Eurodollar Futures

09 June 2021

Specification

TITLE:	Eurodollar Future
LIBRARY:	Risk Engine Migration
OWNER:	Traded Market Risk
SUMMARY:	Market risk currently uses IBM's Riskwatch[1] to manage the traded market risks. They are considering moving to FIS's Front Arena Prime[2] for managing the risk of all traded positions. This potential change in software models prompted an understanding of the valuation differences of the Eurodollar futures.
STEWARDS:	Quantitative Analysts
START DATE:	March 2021
END DATE:	June 2021

Revision History

REVISION	DATE	AUTHOR	COMMENT
1.0	23 Mar 2021	Angelo Joseph	Initial Version
1.1	09 June 2021	Angelo Joseph	Includes direct comparisons of the forward rate and error attribution

[1] Herein referred to as the market risk engine
[2] Herein referred to as the source system

Contents

1.Introduction

Eurodollar futures provide an effective means for banks to participate in the interest rate derivative market. Market risk currently uses IBM's Riskwatch to manage the traded market risks. They are considering moving to FIS's Front Arena Prime for managing the risk of all traded positions. This potential change in software models prompted an understanding of the valuation differences of the Eurodollar futures.

1.1 Scope

A Eurodollar future is a cash-settled futures contract where the price dynamics is exposed to only the Libor interest rate risk factor. The price of a Eurodollar future is determined as (CME, 2021):

$$100 - R$$

Here R is the absolute three-month London interbank offered rate (Libor). For example, a Eurodollar future price of 99.7475 assumes a Libor rate of 0.2525%

For all practical purposes, we consider six Eurodollar futures expiring in 2 years' time[3] in the portfolio FWT_ED Futures. See Annexure A.

2. Interpolation

3.1 Pillar preservation

The market risk historical data base[4] stores all the market data, extracted daily from the source system, used in the VaR migration. The rate data is stored in the data base up to 6 decimal places. It therefore makes sense to confide (as far as possible) all the comparisons to no more than 6 decimal places.

Given that the extracted interest rate data are interpolated for a predefined market risk tenor, we perform a test to confirm that the source pillar rates are used in the interpolation.

We compared a linearly interpolated curve point to that in risk engine. In short, we choose an interpolant of 363 days such that the one year 365 tenor rates are the point to the right of the interpolant[5].

3.1.1 Finding

According to Exhibit 1, we found that for the 363-day interpolant, the risk engine interpolated rate is identical to that found using the 180 days. Since the 180-day rate corresponds to a source

[3] Long term interest rate convexity effects are therefore negligible for the Eurodollar futures in scope.

[4] At present the Algo Scenario Engine (ASE) is used as storage for historical data in the VaR scenario engine.

[5] Suppose we have an interpolant of d days. The interpolated rate is therefore $y_{right} + \frac{(y_{left} - y_{right})}{(d_{left} - d_{right})}(d - d_{right})$ where y_{right}, y_{left} are the rates and d_{right} and d_{left} are the days right and left of the interpolant, respectively.

pillar rate, we can safely conclude that the risk engine, therefore, does use the pillar tenors and rates. In other words, it appears like the source pillar tenors are preserved in the interpolation.

Left Tenors (days)	Interpolated (USD-swap rate)	Bps Change*
180	0.00170195	0.000
181	0.00170192	-0.001
182	0.00170191	-0.001
212	0.00170168	-0.003
242	0.00170135	-0.006
273	0.00170080	-0.012
303	0.00170081	-0.012
334	0.00170081	-0.012

*Bps Change is the (Interpolated rate – risk engine interpolated rate) x 10000

Exhibit 1: Risk engine interpolation using different tenors

3.2 Type

It is well established that the source system uses Hermite interpolation while the risk engine uses linear interpolation. To appraise the difference we perform a non-extensive test using a 5-point (6month to 10-year tenor) sample of the USD swap curve. Next, interpolate for the 1yr, 3yr and 7yr points using the linear and Hermite interpolation, respectively. See Exhibit 2.

