy program.

The paper is organized as follows. Section 2 presents the related literature and places our contribution. Section 3 presents the empirical evidence that motivates our theory. Section 4 contains our main environment with heterogeneous entrepreneurial productivity and the characterization of the equilibrium. Section 5 derives theoretical results about the effects of sectoral distortions on output and entrepreneurship. We also derive analytically welfare losses from distortions and identify the amplification/diminish role of input-output linkages. Section 6 analyzes the effect of distortions in two simple network examples. Section 7 calibrates model parameters and quantifies the losses from distortionary taxes in the US. We decompose these losses from misallocation at the intensive and extensive margins. Section 8 analyzes a targeted subsidy program. Section 9 concludes.

2. Related Literature

Since the paper by Long Jr and Plosser (1983), there is a growing literature in macroeconomics studying multisector models to understand the importance of sectoral shocks and their transmission mechanism through input-output linkages (e.g., Acemoglu et al., 2012). Baqaee and Farhi (2018, 2019) are general theoretical references for the production network literature. The first paper characterizes a class of models with heterogeneous agents and input-output linkages, showing that propagation patterns are constrained by the assumption of representative-agent models. The second paper extends the results from Hulten (1978), deriving a decomposition of the first-order effects at and away from efficiency.[2]

Our paper is closely related to a subset of this literature which investigates how misallo-

[2] Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) provide an extensive review on this literature.

cation can be propagated through the input-output structure of the economy. Jones (2011) studies the same issue in a standard growth model with neoclassical production functions.[3] Bigio and La'o (2020) investigate the effect of wedges between prices and marginal costs and use their model to analyze the role of financial frictions in business cycle fluctuations. Their framework includes elastic labor supply but abstracts from endogenous entrepreneurship and entry of firms.

The paper by Liu (2019) is also related to ours, especially for the analysis of optimal subsidies. He investigates industrial policies in a constant returns to scale, nonparametric production network with market distortions and subsidies. He studies the aggregate effects of sectoral subsidies targeting specific sectors and proposes a measure of "distortion centrality" to identify which sectors should be subsidized. Our paper considers a parametric production structure but adds endogenous occupational choice. Our subsidy program is slightly different. We subsidize entry while his analysis is based on a production subsidy proportional to input expenditure. In our model, we show that an entry subsidy should target those sectors where marginal entrepreneurs suffer larger profit losses relative to the undistorted economy. Our measure to rank sectors partially resembles the "distortion centrality" by Liu (2019).

Baqaee and Farhi (2020b) investigate the effects of misallocation in economies with production networks and non-parametric input-output structure. They do not consider the role of endogenous firm entry. Baqaee and Farhi (2020a) study the effects of distortions in a framework with endogenous entry from a separate set of potential entrants. They decompose changes in aggregate productivity into changes in technical and allocative efficiency, showing the importance of endogenous entry.[4] Their model allows for both decreasing and increasing production functions, and different types of distorting wedges. We model entry differently, as an occupational choice decision: an increase in the mass of firms mechanically reduces the labor force;

[3]See also Fadinger et al. (2022) who use the model to understand how the production structure and sectoral productivity differences influence disparities in income levels across countries.

[4]Baqaee (2018) considers a model with firm entry/exit with production networks, but the focus of his analysis is on the amplification of productivity shocks.

and a higher mass of entrepreneurs in one sector might lead to a lower mass in other sectors. Variations of our model have also been used by researchers to study different macro development questions, such as those related to the implication of regulations and taxes on informal entrepreneurship (e.g., Antunes and Cavalcanti, 2007; Rauch, 1991) or the impact of credit market imperfections on development (e.g., Antunes et al., 2008, 2015; Buera et al., 2011). Our environment with input-output linkages could be clearly adapted to investigate these and related issues.

By modelling firms' entry as an occupational decision, our model relates to the macro-entrepreneurship literature. Recent papers have investigated the factors behind the decline in entrepreneurship occurred in the last decades in many advanced economies (e.g., Akcigit and Ates, 2021; Karahan et al., 2019)). In our model a decline in the number of entrepreneurs can be explained by distortions relatively hitting sectors with low labor intensity or sectors that largely use inputs produced by low labor intensity sectors.

3. Empirical motivation

This section provides evidence of how sectoral shocks affect the economy through the production network. This section should be viewed as an empirical illustration of the mechanisms we explore in our model, rather than a causal analysis of how financial shocks are propagated in the economy.

We use United States data from different sources. We identify sectoral shocks to financial frictions computing the interaction between the growth in aggregate credit to non-financial corporations from the BIS statistics and the measure of External Finance Dependence (EFD) provided by Rajan and Zingales (1998) for manufacturing industries. The measure of EFD is computed from Compustat firm-level data as the average difference between capital expenditure and operating cash flow divided by total capital expenditure for each sector from 1980 to 1989. To analyze the propagation of a shock in the production network, we use the Input-Output tables from the Bureau of Economic Analysis. Finally our dependent variables, number of employees and number of firms at the sectoral level, are from the Bureau of Labor Statistics. Our sample is restricted to 13 manufacturing industries for which all data are available from 1999 to 2020.[5]

Shocks to sectoral distortions are not easy to be identified in the data. We rely on the heterogeneous effects of aggregate credit shocks across manufacturing industries which are different in their external finance dependence (e.g., Manova, 2013). Sectoral distortions are clearly not only driven by financial frictions. However, our focus here is on the propagation of these distortions or identifying the mechanisms explored in the paper, rather than an evaluation of different types of frictions. We also construct a measure of how these distortions propagate upstream and downstream. Specifically, we estimate the following equation:

$$\Delta y_{i,t} = \alpha FinShock_{i,t} + \beta UpstreamExposure_{i,t} + \gamma DownstreamExposure_{i,t} + d_t + \epsilon_{i,t}. \tag{1}$$

Our dependent variables are the changes in the log-number of workers and the log-number of firms in sector i in year t. The variable $FinShock_{i,t}$ represents a change in the financial friction and it is given by:

$$FinShock_{i,t} = \Delta \log(Credit)_t * EFD_i.$$

d_t controls for common factors. We build measures of upstream and downstream exposure to financial shocks, similar to Acemoglu et al. (2016):

$$UpstreamExposure_{i,t} = \sum_j \sigma_{ji} FinShock_{j,t},$$

$$DownstreamExposure_{i,t} = \sum_j \sigma_{ij} FinShock_{j,t}.$$

The variable σ_{ij} represents the share of industry i total sales used to purchase inputs from industry j in 1999. The variable $UpstreamExposure_{i,t}$ captures how the shock propagates from

[5] The industries are Food and Beverages, Textile, Apparel, Wood products, Paper, Printing and Publishing, Petroleum refineries, Industrial chemicals, Fabricated metal, Machinery, Professional equipment, Transport equipment, and Furniture.

sectors that purchase the output produced by sector i. Similarly, the variable $DownstreamExposure_{i,t}$ captures how the shock propagates from sectors that sell their inputs to sector i.

The regression results are presented in Table 1. The first two columns refer to the effect on total employment growth, while the other two columns refer to the growth of the number of firms. The direct effect of the shock is always positive. We also find a positive and significant effect from upstream exposure to the shock. Both the number of firms and workers in a sector increase if the shock hits sectors that purchase inputs from i. Interestingly the sign is reversed once we look at the effect from downstream exposure. The sector i is negatively affected by a positive shock hitting sectors that are more upstream in the production chain.

Table 1: Effect of financial shocks on employment and number of firms

	$\Delta log(employment)_{i,t}$		$\Delta log(firms)_{i,t}$	
	(1)	(2)	(3)	(4)
$FinShock_{i,t}$	1.385***	1.543***	0.368*	0.427**
	(0.426)	(0.331)	(0.217)	(0.203)
$UpstreamExposure_{i,t}$	1.42*	1.124*	1.36***	1.249***
	(0.804)	(0.624)	(0.41)	(0.383)
$DownstreamExposure_{i,t}$	-3.798***	-6.288***	-1.933***	-2.862***
	(1.074)	(0.886)	(0.547)	(0.543)
Year FE	N	Y	N	Y
Obs	286	286	286	286

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

In the next section, we present a model of production network with endogenous occupational choice that can rationalize these facts. While the positive upstream effect can be explained by an increase in the demand for the industry output, the negative downstream effect is consistent with a reallocation of workers and entrepreneurs to sectors that are relatively expanding. In Section 5 we will discuss how the model relates to our empirical results.

4. Model

The economy is static. There are N sectors producing intermediate goods indexed by $i \in S = \{1, ..., N\}$. Each intermediate good is used as a production input for a final consumption good and other intermediate goods. There is also a continuum of individuals of measure 1. The utility function of each individual is strictly increasing and strictly concave on the final good consumption.

Individuals are endowed with one unit of time that can be supplied to firms or used to manage a business. Individuals can either work in any sector or they can open a business in a specific sector i. They first draw from a discrete uniform distribution which sector they can operate. They then draw a managerial productivity v from a Pareto distribution $\mu_i(v)$, with scale parameter 1 and a sector specific shape parameter ξ_i.[6] A lower ξ_i denotes a higher degree of heterogeneity. The production function of an entrepreneur in sector i with productivity v is given by

$$y_i(v) = v a_i l_i(v)^{\theta_i} \prod_{j \in S} x_{ij}(v)^{\sigma_{ij}}, \quad \text{with} \quad \theta_i \geq 0, \quad \sigma_{ij} \geq 0 \qquad \text{and} \quad \eta_i \equiv \theta_i + \sigma_i = \theta_i + \sum_{j \in S} \sigma_{ij} < 1,$$

where y_i denotes the output in sector i, l_i is the labor input, x_{ij} is the quantity of good j used for production of good i, and a_i is a Hicks-neutral productivity factor common to all firms in sector i. Returns to scale are decreasing.

[6]The assumption that individuals can only be an entrepreneur in a specific sector is made for tractability reasons and implies a simple binary decision between employment and entrepreneurship. Our main derivations are similar if we assumed that individuals could choose among all sectors, but entrepreneurial productivities were sector-specific and the probability of high productivity in more than one sector was negligible.

A representative firm aggregates the sectoral goods into a single final consumption good according to

$$Q = \prod_{i \in S} c_i^{\psi_i}, \quad \text{with} \quad \psi_i \geq 0 \quad \text{and} \quad \sum_{i \in S} \psi_i = 1.$$

The price of this final good is normalized to 1.

Individuals who choose to be workers earn the equilibrium wage w. Entrepreneurs make positive profits given decreasing returns to scale. The input choice they make is distorted by sectoral wedges. An entrepreneur in sector i pays a variable cost $(1-\phi_i)$ per unit of revenue. This cost creates a wedge between the marginal productivity of each input used in the production of sector i and its rental price. Parameter ϕ_i affects directly the optimal scale of firms in sector i and distorts the optimal occupational choice.

An entrepreneur with productivity v in sector i takes prices as given and chooses $l_i(v)$ and $x_{ij}(v)$, for $j \in S$, to maximize profits:

$$\pi_i(v) \equiv \phi_i p_i a_i v l_i(v)^{\theta_i} \prod_{j \in S} x_{ij}(v)^{\sigma_{ij}} - w l_i(v) - \sum_{j \in S} p_j x_{ij}(v). \tag{2}$$

Given the optimal input decisions in each sector i, an individual chooses to be an entrepreneur in sector i if and only if

$$\pi_i \equiv (1-\eta_i)\phi_i p_i y_i(v) \geq w.$$

The previous condition implies a productivity cutoff $\hat{v}_i$ for each sector i, which is given by:

$$\hat{v}_i = \left[\frac{w}{(1-\eta_i)\phi_i p_i a_i} \frac{\left(\frac{\xi_i(1-\eta_i)}{\xi_i(1-\eta_i)-1}\right)^{\eta_i}}{N^{\eta_i} L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}}} \right]^{\frac{1}{1+\xi_i \eta_i}},$$

where L_i and X_{ji} are respectively the aggregate labor in sector i and the aggregate demand of good i from sector j.[7]

[7]See the Appendix for details.

