#### STAT 444 FINAL PROJECT PROPOSAL

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This paper contains our proposal for the STAT 444 final project. It outlines our dataset,

1. Introduction to our chosen dataset. Our proposal primarily aims to assess the feasibility of utilizing regression techniques for interpolating and modeling housing prices. To achieve this objective, we have selected a dataset from the Journal of Statistics, which includes housing prices in Ames, Iowa, along with other pertinent features. In today's dynamic real estate market, precise and dependable house price predictions hold immense significance for homeowners, buyers, and real estate professionals alike. By delving into the potential of these advanced modeling techniques, our goal is to augment the accuracy and reliability of house price predictions, contributing to more informed decision-making in the industry.

## 2. Exploratory Data Analysis.

- 2.1. Summary Statistics.
- the dataset contains 2930 rows and 82 columns
- 80 explanatory variables, 23 nominal, 23 ordinal, 14 discrete, and 20 continuous variates.
- ## [1] 1460 ## [1] 81
  - 2.2. Missing/duplicate Values.
- some columns contain a large number of NAs
- 0 duplicates

```
## [1] 91
## Warning: Unknown or uninitialised column: 'Pool.QC'.
## [1] 0
## Warning: Unknown or uninitialised column: 'Misc.Feature'.
## [1] 0
## [1] 281
## Warning: Unknown or uninitialised column: 'Fireplace.Qu'.
## [1] 0
## [1] 0 81
```

Keywords and phrases: We love regression.

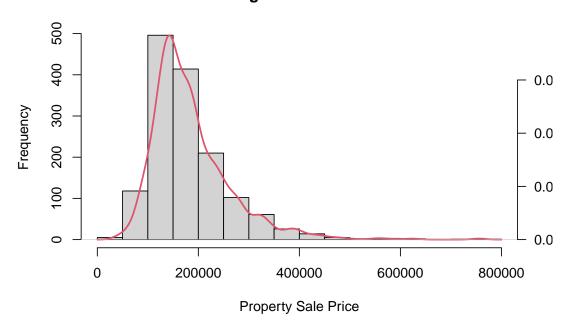
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## 2.3. Distribution of dependent variable.

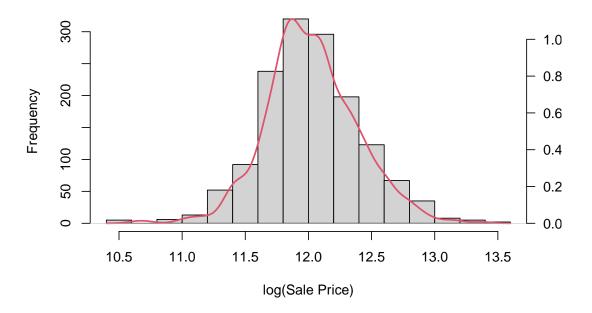
- the distribution of Sale Price looks very right-skewed: most values are clustered around the left tail while the right tail is longer (extreme high values)
- The sale prices range from \$12,789 to \$755,000 with a mean of \$180,796 and a standard deviation of \$79,886.69.
- Need to apply natural log transformation to address the non-normal distribution

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 34900 129975 163000 180921 214000 755000
```

# **Histogram of Sale Price**



# **Histogram of log(Sale Price)**