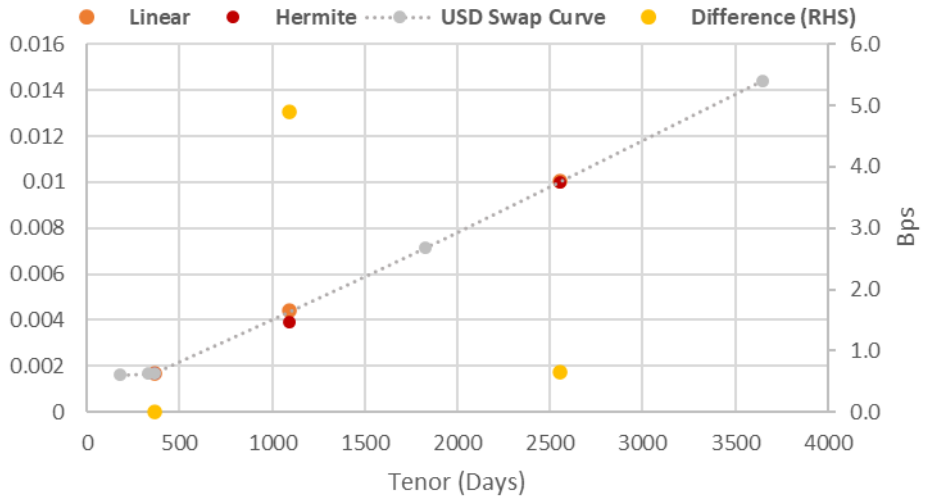

Exhibit 2: Risk engine and source system interpolation

3.2.1 Finding

For relatively linear rates in dates, the difference between the linear and Hermite interpolation is low (not more than 1bps). See the right-hand side axis in Exhibit 2. However, when there is a

potential curve in the rate tenor profile, the difference appears slightly higher (5bps). See the front end of the plot in Exhibit 2. The Hermite interpolator is designed to capture curvature or oscillatory behaviour in curves. Consequently, a difference between the Hermite and the linear interpolator especially on yield curves can be expected.

4. Forward Rates

4.1 Spots

In the source system the Libor rates are mapped to the USD-Swap curve. As a first step, the source USD-Swap curve are compared to that extracted and used in the risk engine. See Exhibit 3.

Exhibit 3: USD-Swap curve comparison between the risk engine (RW) and source system (FA).

4.1.1 Findings

In the near term, the points on the curve are very similar. See Exhibit 3. Given that the risk engine has a predefined set of tenor points (e.g. a tenor of 2555 days) that are not always in the source tenor grid (e.g. the nearest source tenor is at 2561) introduces interpolation[6] errors. This interpolation error is further enhanced by the interpolation type differences (described in section 3.2). Furthermore, the predefined risk engine tenor points beyond that of the source introduce extrapolation biases (e.g. the risk engine 35-year rate is obtained by extrapolation because the source only goes to 30-years). See the 12775-day rate in Exhibit 3.

[6] The interpolation differences are amplified, primarily when the source uses Hermite interpolation, but the risk engine uses linear interpolation.

4.2 Forwards

To confirm the system forward rate differences, we replicate the risk engine and the forward source rates, respectively. The nearest contract in the Eurodollar future population (randomly chosen) is used to illustrate the forward rate calculations. The nearest Eurodollar future contract details are given below.

Euro Dollar Future	
Value Day	22-Feb-21
Instrument Expiry	13-Dec-21
Underlying Maturity	14-Mar-22
Nominal Amount (USD)	1,000,000
Strike	99.7875

Exhibit 4: Contract specification of the near dated Eurodollar future.

In all the analysis to follow this Eurodollar future is used.

4.2.1 Risk engine

The risk engine linearly interpolates the spot curve for the instrument expiry rate as well as the underlying maturity rate. Using the two rates, the forward rate is determined using the no-arbitrage quantitative formulae. The interpolated rates and the calculated forward rate (see Annexure B) are given in Exhibit 5.