Revenues from distortions are equally rebated back to individuals. Therefore, the market clearing condition for good i is

$$Y_i = c_i + \sum_{j \in S} X_{ji}, \tag{3}$$

where Y_i is the aggregate output in sector i. Finally, the labor market equilibrium condition requires

$$\sum_{i \in S} M_i + L = 1,$$

where M_i is the equilibrium share of entrepreneurs in sector i and L is the equilibrium total share of workers.

Under the assumption that $\xi_i(1-\eta_i) > 1$ for any i, aggregate output in sector i can be approximated by

$$Y_i \approx A_i \left(L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}} \right)^{\frac{\xi_i}{1+\xi_i \eta_i}},$$

with

$$A_i = \left[a_i \left(\frac{(1-\eta_i)\phi_i p_i}{w} \right)^{\frac{\xi_i(1-\eta_i)-1}{\xi_i}} \left(\frac{\xi_i(1-\eta_i)}{\xi_i(1-\eta_i)-1} \right)^{\frac{1}{\xi_i}} \left(\frac{1}{N} \right)^{\frac{1}{\xi_i}} \right]^{\frac{\xi_i}{1+\xi_i \eta_i}}.$$

The complete derivation is presented in the Appendix.

4.1. Equilibrium

Let the Domar weight of sector i be the industry's sales as a fraction of GDP:

$$\lambda(\phi)_i \equiv \frac{p_i Y_i}{Q}.$$

From the market clearing condition of all goods, it is possible to derive the vector of equilibrium Domar weights as a function of model primitives:[8]

$$\lambda(\phi) = \left(\mathbb{I}_N - \Sigma' \circ (1\phi') \right)^{-1} \psi. \tag{4}$$

[8]See the Appendix for the full derivation.

The vector of Domar weights describes the centrality of each sector in the production network. The weights depend on the vector of final shares, ψ, and the linkages between sectors described by the matrix Σ. Economic distortions affect the Domar weights through these linkages. In particular, distortions in sector i reduce the sales of those other sectors supplying intermediate goods to i.

In the remaining of the paper, we will express our solutions in terms of shares of income of workers and entrepreneurs in each sector. The share of labor income is denoted by $s_L(\phi) \equiv \frac{wL}{Q}$. All workers earn the same wage, given that labor productivity is homogeneous and there is perfect labor mobility across sectors. Differently, entrepreneurs in one sectors earn different profits depending on their managerial productivity. It is convenient to define the share of profits of each sector as if all entrepreneurs were marginal, $s_\Pi(\phi)_i \equiv \frac{wM_i}{Q}$. Finally, we define $s_T(\phi)$ as the total sum of $s_L(\phi)$ and $s_\Pi(\phi)_i$ across all sectors. Notice that all defined shares are after tax and before rebate. We can write these shares as:

$$s_L(\phi) = \sum_j \theta_j \phi_j \lambda(\phi)_j,$$

$$s_\Pi(\phi)_i = \left[(1-\eta_i) - \frac{1}{\xi_i}\right] \phi_i \lambda(\phi)_i \ \forall i \in S, \text{ and}$$

$$s_T(\phi) = \sum_j \left[(1-\sigma_j) - \frac{1}{\xi_j}\right] \phi_j \lambda(\phi)_j.$$

We can observe that distortions affect theses shares directly and through the Domar weights. Therefore, we also define the following wedges, which present the labor, profit, and total shares relative to an economy without distortions:

$$\tau_L(\phi) \equiv \frac{s_L(\phi)}{s_L(1)},$$

$$\tau_\Pi(\phi)_i \equiv \frac{s_\Pi(\phi)_i}{s_\Pi(1)_i},$$

$$\tau_T(\phi) \equiv \frac{s_T(\phi)}{s_T(1)}.$$

We can now characterize the equilibrium.

Proposition 4.1 *The equilibrium in the economy can be described by an aggregate production function*

$$\log Q = \sum_j \psi_j \log \psi_j + \lambda(1)' \log A(\phi) + \lambda(1)' \left\{ \left[(1-\eta) - \frac{1}{\xi} \right] \circ \log M + \theta \log L \right\} \tag{5}$$

with

$$A(\phi)_i = a_i \phi_i \left(\frac{(1-\eta_i)}{s_\Pi(\phi)_i} \right)^{1-\eta_i} \left(\frac{\theta_i}{s_L(\phi)} \right)^{\theta_i} \prod_j \left(\sigma_{ji} \right)^{\sigma_{ji}} \left(\frac{1}{N} \right)^{\frac{1}{\xi_i}}, \tag{6}$$

the equilibrium shares of entrepreneurs in each sector

$$M_i = \tau_\Pi(\phi)_i \frac{s_\Pi(1)_i Q}{w} = \frac{s_\Pi(\phi)_i}{s_T(\phi)}, \tag{7}$$

and the equilibrium share of workers

$$L = \tau_L(\phi) \frac{s_L(1) Q}{w} = \frac{s_L(\phi)}{s_T(\phi)}. \tag{8}$$

Proposition 4.1 is proved in the Appendix. Equation (52) describes the aggregate production as a function of two components.[9] The first one, $\sum_i \lambda(1)_i \log A(\phi)_i$, describes the allocation of workers and intermediate goods across sectors.[10] From now on, we will refer to it as the intensive margin component. The second component, $\sum_i \lambda(1)_i \left\{ \left[(1-\eta_i) - \frac{1}{\xi_i} \right] \circ \log M_i + \theta_i \log L \right\}$, represents the allocation of individuals between paid jobs and entrepreneurship. Given that the sum of all workers and entrepreneurs is fixed, this component describes misallocation at the extensive margin. In order to distinguish from the previous one, we will refer to the latter as the occupational choice component.

The mass of firms in each sector is an endogenous object. Equation (7) describes the selection of entrepreneurs into sector i: in an undistorted economy the opportunity cost from opening an additional firm, w, must equalize the profits of the marginal entrepreneur, $\frac{s_\Pi(1)_i Q}{M_i}$.[11]

[9] There is also the component $\sum_j \psi_j \log \psi_j$, but this component is invariant to distortions and equilibrium objects.

[10] When $\phi_i = 1$, then TFP of sector i is only a function of primitives, i.e., $A(1)_i = a_i \left(\frac{(1-\eta_i)}{s_\Pi(1)_i} \right)^{1-\eta_i} \left(\frac{\theta_i}{s_L(1)} \right)^{\theta_i} \prod_j \left(\sigma_{ji} \right)^{\sigma_{ji}} \left(\frac{1}{N} \right)^{\frac{1}{\xi_i}}$.

[11] Remember that $s_\Pi(\phi)_i$ represents the share of profits as if all entrepreneurs were marginal, so $\frac{s_\Pi(\phi)_i Q}{M_i}$ are exactly the profits of the marginal entrepreneur.

The term $\tau_{\Pi,i}$ is therefore the deviation in the (marginal) profit share of sector i relative to an undistorted economy. It represents an "entrepreneurship" wedge distorting the allocation of entrepreneurs into sector i. Equation (8) describes the aggregate selection into the labor force: in an undistorted economy, the marginal cost of labor w must approximately equalize its marginal productivity, $\frac{s_L(1)Q}{L}$. The term τ_L is the deviation in total labor share and represents a wedge in the allocation of individuals between entrepreneurship and paid work. In the next sections, we will generally refer to the $N+1$ wedges τ (N "entrepreneurship" wedges plus the "labor" wedge) as occupational wedges.

5. The effect of distortions

In this section, we analyze the marginal effect of productivity and distortion shocks in our economy.

We start stating the following Hulten's Theorem result:

Theorem 5.1 *The first-order effect of a sectoral productivity shock on aggregate TFP and total output is equal to the efficient-economy Domar weight of the sector:*

$$\frac{d\sum_j \lambda(1)_j \log A(\phi)_j}{d\log a_i} = \frac{d\log Q}{d\log a_i} = \lambda(1)_i\,. \tag{9}$$

The shock does not induce any change in the mass of firms.

The Theorem is proved in the Appendix. At efficiency, the effect of a sectoral productivity shock on TFP can be summarized by the Domar weight of the sector. In a Cobb-Douglas economy, once we depart from efficiency, the effect of the shock is still equal to the efficient-economy Domar weight. However, the actual industry's sales shares are modified by distortions.[12]

[12]In our model with Cobb-Douglas production functions and no fixed costs, second-order effects from productivity shocks are irrelevant. This is not the case if fixed technological costs are introduced. See Baqaee and Farhi (2019) for an analysis of second-order productivity shocks in a more general class of models.

The Theorem also states that a productivity shock does not alter the allocation of workers and entrepreneurs. This is because the shock in sector i does not change the marginal conditions between paid work and entrepreneurship.

The total first-order effect of distortions on aggregate output is also a positive function of the efficient-economy Domar weights:[13]

$$\frac{d\log Q}{d\log\phi_i} = \lambda(1)_i - \frac{ds_T(\phi)}{d\phi_i} - \sum_j \frac{1}{\xi_j(1-\eta_j)-1}\frac{ds_\Pi(\phi)_j}{d\phi_i}. \tag{10}$$

The direct effect of a tax on revenues (lower ϕ_i) is equivalent to the effect of a negative productivity shock. However, resources are not destroyed so the direct effect must be adjusted by two additional terms. These terms represent the (positive) change in tax rebates which diminish the direct drop in firms' revenues. Notice that the reduction in output is lessened for a higher level of heterogeneity (lower ξ_j). In this case, a marginal change in the number of firms does not produce a big impact on total production since most of the output is produced by a small number of very productive firms.

From Equation (8), we also derive the effects of distortions ϕ_i on labor supply:

$$\frac{d\log L}{d\log\phi_i} = \frac{d\log s_L(\phi)}{d\log\phi_i} - \frac{d\log s_T(\phi)}{d\log\phi_i}. \tag{11}$$

The change in the mass of workers is given by the difference between the changes in the labor share and the total share.

We can get further insights focusing on the efficient economy. From Equations (10) and (11) and considering the case in which the ϕ_is are close to one, we can state the following Proposition:

Proposition 5.2 *Starting from the efficient equilibrium, the first-order effect of a sectoral distortion shock on total output is 0. The shock changes the mass of firms according to:*

$$\left.\frac{d\log\left(\sum M_i\right)}{d\log\phi_i}\right|_{\phi=1} = \left(\frac{d\log s_T(\phi)}{d\log\phi_i} - \frac{d\log s_L(\phi)}{d\log\phi_i}\right)_{\phi=1}$$

[13] See the Appendix for the full derivation.

$$= \frac{\left(1-\frac{1}{\xi_i}\right)\lambda(1)_i - \sum_j \frac{1}{\xi_j}\frac{d\lambda(\phi)_j}{d\phi_i}\Big|_{\phi=1}}{1-\sum_j \frac{1}{\xi_j}\lambda(1)_j} - \frac{\theta_i \lambda(1)_i + \sum_j \theta_j \frac{d\lambda(\phi)_j}{d\phi_i}\Big|_{\phi=1}}{\sum_j \theta_j \lambda(1)_j}. \tag{12}$$

The Proposition is proved in the Appendix. The first result of this Proposition is expected since $\phi_i = 1, \forall i \in S$, corresponds to the point in which $\log Q$ attains its maximum and the derivative of $\log Q$ with respect to each ϕ_i must all be equal to 0. Similar result is also shown in Baqaee and Farhi (2020b).

Equation (12) contains one of the main analytical contributions of this paper. The first term, represents the positive change in the total share of income when distortions are reduced (higher ϕ). This change is larger when entrepreneurs are more homogeneous (higher ξ_i). This positive effect is counteracted by the second term, which captures the increase in the labor share. Lower distortions in sector i (a higher ϕ_i) reduces the number of entrepreneurs through the higher demand of workers by the sector. This direct effect depends on the labor intensity θ_i. Intuitively, if sector i is intensive in the use of labor, then distortions in this sector will reduce its labor demand and increase the number of entrepreneurs in the economy. A similar effect occurs through the other sectors in the production network. If the positive shock to sector i increases the Domar weights of labor intensive sectors, then the number of entrepreneurs is reduced. The opposite occurs if the shock reduces the size of these labor intensive sectors.

