Verification

Outline

- What are the goals of verification?
- What are the main approaches to verification?
 - What kind of assurance do we get through testing?
 - How can testing be done systematically?
 - How can we remove defects (debugging)?
- What are the main approaches to software analysis?
 - informal vs. formal

Need for verification

- Designers are fallible even if they are skilled and follow sound principles
- Everything must be verified, every required quality, process and products
 - even verification itself...

Properties of verification

- May not be binary (OK, not OK)
 - severity of defect is important
 - some defects may be tolerated
- May be subjective or objective
 - e.g., usability
- Even implicit qualities should be verified
 - because requirements are often incomplete
 - e.g., robustness

Approaches to verification

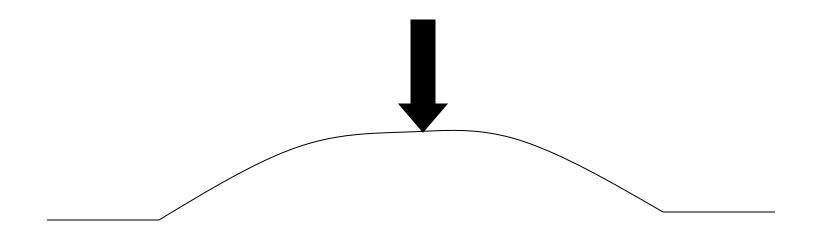
- Experiment with behavior of product
 - sample behaviors via testing
 - goal is to find "counterexamples"
 - dynamic technique
- Analyze product to deduce its adequacy
 - analytic study of properties
 - static technique

Testing and lack of "continuity"

- Testing samples behaviors by examining "test cases"
- Impossible to extrapolate behavior of software from a finite set of test cases
- No continuity of behavior
 - it can exhibit correct behavior in infinitely many cases, but may still be incorrect in some cases

Verification in engineering

- Example of bridge design
- One test assures infinite correct situations



```
procedure binary-search (key: in element;
                 table: in elementTable; found: out Boolean) is
begin
   bottom := table'first; top := table'last;
   while bottom < top loop
       if (bottom + top) rem 2 \neq 0 then
          middle := (bottom + top - 1) / 2;
       else
          middle := (bottom + top) / 2;
       end if;
       if key \leq table (middle) then
          top := middle;
       else
          bottom := middle + 1;
       end if;
   end loop;
   found := key = table (top);
end binary-search
```

if we omit this the routine works if the else is never hit! (i.e. if size of table is a power of 2)

Goals of testing

- To show the *presence* of bugs (Dijkstra, 1987)
- If tests do detect failures, we cannot conclude that software is defect-free
- Still, we need to do testing
 - driven by sound and systematic principles

Goals of testing (cont.)

- Should help isolate errors
 - to facilitate debugging
- Should be repeatable
 - repeating the same experiment, we should get the same results
 - this may not be true because of the effect of execution environment on testing
 - because of nondeterminism
- Should be accurate

Theoretical foundations of testing

Definitions (1)

- P (program), D (input domain), R (output domain)
 - $-P: D \rightarrow R$ (may be partial)
- Correctness defined by OR ⊆ D × R
 - -P(d) correct if <d, $P(d)> \in OR$
 - P correct if all P(d) are correct

Definitions (2)

FAILURE

- P(d) is not correct
 - may be undefined (error state) or may be the wrong result
- ERROR (DEFECT)
 - anything that may cause a failure
 - typing mistake
 - programmer forgot to test "x = 0"

FAULT

incorrect intermediate state entered by program

Definitions (3)

- Test case t
 - an element of D
- Test set T
 - a finite subset of D
- Test is successful if P(t) is correct
- Test set successful if P correct for all t in

Definitions (4)

- Ideal test set T
 - if P is incorrect, there is an element of T such that P(d) is incorrect
- if an ideal test set exists for any program, we could prove program correctness by testing

Test criterion

 A criterion C defines finite subsets of D (test sets)

$$-C \subset 2^{D}$$

 A test set T satisfies C if it is an element of C

Example

<-5, 0, 22> is a test set that satisfies C

Properties of criteria (1)

- C is consistent
 - for any pairs T1, T2 satisfying C, T1 is successful iff T2 is successful
 - so either of them provides the "same" information
- C is complete
 - if P is incorrect, there is a test set T of C that is not successful
- C is complete and consistent
 - identifies an ideal test set
 - allows correctness to be proved!

Properties of criteria (2)

- C1 is finer than C2
 - for any program P
 - for any T1 satisfying C1 there is a subset T2 of T1 which satisfies C2

Properties of definitions

- None is effective, i.e., no algorithms exist to state if a program, test set, or criterion has that property
- In particular, there is no algorithm to derive a test set that would prove program correctness
 - there is no constructive criterion that is consistent and complete

Empirical testing principles

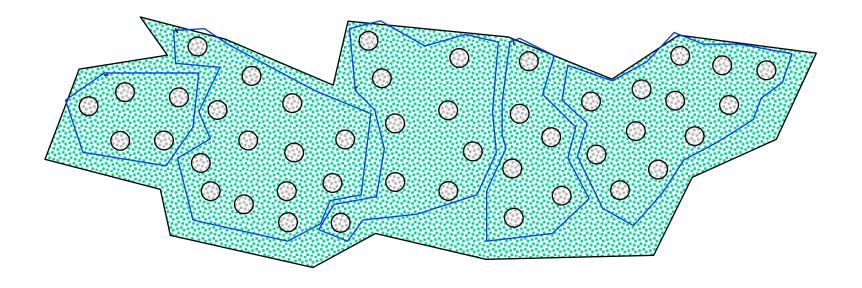
- Attempted compromise between the impossible and the inadequate
- Find strategy to select significant test cases
 - significant=has high potential of uncovering presence of error