## 2.4. Correlations with the dependent variable.

- Top 5 variables with the highest correlations:
  - Overall.Qual: Rates the overall material and finish of the house
  - Gr.Liv.Area: Above grade (ground) living area square feet
  - Garage.Cars: Size of garage in car capacity

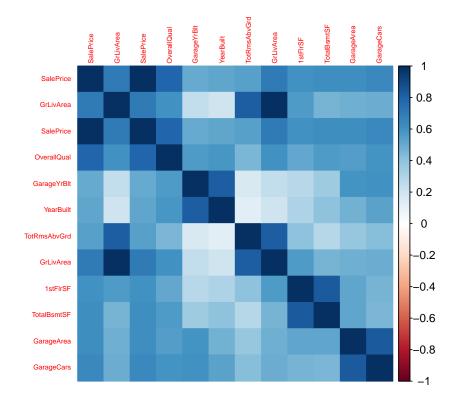
- Garage. Area: Size of garage in square feet
- Total.Bsmt.SF: Total square feet of basement area

```
##
    OverallQual
                   GrLivArea
                                GarageCars
                                             GarageArea
                                                         TotalBsmtSF
                                                                           1stFl:
##
     0.79788068
                  0.70515357
                                0.64703361
                                             0.61932962
                                                           0.61561224
                                                                         0.607969
##
       FullBath TotRmsAbvGrd
                                 YearBuilt YearRemodAdd GarageYrBlt
                                                                         MasVnrA
##
     0.56662744
                  0.54706736
                                0.52539360
                                             0.52125327
                                                           0.50475302
                                                                         0.488658
##
     Fireplaces
                  BsmtFinSF1
                              LotFrontage OpenPorchSF
                                                           WoodDeckSF
                                                                           2ndFl:
##
     0.46187269
                  0.39030052
                                0.34426977
                                             0.34335381
                                                           0.33685512
                                                                         0.306879
##
        LotArea
                    HalfBath BsmtFullBath
                                              BsmtUnfSF BedroomAbvGr
                                                                        ScreenPo
##
     0.29996221
                                0.23673741
                                             0.21312868
                                                           0.16681389
                                                                         0.11042
                  0.26856030
##
       PoolArea
                                 3SsnPorch
                      MoSold
##
     0.09248812
                  0.05156806
                                0.03077659
## EnclosedPorch KitchenAbvGr
                                  OverallCond
                                                 MSSubClass
                                                                         Id
##
   -0.154843204
                  -0.140497445
                                 -0.124391232
                                               -0.088031702
                                                              -0.047121850
##
    BsmtHalfBath
                       MiscVal
                                   BsmtFinSF2
                                                      YrSold LowQualFinSF
    -0.036512665 \quad -0.036041237
                                 -0.028021366
                                               -0.011868823
##
                                                              -0.001481983
```

# 2.5. Check if there is a high degree of correlation or linear association among independent variables.

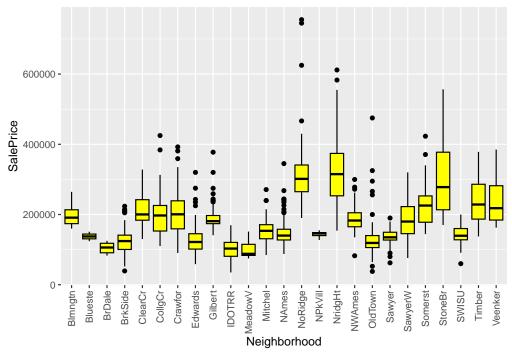
```
## Using var as id variables
```

```
##
                         variable
                                       value
                 var
## 1016
          GarageArea
                       GarageCars 0.8394149
## 1053
          GarageCars
                       GarageArea 0.8394149
                      TotalBsmtSF 0.8359994
## 470
            1stFlrSF
## 507
         TotalBsmtSF
                        X1stFlrSF 0.8359994
## 632 TotRmsAbvGrd
                        GrLivArea 0.8243121
## 891
           GrLivArea TotRmsAbvGrd 0.8243121
## 254
         GarageYrBlt
                        YearBuilt 0.8235195
## 957
           YearBuilt
                      GarageYrBlt 0.8235195
## 190
           SalePrice OverallQual 0.7978807
## 1411 OverallQual
                        SalePrice 0.7978807
## 646
           SalePrice
                        GrLivArea 0.7051536
                        SalePrice 0.7051536
## 1423
           GrLivArea
## Using var as id variables
                variable value
## [1] var
## <0 rows> (or 0-length row.names)
## Warning: package 'corrplot' was built under R version 4.3.1
## corrplot 0.92 loaded
```



2.6. Other interesting but non-numeric variables.

- Neighborhood
- Street: Type of road access to property
  - no predictive power; most likely irrelevant to house prices
  - can be dropped
- Utilities: Type of utilities available
  - There is not much variability in the distribution of data. All observations but 3 have "AllPub" as their values for Utilities
  - can be dropped
- Roof.Matl: Roof material
  - The most common data item "CompShg" occurs 2887 times out of 2930 observations



## - can be dropped

```
## Var1 Freq
## 1 Grvl 6
## 2 Pave 1454

## Var1 Freq
## 1 AllPub 1459
## 2 NoSeWa 1
## Warning: Unknown or uninitialised column: 'Roof.Matl'.
## [1] Freq
## <0 rows> (or 0-length row.names)
```

- **3. Introduction.** This template helps you to create a properly formatted LATEX  $2_{\mathcal{E}}$  manuscript. Prepare your paper in the same style as used in this sample .pdf file. Try to avoid excessive use of italics and bold face. Please do not use any LATEX  $2_{\mathcal{E}}$  or TeX commands that affect the layout or formatting of your document (i.e., commands like \textheight, \textwidth, etc.).