We now relate some analytical results of our model to the empirical results found in Section 3. The mass of workers and entrepreneurs in sector i can be expressed as:

$$\log(L_i) = \log\theta_i + \log(\phi_i \lambda(\phi)_i) - \log\left(\sum_j \left[(1-\sigma_j) - \frac{1}{\xi_j}\right]\phi_j \lambda(\phi)_j\right) \tag{13}$$

and

$$\log(M_i) = \log\left[(1-\eta_i) - \frac{1}{\xi_i}\right] + \log(\phi_i \lambda(\phi)_i) - \log\left(\sum_j \left[(1-\sigma_j) - \frac{1}{\xi_j}\right]\phi_j \lambda(\phi)_j\right). \tag{14}$$

Both quantities positively depend on the direct effect of distortion ϕ_i. Moreover, they also increase if a reduction of distortions in other sectors raises the Domar weight $\lambda(\phi)_i$ of sector i.

From the intermediate goods market clearing (3), we can derive the marginal change in sales share of sector i after a distortion reduction in sector j:

$$\frac{d\lambda(\phi)_i}{d\phi_j} = \sigma_{ji}\lambda(\phi)_j + \sum_n \phi_n \sigma_{ni} \frac{d\lambda(\phi)_n}{d\phi_j}. \tag{15}$$

In particular, near efficiency, it must be:

$$\frac{d\lambda(\phi)}{d\phi_i} = (\mathbb{I}_N - \Sigma)^{-1} \boldsymbol{\sigma_i} \lambda(1)_i. \tag{16}$$

A sector i is positively affected by a positive shock to sector j if j is a large direct or indirect buyer of sector i's output. In other words, the model predicts a positive upstream propagation of distortions. Given the limited measure of agents in our economy, a shock to other sectors may reduce the number of workers and entrepreneurs in sector i by the reallocation of agents in the economy. This effect is represented by the term $-\log\left(\sum_j \left[(1-\sigma_j) - \frac{1}{\xi_j}\right]\phi_j \lambda(\phi)_j\right)$ in both (13) and (14). This negative effect emerges when the shock hits sectors that are not direct or indirect buyers of sector i's output.

5.1. The welfare cost of distortions

Next, we analytically derive the output loss from distortions, which also corresponds to the welfare loss from such distortions. The log difference between aggregate output with and without distortions is given by:

$$\log Q(\phi) - \log Q(1) = \sum_i \lambda(1)_i (\log \phi_i) - s_T(1)(\log \tau_T(\phi)) - \sum_i \frac{\lambda(1)_i}{\xi_i}\left(\log \tau_\Pi(\phi)_i\right). \tag{17}$$

Notice that this formula depends only on model primitives and is not a function of any endogenous objects. The welfare is directly reduced by original distortions, $\log \phi_i$. The loss is adjusted considering the change in tax rebates, captured by $\log \tau_T(\phi)$ and $\log \tau_\Pi(\phi)_i$s.

In order to gather further insights over the impact of distortions and relate this to the misallocation literature, we proceed by taking a second-order approximation of Equation (17) around the efficient equilibrium. Following this approximation, we can express the welfare loss as a function of sectoral wedges and identify the propagation effect from network linkages.

Proposition 5.3 *The second-order approximation around efficiency of the output loss from distortions is given by*

$$\log Q(\phi) - \log Q(1) \approx -\frac{1}{2}\left[s_T(1)Var(\log \tau_{\Pi}(\phi)) + \sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2\right], \tag{18}$$

with

$$Var(\log \tau_{\Pi}(\phi)) = \sum_i [M(1)_i + L(1)_i](\log \tau_{\Pi,i})^2 - \left(\sum_i [M(1)_i + L(1)_i] \log \tau_{\Pi,i}\right)^2. \tag{19}$$

The derivation is reported in the Appendix. The total loss is given by two misallocation components. The first one, represented by $s_T(1)Var(\log \tau_{\Pi}(\phi))$, refers to the aggregate misallocation of individuals, both as entrepreneurs and workers in different sectors. It reminds the measurement of misallocation by Hsieh and Klenow (2009). The loss from distorting the optimal allocation of individuals is given by the dispersion of the log wedges of each sector. Remember that those wedges represent how profit shares deviate from the efficient equilibrium. The weights are the sum of workers and entrepreneurs in each sector, $M(1)_i + L(1)_i$. The variance is multiplied by the total share of income at efficiency, $s_T(1)$: this term is always ≤ 1 and converges to 1 in the case with no entrepreneurial heterogeneity, or $\xi_i \to \infty$ for all i. Intuitively, when most of the production is made by a small number of big firms, a marginal change in the extensive margin does not have a big impact on final output.

The second component, $\sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$, refers to the misallocation of intermediate inputs and it is zero when $\sigma_i = 0$ for all i.

Next, we want to decompose the total output loss into the intensive margin loss and the loss generated by the occupational component. First, we characterize the intensive margin loss of distortions.

Proposition 5.4 *The second-order approximation around efficiency of the intensive margin loss from distortions is given by*

$$\log A(\phi) - \log A(1) \approx -\frac{1}{2}\left[s_L(1)Var_L(\log \tau_\Pi(\phi)) + \sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2\right] \tag{20}$$

with

$$Var_L(\log \tau_\Pi(\phi)) = \sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} (\log \tau_{\Pi,i})^2 - \left(\sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} \log \tau_{\Pi,i}\right)^2. \tag{21}$$

The steps to obtain the solution are reported in the Appendix. The variance component here is related to the allocation of workers - depends on θ_i and the labor share. The second part referring to the allocation of intermediate inputs is identical to the one presented in Proposition 5.3.

Finally, by subtracting Equation (18) from Equation (20) we can identify the loss associated to the misallocation of individuals between labor and entrepreneurship in the different sectors.

Proposition 5.5 *The occupational loss can be represented by the variance of the log occupational wedges, $\tau_{\Pi,i}$ and τ_L:*

$$[\log Q(\phi) - \log Q(1)] - [\log A(\phi) - \log A(1)] \approx -\frac{1}{2} s_T(1) Var_{Occ}(\log \tau(\phi)), \tag{22}$$

with

$$Var_{Occ}(\log \tau(\phi)) = \sum_i M(1)_i (\log \tau_{\Pi,i})^2 + L(1)(\log \tau_L)^2 - \left(\sum_i M(1)_i \log \tau_{\Pi,i} + L(1) \log \tau_L\right)^2. \tag{23}$$

The misallocation of entrepreneurs can be approximated by the variance of profit share wedges $\tau_{\Pi,i}$ and the labor share wedge τ_L, describing the deviations with respect to an economy without distortions. Notice that the wedges $\tau_{\Pi,i}$ only refer to marginal entrepreneurs: these are the individuals who change their occupational choice and affect misallocation at the extensive margin.

5.2. Network linkages and misallocation

In order to evaluate the role of network linkages in propagating distortions for a given network structure Σ, we define an equivalent horizontal one.

Definition 5.6 *The equivalent horizontal economy of an economy with a given production network structure Σ is represented by the following characteristics:*

1. *no input-output linkages, i.e.* $\sigma^H_{i,j} = 0 \; \forall \; i, j$;
2. *same profit shares at efficiency:* $\left[(1-\eta_i) - \frac{1}{\xi_i}\right]\lambda(1)_i = \left[(1-\theta^H_i) - \frac{1}{\xi_i}\right]\psi^H_i \; \forall \; i$; *and*
3. *same labor income shares at efficiency:* $\theta_i \lambda(1)_i = \theta^H_i \psi^H_i \; \forall \; i$.

Given conditions 2 and 3, then the allocation of workers and entrepreneurs are identical at efficiency in the network economy and the equivalent horizontal economy. The three conditions also imply:

$$\left[(1-\sigma_i) - \frac{1}{\xi_i}\right]\lambda(1)_i = \left(1 - \frac{1}{\xi_i}\right)\psi^H_i,$$

for any i. Note that, for a given horizontal structure identified by θ^H and ψ^H, there exist infinite combinations of matrices Σ, shares θ, and ψ respecting the three conditions above.

Having defined the equivalent horizontal structure of a network, we can investigate how distortions are amplified or damped through the network.

Proposition 5.7 *The intensive margin and occupational loss from distortions in the equivalent horizontal structure of a given network economy Σ are summarized by*

$$\frac{1}{2} s_L(1) Var_L(\log\phi) \quad \textit{and} \quad \frac{1}{2} s_T(1) Var_{Occ}(\log\phi), \;\; \textit{respectively.}$$

With

$$Var_L(\log\phi) = \sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} (\log\phi_i)^2 - \left(\sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} \log\phi_i\right)^2$$

and

$$Var_{Occ}(\log\phi) = \sum_i M(1)_i (\log\phi_i)^2 + L(1)\left(\sum_i \frac{\theta_i \lambda(1)_i}{s_L(1)} \log\phi_i\right)^2 - \left(\sum_i [M(1)_i + L(1)_i] \log\phi_i\right)^2.$$

In a horizontal economy, the dispersion of original distortions is a sufficient object to describe the intensive margin loss. The $\phi_i s$ directly distort the optimal allocation of workers and entrepreneurs through the reduction in firm revenues. The presence of input-output linkages alters this result through two channels. First, intensive margin losses are amplified through the additional misallocation of intermediate inputs. This is captured by the component $\sum_i \sigma_i \lambda(1)_i (\log\tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$ appearing in (18) and (20). This component is always positive.

In addition, in a network economy, the allocation of workers and entrepreneurs in a sector is indirectly influenced by the variation in relative centrality of that sector. This effect is captured by the change in the Domar weights. Specifically, the occupational loss in (22) can be expressed as:

$$\frac{1}{2}s_T(1)\left[Var_{Occ}(\log\phi) + Var_{Occ}\left(\log\frac{\lambda(\phi)}{\lambda(1)}\right) + 2Cov_{Occ}\left(\log\phi, \log\frac{\lambda(\phi)}{\lambda(1)}\right)\right].$$

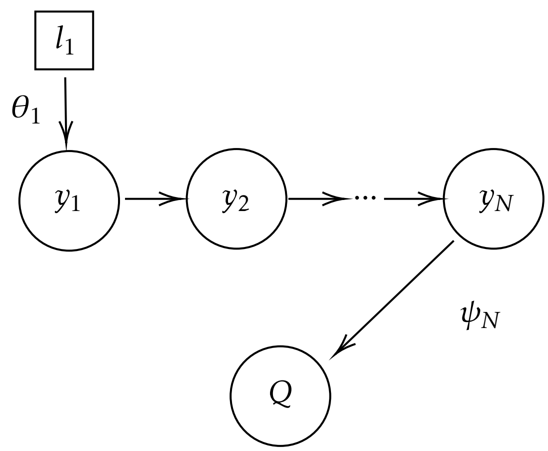
The term $Var_{Occ}\left(\log\frac{\lambda(\phi)}{\lambda(1)}\right) + 2Cov_{Occ}\left(\log\phi, \log\frac{\lambda(\phi)}{\lambda(1)}\right)$ may be positive or negative, amplifying or damping the direct effect of distortions. In particular, a direct effect of distortion ϕ_i to a sector i may be counteracted by the reduction in sales of the main suppliers of i.

6. Loss from distortions in two network examples

In order to get some additional intuitions about the role of linkages in amplifying or damping the effect of distortions, in this section we analyze two simple network structures. As it will be clear, network linkages amplify losses when distortions hit more upstream sectors.

6.1. The case of a pure vertical economy

Figure 1: A pure vertical network



Let us consider the example of a pure vertical economy depicted in Figure 1. Labor is used as an input only by the first sector ($\theta_1 > 0$ and $\theta_j = 0$ for $j > 1$). All remaining sectors are chained in a sequence, until a last intermediate sector that supplies inputs to the final consumption good firms ($\psi_N = 1$). In such a network structure, there cannot be any misallocation of intermediate goods and workers across sectors: the variance $Var_L(\log \tau_\Pi(\phi))$ and the component $\sum_i \sigma_i \lambda(1)_i (\log \tau_{\Pi,i})^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$ are always equal to 0. The reason is that only one sector uses labor and each sector uses the inputs produced by only one sector. Therefore, the difference in welfare loss between the network economy and its equivalent horizontal only depends on the variances of occupational wedges.