	Calendar days to Maturity	Interpolated NACA Rates		
13-Dec-21	294	0.001619401		
14-Mar-22	385	0.001716322		
		NACA Fwd Rate:	0.0020295163	
			Simple Fwd Rate:	0.0020279793
			Risk Watch:	0.0020279793

Exhibit 5: Replication of the risk engine Forward Rate

4.2.1.1 Findings

There are no findings. The calculated forward rate and the risk engine forward rate are the same. See Exhibit 5.

4.2.2 Source system

The source system does not interpolate the spot curve for the forward rates. The source system interpolates the forward curve for the forward rates. Based on the date when the underlying matures, the source system gets the spot rates for the two nearest dates, ensuring the ultimate period is one year apart. For example, for the near dated Eurodollar future contract with Mar 2022 maturity, it will use the Feb 2022 and the Aug 2022 spot rates. See Exhibit 6. Then together with the rates 3months prior and after these dates (Nov 2021 and Nov 2022), it will, in total, use four points for the forward rate calculation. Next, the source system will determine the two forwards implied by these four points.

Finally, the desired forward rate will be obtained by linear interpolating the predetermined forwards. For another theoretical example of how the source determines the forward rate, kindly see Annexure C.

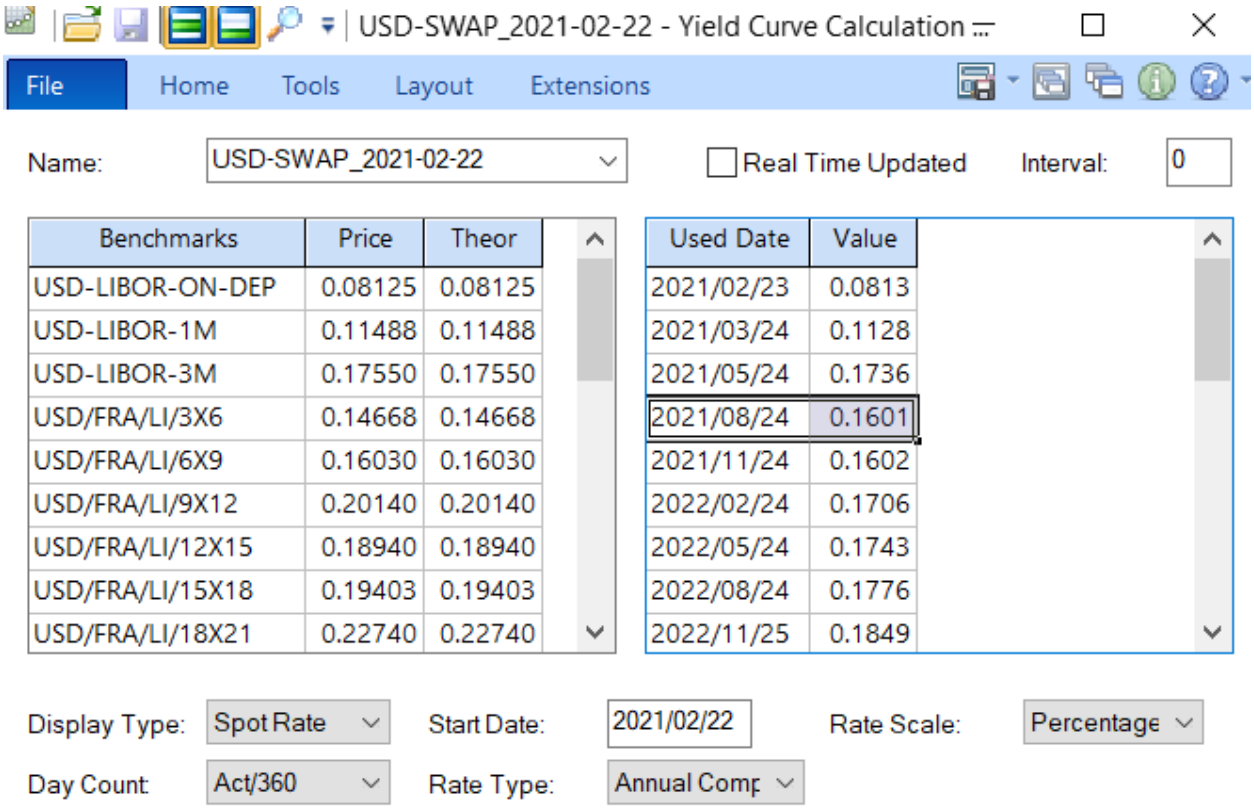

Exhibit 6: Source system spot interest rate curve

4.2.2.1 Findings

In reproducing the source system forward rate, we find a difference between the replication and the source of 0.01 basis point. We conclude that the main difference between the forward replication and that determined by the system is the rounding. For example, the values used in the replication had only four decimals (see the value decimals in Exhibit 6), even though the source system probably use much more decimals – leading to more precise forward rates. In fact, the spot inputs' decimal differences cause the replicated forward to differ from the source forward after the 4th decimal. See Exhibit 7.

	Calendar days to Maturity	NACA Rates		
22-Nov-21	273	0.001601978		
22-Feb-22	367	0.001706000		
		Fwd Rate (Left):	0.002008167	
			Simple Fwd Rate (Left):	0.002006679
22-Aug-22	546	0.001775283		