Complete-Coverage Principle

 Try to group elements of D into subdomains D1, D2, ..., Dn where any element of each Di is likely to have similar behavior

- $-D = D1 \cup D2 \cup ... \cup Dn$
- Select one test as a representative of the subdomain
- If Dj \cap Dk $\neq \emptyset$ for all j, k (partition), any element can be chosen from each subdomain
- Otherwise choose representatives to minimize number of tests, yet fulfilling the principle

Complete-Coverage Principle example of a partition



Testing in the small

We test individual modules

- BLACK BOX (functional) testing
 - partitioning criteria based on the module's specification
 - tests what the program is supposed to do
- WHITE BOX (structural) testing
 - partitioning criteria based on module's internal code
 - tests what the program does

White box testing

derives test cases from program code

Structural Coverage Testing

- (In)adequacy criteria
 - If significant parts of program structure are not tested, testing is inadequate
- Control flow coverage criteria
 - Statement coverage
 - Edge coverage
 - Condition coverage
 - Path coverage

Statement-coverage criterion

- Select a test set T such that every elementary statement in P is executed at least once by some d in T
 - an input datum executes many statements → try to minimize the number of test cases still preserving the desired coverage

Example

```
read (x); read (y);
if x > 0 then
       write ("1");
else
       write ("2");
end if;
if y > 0 then
       write ("3");
else
       write ("4");
end if;
```

```
{<x = 2, y = 3>, <x = -13, y = 51>,
<x = 97, y = 17>, <x = -1, y = -1>}
covers all statements
```

$$\{, \}$$
 is minimal

Weakness of the criterion

{<x=-3} covers all statements

it does not exercise the case when x is positive and the then branch is not entered

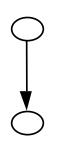
Edge-coverage criterion

 Select a test set T such that every edge (branch) of the control flow is exercised at least once by some d in T

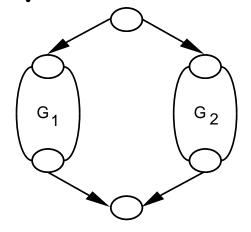
this requires formalizing the concept of the control graph, and how to construct it

- edges represent statements
- nodes at the ends of an edge represent entry into the statement and exit

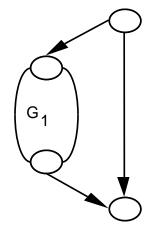
Control graph construction rules



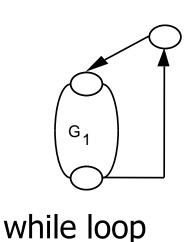
I/O, assignment, or procedure call

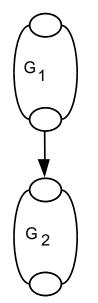


if-then-else



if-then



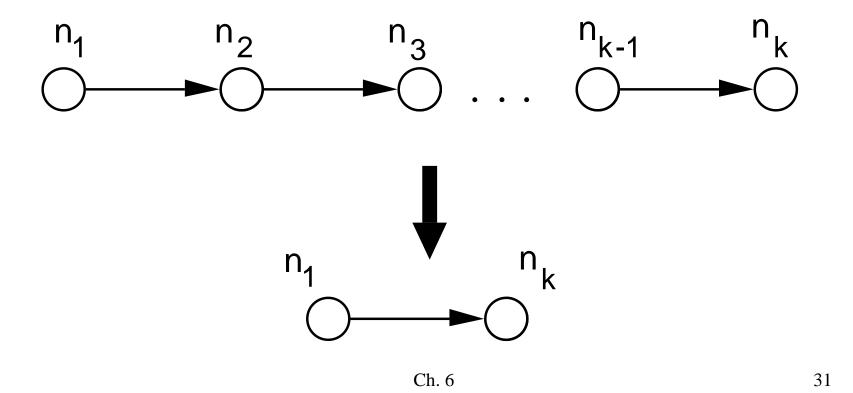


two sequential statements

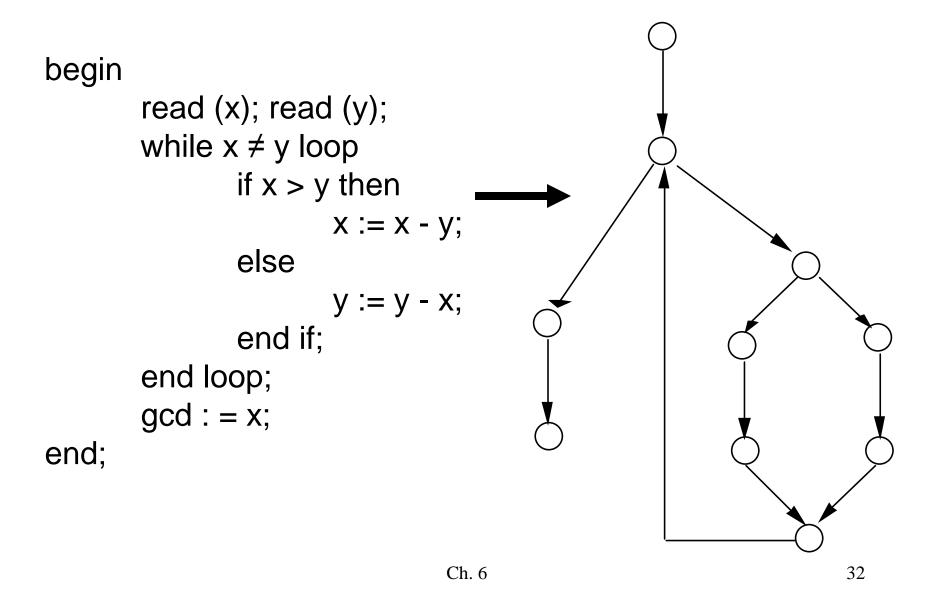
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Simplification

a sequence of edges can be collapsed into just one edge



Exemple: Euclid's algorithm



Weakness

```
found := false; counter := 1;
  while (not found) and counter < number_of_items loop
         if table (counter) = desired_element then
                found := true;
         end if;
         counter := counter + 1;
  end loop;
  if found then
         write ("the desired element is in the table");
  else
         write ("the desired element is not in the table");
  end if;
test cases: (1) empty table, (2) table with 3 items, second of
which is the item to look for
do not discover error (< instead of \le )
                                                             33
```