For simplicity, suppose we only distort one sector at the time. In such a simple structure, it is easy to show that the Domar weights of the economy are unaffected if we distort the first (more upstream) sector ($\phi_1 < 1$ and $\phi_j = 1$ for $j > 1$).[14] Since this sector does not purchase inputs from any other sector, there is no change in the relative industry sales. In this scenario, the welfare loss in the network economy and the equivalent horizontal economy are identical and equal to $\frac{1}{2}s_T(1)Var(\log\phi)$.

Results are different if we distort downstream sectors. In the extreme case of distortions only in the last sector ($\phi_N < 1$ and $\phi_j = 1$ for $j < N$), only the Domar weights of upstream sectors would be affected, so that $\frac{1}{2}s_T(1)Var(\log\tau_\Pi(\phi)) = 0$. Intuitively, by distorting the most downstream sector we indirectly reduce the sales of the previous sectors and offset the outflow of entrepreneurs from sector N. In this scenario, the loss in the network economy would be lower than in the equivalent horizontal one (which is still equal to $\frac{1}{2}s_T(1)Var(\log\phi)$).

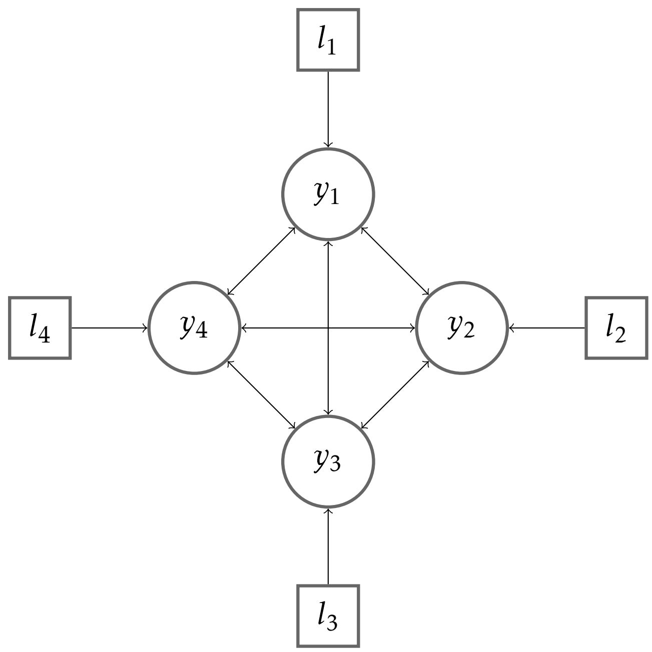
[14] We can use the Neumann series to analytically solve for

$$\left(\mathbb{I}_N - \Sigma' \circ (1\phi')\right)^{-1}\psi = \left(\mathbb{I}_N - \begin{bmatrix} 0 & 0 & \dots & 0 \\ \phi_2\sigma_{21} & 0 & \dots & 0 \\ 0 & \phi_3\sigma_{32} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \phi_N\sigma_{N(N-1)} & 0 \end{bmatrix}'\right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} \prod_{j>1}\phi_j \prod_{j>1}\sigma_{j(j-1)} \\ \prod_{j>2}\phi_j \prod_{j>2}\sigma_{j(j-1)} \\ \prod_{j>3}\phi_j \prod_{j>3}\sigma_{j(j-1)} \\ \vdots \\ 1 \end{bmatrix}.$$

The solution does not depend on ϕ_1, so the weights do not change if we only distort the first sector. If instead we only distort the last sector, the weights are $\begin{bmatrix} \phi_N \prod_{j>1}\sigma_{j(j-1)} \\ \phi_N \prod_{j>2}\sigma_{j(j-1)} \\ \phi_N \prod_{j>3}\sigma_{j(j-1)} \\ \vdots \\ 1 \end{bmatrix}$, so $\tau_\Pi(\phi) = \phi \circ \frac{\lambda(\phi)}{\lambda(1)} = \phi_N \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$.

6.2. The case of a symmetric economy

Figure 2: A symmetric economy



Another simple production network economy, depicted in Figure 2, is one in which all sectors are identically connected and have the same weights in the undistorted economy. Specifically, let us consider the case in which $\sigma_{ij} = \sigma$ for any i and j, and $\psi_i = \frac{1}{N}$ for any i. In this economy, distorting any one of the sectors will induce exactly the same change in all Domar weights. Therefore, the dispersion in wedges τ_Π is equal to the dispersion of original distortions ϕ, which implies that the occupational loss is the same in the network and equivalent horizontal economies. However, the total output loss is still larger because of the misallocation of intermediate inputs captured by the terms $\sum_i \sigma_i \lambda(1)_i \left(\log \tau_{\Pi,i}\right)^2 - \sum_i \lambda(1)_i \left(\log \frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2$.

7. Quantitative analysis

In this section, we calibrate and use our model to evaluate the cost of sectoral taxation in the US. Since our framework neglects many relevant aspects of the tax system, it should be intended as an exercise to assess the role of industry linkages in the misallocation at the intensive and extensive margin rather than a conclusive estimate of the welfare loss from taxation.

We compare the results obtained for our main model to the ones from a modified version that includes fixed technological entry costs. Specifically, we assume that an individual who wants to open a business in sector i must pay a cost f_i in units of final output. The derivation of the equilibrium for this second model is presented in the Appendix.

We consider a seven-sectors economy. The sectors are: 1) Agriculture, Utilities and Mining (AMU); 2) Construction; 3) Manufacturing; 4) Trade; 5) Transportation; 6) Finance, Insurance, and Real Estate (FIRE)[15]; and 7) Other[16]. We normalize the productivity parameters a_i to unity. The remaining parameters to be calibrated are: (i) intermediate input shares, σ_{ij} (49 parameters); (ii) labor shares , θ_i (7 parameters); (iii) final good shares ψ_i (7 parameters); (iv) the shape parameters of the entrepreneurial ability distributions, ξ_i (7 parameter). Therefore, there are 63 parameters to be set in the main model. In the model with fixed costs, we also need to calibrate the f_is, so we have 7 additional parameters.

We calibrate intermediate input shares σ_{ij} using data from the input-output tables of the Bureau of Economic Analysis (BEA).[17] We calibrate θ_i using the labor share of income as a fraction of total industry output. Similarly we calibrate final good shares ψ_is, using the share of final use of industry outputs. These data are from the Bureau of Labor Statistics (BLS). In order to calibrate the ξ_is, we target the share of entrepreneurs in each sector out of the total

[15] Since we do not explicit modeled financial intermediaries we choose to include financial services in the list of calibrated sectors.

[16] Information, Business services, Education, and Entertainment.

[17] All data refers to the year 2019.

population. We compute these moments using data from the Panel Study of Income Dynamics. This is a survey following a representative sample of 5000 American families. Finally, for the model with fixed costs, we can express the f_is as a function of the ξ_is and the data average firms' size in each sector (See the Appendix). Therefore, we still calibrate the ξ_is targeting the shares of entrepreneurs.

For tax distortions ϕ_is, we use independent estimates computed and publicly provided by Aswath Damodaran.[18]. We use the aggregate tax rates by industry and take a simple average to obtain distortions for our 7-sectors economy. The distortions are: $\phi_1 = 0.7$, $\phi_2 = 0.76$, $\phi_3 = 0.64$, $\phi_4 = 0.78$, $\phi_5 = 0.7$, $\phi_6 = 0.83$, $\phi_7 = 0.74$.

The values from our calibration of the main model and the model with fixed costs are reported in Table 3, 4, and 5, in the Data and Calibration Appendix. The tables also report the computed parameters of the equivalent horizontal economies. Given parameter values, we can compute the losses from distortions.

Table 2 reports the estimated welfare, intensive margin, and occupational losses. We compare the losses in our network economy to those of the equivalent horizontal economy in our main model and the modified version with fixed costs.[19] Notice that, in our main model, total welfare and GDP are equivalent. Differently, in the model with fixed costs, total GDP includes the production of fixed capital, so it is not equivalent to aggregate welfare.

Network linkages always amplify the effect of distortions and this amplification is larger for the intensive margin component. The estimated occupational misallocation is quite small in our main model, while it becomes an important source of welfare loss in the model with entry costs. The reason is that, in this second model, the small fraction of entrepreneurs in the economy is not only imputed to a high heterogeneity (low ξ_is), but also to high fixed costs f_is. Higher estimates for ξ_is imply a higher level of misallocation at the extensive margin. However,

[18] https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/taxrate.html

[19] Details about the definition of the equivalent horizontal of the network economy with fixed costs are included in the Data and Calibration Appendix.

in both models, network linkages amplify (and not reduce) the misallocation of entrepreneurs, respectively by a factor of 4 and 12.

Table 2: Welfare losses from distortions (baseline) relative to the efficient (undistorted) economy

	Network economy	Equivalent horizontal
Main Model		
Welfare (GDP)	-8.23%	-0.27%
Intensive Margin	-8.15%	-0.25%
Occupational	-0.08%	-0.02%
Model with fixed costs		
Welfare	-15.69%	-0.86%
Intensive Margin	-8.16%	-0.24%
Occupational	-7.53%	-0.62%

To sum up, the network structure is quantitatively important to propagate sectoral distortions. This amplification occurs not just at the intensive margin (as shown by Baqaee and Farhi (2020b) and Bigio and La'o (2020)), but also at the extensive one.

8. The aggregate output effects from entry subsidies

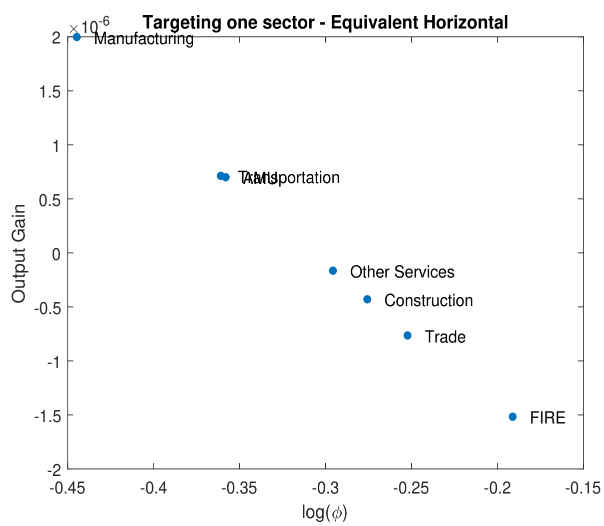
We now study the effect of entry subsidies in our calibrated model. We still assume distortions are represented by the uneven sectoral tax rates in the US.[20] We run few exercises trying to address the following questions: what is the output gain/loss of an entry subsidy program (involving the same total transfer) targeting one sector at the time? What is the best statistics to identify which sectors should be targeted?

[20]The exercise should not be intended as an exact derivation of the optimal sectoral subsidies in the US, but rather as a way to assess the role of network linkages in the designing of such subsidies.

We add to our model a fixed (positive) subsidy to any individual who open a business in a targeted sector i. The subsidy is financed with a lump-sum tax on consumers. In the absence of distortions ϕ, it would definitely misallocate resources and reduce output. However, as distortions ϕ may create a barrier for entrepreneurship in specific sectors, the subsidy might relax this barrier.

We calibrate the size of the aggregate subsidy so that the total equilibrium tax transfer is always equal to 0.001% of the initial aggregate GDP. Therefore, the aggregate size of the entry subsidy is the same relative to the baseline GDP. This allows comparison of the policy of targeting different sectors.

Figure 3: Output gain/loss from subsidizing entry in the equivalent horizontal economy. The horizontal axis is the sectoral log of distortions. The vertical axis displays the percentage deviation of output of the economy with subsidy relative to the baseline output of the horizontal economy. Each dot in the graph corresponds to the change in aggregate output of subsidizing entry only in the respective sector of production. The cost of the policy always amounts to 0.001% of the baseline GDP.