  - **4. Section headings.** Here are some sub-sections:
  - 4.1. A sub-section. Regular text.
  - 4.1.1. A sub-sub-section. Regular text.

#### 5. Text.

- 5.1. Lists. The following is an example of an itemized list, two levels deep.
- This is the first item of an itemized list. Each item in the list is marked with a "tick." The document style determines what kind of tick mark is used.
- This is the second item of the list. It contains another list nested inside it.
  - This is the first item of an itemized list that is nested within the itemized list.
  - This is the second item of the inner list. LATEX allows you to nest lists deeper than you really should.
- This is the third item of the list.

The following is an example of an *enumerated* list of one level.

- (i) This is the first item of an enumerated list.
- (ii) This is the second item of an enumerated list.

The following is an example of an *enumerated* list, two levels deep.

- 1. This is the first item of an enumerated list. Each item in the list is marked with a "tick.". The document style determines what kind of tick mark is used.
  - 2. This is the second item of the list. It contains another list nested inside of it.
    - (i) This is the first item of an enumerated list that is nested within.
  - (ii) This is the second item of the inner list. LaTeX allows you to nest lists deeper than you really should.

This is the rest of the second item of the outer list.

3. This is the third item of the list.

ird item of the list. \end{longlist}

5.2. Punctuation. Dashes come in three sizes: a hyphen, an intra-word dash like "U-statistics" or "the time-homogeneous model"; a medium dash (also called an "en-dash") for number ranges or between two equal entities like "1–2" or "Cauchy–Schwarz inequality"; and a punctuation dash (also called an "em-dash") in place of a comma, semicolon, colon or parentheses—like this.

Generating an ellipsis ... with the right spacing around the periods requires a special command.

5.3. Citation. Simple author and year cite: Billingsley (1999). Multiple bibliography items cite: Billingsley (1999); Bourbaki (1966) or (Billingsley, 1999; Bourbaki, 1966). Author only cite: Ethier and Kurtz. Year only cite: 1956 or (1956).

**6. Fonts.** Please use text fonts in text mode, e.g.:

Roman

Italic

**Bold** 

SMALL CAPS

Sans serif

Typewriter

Please use mathematical fonts in mathematical mode, e.g.:

ABCabc123

ABCabc123

ABCabc123

 $ABCabc123lphaeta\gamma$ 

ABC

 $\mathbb{ABC}$ 

ABCabc123

ABCabc123

ABCabc123

Note that \mathcal, \mathbb belongs to capital letters-only font typefaces.

- **7. Notes.** Footnotes<sup>1</sup> pose no problem.<sup>2</sup>
- **8. Quotations.** Text is displayed by indenting it from the left margin. There are short quotations

This is a short quotation. It consists of a single paragraph of text. There is no paragraph indentation.

and longer ones.

This is a longer quotation. It consists of two paragraphs of text. The beginning of each paragraph is indicated by an extra indentation.

This is the second paragraph of the quotation. It is just as dull as the first paragraph.

#### 9. Environments.

9.1. *Examples for* plain-style environments.

AXIOM 1. This is the body of Axiom 1.

PROOF. This is the body of the proof of the axiom above.

CLAIM 2. This is the body of Claim 2. Claim 2 is numbered after Axiom 1 because we used [axiom] in \newtheorem.

THEOREM 9.1. This is the body of Theorem 9.1. Theorem 9.1 numbering is dependent on section because we used [section] after \newtheorem.

<sup>&</sup>lt;sup>1</sup>This is an example of a footnote.

<sup>&</sup>lt;sup>2</sup>Note that footnote number is after punctuation.

THEOREM 9.2 (Title of the theorem). This is the body of Theorem 9.2. Theorem 9.2 has additional title.