Condition-coverage criterion

- Select a test set T such that every edge of P's control flow is traversed and all possible values of the constituents of compound conditions are exercised at least once
 - it is finer than edge coverage

Weakness

```
if x \neq 0 then
        y := 5;
else
        Z := Z - X;
end if;
if z > 1 then
        z := z / x;
else
        z := 0;
end if;
```

{<x = 0, z = 1>, <x = 1, z = 3>} causes the execution of all edges, but fails to expose the risk of a division by zero

Path-coverage criterion

- Select a test set T which traverses all paths from the initial to the final node of P's control flow
 - it is finer than previous kinds of coverage
 - however, number of paths may be too large, or even infinite (see while loops)
 - additional constraints must be provided

The infeasibility problem

- Syntactically indicated behaviors (statements, edges, etc.) are often impossible
 - unreachable code, infeasible edges, paths, etc.
- Adequacy criteria may be impossible to satisfy
 - manual justification for omitting each impossible test case
 - adequacy "scores" based on coverage
 - example: 95% statement coverage

Further problem

- What if the code omits the implementation of some part of the specification?
- White box test cases derived from the code will ignore that part of the specification!

Black box testing

derives test cases from specifications

The specification

The program receives as input a record describing an invoice. (A detailed description of the format of the record is given.) The invoice must be inserted into a file of invoices that is sorted by date. The invoice must be inserted in the appropriate position: If other invoices exist in the file with the same date, then the invoice should be inserted after the last one. Also, some consistency checks must be performed: The program should verify whether the customer is already in a corresponding file of customers, whether the customer's data in the two files match, etc.

Did you consider these cases?

- An invoice whose date is the current date
- An invoice whose date is before the current date (This might be even forbidden by law)
 This case, in turn, can be split into the two following subcases:
- An invoice whose date is the same as that some existing invoice
- An invoice whose date does not exist in any previously recorded invoice
- Several incorrect invoices, checking different types of inconsistencies

Systematic black-box techniques

- Testing driven by logic specifications (pre and postconditions)
- Syntax-driven testing
- Decision table based testing
- Cause-effect graph based testing

Logic specification of insertion of invoice record in a file

```
for all x in Invoices, f in Invoice_Files
{sorted by date(f) and not exist j, k (j \neq k and f(j) =f(k)}
insert(x, f)
{sorted_by_date(f) and
for all k (old_f(k) = z implies exists j (f(j) = z)) and
for all k (f(k) = z and z \neq x) implies exists j (old f(j) = z) and
exists j (f(j). date = x. date and f(j) \neq x) implies j < pos(x, f) and
result = x.customer belongs_to customer_file and
warning = (x belongs_to old_f or x.date < current_date or ....)
```

```
Apply coverage criterion to postcondition...
Rewrite in a more convenient way...
```

```
TRUE implies
       sorted_by_date(f) and for all k old_f(k) = z
       implies exists j(f(j) = z) and
       for all k (f(k) = z and z \neq x) implies exists j (old f(j) = z)
and
(x.customer belongs_to customer_file) implies result
and
not (x.customer belongs_to customer_file and ...)
       implies not result
and
x belongs_to old_y implies warning
and
x.date < current_date implies warning
and
```

. . . .

Syntax-driven testing (1)

Consider testing an interpreter of the following language

Syntax-driven testing (2)

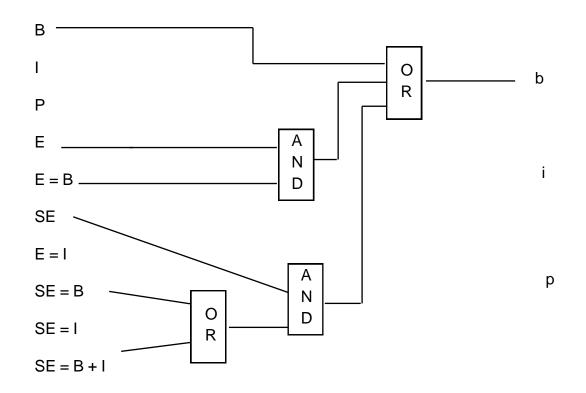
- Apply complete coverage principle to all grammar rules
- Generate a test case for each rule of the grammar
 - note, however that the test case might also cover other rules
- Note: the specification is formal, and test generation can be automated

Decision-table-based testing

"The word-processor may present portions of text in three different formats: plain text (p), boldface (b), italics (i). The following commands may be applied to each portion of text: make text plain (P), make boldface (B), make italics (I), emphasize (E), super emphasize (SE). Commands are available to dynamically set E to mean either B or I (we denote such commands as E=B and E=I, respectively.) Similarly, SE can be dynamically set to mean either B (command SE=B) or I (command SE=I), or B and I (command SE=B+I.)"