We start by considering the effects of entry subsidy on aggregate output in the equivalent horizontal economy of our baseline model. Figure 3 shows the percentage deviation in output relative to the baseline calibrated economy from subsidizing one sector at the time in the economy without Input-Output linkages. The sectors are ranked in the x-axis from the most distorted to the least distorted. Entry Subsidy increase output only when they target the three most distorted sectors. In particular, distortions ϕ are a sufficient statistic to rank the sectors in terms of gains from entry subsidies.

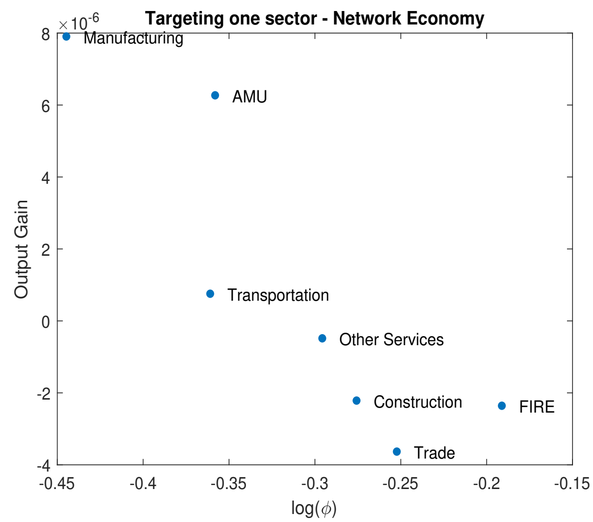
Results are different once we consider our original network economy. Figure 4(a) presents the relation between distortions and output gains from entry subsidies when the observed production network is taken into account. While Manufacturing once more is rightly identified as the sector which generates the highest output rise from entry subsidies, sectoral distortions are not anymore a good measure to rank the remaining sectors in terms of changes in aggregate output. In particular, while Transportation is slightly more distorted than AMU, targeting entry subsidies on the latter would generate a much larger effect on aggregate output.

The measure of occupational loss presented in Equation (22) helps us to understand the difference of the results of subsidizing entry in the horizontal economy and in the production network economy. The dispersion of wedges $\tau_{\Pi,i}$ summarizes this loss. An entry subsidy program should target sectors with the largest marginal profit share reduction. In Figure 4(b), we represent the same output loss/gain of an entry subsidy program against $\log \tau_{\Pi,i} = \log \phi_i + \log\left(\frac{\lambda(\phi)_i}{\lambda(1)}_i\right)$. The gains are now monotonically ranked.

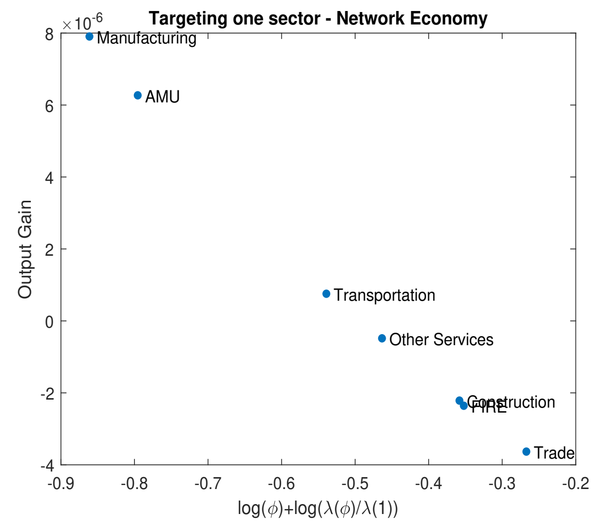
Consequently, in a network economy, knowing the direct distortions may not be sufficient to design a subsidy program for firms' entry. The profit losses of the marginal entrepreneurs are a superior measure to identify which industries should be targeted. A correct computation of such measure requires not only knowledge of distortions but also information on the production network structure.

In Appendix C, we run the same exercise for the model with fixed costs. We show how the profit losses of marginal entrepreneurs are still a good statistics to optimally subsidize targeted

Figure 4: Output gain/loss from subsidizing entry in in the baseline network economy. The horizontal axis in graph (a) is the sectoral log of distortions, $\log(\phi_i)$; the horizontal axis in graph (b) is the sectoral log of profit share relative to the efficient economy, $\log(\tau_{\Pi,i}) = \log(\phi_i) + \log\left(\frac{\lambda(\phi)}{\lambda(1)}\right)$. The vertical axis displays the percentage deviation of output of the economy with subsidy relative to the baseline output. Each dot in the graph corresponds to the change in aggregate output of subsidizing entry only in the respective sector of production. The cost of the policy always amounts to 0.001% of the baseline GDP.



(a) x-axis: original distortions, $\log(\phi_i)$



(b) x-axis: total wedges, $\log(\tau_{\Pi,i})$

sectors.[21]

9. Conclusions

We studied the effect of distortions in a multisector general equilibrium model with production network and endogenous occupational choice. Individuals can be workers or they can run a business in one of the production sectors. The environment is an extension of Lucas

[21] In the model with fixed costs, the wedges $\tau_{\Pi,i}$ include not only the direct distortions and the changes in Domar Weights, but also the changes in the relative size of entry costs.

span of control model (c.f., Lucas Jr, 1978), which has been used to investigate different issues in the macro economic development literature. At the aggregate level, distortions reduce TFP by misallocating labor and intermediate inputs. In addition, they also misallocate the occupational decision of individuals manifested into two additional wedges: a “labor” wedge and an “entrepreneurship” wedge.

We showed that shocks to sectoral distortions induce a reallocation of individuals between the labor force and entrepreneurship in a non-trivial manner. A raise in distortions in a sector i reduces (increases) the mass of firms in the same sector if the sector has low (high) labor intensity. In addition, the mass of firms in other sectors is also reduced if they supply inputs to sector i and they have low labor intensity.

We analytically derived the output loss from distortions, identifying the role of sectoral linkages and endogenous firm entry. Network linkages amplify (diminish) losses if distortions hit more upstream (downstream) sectors. Therefore, we calibrated our model to US data and calculated that the misallocation of entrepreneurs caused by sectoral taxes is amplified by network linkages by at least a factor of three.

Finally, we also studied the effects of entry subsidies. We found that subsidies should target sectors in which the marginal entrepreneurs suffer larger profit losses from distortions. These sectors are not necessarily the ones directly distorted. Sectoral distortions are usually not a good measure to represent profit losses, as they do not include the indirect effect of reduction in intermediate goods demand, which depends on the production network.

We believe that our framework could be extended to investigate other specific distortions in models with input-output linkages and entrepreneurial decisions, such as credit market imperfections, labor frictions, and entry regulations.

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A. Mathematical Appendix

A.1. Aggregate Sectoral Output

The first order conditions from the firm's problem (2) are:

$$l_i(v) = \theta_i \phi_i \frac{p_i y_i(v)}{w} \tag{24}$$

and

$$x_{ij}(v) = \sigma_{ij} \phi_i \frac{p_i y_i(v)}{p_j}, \tag{25}$$

which implies:

$$p_i y_i(v) = \left[p_i a_i v \left(\frac{\theta_i \phi_i}{w} \right)^{\theta_i} \prod_j \left(\frac{\sigma_{ij} \phi_i}{p_j} \right)^{\sigma_{ij}} \right]^{\frac{1}{1-\eta_i}}.$$

The aggregate output in sector i is

$$Y_i \equiv \frac{1}{N} \int_{\hat{v}_i}^{\infty} a_i v \left(l_i(v) \right)^{\theta_i} \prod_j \left(x_{ij}(v) \right)^{\sigma_{ij}} \mu(dv).$$

We define:

$$L_i \equiv \frac{1}{N} \int_{\hat{v}_i}^{\infty} l_i(v) \mu(dv) = \frac{1}{N} \left(\frac{V_i}{\hat{v}_i} \right)^{\frac{1}{1-\eta_i}} l_i\left(\hat{v}_i\right),$$

$$X_{ij} \equiv \frac{1}{N} \int_{\hat{v}_i}^{\infty} x_{ij}(v) \mu(dv) = \frac{1}{N} \left(\frac{V_i}{\hat{v}_i} \right)^{\frac{1}{1-\eta_i}} x_{ij}\left(\hat{v}_i\right),$$

and

$$V_i \equiv \left[\int_{\hat{v}_i}^{\infty} v^{\frac{1}{1-\eta_i}} \mu(dv) \right]^{1-\eta_i}.$$

The optimal choice of inputs by a firm with productivity v can be expressed as a function of v and aggregate variables:

$$l_i(v) = N \left(\frac{v}{V_i} \right)^{\frac{1}{1-\eta_i}} L_i,$$

$$x_{ij}(v) = N \left(\frac{v}{V_i} \right)^{\frac{1}{1-\eta_i}} X_{ij}.$$

Therefore, we can re-express the aggregate output as:

$$Y_i = \frac{1}{N^{1-\eta_i}} a_i L_i^{\theta} \prod_j X_{ij}^{\sigma_{ij}} V_i. \tag{26}$$

The entrepreneurial productivity $\hat{v}_i$ of the marginal entrepreneur in sector i is such that:

$$w = (1-\eta_i)\,\phi_i \hat{v}_i^{\frac{1}{1-\eta_i}} \left[p_i a_i \left(\frac{\theta_i \phi_i}{w} \right)^{\theta_i} \prod_j \left(\frac{\sigma_{ij}\phi_i}{p_j} \right)^{\sigma_{ij}} \right]^{\frac{1}{1-\eta_i}}. \quad \forall i \in S.$$

Solving further, we obtain

$$\hat{v}_i = \left[\frac{w}{(1-\eta_i)\phi_i p_i a_i} \frac{\left(\frac{\xi_i(1-\eta_i)}{\xi_i(1-\eta_i)-1} \right)^{\eta_i}}{N^{\eta_i} L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}}} \right]^{\frac{1}{1+\xi_i\eta_i}},$$

where we used[22]

$$V_i \approx \left[\int_{\hat{v}_i}^{\infty} v_i^{\frac{1}{1-\eta_i}} \mu(dv) \right]^{1-\eta_i} = \left[\frac{\xi_i(1-\eta_i)}{\xi_i(1-\eta_i)-1} \hat{v}_i^{\frac{1}{1-\eta_i}-\xi_i} \right]^{1-\eta_i}.$$

Finally, by plugging V_i into (26) and using $\int_{\hat{v}_i}^{\infty} \mu(dv) \approx \hat{v}_i^{-\xi_i}$, we obtain:

$$Y_i \approx A_i \left(L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}} \right)^{\frac{\xi_i}{1+\xi_i\eta_i}}, \tag{27}$$

with

$$A_i = \left[a_i \left(\frac{(1-\eta_i)\phi_i p_i}{w} \right)^{\frac{\xi_i(1-\eta_i)-1}{\xi_i}} \left(\frac{\xi_i(1-\eta_i)}{\xi_i(1-\eta_i)-1} \right)^{\frac{1}{\xi_i}} \left(\frac{1}{N} \right)^{\frac{1}{\xi_i}} \right]^{\frac{\xi_i}{1+\xi_i\eta_i}}.$$

A.2. Computing Domar Weights

The Domar weight of sector i corresponds to the industry's sales as a fraction of GDP, i.e.,

$$\frac{p_i Y_i}{Q}.$$

[22] We also assume $\xi_i(1-\eta_i) > 1$ in order to have a finite integral.