LEMMA 9.3. This is the body of Lemma 9.3. Lemma 9.3 is numbered after Theorem 9.2 because we used [theorem] in \newtheorem.

PROOF OF LEMMA 9.3. This is the body of the proof of Lemma 9.3.

9.2. Examples for remark-style environments.

DEFINITION 9.4. This is the body of Definition 9.4. Definition 9.4 is numbered after Lemma 9.3 because we used [theorem] in \newtheorem.

EXAMPLE. This is the body of the example. Example is unnumbered because we used \newtheorem\* instead of \newtheorem.

FACT. This is the body of the fact. Fact is unnumbered because we used \newtheorem\* instead of \newtheorem.

**10. Tables and figures.** Cross-references to labeled tables: As you can see in Table1 and also in Table2.

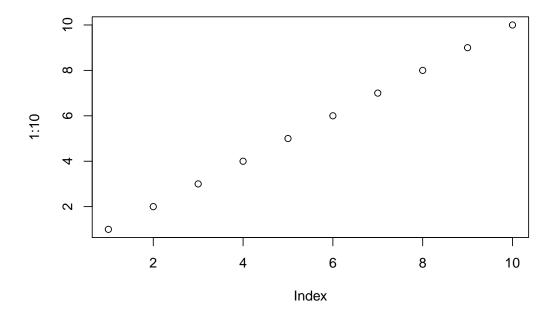


FIGURE 1. Figure caption

Sample of cross-reference to figure. Figure 1 shows that it is not easy to get something on paper.

TABLE 1
Table caption

Mazda RX4         21.0         6         160.0         110         3.90         2.620         16.46         0         1         4         4           Mazda RX4 Wag         21.0         6         160.0         110         3.90         2.620         16.46         0         1         4         4           Mazda RX4 Wag         21.0         6         160.0         110         3.90         2.875         17.02         0         1         4         4           Datsun 710         22.8         4         108.0         93         3.85         2.320         18.61         1         1         1         4         1           Hornet Porive         21.4         6         258.0         110         3.08         3.215         19.44         1         0         3         2           Valiant         18.1         6         225.0         105         2.76         3.460         20.22         1         0         3         1           Waliant         18.1         6         225.0         105         2.76         3.460         20.22         1         0         3         1           Waliant         18.2         4         140.8			1	1.	- 1	1 4						
Mazda RX4 Wag         21.0         6         160.0         110         3.90         2.875         17.02         0         1         4         4           Datsun 710         22.8         4         108.0         93         3.85         2.320         18.61         1         1         4         1           Hornet 4 Drive         21.4         6         258.0         110         3.08         3.215         19.44         1         0         3         1           Hornet Sportabout         18.7         8         360.0         175         3.15         3.440         17.02         0         0         3         2           Valiant         18.1         6         225.0         105         2.76         3.460         20.22         1         0         3         1           Duster 360         14.3         8         360.0         245         3.21         3.570         15.84         0         0         3         4           Merc 240D         24.4         4         146.7         62         3.69         3.190         20.00         1         0         4         2           Merc 280         19.2         6         167.6	M 1 DX/4											
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Hornet Sportabout 18.7 8 360.0 175 3.15 3.440 17.02 0 0 3 2 2 Valiant 18.1 6 225.0 105 2.76 3.460 20.22 1 0 3 1 Duster 360 14.3 8 360.0 245 3.21 3.570 15.84 0 0 3 4 Merc 240D 24.4 4 146.7 62 3.69 3.190 20.00 1 0 4 2 Merc 230 22.8 4 140.8 95 3.92 3.150 22.90 1 0 4 2 Merc 280 19.2 6 167.6 123 3.92 3.440 18.30 1 0 4 4 Merc 280C 17.8 6 167.6 123 3.92 3.440 18.30 1 0 4 4 Merc 450SE 16.4 8 275.8 180 3.07 4.070 17.40 0 0 3 3 Merc 450SL 17.3 8 275.8 180 3.07 3.730 17.60 0 0 3 3 3 Merc 450SLC 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 3 Merc 450SLC 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 3 Merc 450SLC 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 3 Merc 450SLC 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 4 Merc 450SL 17.3 8 440.0 215 3.00 5.424 17.82 0 0 3 4 Merc 450SL 17.3 8 472.0 205 2.93 5.250 17.98 0 0 3 4 Merc 450SL 10.4 8 460.0 215 3.00 5.424 17.82 0 0 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 4 Merc 450SL 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 4 Merc 450SL 15.2 8 30.4 15.2 15.2 15.2 15.2 15.2 15.2 15.2 15.2									_	_	-	
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Merc 280         19.2         6         167.6         123         3.92         3.440         18.30         1         0         4         4           Merc 280C         17.8         6         167.6         123         3.92         3.440         18.90         1         0         4         4           Merc 450SE         16.4         8         275.8         180         3.07         4.070         17.40         0         0         3         3           Merc 450SL         17.3         8         275.8         180         3.07         3.730         17.60         0         0         3         3           Cadillac Fleetwood         10.4         8         472.0         205         2.93         5.250         17.98         0         0         3         4           Lincoln Continental         10.4         8         460.0         215         3.00         5.424         17.82         0         0         3         4           Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4         78.			-						_		-	