Р	*								
В		*							*
ı			*						*
E				*	*				
SE						*	*	*	
E = B				*					
E = I					*				
SE = B						*			
SE = I							*		
SE = B + I								*	
action	р	b	i	b	i	b	i	b,i	b,i

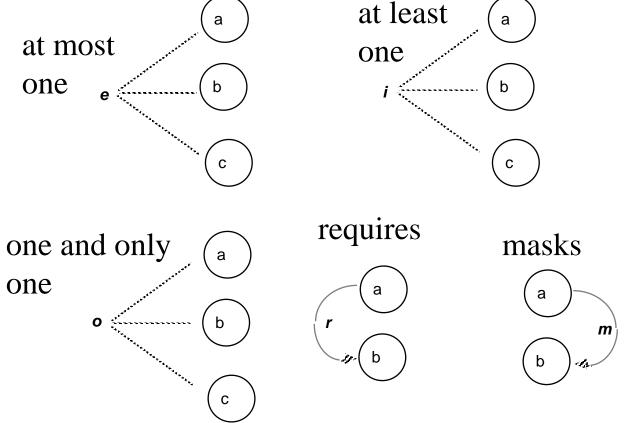
Cause effect graphs

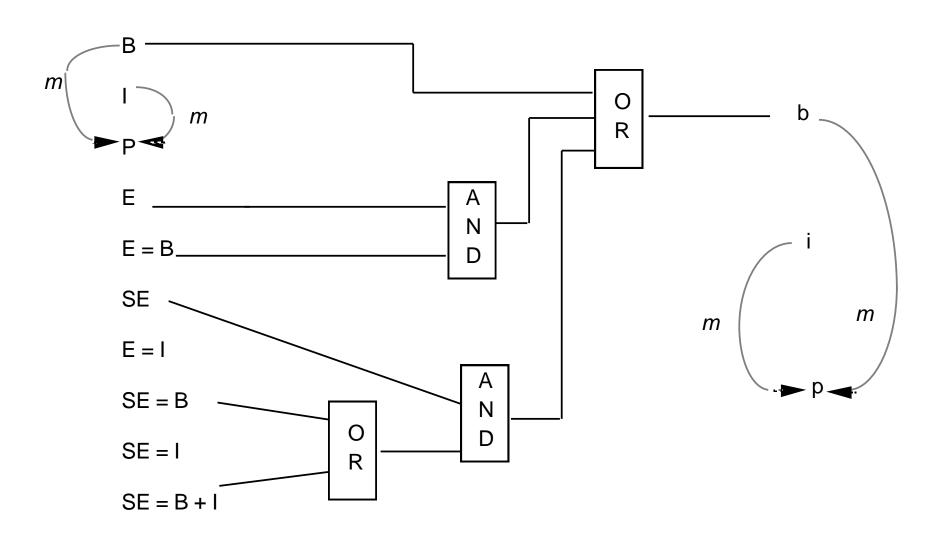


The AND/OR graph represents the correspondence between causes and effects

Further constraints

"Both B and I exclude P (i.e., one cannot ask both for plain text and, say, italics for the same portion of text.) E and SE are mutually exclusive."





X m Y = X implies not Y

Coverage criterion

- Generate all possible input combinations and check outputs
- May reduce the number by going backwards from outputs
 - OR node with true output:
 - use input combinations with only one true input
 - AND node with false output:
 - use input combinations with only one false input

Testing boundary conditions

- Testing criteria partition input domain in classes, assuming that behavior is "similar" for all data within a class
- Some typical programming errors, however, just happen to be at the boundary between different classes

Criterion

- After partitioning the input domain D into several classes, test the program using input values not only "inside" the classes, but also at their boundaries
- This applies to both white-box and black-box techniques

The oracle problem

How to inspect the results of test executions to reveal failures

- Oracles are required at each stage of testing
- Automated test oracles are required for running large amounts of tests
- Oracles are difficult to design no universal recipe

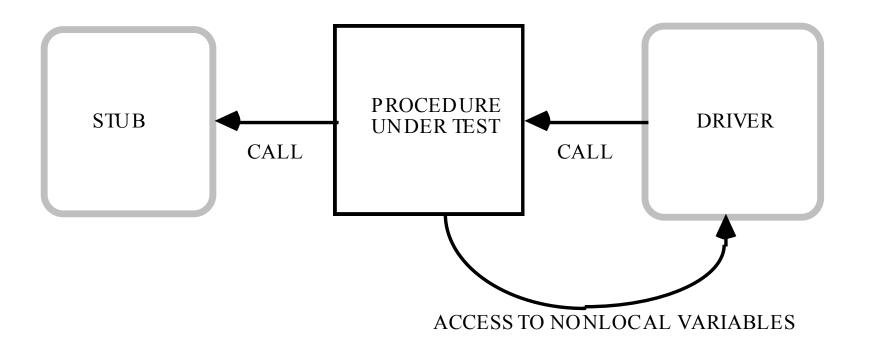
Testing in the large

- Module testing
 - testing a single module
- Integration testing
 - integration of modules and subsystems
- System testing
 - testing the entire system
- Acceptance testing
 - performed by the customer

Module testing

- Scaffolding needed to create the environment in which the module should be tested
 - stubs
 - modules used by the module under test
 - driver
 - module activating the module under test

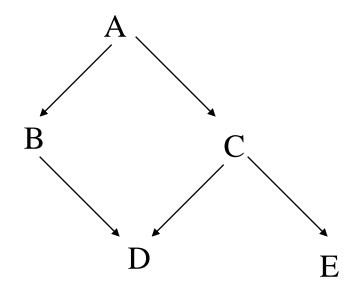
Testing a functional module



Integration testing

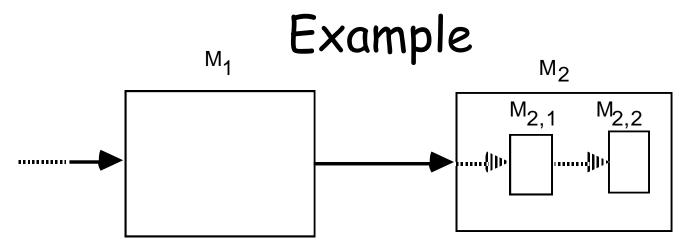
- Big-bang approach
 - first test individual modules in isolation
 - then test integrated system
- Incremental approach
 - modules are progressively integrated and tested
 - can proceed both top-down and bottom-up according to the USES relation