23-Nov-22	641	0.001849000		
		Fwd Rate (Right)	0.002272786	
			Simple Fwd Rate (Right):	0.002270887
			Interpolated Fwd Rate:	0.002027003
			Front Arena:	0.002037638

Exhibit 7: Replication of the source system Forward Rate

4.2.2.2 Lag

In the replication of the forward rate, it was noticed that the replicated forward is much closer to the source when a lag of 2 days is introduced in the forward starting date. The replicated forward rate is 0.08bps closer to the source value when a delay of 2 days is used in the calculation of the two forward starting rates. See Exhibit 8.

Source Replication Test	Forward Rate	Difference (Bps)
With 2-day Libor Lag	0.002027003	0.084196074
Without 2-day Libor Lag	0.002035301	

Exhibit 8: Impact of the the 2-day Libor lag on the replicated source forward calculation

4.3 System comparisons

To demonstrate the understanding of the differences, the various system forward rates are tabulated together with the source replication. See Exhibit 9.

	Forward Rates	**Differences**	**Bps Change**
Risk Engine	0.0020279793		
Source	0.0020376381	Risk Engine – Source	**-0.0965885846**
Source (Replicate)	0.0020270025	Source Replicate - Source	0.0110650866

Exhibit 9: Summary of the differences in forward rates between the source, risk engine and replication.

4.3.1 Findings

The risk engine and source Eurodollar forward difference are approximately 0.1bps. This difference can be ascribed mainly to the 2-day lag that is not incorporated in the market risk engine. The second most significant source of the error is that the risk engine interpolates on the spot rates, not on the forward (consistent with the source). The other error contributors in line with the findings are given in Exhibits 10 and 11.

Type	**Risk Watch**	**Front Arena**	**Differences (bps)**
2-day Libor Lag	Exclude	Include	0.084
Spot rate Interpolation*	Linear	Hermite	0.014
Rounding**	Include	Exclude	0.010
Forward Rate Method	Interpolate on Spot	Interpolate on Forward	0.060
Total* **			0.105

*Inferred from Exhibit 2.
**Limited to four decimals.
***Square root of the sum squared differences

Exhibit 10: Summary of the potential sources of differences between the risk engine and source forward rate determination.