Merc 280C         17.8         6         167.6         123         3.92         3.440         18.90         1         0         4         4           Merc 450SE         16.4         8         275.8         180         3.07         4.070         17.40         0         0         3         3           Merc 450SL         17.3         8         275.8         180         3.07         3.730         17.60         0         0         3         3           Merc 450SLC         15.2         8         275.8         180         3.07         3.780         18.00         0         0         3         3           Cadillac Fleetwood         10.4         8         472.0         205         2.93         5.250         17.98         0         0         3         4           Lincoln Continental         10.4         8         460.0         215         3.00         5.424         17.82         0         0         3         4           Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4									_		-	
Merc 450SE         16.4         8         275.8         180         3.07         4.070         17.40         0         0         3         3           Merc 450SL         17.3         8         275.8         180         3.07         3.730         17.60         0         0         3         3           Merc 450SLC         15.2         8         275.8         180         3.07         3.780         18.00         0         0         3         3           Cadillac Fleetwood         10.4         8         472.0         205         2.93         5.250         17.98         0         0         3         4           Lincoln Continental         10.4         8         460.0         215         3.00         5.424         17.82         0         0         3         4           Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4         78.7         66         4.08         2.200         19.47         1         1         4         1           Honda Civic         30.4         4									_		-	-
Merc 450SL         17.3         8         275.8         180         3.07         3.730         17.60         0         0         3         3           Merc 450SLC         15.2         8         275.8         180         3.07         3.780         18.00         0         0         3         3           Cadillac Fleetwood         10.4         8         472.0         205         2.93         5.250         17.98         0         0         3         4           Lincoln Continental         10.4         8         460.0         215         3.00         5.424         17.82         0         0         3         4           Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4         78.7         66         4.08         2.200         19.47         1         1         4         1           Honda Civic         30.4         4         75.7         52         4.93         1.615         18.52         1         1         4         2           Toyota Corolla         33.9         4 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>_</td><td></td><td></td><td></td></t<>									_			
Merc 450SLC         15.2         8         275.8         180         3.07         3.780         18.00         0         0         3         3           Cadillac Fleetwood         10.4         8         472.0         205         2.93         5.250         17.98         0         0         3         4           Lincoln Continental         10.4         8         460.0         215         3.00         5.424         17.82         0         0         3         4           Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4         78.7         66         4.08         2.200         19.47         1         1         4         1           Honda Civic         30.4         4         75.7         52         4.93         1.615         18.52         1         1         4         2           Toyota Corolla         33.9         4         71.1         65         4.22         1.835         19.90         1         1         4         1           Toyota Corona         21.5         4         <									-			