Integration testing and USES relation



If integration and test proceed bottom-up only need drivers

Otherwise, if we proceed top-down only stubs are needed



M1 USES M2 and M2 IS_COMPOSED_OF {M2,1, M2,2}

CASE 1

Test M1, providing a stub for M2 and a driver for M1 Then provide an implementation for M2,1 and a stub for M2,2

CASE 2

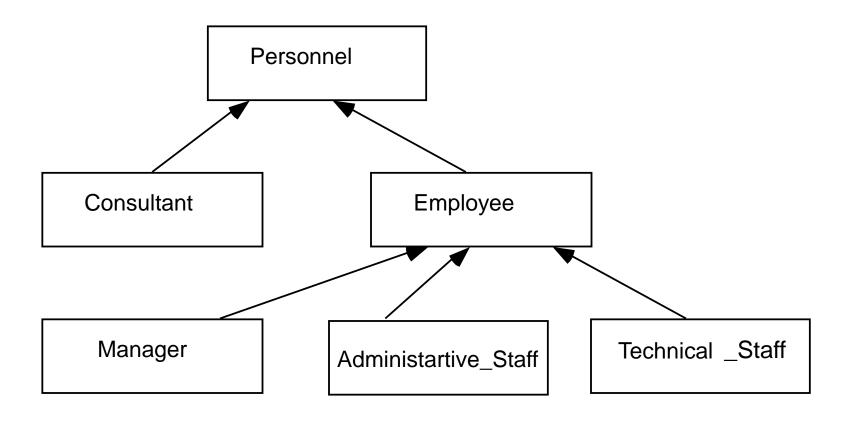
Implement M2,2 and test it by using a driver, Implement M2,1 and test the combination of M2,1 and M2,2 (i.e., M2) by using a driver

Finally, implement M1 and test it with M2, using a driver for M1

Testing 00 programs

- New issues
 - inheritance
 - genericity
 - polymorphism
 - dynamic binding
- Open problems still exist

Inheritance



How to test classes of the hierarchy?

- "Flattening" the whole hierarchy and considering every class as a totally independent component
 - does not exploit incrementality
- Finding an ad-hoc way to take advantage of the hierarchy

A sample strategy

- A test that does not have to be repeated for any heir
- A test that must be performed for heir class X and all of its further heirs
- A test that must be redone by applying the same input data, but verifying that the output is not (or is) changed
- A test that must be modified by adding other input parameters and verifying that the output changes accordingly

Separate concerns in testing

- Testing for functionality is not enough
- Overload testing
- Robustness testing
- Regression testing
 - organize testing with the purpose of verifying possible *regressions* of software during its life—that is, degradations of correctness or other qualities due to later modifications

Testing concurrent and realtime systems

- Nondeterminism inherent in concurrency affects repeatability
- For real-time systems, a test case consists not only of input data, but also of the times when such data are supplied

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Analysis

Analysis vs. testing

- Testing characterizes a single execution
- Analysis characterizes a *class* of executions; it is based on a *model*
- They have complementary advantages and disadvantages

Informal analysis techniques Code walkthroughs

- Recommended prescriptions
 - Small number of people (three to five)
 - Participants receive written documentation from the designer a few days before the meeting
 - Predefined duration of meeting (a few hours)
 - Focus on the discovery of errors, not on fixing them
 - Participants: designer, moderator, and a secretary
 - Foster cooperation; no evaluation of people
 - Experience shows that most errors are discovered by the designer during the presentation, while trying to explain the design to other people.

Informal analysis techniques Code inspection

- A reading technique aiming at error discovery
- Based on checklists; e.g.:
 - use of uninitialized variables;
 - jumps into loops;
 - nonterminating loops;
 - array indexes out of bounds;

— ...

Correctness proofs

A program and its specification (Hoare notation)

```
{true}
begin
    read (a); read (b);
    x := a + b;
    write (x);
end
{output = input1 + input2}
```

proof by backwards substitution

Proof rules

Notation:

If Claim 1 and Claim 2 have been proven, one can deduce Claim3

Claim1, Claim2
Claim3

Proof rules for a language

```
{F1}S1{F2}, {F2}S2{F3} sequence {F1}S1;S2{F3}
```

if-then-else

```
{Pre and cond} S1 {Post},{Pre and not cond} S2 {Post} {Pre} if cond then S1; else S 2; end if; {Post}
```

{I and cond} S {I}

while-do

{I} while cond loop S; end loop; {I and not cond}

I *loop invariant*

Correctness proof

- Partial correctness
 - validity of {Pre} Program {Post}
 guarantees that if the Pre holds before the execution of Program, and if the program ever terminates, then Post will be achieved
- Total correctness
 - Pre guarantees Program's termination and the truth of Post

These problems are undecidable!!!

Example

```
\{input1 > 0 \text{ and input2} > 0\}
begin
       read (x); read (y);
       div := 0;
       while x = y loop
              div := div + 1;
              x := x - y;
       end loop;
       write (div); write (x);
end;
{input1 = output1 * input2 + output2 and
0 = output2 < input2 }
```

Invention of loop invariant

- Difficult and creative step
- Cannot be constructed automatically
- In the example
 input1 = div * y + x and x = 0 and y = input2

Programs with arrays

```
{Pre} a(i) := expression; {Post}
```

Pre denotes the assertion obtained from Post by substituting every occurrence of an indexed variable a(j) by the term if j = i then expression else a(j);

Example

```
{n = 1}
i := 1; j := 1;
found := false;
                                 : old_table, old_n
while i =n loop
                                 constants denoting the
       if table (i) = x then
                                 values of table and of n
               found := true; !before execution
               i := i + 1
                                 of the program fragment
       else
               table(j) := table(i);
               i := i + 1; j := j + 1;
       end if;
end loop;
n := j - 1;
{not exists m (1 = m = n \text{ and table } (m) = x) \text{ and}
found = exists m (1 = m = old_n \text{ and } old_t \text{ able } (m) = x)
```