Observe that we can rewrite the market clearing conditions for intermediate good j as:

$$c_j + \sum_{i \in S} X_{ij} = Y_j. \tag{28}$$

Multiplying both sides by p_j:

$$p_j c_j + \sum_{i \in S} p_j X_{ij} = p_j Y_j. \tag{29}$$

By firms' first order condition:

$$p_j c_j + \sum_{i \in S} \phi_i \sigma_{ij} p_i Y_i = p_j Y_j. \tag{30}$$

Using the fact that $p_i = \psi_i Q / c_i$, which follows from final producers' first order condition, we have:

$$\psi_j Q + \sum_{i \in S} \phi_i \sigma_{ij} p_i Y_i = p_j Y_j \quad (\div Q), \tag{31}$$

$$\psi_j + \sum_{i \in S} \phi_i \sigma_{ij} \frac{\psi_i Y_i}{c_i} = \frac{\psi_j Y_j}{c_j}. \tag{32}$$

Now, define $v_i = \frac{\psi_i Y_i}{c_i}$. Then:

$$\psi + \Sigma' [\phi \circ v] = v \Longrightarrow v^* = \left(\mathbb{I}_S - \Sigma' \circ (\mathbf{1} \phi') \right)^{-1} \psi =: \lambda(\phi), \tag{33}$$

where $\lambda_i = p_i Y_i / Q$ is the Domar weight of sector i, Σ is the firm I-O matrix.

A.3. Proof of Proposition 4.1

Aggregating both sides of (24) and (25), we obtain:

$$L_i = \theta_i \phi_i \frac{p_i}{w} Y_i = \theta_i \phi_i \lambda(\phi)_i \frac{Q}{w},$$

$$X_{ij} = \sigma_{ij} \phi_i \frac{p_i}{p_j} Y_i = \sigma_{ij} \phi_i \lambda(\phi)_i \frac{Q}{p_j}.$$

The share of entrepreneurs in sector i can be approximated as:

$$M_i = \int_{\hat{v}_i}^{\infty} \mu(dv) = \hat{v}_i{}^{-\xi_i} = \frac{\xi_i(1-\eta_i)-1}{\xi_i}\phi_i\lambda(\phi)_i\frac{Q}{w}.$$

Using the labor market clearing condition, $\sum_i M_i + \sum_i L_i = 1$, we obtain the equilibrium solution for M_i and L as presented in (7) and (8).

Using the definition of Domar weights and $p_i c_i = \psi_i Q$, we can rewrite (51) as:

$$\frac{c_i}{\psi_i} = \left(\frac{1}{N}\right)^{\frac{1}{\xi_i}} a_i\phi_i\left(\frac{(1-\eta_i)}{s_\Pi(\phi)_i}\right)^{1-\eta_i}\left(\frac{\theta_i}{s_L(\phi)}\right)^{\theta_i} M_i^{\frac{\xi_i(1-\eta_i)-1}{\xi_i}} L^{\theta_i}\prod_j\left(\sigma_{ij}\frac{c_j}{\psi_j}\right)^{\sigma_{ij}}. \tag{34}$$

Taking logs and plugging into

$$\log Q = \sum_i \psi_i \log\psi_i + \sum_i \psi_i \log\left(\frac{c_i}{\psi_i}\right), \tag{35}$$

we obtain the solution for the aggregate production function.

A.4. Proof of Theorem 5.1

Let us plug Equations (7) and (8) into the aggregate production function (52):

$$\log Q = \sum_j \psi_j \log\psi_j - s_T(1)\log s_T(\phi)$$
$$+\lambda(1)'\left\{\log A(\phi) + \left(1-\eta-\frac{1}{\xi}\right)\circ\log\left[\left(1-\eta-\frac{1}{\xi}\right)\phi\lambda(\phi)\right] + \theta\log\left(\sum_j\theta_j\phi_j\lambda(\phi)_j\right)\right\}. \tag{36}$$

We now plug (6) and obtain:

$$\log Q = \sum_j \psi_j \log\psi_j - s_T(1)\log s_T(\phi)$$
$$+\sum_j\lambda(1)_j\left\{\log a_j + \log\phi_j - \frac{1}{\xi_j}\log N + (1-\eta_j)\log(1-\eta_j) + \theta_j\log\theta_j + \sum_i\sigma_{ij}\log\sigma_{ij} - \frac{1}{\xi_j}\log s_\Pi(\phi)\right\}. \tag{37}$$

Therefore, we can conclude that $\frac{d\log Q}{d\log a_i} = \lambda(1)_i$.

A.5. Proof of Proposition 5.2

By taking the first derivative of (37) with respect to ϕ_i, we obtain (10). Starting from efficiency it is:

$$\left.\frac{ds_T(\phi)}{d\phi_i}\right|_{\phi=1}+\sum_j \frac{1}{\xi_j(1-\eta_j)-1}\left.\frac{ds_\Pi(\phi)_j}{d\phi_i}\right|_{\phi=1}=(1-\sigma_i)\lambda(1)_i+\sum_j(1-\sigma_j)\left.\frac{d\lambda(\phi)_j}{d\phi_i}\right|_{\phi=1}. \tag{38}$$

We can solve for $\frac{d\lambda(\phi)_j}{d\phi_i}$ starting from the intermediate goods market clearing (3):

$$\frac{d\lambda(\phi)_j}{d\phi_i}=\sigma_{ij}\lambda(\phi)_i+\sum_n \phi_n\sigma_{nj}\frac{d\lambda(\phi)_n}{d\phi_i}. \tag{39}$$

The solution of the system is:

$$\frac{d\lambda(\phi)}{d\phi_i}=(\mathbb{I}_N-\Sigma)^{-1}\boldsymbol{\sigma_i}\lambda(1)_i. \tag{40}$$

Therefore, it is $\sum_j(1-\sigma_j)\left.\frac{d\lambda(\phi)_j}{d\phi_i}\right|_{\phi=1}=\sigma_i\lambda(1)_i$, which implies $\left.\frac{d\log Q}{d\log\phi_i}\right|_{\phi=1}=0$.

To obtain equation (12), notice that:

$$\frac{d\log s_T(\phi)}{d\log\phi_i}=\frac{\left(1-\sigma_i-\frac{1}{\xi_i}\right)\lambda(1)_i+\sum_j\left(1-\sigma_j-\frac{1}{\xi_j}\right)\left.\frac{d\lambda(\phi)_j}{d\phi_i}\right|_{\phi=1}}{\sum_j\left(1-\sigma_j-\frac{1}{\xi_j}\right)\lambda(1)_j}=\frac{\left(1-\frac{1}{\xi_i}\right)\lambda(1)_i-\sum_j\frac{1}{\xi_j}\left.\frac{d\lambda(\phi)_j}{d\phi_i}\right|_{\phi=1}}{1-\sum_j\frac{1}{\xi_j}\lambda(1)_j}. \tag{41}$$

A.6. Proof of Proposition 5.3

By taking the second-order Taylor expansion with respect to all $\log\phi_j$ of Equation (17), we obtain:

$$\log Q(\phi) - \log Q(1) \approx$$

$$\sum_j \lambda(1)_j (\log\phi_j) - s_T(1)\sum_j \frac{ds_T(\phi)}{d\phi_j}\frac{\phi_j}{s_T(\phi)}\bigg|_{\phi=1}\left(\log\phi_j\right) - \sum_j\sum_i \frac{\lambda(1)_i}{\xi_i}\frac{ds_\Pi(\phi)_i}{d\phi_j}\frac{\phi_j}{s_\Pi(\phi)_i}\bigg|_{\phi=1}\left(\log\phi_j\right)$$

$$-\frac{1}{2}\sum_j \frac{ds_T(\phi)}{d\phi_j}\bigg|_{\phi=1}\left(\log\phi_j\right)^2 + \frac{1}{2}s_T(1)\left[\sum_j \frac{ds_T(\phi)}{d\phi_j}\frac{\phi_j}{s_T(\phi)}\bigg|_{\phi=1}(\log\phi_j)\right]^2$$

$$-\frac{1}{2}\sum_j\sum_i \frac{ds_\Pi(\phi)_i}{[\xi_i(1-\eta_i)-1]d\phi_j}\bigg|_{\phi=1}\left(\log\phi_j\right)^2 + \frac{1}{2}\sum_k \frac{\lambda(1)_k}{\xi_k}\left[\sum_j \frac{ds_\Pi(\phi)_k}{d\phi_j}\frac{\phi_j}{s_\Pi(\phi)_k}\bigg|_{\phi=1}(\log\phi_j)\right]^2$$

$$-\frac{1}{2}\sum_j\sum_i \frac{d^2 s_T(\phi)}{d\phi_j d\phi_i}\bigg|_{\phi=1}(\log\phi_j)(\log\phi_i) - \frac{1}{2}\sum_k \frac{1}{\xi_k(1-\eta_k)-1}\sum_j\sum_i \frac{d^2 s_\Pi(\phi)_k}{d\phi_j d\phi_i}\bigg|_{\phi=1}(\log\phi_j)(\log\phi_i). \tag{42}$$

From the result in A.5, the total first-order effect in the first line is zero.

In order to simplify, we use the following result:

$$\sum_j \frac{ds_T(\phi)}{d\phi_j}\bigg|_{\phi=1}(\log\phi_j) = \sum_j\left(1-\sigma_j-\frac{1}{\xi_j}\right)\lambda(1)_j(\log\phi_j) + \sum_i\left(1-\sigma_i-\frac{1}{\xi_i}\right)\sum_j \frac{d\lambda(\phi)_i}{d\phi_j}\bigg|_{\phi=1}(\log\phi_j)$$

$$\approx \sum_j\left(1-\sigma_j-\frac{1}{\xi_j}\right)\lambda(1)_j(\log\phi_j) + \sum_j\left(1-\sigma_j-\frac{1}{\xi_j}\right)\lambda(1)_j\left(\log\frac{\lambda(\phi)_j}{\lambda(1)_j}\right) = \sum_j\left(1-\sigma_j-\frac{1}{\xi_j}\right)\lambda(1)_j\log\tau_{\Pi,j}. \tag{43}$$

Therefore, we can write:

$$\left[\sum_j \frac{ds_T(\phi)}{d\phi_j}\frac{\phi_j}{s_T(\phi)}\bigg|_{\phi=1}(\log\phi_j)\right]^2 = \left[\sum_j \frac{\left(1-\sigma_j-\frac{1}{\xi_j}\right)\lambda(1)_j}{s_T(1)}\log\tau_{\Pi,j}\right]^2 = \left(\sum_i [M(1)_i + L(1)_i]\log\tau_{\Pi,i}\right)^2. \tag{44}$$

Moreover, we can simplify

$$\left[\sum_j \frac{ds_T(\phi)}{d\phi_j}\bigg|_{\phi=1} + \sum_j \sum_i \frac{ds_\Pi(\phi)_i}{[\xi_i(1-\eta_i)-1]d\phi_j}\bigg|_{\phi=1}\right]\left(\log\phi_j\right)^2 = \sum_j \lambda(1)_j \left(\log\phi_j\right)^2, \tag{45}$$

and

$$\sum_k \frac{\lambda(1)_k}{\xi_k}\left[\sum_j \frac{ds_\Pi(\phi)_k}{d\phi_j}\frac{\phi_j}{s_\Pi(\phi)_k}\bigg|_{\phi=1}(\log\phi_j)\right]^2 \approx \sum_j \frac{\lambda(1)_j}{\xi_j}\left(\log\tau_{\Pi,j}\right)^2. \tag{46}$$

From the intermediate market clearing (3), we can solve for the second-order effect on Domar weights:

$$\frac{d^2\lambda(\phi)_k}{d\phi_i d\phi_j} = \sigma_{ik}\frac{d\lambda(\phi)_i}{d\phi_j} + \sigma_{jk}\frac{d\lambda(\phi)_j}{d\phi_i} + \sum_n \phi_n\sigma_{nk}\frac{d^2\lambda(\phi)_n}{d\phi_i d\phi_j}.$$

The solutions are:

$$\frac{d^2\lambda(\phi)}{d\phi_i d\phi_j} = (\mathbb{I}_N - \Sigma)^{-1}\left(\boldsymbol{\sigma_i}\frac{d\lambda(\phi)_i}{d\phi_j} + \boldsymbol{\sigma_j}\frac{d\lambda(\phi)_j}{d\phi_i}\right).$$