Cadillac Fleetwood         10.4         8         472.0         205         2.93         5.250         17.98         0         0         3         4           Lincoln Continental         10.4         8         460.0         215         3.00         5.424         17.82         0         0         3         4           Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4         78.7         66         4.08         2.200         19.47         1         1         4         1           Honda Civic         30.4         4         75.7         52         4.93         1.615         18.52         1         1         4         2           Toyota Corolla         33.9         4         71.1         65         4.22         1.835         19.90         1         1         4         1           Toyota Corona         21.5         4         120.1         97         3.70         2.465         20.01         1         0         3         1           Dodge Challenger         15.5         8									-			
Lincoln Continental         10.4         8         460.0         215         3.00         5.424         17.82         0         0         3         4           Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4         78.7         66         4.08         2.200         19.47         1         1         4         1           Honda Civic         30.4         4         75.7         52         4.93         1.615         18.52         1         1         4         2           Toyota Corolla         33.9         4         71.1         65         4.22         1.835         19.90         1         1         4         1           Toyota Corona         21.5         4         120.1         97         3.70         2.465         20.01         1         0         3         1           Dodge Challenger         15.5         8         318.0         150         2.76         3.520         16.87         0         0         3         2           Camaro Z28         13.3         8         3			8	275.8	180	3.07	3.780	18.00	0	0		3
Chrysler Imperial         14.7         8         440.0         230         3.23         5.345         17.42         0         0         3         4           Fiat 128         32.4         4         78.7         66         4.08         2.200         19.47         1         1         4         1           Honda Civic         30.4         4         75.7         52         4.93         1.615         18.52         1         1         4         2           Toyota Corolla         33.9         4         71.1         65         4.22         1.835         19.90         1         1         4         1           Toyota Corona         21.5         4         120.1         97         3.70         2.465         20.01         1         0         3         1           Dodge Challenger         15.5         8         318.0         150         2.76         3.520         16.87         0         0         3         2           AMC Javelin         15.2         8         304.0         150         3.15         3.435         17.30         0         0         3         2           Camaro Z28         13.3         8         350.0 <td>Cadillac Fleetwood</td> <td></td> <td>8</td> <td>472.0</td> <td>205</td> <td>2.93</td> <td>5.250</td> <td>17.98</td> <td>0</td> <td>0</td> <td></td> <td>4</td>	Cadillac Fleetwood		8	472.0	205	2.93	5.250	17.98	0	0		4
Fiat 128         32.4         4         78.7         66         4.08         2.200         19.47         1         1         4         1           Honda Civic         30.4         4         75.7         52         4.93         1.615         18.52         1         1         4         2           Toyota Corolla         33.9         4         71.1         65         4.22         1.835         19.90         1         1         4         1           Toyota Corona         21.5         4         120.1         97         3.70         2.465         20.01         1         0         3         1           Dodge Challenger         15.5         8         318.0         150         2.76         3.520         16.87         0         0         3         2           AMC Javelin         15.2         8         304.0         150         3.15         3.435         17.30         0         0         3         2           Camaro Z28         13.3         8         350.0         245         3.73         3.840         15.41         0         0         3         4           Pontiac Firebird         19.2         8         400.0	Lincoln Continental	10.4	8	460.0	215	3.00	5.424	17.82	0	0	3	4