Correctness proof

Can be done by using the following loop invariant

```
\{(j = i) \text{ and } (i = old\_n + 1) \text{ and}

\{(not \text{ exists } m (1 = m < j \text{ and}

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```

Correctness proofs in the large

- We can prove correctness of operations (e.g., operations on an abstract data type)
- Then use the result of the proof in proving fragments that operate on objects of the ADT

Example

```
module TABLE;
exports
      type Table_Type (max_size: NATURAL): ?;
      no more than max_size entries may be
      stored in a table; user modules must quarantee this
      procedure Insert (Table: in out TableType;
             ELEMENT: in ElementType);
      procedure Delete (Table: in out TableType;
             ELEMENT: in ElementType);
      function Size (Table: in Table_Type) return NATURAL;
      provides the current size of a table
```

end TABLE

Having proved these

```
{true}
Delete (Table, Element);
{Element ∉ Table};

{Size (Table) < max_size}
Insert (Table, Element)
{Element ∈ Table};</pre>
```

We can then prove properties of programs using tables For example, that after executing the sequence

```
Insert(T, x);
Delete(T, x);
x is not present in T
```

An assessment of correctness proofs

- Still not used in practice
- However
 - may be used for very critical portions
 - assertions may be the basis for a systematic way of inserting runtime checks
 - proofs may become more practical as more powerful support tools are developed
 - knowledge of correctness theory helps programmers being rigorous

Symbolic execution

- Can be viewed as a middle way between testing and analysis
- Executes the program on symbolic values
- One symbolic execution corresponds to many actual executions

Example(1)

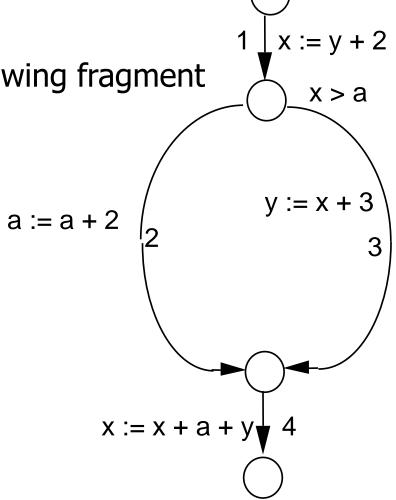
Consider executing the following fragment

with x=X, y=Y, a=A

$$y := x + 3;$$

end if;

$$x := x + a + y;$$



Example(2)

- When control reaches the conditional, symbolic values do not allow execution to select a branch
- One can choose a branch, and record the choice in a path condition
- Result:

$$<$$
{a = A, y = Y + 5, x = 2 * Y + A + 7},
 $<$ 1, 3, 4>, $Y + 2 \le A$ >
execution path condition
path

Symbolic execution rules (1)

symbolic state:

<symbolic_variable_values, execution_path, path_condition>

- read (x)
 - removes any existing binding for x and adds binding x = X, where X is a *newly introduced* symbolic value
- Write (expression)
 - output(n) = computed_symbolic_value (n counter initialized to 1 and automatically incremented after each output statement)

Symbolic execution rules (2)

- x:= expression
 - construct symbolic value of expression, SV; replace previous binding for x with x = SV
- After execution of the last statement of a sequence that corresponds to an edge of control graph, append the edge to execution path

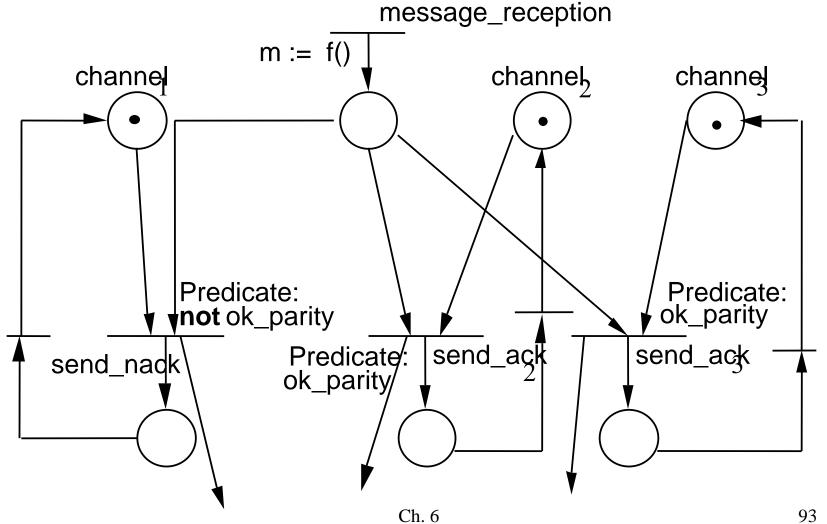
Symbolic execution rules (3)

- if cond then S1; else S2; endif
- while cond loop...endloop
 - condition is symbolically evaluated
 - eval (cond)
 - if eval (cond) \Rightarrow true or false then execution proceeds by following appropriate branch
 - otherwise, make nondeterministic choice of true or false, and conjoin eval (cond) (resp., not eval (cond)) to the path condition Ch. 6 91

Programs with arrays

- Let A1 be the symbolic value of array a when statement a(i)= exp is executed
- Then, after execution of the statement, a receives the new symbolic value A2, denoted as A2 = A1<i, exp>, a shorthand for
 - for all k if k = i then A2(k) = expelse A2(k) = A1(k)

Symbolic execution of concurrent programs



Assumptions

- Simplifying assumption: no more than one token in a place
- A sequence of atomic steps can be modeled by a firing sequence
 - this resolves the nondeterminism that is due to several transitions being enabled
- The triple <symbolic_variable_values, execution_path, path_condition> can be used to model the symbolic state of the interpreter (execution_path is the firing sequence)

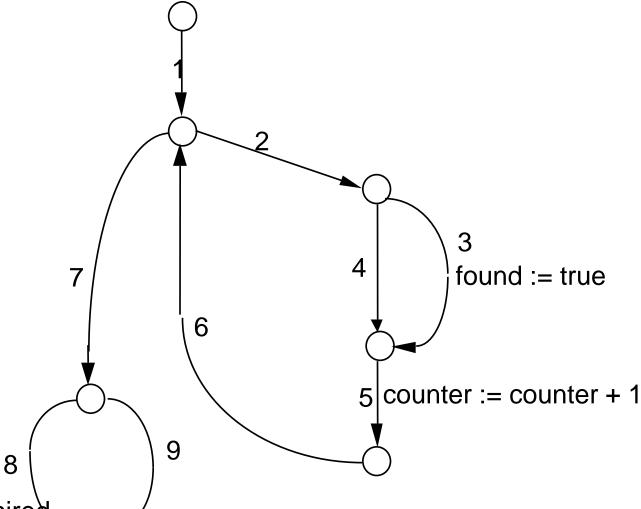
Symbolic execution and testing

- The path condition describes the data that traverse a certain path
- Use in testing:
 - select path
 - symbolically execute it
 - synthesize data that satisfy the path condition
 - they will execute that path

Example (1)