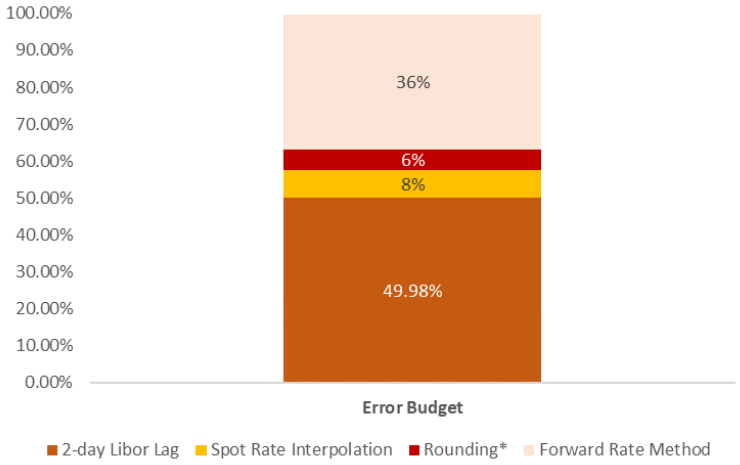

Exhibit 11: Stack chart showing the relative contribution of the differences.

5. Conclusion

We have randomly picked a Eurodollar future and found the forward rate difference between the systems in the order of 0.1bps. An attribution analysis showed that the main driver of the difference is the 2-day lag not incorporated in the risk engine, followed by the fact that the source (risk engine) interpolates the forward (spot) curve.

Given the source population of Eurodollar future, it will be interesting to check whether the low difference is not merely because the sample analysed was the near dated future. In this light, we plotted the differences for longer-dated (e.g. Jun 2023) expiry futures. Overall the Eurodollar future population, we find that the max forward differences are 0.3 bps. See Exhibit 12. It does look like the system forward rate differences increases with instrument expiry.

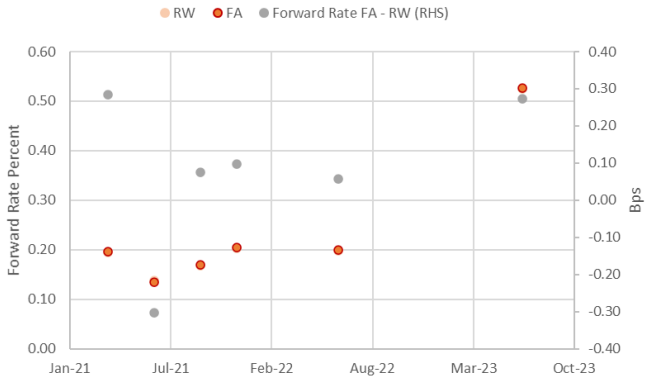

Exhibit 12: Forward rates and the system differences over the Eurodollar future source population.

For completeness, we also plotted the dollar impact over the entire EuroDollar future source population. See Exhibit 13. It appears like the maximum difference is about 800 USD per Eurodollar future contract. Since each future contract has a standard notional of 1M USD, this monetary amount due to the system forward difference is insignificant relative to the 3% tolerance.

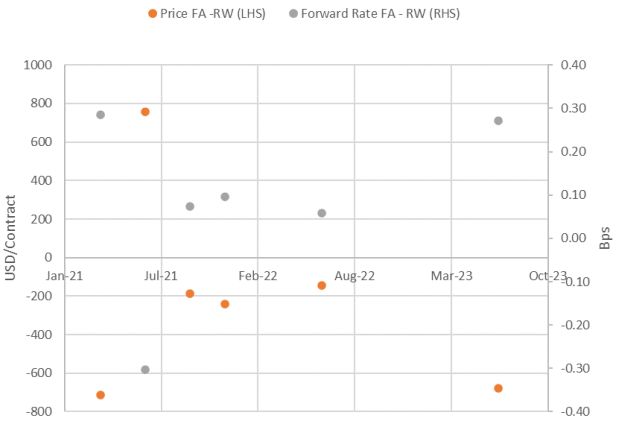

Exhibit 13: Forward rates and the system price differences over the Eurodollar future source population.