Honda Civic         30.4         4         75.7         52         4.93         1.615         18.52         1         1         4         2           Toyota Corolla         33.9         4         71.1         65         4.22         1.835         19.90         1         1         4         1           Toyota Corona         21.5         4         120.1         97         3.70         2.465         20.01         1         0         3         1           Dodge Challenger         15.5         8         318.0         150         2.76         3.520         16.87         0         0         3         2           AMC Javelin         15.2         8         304.0         150         3.15         3.435         17.30         0         0         3         2           Camaro Z28         13.3         8         350.0         245         3.73         3.840         15.41         0         0         3         4           Pontiac Firebird         19.2         8         400.0         175         3.08         3.845         17.05         0         0         3         2           Fiat X1-9         27.3         4         79.0 <td>Chrysler Imperial</td> <td>14.7</td> <td>8</td> <td>440.0</td> <td>230</td> <td>3.23</td> <td>5.345</td> <td>17.42</td> <td>0</td> <td>0</td> <td>3</td> <td>4</td>	Chrysler Imperial	14.7	8	440.0	230	3.23	5.345	17.42	0	0	3	4
Toyota Corolla         33.9         4         71.1         65         4.22         1.835         19.90         1         1         4         1           Toyota Corona         21.5         4         120.1         97         3.70         2.465         20.01         1         0         3         1           Dodge Challenger         15.5         8         318.0         150         2.76         3.520         16.87         0         0         3         2           AMC Javelin         15.2         8         304.0         150         3.15         3.435         17.30         0         0         3         2           Camaro Z28         13.3         8         350.0         245         3.73         3.840         15.41         0         0         3         4           Pontiac Firebird         19.2         8         400.0         175         3.08         3.845         17.05         0         0         3         2           Fiat X1-9         27.3         4         79.0         66         4.08         1.935         18.90         1         1         4         1           Porsche 914-2         26.0         4         120.3	Fiat 128	32.4	4	78.7	66	4.08	2.200	19.47	1	1	4	1
Toyota Corona         21.5         4         120.1         97         3.70         2.465         20.01         1         0         3         1           Dodge Challenger         15.5         8         318.0         150         2.76         3.520         16.87         0         0         3         2           AMC Javelin         15.2         8         304.0         150         3.15         3.435         17.30         0         0         3         2           Camaro Z28         13.3         8         350.0         245         3.73         3.840         15.41         0         0         3         4           Pontiac Firebird         19.2         8         400.0         175         3.08         3.845         17.05         0         0         3         2           Fiat X1-9         27.3         4         79.0         66         4.08         1.935         18.90         1         1         4         1           Porsche 914-2         26.0         4         120.3         91         4.43         2.140         16.70         0         1         5         2           Lotus Europa         30.4         4         95.1 </td <td>Honda Civic</td> <td>30.4</td> <td>4</td> <td>75.7</td> <td>52</td> <td>4.93</td> <td>1.615</td> <td>18.52</td> <td>1</td> <td>1</td> <td>4</td> <td>2</td>	Honda Civic	30.4	4	75.7	52	4.93	1.615	18.52	1	1	4	2
Dodge Challenger         15.5         8         318.0         150         2.76         3.520         16.87         0         0         3         2           AMC Javelin         15.2         8         304.0         150         3.15         3.435         17.30         0         0         3         2           Camaro Z28         13.3         8         350.0         245         3.73         3.840         15.41         0         0         3         4           Pontiac Firebird         19.2         8         400.0         175         3.08         3.845         17.05         0         0         3         2           Fiat X1-9         27.3         4         79.0         66         4.08         1.935         18.90         1         1         4         1           Porsche 914-2         26.0         4         120.3         91         4.43         2.140         16.70         0         1         5         2           Lotus Europa         30.4         4         95.1         113         3.77         1.513         16.90         1         1         5         2	Toyota Corolla	33.9	4	71.1	65	4.22	1.835	19.90	1	1	4	1
AMC Javelin       15.2       8       304.0       150       3.15       3.435       17.30       0       0       3       2         Camaro Z28       13.3       8       350.0       245       3.73       3.840       15.41       0       0       3       4         Pontiac Firebird       19.2       8       400.0       175       3.08       3.845       17.05       0       0       3       2         Fiat X1-9       27.3       4       79.0       66       4.08       1.935       18.90       1       1       4       1         Porsche 914-2       26.0       4       120.3       91       4.43       2.140       16.70       0       1       5       2         Lotus Europa       30.4       4       95.1       113       3.77       1.513       16.90       1       1       5       2	Toyota Corona	21.5	4	120.1	97	3.70	2.465	20.01	1	0	3	1