```
found := false; counter := 1;
while (not found) and counter < number_of_items loop
      if table (counter) = desired_element then
             found := true;
      end if;
      counter := counter + 1;
end loop;
if found then
      write ("the desired element exists in the table");
else
      write ("the desired element does not exist
                                         in the table");
end if;
```

Example (2)



write "the desired element does not exist in the table" write "the desired element exists in the table" Ch. 6

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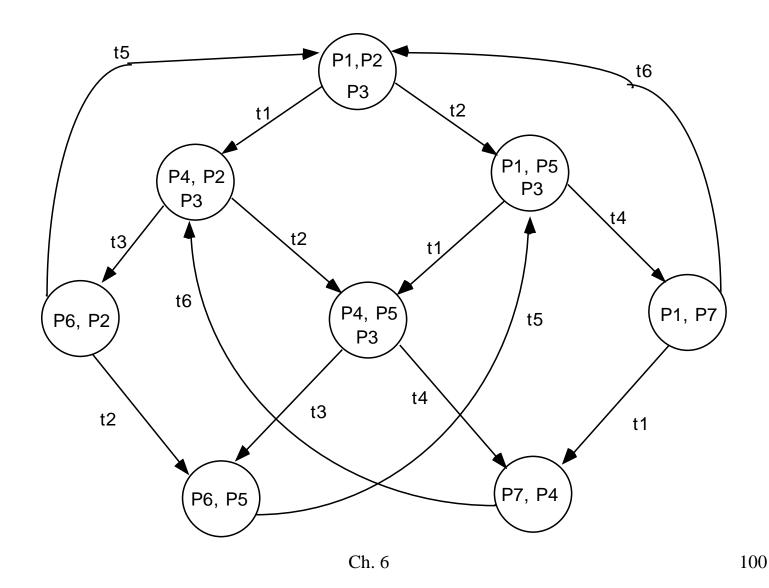
Model checking

- Correctness verification, in general, is an undecidable problem
- Model checking is a rather recent verification technique based on the fact that most interesting system properties become decidable (i.e., algorithmically verifiable) when the system is modeled as a finite state machine

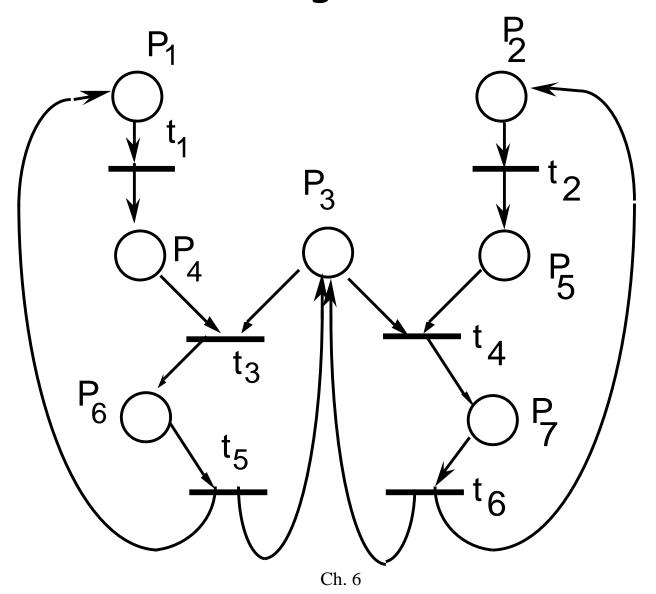
Principles

- Describe a given system—software or otherwise—as an FSM
- Express a given property of interest as a suitable formula
- Verify whether the system's behavior does indeed satisfy the desired property
 - this step can be performed automatically
 - the model checker either provides a proof that the property holds or gives a counterexample in the form of a test case that exposes the system's failure to behave according to the property

FSM representing markings of a PN



The original PN



Properties and proofs

- Property to be verified given through a formula (in temporal logic)
- In the example, one can prove
 - there is always a computation that allows the left process to enter the critical region
 - there is no guarantee that the left process accesses the shared resource unless it already owns it

Why so many approaches to testing and analysis?

- Testing versus (correctness) analysis
- Formal versus informal techniques
- White-box versus black-box techniques
- Techniques in the small/large
- Fully automatic vs. semiautomatic techniques (for undecidable properties)

• ...

view all these as complementary

Debugging

- The activity of locating and correcting errors
- It can start once a failure has been detected
- The goal is closing up the gap between a fault and failure
 - memory dumps, watch points
 - intermediate assertions can help

Verifying other qualities

Performance

- Worst case analysis
 - focus is on proving that the system response time is bounded by some function of the external requests
 - vs. average behavior
- Standard deviation
- Analytical vs. experimental approaches

Reliability (1)

- There are approaches to measuring reliability on a probabilistic basis, as in other engineering fields
- Unfortunately there are some difficulties with this approach
- Independence of failures does not hold for software

Reliability (2)

- Reliability is concerned with measuring the probability of the occurrence of failure
- Meaningful parameters include:
 - average total number of failures observed at time t: AF(t)
 - failure intensity: FI(t)=AF'(t)
 - mean time to failure at time t: MTTF(t)=1/FI(t)
- Time in the model can be execution or clock or calendar time

Basic reliability model

- Assumes that the decrement per failure experienced (i.e., the derivative with respect to the number of detected failures) of the failure intensity function is constant
 - i.e., FI is a function of AF

$$FI(AF) = FI_0 (1 - AF/AF_{\infty})$$

where FI_0 is the initial failure intensity and AF_{∞} is the total number of failures

 The model is based on optimistic hypothesis that a decrease in failures is due to the fixing of the errors that were sources of failures

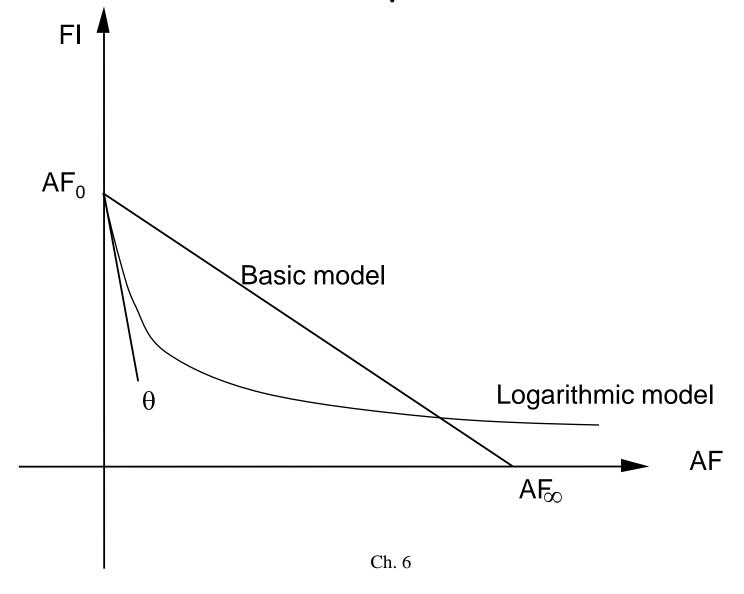
Logarithmic model

 Assumes, more conservatively, that the decrement per failure of FI decreases exponentially