Given this result, we can re-express:

$$\begin{aligned}\sum_j\sum_i \frac{d^2 s_T(\phi)}{d\phi_j d\phi_i}\bigg|_{\phi=1}(\log\phi_j)(\log\phi_i) + \sum_k \frac{1}{\xi_k(1-\eta_k)-1}\sum_j\sum_i \frac{d^2 s_\Pi(\phi)_k}{d\phi_j d\phi_i}\bigg|_{\phi=1}(\log\phi_j)(\log\phi_i)\\ \approx 2\sum_j \lambda(1)_j\left(\log\frac{\lambda(\phi)_j}{\lambda(1)_j}\right)\left(\log\phi_j\right).\end{aligned}$$

We can now substitute into (42):

$$\begin{aligned}&\log Q(\phi) - \log Q(1) \approx\\ &\quad -\frac{1}{2}s_T(1)\left\{\sum_i [M(1)_i + L(1)_i](\log\tau_{\Pi,i})^2 - \left(\sum_i [M(1)_i + L(1)_i]\log\tau_{\Pi,i}\right)^2\right\}\\ &\quad +\frac{1}{2}\left\{\sum_i\left(1-\sigma_i-\frac{1}{\xi_i}\right)\lambda(1)_i(\log\tau_{\Pi,i})^2 + \sum_j\frac{\lambda(1)_j}{\xi_j}\left(\log\tau_{\Pi,j}\right)^2 - \sum_j\lambda(1)_j\left(\log\phi_j\right)^2\right\}\\ &\quad -\sum_j\lambda(1)_j\left(\log\frac{\lambda(\phi)_j}{\lambda(1)_j}\right)\left(\log\phi_j\right)\\ &= -\frac{1}{2}\left\{s_T Var(\log\tau_\Pi) + \sum_i\sigma_i\lambda(1)_i(\log\tau_{\Pi,i})^2 - \sum_i\lambda(1)_i\left(\log\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)^2\right\}.\end{aligned} \tag{47}$$

A.7. Proof of Propositions 5.4 and 5.5

From (17) and (6), we can write:

$$[\log Q(\phi) - \log Q(1)] - [\log A(\phi) - \log A(1)] =$$

$$-s_T(1)[\log s_T(\phi) - \log s_T(1)] + \sum_i s_\Pi(1)_i [\log s_\Pi(\phi)_i - \log s_\Pi(1)_i] + s_L(1)[\log s_L(\phi) - \log s_L(1)]. \quad (48)$$

Deriving a second-order Taylor expansion, we obtain:

$$-\sum_j \left[\frac{ds_T(\phi)}{d\phi_j} - \sum_i \frac{ds_\Pi(\phi)_i}{d\phi_j} - \frac{ds_L(\phi)}{d\phi_j} \right]_{\phi=1} \left(\log \phi_j\right) - \frac{1}{2} \sum_j \left[\frac{ds_T(\phi)}{d\phi_j} - \sum_i \frac{ds_\Pi(\phi)_i}{d\phi_j} - \frac{ds_L(\phi)}{d\phi_j} \right]_{\phi=1} \left(\log \phi_j\right)^2$$

$$-\frac{1}{2} \sum_j \sum_i \left[\frac{d^2 s_T(\phi)}{d\phi_j d\phi_i} - \sum_k \frac{d^2 s_\Pi(\phi)_k}{d\phi_j d\phi_i} - \frac{d^2 s_L(\phi)}{d\phi_j d\phi_i} \right]_{\phi=1} \left(\log \phi_j\right)(\log \phi_i)$$

$$-\frac{1}{2} \left\{ \sum_i s_\Pi(1)_i (\log \tau_{\Pi,i})^2 + s_L(1)(\log \tau_L)^2 - s_T(1) \left(\sum_i M(1)_i \log \tau_{\Pi,i} + L(1) \log \tau_L \right)^2 \right\}. \quad (49)$$

The first two lines are equal to 0. Then, we can express the third line as:

$$-\frac{1}{2} s_T(1) Var_{Occ} = \frac{1}{2} s_T(1) \left[\sum_i M(1)_i (\log \tau_{\Pi,i})^2 + L(1)(\log \tau_L)^2 - \left(\sum_i M(1)_i \log \tau_{\Pi,i} + L(1) \log \tau_L \right)^2 \right]. \quad (50)$$

The result in (20) is obtained by subtracting (50) from (18).

A.8. Model with fixed entry costs

We derive the aggregate output in a modified version of our model where each entrepreneur in sector i must pay a fixed cost f_i. This cost is assumed to be in terms of final output.

Derivations from (24) to (26) still hold. A main difference in this model appears in the occupational choice of the entrepreneurs, which is now:

$$(1 - \eta_i) \phi_i p_i y_i(v) - f_i \geq w.$$

Therefore, the productivity of the marginal entrepreneur in sector i is:

$$\hat{v}_i = \left[\frac{w}{(1-\eta_i)\phi_i \kappa(\phi)_i p_i a_i} \frac{\left(\frac{\xi_i(1-\eta_i)}{\xi_i(1-\eta_i)-1}\right)^{\eta_i}}{N^{\eta_i} L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}}} \right]^{\frac{1}{1+\xi_i\eta_i}},$$

where we defined:

$$\kappa(\phi)_i = \frac{w}{w+f_i}.$$

The ratio κ_i is a measure of the relative size of entry costs, f_i, with respect to the final profits of the marginal entrepreneur, w.

The sectoral output is now:

$$Y_i \approx A_i \left(L_i^{\theta_i} \prod_j X_{ij}^{\sigma_{ij}} \right)^{\frac{\xi_i}{1+\xi_i\eta_i}}, \tag{51}$$

with

$$A_i = \left[a_i \left(\frac{(1-\eta_i)\phi_i \kappa(\phi)_i p_i}{w} \right)^{\frac{\xi_i(1-\eta_i)-1}{\xi_i}} \left(\frac{\xi_i(1-\eta_i)}{\xi_i(1-\eta_i)-1} \right)^{\frac{1}{\xi_i}} \left(\frac{1}{N} \right)^{\frac{1}{\xi_i}} \right]^{\frac{\xi_i}{1+\xi_i\eta_i}}.$$

The solution for the equilibirum Domar weights is the same as in A.2. Therefore, we can follow the same steps in A.3 to obtain the aggregate output:

$$\log Q = \sum_j \psi_j \log \psi_j + \lambda(1)' \log A(\phi) + \lambda(1)' \left\{ \left[(1-\eta) - \frac{1}{\xi} \right] \circ \log M + \theta \log L \right\} \tag{52}$$

with

$$A(\phi)_i = a_i \phi_i \left(\frac{(1-\eta_i)\kappa(\phi)_i}{s_\Pi(\phi)_i} \right)^{1-\eta_i} \left(\frac{\theta_i}{s_L(\phi)} \right)^{\theta_i} \prod_j \left(\sigma_{ji}\right)^{\sigma_{ji}} \left(\frac{1}{N} \right)^{\frac{1}{\xi_i}}, \tag{53}$$

$$M_i = \frac{s_\Pi(\phi)_i}{s_T(\phi)}, \tag{54}$$

and

$$L = \frac{s_L(\phi)}{s_T(\phi)}, \tag{55}$$

given

$$s_L(\phi) = \sum_j \theta_j \phi_j \lambda(\phi)_j,$$

$$s_{\Pi}(\phi)_i = \left[1-\eta_i-\frac{1}{\xi_i}\right]\phi_i\kappa(\phi)_i\lambda(\phi)_i \ \forall i\in S, \text{ and} \tag{56}$$
$$s_T(\phi) = \sum_j\left[\theta_j+\left(1-\eta_j-\frac{1}{\xi_j}\right)\kappa(\phi)_j\right]\phi_j\lambda(\phi)_j.$$

Using (54) and (55), we can express the fixed costs as a function of the average size and the ξ_i of each sector:

$$f_i = w\left[\left(1-\eta_i-\frac{1}{\xi_i}\right)\frac{L_i}{\theta_i M_i}-1\right]. \tag{57}$$

Notice that the aggregate final welfare is given by $Q-\sum_i f_i M_i$. In order to decompose the welfare loss between intensive margin and occupational loss, we consider the following approximated decomposition:

$$\log W(\phi)-\log W(1)\approx$$
$$\frac{Q(1)}{Q(1)-\sum_i f_i M(1)_i}\left[\log Q(\phi)-\log Q(1)\right]-\sum_j\frac{f_j M(1)_j}{Q(1)-\sum_i f_i M(1)_i}\left[\log M(\phi)_j-\log M(1)_j\right].$$

Therefore, the intensive margin loss is given by:

$$\frac{Q(1)}{Q(1)-\sum_i f_i M(1)_i}\lambda(1)'\left[\log A(\phi)-\log A(1)\right],$$

while the residual represents the occupational loss.

B. Data and Calibration Appendix

We analyze a seven-sectors economy. The sectors are: 1) Agriculture, Utilities and Mining (AMU) (sector 1); 2) Construction (sector 2); 3) Manufacturing (sector 3); 4) Trade (sector 4); 5) Transportation (sector 5); 6) Finance, Insurance, and Real Estate (FIRE) (sector 6); and 7) Other (sector 7)[23]. We normalize the productivity parameters a_i to unity.

We calibrate parameters for the main model and the one with fixed entry costs. We start describing the estimation of those parameters that are in common for the two models.

We use the input-output tables from the Bureau of Economic Analysis to calibrate the structure of the network. The estimated matrix of intermediate goods shares σ_{ij} is for the seven sectors we are studying is:

$$\Sigma = \begin{bmatrix} 0.1802 & 0.0092 & 0.1624 & 0.0019 & 0.0087 & 0.0647 & 0.0714 \\ 0.0209 & 0.0001 & 0.3635 & 0.0000 & 0.0005 & 0.0322 & 0.0626 \\ 0.1323 & 0.0025 & 0.4000 & 0.0043 & 0.0085 & 0.0167 & 0.0675 \\ 0.0169 & 0.0019 & 0.0556 & 0.0189 & 0.0447 & 0.1194 & 0.1957 \\ 0.0111 & 0.0045 & 0.1267 & 0.0002 & 0.1181 & 0.0994 & 0.1190 \\ 0.0169 & 0.0241 & 0.0187 & 0.0012 & 0.0056 & 0.2103 & 0.1221 \\ 0.0086 & 0.0011 & 0.0772 & 0.0003 & 0.0103 & 0.0866 & 0.2149 \end{bmatrix}.$$

We calibrate θ_i using the labor share of income as a fraction of total industry output. Similarly we calibrate final good shares ψ_is, using the share of final use of industry outputs.

In order to calibrate the ξ_is of the main model, we match the share of entrepreneurs in each sector using data from the PSID in 2019 (same year as the I-O table used). The estimated values for our main model are: $\xi_1 = 3.25$, $\xi_2 = 22.63$, $\xi_3 = 5.87$, $\xi_4 = 5.23$, $\xi_5 = 5.53$, $\xi_6 = 2.23$, $\xi_7 = 6.48$.

[23] Information, Business services, Education, and Entertainment.

The remaining calibrated parameters for the main model are reported in Table 3. The table also reports the parameters of the equivalent horizontal economy.