Camaro Z28       13.3       8       350.0       245       3.73       3.840       15.41       0       0       3       4         Pontiac Firebird       19.2       8       400.0       175       3.08       3.845       17.05       0       0       3       2         Fiat X1-9       27.3       4       79.0       66       4.08       1.935       18.90       1       1       4       1         Porsche 914-2       26.0       4       120.3       91       4.43       2.140       16.70       0       1       5       2         Lotus Europa       30.4       4       95.1       113       3.77       1.513       16.90       1       1       5       2	Dodge Challenger	15.5	8	318.0	150	2.76	3.520	16.87	0	0	3	2
Pontiac Firebird         19.2         8         400.0         175         3.08         3.845         17.05         0         0         3         2           Fiat X1-9         27.3         4         79.0         66         4.08         1.935         18.90         1         1         4         1           Porsche 914-2         26.0         4         120.3         91         4.43         2.140         16.70         0         1         5         2           Lotus Europa         30.4         4         95.1         113         3.77         1.513         16.90         1         1         5         2	AMC Javelin	15.2	8	304.0	150	3.15	3.435	17.30	0	0	3	2
Fiat X1-9       27.3       4       79.0       66       4.08       1.935       18.90       1       1       4       1         Porsche 914-2       26.0       4       120.3       91       4.43       2.140       16.70       0       1       5       2         Lotus Europa       30.4       4       95.1       113       3.77       1.513       16.90       1       1       5       2	Camaro Z28	13.3	8	350.0	245	3.73	3.840	15.41	0	0	3	4
Porsche 914-2 26.0 4 120.3 91 4.43 2.140 16.70 0 1 5 2 Lotus Europa 30.4 4 95.1 113 3.77 1.513 16.90 1 1 5 2	Pontiac Firebird	19.2	8	400.0	175	3.08	3.845	17.05	0	0	3	2
Lotus Europa 30.4 4 95.1 113 3.77 1.513 16.90 1 1 5 2	Fiat X1-9	27.3	4	79.0	66	4.08	1.935	18.90	1	1	4	1
*	Porsche 914-2	26.0	4	120.3	91	4.43	2.140	16.70	0	1	5	2
	Lotus Europa	30.4	4	95.1	113	3.77	1.513	16.90	1	1	5	2
	Ford Pantera L	15.8	8	351.0	264	4.22	3.170	14.50	0	1	5	4
Ferrari Dino 19.7 6 145.0 175 3.62 2.770 15.50 0 1 5 6	Ferrari Dino	19.7	6	145.0	175	3.62	2.770	15.50	0	1	5	6
Maserati Bora 15.0 8 301.0 335 3.54 3.570 14.60 0 1 5 8	Maserati Bora	15.0	8	301.0	335	3.54	3.570	14.60	0	1	5	8
Volvo 142E 21.4 4 121.0 109 4.11 2.780 18.60 1 1 4 2	Volvo 142E	21.4	4		109	4.11	2.780	18.60	1	1	4	2

TABLE 2 Sample posterior estimates for each model

				Quantile				
Model	Parameter	Mean	Std. dev.	2.5%	50%	97.5%		
Model 0	$\beta_0$	-12.29	2.29	-18.04	-11.99	-8.56		
	$eta_1$	0.10	0.07	-0.05	0.10	0.26		
	$eta_2$	0.01	0.09	-0.22	0.02	0.16		
Model 1	$\beta_0$	-4.58	3.04	-11.00	-4.44	1.06		
	$\beta_1$	0.79	0.21	0.38	0.78	1.20		
	$eta_2$	-0.28	0.10	-0.48	-0.28	-0.07		
Model 2	$\beta_0$	-11.85	2.24	-17.34	-11.60	-7.85		
	$eta_1$	0.73	0.21	0.32	0.73	1.16		
	$eta_2$	-0.60	0.14	-0.88	-0.60	-0.34		
	$\beta_3$	0.22	0.17	-0.10	0.22	0.55		

# **11. Equations and the like.** Two equations:

(11.1) 
$$C_s = K_M \frac{\mu/\mu_x}{1 - \mu/\mu_x}$$

and

(11.2) 
$$G = \frac{P_{\text{opt}} - P_{\text{ref}}}{P_{\text{ref}}} 100(\%).$$

Equation arrays:

(11.3) 
$$\frac{dS}{dt} = -\sigma X + s_F F,$$

$$\frac{dX}{dt} = \mu X,$$

$$\frac{dP}{dt} = \pi X - k_h P,$$

$$\frac{dV}{dt} = F.$$

One long equation:

(11.7) 
$$\mu_{\text{normal}} = \mu_x \frac{C_s}{K_x C_x + C_s}$$
$$= \mu_{\text{normal}} - Y_{x/s} (1 - H(C_s)) (m_s + \pi/Y_{p/s})$$
$$= \mu_{\text{normal}} / Y_{x/s} + H(C_s) (m_s + \pi/Y_{p/s}).$$

#### APPENDIX: TITLE

Appendices should be provided in {appendix} environment, before Acknowledgements.

If there is only one appendix, then please refer to it in text as ... in the Appendix.

#### APPENDIX A: TITLE OF THE FIRST APPENDIX

If there are more than one appendix, then please refer to it as ... in Appendix A, Appendix B, etc.

#### APPENDIX B: TITLE OF THE SECOND APPENDIX

**B.1. First subsection of Appendix B.** Use the standard LATEX commands for headings in {appendix}. Headings and other objects will be numbered automatically.

(B.1) 
$$\mathcal{P} = (j_{k,1}, j_{k,2}, \dots, j_{k,m(k)}).$$

Sample of cross-reference to the formula (B.1) in Appendix B.

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#### SUPPLEMENTARY MATERIAL

#### **Title of Supplement A**

Short description of Supplement A.

## Title of Supplement B

Short description of Supplement B.

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