```
FI(AF) = FI_0 \exp(-\theta AF)
```

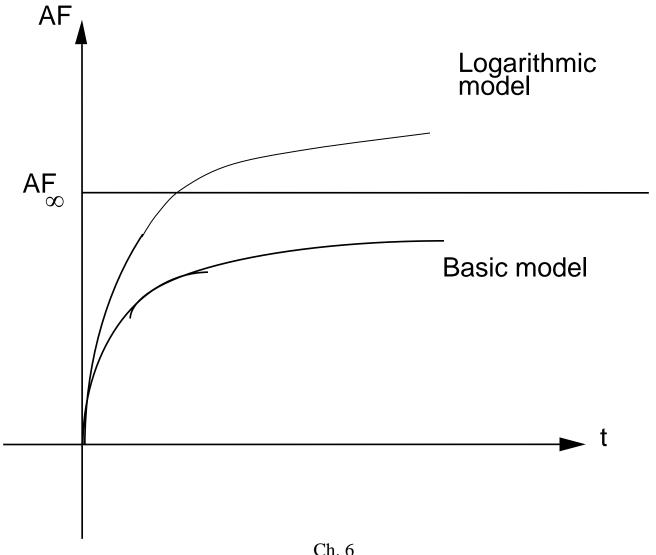
θ: failure intensity decay parameter

Model comparison



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Model comparison



Verifying subjective qualities

- Consider notions like simplicity, reusability, understandability ...
- Software science (due to Halstead) has been an attempt

Halstead's software science

- Tries to measure some software qualities, such as
 - abstraction level, effort, ...
- by measuring some quantities on code, such as
 - $-\eta$ 1, number of distinct operators in the program
 - $-\eta$ 2, number of distinct operands in the program
 - N1, number of occurrences of operators in the program
 - N2, number of occurrences of operands in the program

McCabe's source code metric

Cyclomatic complexity of the control graph

```
-C = e - n + 2 p
```

- e is # edges, n is # nodes, p is # connected components
- McCabe contends that well-structured modules have C in range 3 .. 7, and C = 10 is a reasonable upper limit for the complexity of a single module
 - confirmed by empirical evidence

Goal-question-metric (GQM)

Premise

- software metrics must be used to analyze software qualities, not to evaluate people
- quality evaluation must be of end product, intermediate products, and process
- metrics must be defined in the context of a complete and well-designed quality improvement paradigm (QIP)

GQM

- Not concerned with measuring a single quantity or group of quantities
 - e.g., cyclomatic complexity
- A method that is intended to lead from a precise definition of the objectives of measuring qualities (the *goals*) to the quantities (the *metrics*) whose measures are used to verify the achievement of such qualities.

The method

- Define goal precisely—Example:
 - Analyze the information system with the purpose of estimating the costs from the point of view of the manager in the context of a major software house
- Define suitable set of questions aimed at achieving the stated goal
- Associate a precise metric with every question