6. Annexures

Annexure A: Risk engine extract of the Eurodollar futures

```
BAS,EuroDollar Future,EuroDollar FutureSPEC,EuroDollar Future,USD/IMM/JUN21,insaddr_5764586,,,,USD,1,,,,,USD,1000000.0,,2021/06/14,2021/09/13,,,,,,,,,,,,,,,,,,,,USD,99.8325,,USD-SWAP_2021-03-18,EuroDollar Future,,,
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BAS,EuroDollar Future,EuroDollar FutureSPEC,EuroDollar Future,USD/IMM/SEP21,insaddr_8767224,,,,USD,1,,,,,USD,1000000.0,,2021/09/13,2021/12/13,,,,,,,,,,,,,,,,,,,,USD,99.8125,,USD-SWAP_2021-03-18,EuroDollar Future,,,
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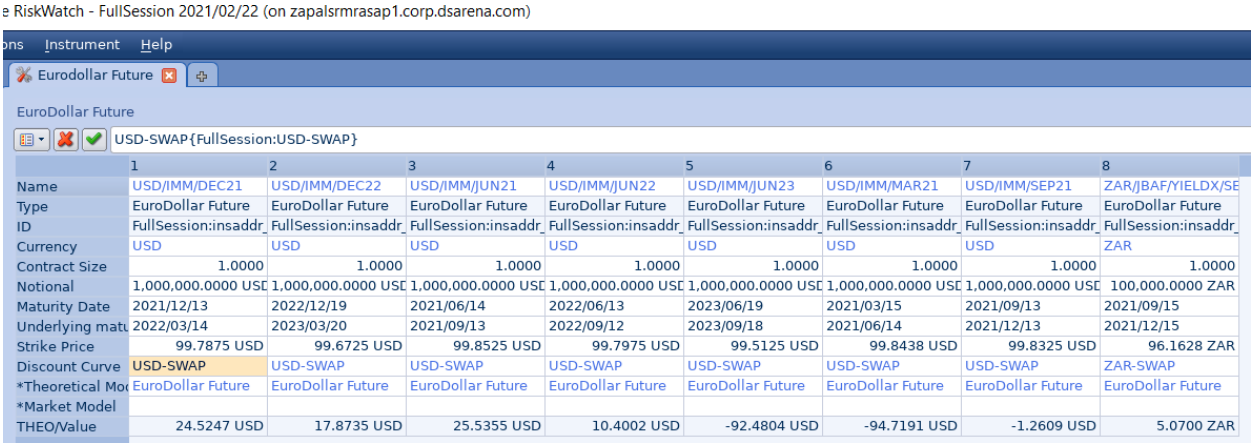
Figure A1: Eurodollar future instruments raw extract from the Front Arena source system.



e RiskWatch - FullSession 2021/02/22 (on zapalsrmrasap1.corp.dsarena.com)

ons Instrument Help

Eurodollar Future

EuroDollar Future

USD-SWAP{FullSession:USD-SWAP}

	1	2	3	4	5	6	7	8
Name	USD/IMM/DEC21	USD/IMM/DEC22	USD/IMM/JUN21	USD/IMM/JUN22	USD/IMM/JUN23	USD/IMM/MAR21	USD/IMM/SEP21	ZAR/JBAF/YIELDX/SE
Type	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future
ID	FullSession:insaddr_	FullSession:insaddr_	FullSession:insaddr_	FullSession:insaddr_	FullSession:insaddr_	FullSession:insaddr_	FullSession:insaddr_	FullSession:insaddr_
Currency	USD	USD	USD	USD	USD	USD	USD	ZAR
Contract Size	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Notional	1,000,000.0000 USD	1,000,000.0000 USD	1,000,000.0000 USD	1,000,000.0000 USD	1,000,000.0000 USD	1,000,000.0000 USD	1,000,000.0000 USD	100,000.0000 ZAR
Maturity Date	2021/12/13	2022/12/19	2021/06/14	2022/06/13	2023/06/19	2021/03/15	2021/09/13	2021/09/15
Underlying matu	2022/03/14	2023/03/20	2021/09/13	2022/09/12	2023/09/18	2021/06/14	2021/12/13	2021/12/15
Strike Price	99.7875 USD	99.6725 USD	99.8525 USD	99.7975 USD	99.5125 USD	99.8438 USD	99.8325 USD	96.1628 ZAR
Discount Curve	USD-SWAP	USD-SWAP	USD-SWAP	USD-SWAP	USD-SWAP	USD-SWAP	USD-SWAP	ZAR-SWAP
*Theoretical Mo	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future	EuroDollar Future
*Market Model								
THEO/Value	24.5247 USD	17.8735 USD	25.5355 USD	10.4002 USD	-92.4804 USD	-94.7191 USD	-1.2609 USD	5.0700 ZAR

Figure A2: Eurodollar future population details

Annexure B: Theoretical snips on the Risk engine valuation

Symbol	RiskWatch Attribute
t_{sim}	<current simulation time>
Υ	Currency
$d_Y(t, t, T_u)$	<calculated internally>
T_u	<internal calculated> [1]
Π	Term of Underlying Curve Index
NB	Net Basis
K	Strike Price
$r_{IX}{}_m{}_{DC}$	<underlying> forward rate[2]
$r_f{}_m{}_{DC}$	<underlying> forward rate[3]
S	Spot Price