Table 3: Calibrated technology parameters for the main model

Network economy													
ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
0.05	0.05	0.13	0.13	0.04	0.23	0.37	0.15	0.35	0.19	0.34	0.30	0.14	0.41
Equivalent horizontal (main model)													
ψ_1^H	ψ_2^H	ψ_3^H	ψ_4^H	ψ_5^H	ψ_6^H	ψ_7^H	θ_1^H	θ_2^H	θ_3^H	θ_4^H	θ_5^H	θ_6^H	θ_7^H
0.08	0.06	0.19	0.12	0.05	0.23	0.26	0.52	0.70	0.8	0.77	0.73	0.52	0.79

In the model with fixed costs, the ψ_is and θ_is are identical. We calibrate the ξ_is matching the same share of entrepreneurs in each sector out of the active population. The estimated values for the ξ_is are: $\xi_1 = 19.26$, $\xi_2 = 12.28$, $\xi_3 = 8.34$, $\xi_4 = 14.18$, $\xi_5 = 26.01$, $\xi_6 = 11.75$, $\xi_7 = 32.95$. The fixed costs f_is are obtained from the following expression:

$$f_i = w\left[(1-\eta_i - 1/\xi_i)\frac{(L_i/M_i)^{Data}}{\theta_i} - 1\right]$$

Where $(L_i/M_i)^{Data}$ is the average firm size in sector i obtained from the data. The equivalent horizontal of the network economy with fixed costs has the following characteristics:

1. no input-output linkages, i.e. $\sigma_{i,j}^H = 0 \ \forall \ i, j$;
2. same profit shares at efficiency: $\left[(1-\eta_i) - \frac{1}{\xi_i}\right]\kappa(1)_i\lambda(1)_i = \left[(1-\theta_i^H) - \frac{1}{\xi_i}\right]\kappa^H(1)_i\psi_i^H \ \forall \ i$;
3. same labor income shares at efficiency: $\theta_i\lambda(1)_i = \theta_i^H\psi_i^H \ \forall \ i$; and
4. same entry costs shares at efficiency: $\frac{f_i M(1)_i}{Q(1)} = \frac{f_i^H M^H(1)_i}{Q^H(1)} \ \forall \ i$.

Table 4 reports the estimated ψ_is and θ_is for the equivalent horizontal economy.

Table 4: Calibrated technology parameters for the model with fixed costs

Network economy													
ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
0.05	0.05	0.13	0.13	0.04	0.23	0.37	0.15	0.35	0.19	0.34	0.30	0.14	0.42
Equivalent horizontal (model with fixed costs)													
ψ_1^H	ψ_2^H	ψ_3^H	ψ_4^H	ψ_5^H	ψ_6^H	ψ_7^H	θ_1^H	θ_2^H	θ_3^H	θ_4^H	θ_5^H	θ_6^H	θ_7^H
0.1	0.05	0.20	0.12	0.05	0.32	0.17	0.40	0.74	0.74	0.74	0.70	0.36	0.76

Table 5 reports the estimated f_i for the network and equivalent horizontal economies.

Table 5: Calibrated fixed costs

Network economy						
f_1	f_2	f_3	f_4	f_5	f_6	f_7
0.0063	0.0005	0.0128	0.0113	0.0121	0.0177	0.0038
Equivalent horizontal (model with fixed costs)						
f_1^H	f_2^H	f_3^H	f_4^H	f_5^H	f_6^H	f_7^H
0.0585	0.0047	0.1199	0.1060	0.1132	0.1659	0.0358

The match between the targeted moments in the data and the model is perfect for the main model. The moments for the model with fixed costs are reported in Table 6.

Table 6: Target Moments - Model with fixed costs

Share of Entrepreneurs		
Sector	Data	Model
AMU	0.91	0.92
Construction	1.58	0.78
Manufacturing	0.36	0.25
Trade	0.47	0.39
Transportation	0.41	0.17
FIRE	0.89	2.03
Others	3.59	4.27
Average Firm Size		
Sector	Data	Model
AMU	3.03	3.03
Construction	5.50	5.50
Manufacturing	38.92	38.92
Trade	26.28	26.28
Transportation	18.85	18.85
FIRE	6.10	6.10
Others	11.36	11.36

C. Entry subsidies in the model with fixed costs

In this section, we run the same exercise of Section 8 in the framework with fixed costs. We still subsidize entry into one sector at a time, so that the total transfer always amounts to 0.001% of final consumption.

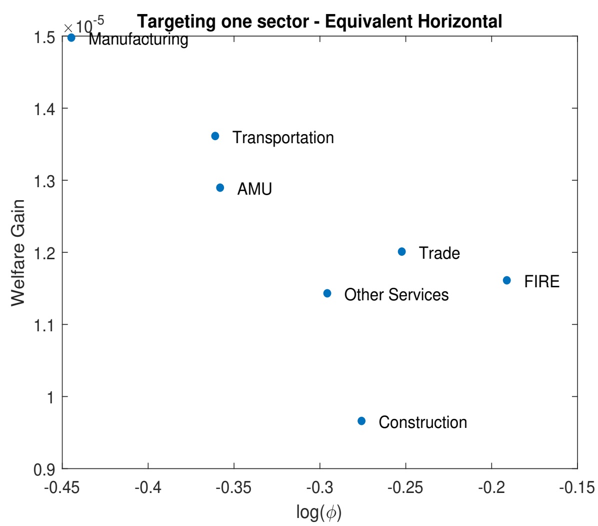
Figure 5(a) reports the welfare gain in the equivalent horizontal economy against the original distortions, $\log(\phi_i)$. Overall, now subsidies always have a positive and larger impact on final welfare. The reason is that the calibration of this model implies a lower level of heterogeneity (higher ξ_is), which means marginal entrepreneurs are relatively more productive. Moreover, differently from our main model, now the direct distortions are not anymore an optimal measure to rank sectors. The reason is that $\log(\phi_i)$s do not represent anymore the total "entrepreneurial" wedges, not even in the horizontal economy. To understand this, we should look at equations (54) and (56). The "entrepreneurial" wedge is now:

$$\tau_{\Pi,i} = \phi_i \frac{\kappa(\phi)_i}{\kappa(1)_i} \frac{\lambda(\phi)_i}{\lambda(1)_i}.$$

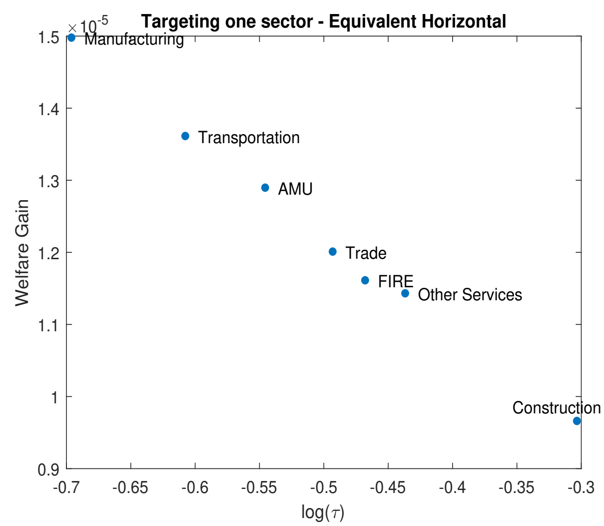
While $\frac{\lambda(\phi)_i}{\lambda(1)_i}$ is equal to 1 in the horizontal economy, the change in relative size of entry cost, $\frac{\kappa(\phi)_i}{\kappa(1)_i}$, is not. Figure 5(b) considers the same welfare gain this time against the total wedges. The sectors are now ranked almost perfectly.

We replicate the same exercise for the network economy in Figure 6. In graph (a), the sectors are ranked by the direct distortions; in graph (b), we add the deviations in Domar Weights; in graph (c), we finally add the change in relative size of entry costs for the marginal entrepreneurs. The second measure, which is sufficient in the economy without fixed costs, greatly improves the ranking based on $\log(\phi_i)$. However, it wrongly ranks Construction (the sector creating the lowest welfare gain) above the FIRE and Trade sectors. In the last graph, we correct this mistake. Computing the total profit losses of the marginal entrepreneurs is still a good way to identify which sectors would generate higher welfare gains from entry subsidies.

Figure 5: Welfare gain/loss from subsidizing entry in the equivalent horizontal economy of the model with fixed costs. The horizontal axis in graph (a) is the sectoral log of distortions, $\log(\phi_i)$; the horizontal axis in graph (b) is the sectoral log of profit share relative to the efficient economy, $\log(\tau_{\Pi,i}) = \log(\phi_i) + \log\left(\frac{\kappa(\phi)}{\kappa(1)}\right)$. The vertical axis displays the percentage deviation in aggregate welfare of the economy with subsidy relative to the baseline output. Each dot in the graph corresponds to the change in welfare of subsidizing entry only in the respective sector of production. The cost of the policy always amounts to 0.001% of the baseline GDP.

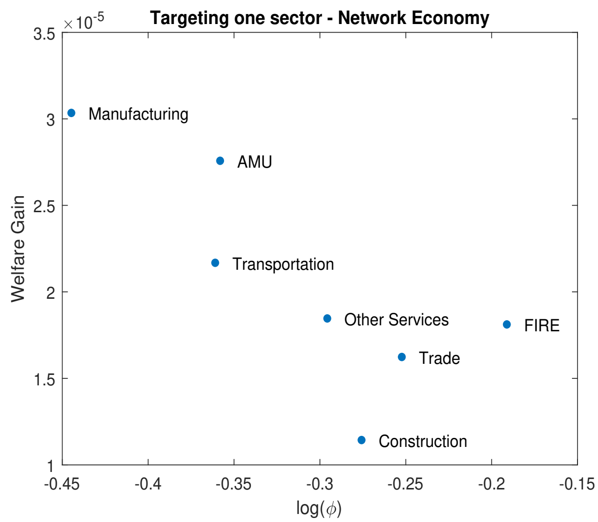


(a) x-axis: original distortions, $\log(\phi_i)$

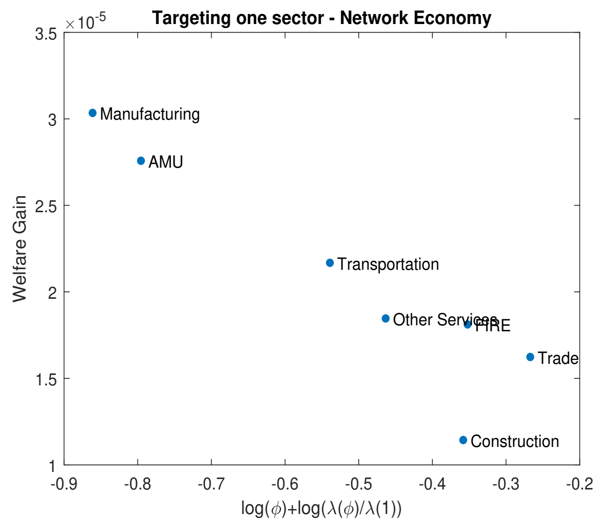


(b) x-axis: total wedges, $\log(\tau_{\Pi,i})$

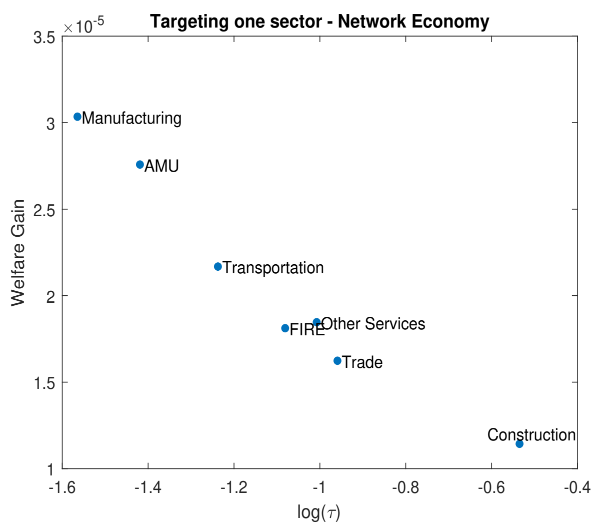
Figure 6: Welfare gain/loss from subsidizing entry in the equivalent horizontal economy of the model with fixed costs. The horizontal axis in graph (a) is the sectoral log of distortions, $\log(\phi_i)$; the horizontal axis in graph (b) is the sectoral log of profit share relative to the efficient economy, $\log(\tau_{\Pi,i}) = \log(\phi_i) + \log\left(\frac{\kappa(\phi)_i}{\kappa(1)_i}\right)$. The vertical axis displays the percentage deviation in aggregate welfare of the economy with subsidy relative to the baseline output. Each dot in the graph corresponds to the change in welfare of subsidizing entry only in the respective sector of production. The cost of the policy always amounts to 0.001% of the baseline GDP.



(a) x-axis: original distortions, $\log(\phi_i)$



(b) x-axis: $\log(\phi_i) + \log\left(\frac{\lambda(\phi)_i}{\lambda(1)_i}\right)$



c) x-axis: total wedges, $\log(\tau_{\Pi,i})$