Euro Dollar Future assumes that the underlying is a CD, and the value of the underlying is defined as follows:

$$V_u = N_u \cdot \left[1 + \left(1 - \frac{K}{100}\right) \cdot TF(T, T_u, DC_Y)\right] \cdot d_Y(t, T, T_u)$$

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@euro dollar future price ♦ $_{THEO}$

$$P = 100 \cdot [1 - r_f(t, T, T_u)] + NB$$

where r_f is the forward rate calculated by

$$r_f(t, T, T_u) = \frac{1}{TF(T, T_u, DC_I)} \cdot \left(\frac{1}{d_u(t, T, T_u)} - 1\right)$$

and DC_I is the instrument's attribute Daycount Basis.

@spot price ♦ $_{MKT}$

- **Value, V**

@cash flow generator

When pricing in the Instrument Module:

$$V = \left(\frac{P - K}{100}\right) \cdot N_u \cdot TF(T, T_u, DC_I)$$

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Annexure C: Theoretical snips on the source system valuation

Applications>Data>Yield Curve Definition>Fields>Yield Curve Definition Benchmark curve fields

PRIME Yield Curve Definition Benchmark curve fields

Use the following guidelines to complete the fields for Benchmark curves in the Yield Curve Definition window.

Curve setup (View Setup Ctrl + 1)

Top section of the Yield Curve Definition window.

Field	Description
Name	Specify the unique name of the curve.
Curve Type	From the drop-down list, select the type of curve. This setting cannot be updated in the GUI once the curve has been saved. (If the setting is updated in the database, the PRIME client must be restarted.)
Interpolation	From the drop-down list, select the interpolation method to use when calculating/interpolating the curve. Interpolation between curve points is performed in the curve's calculation format. Select **User Defined** to use a user-defined interpolation method (see **User Defined** below).
User Defined	This field is available if **User Defined** is selected as the interpolation method (in the **Interpolation** field). From the drop-down list, select the user-defined interpolation method. The list of user-defined interpolation methods is specified in the **Interpolation** choice list in Administration Console. For full details, see the User Defined section in Interpolation methods.
Interpol Rate Type	Specify the rate type in which interpolation will be done, where possible: **Spot Rate**: This is the default setting. If **Spot Rate** is selected, all interpolation will be done in spot rate terms. More specifically the interpolation will always be made in the calculation format of the curve. An exception to the last two points is the method **Piecewise Quadratic (Cont Fwd)**. **Forward Rate**: If **Forward Rate** is selected, a forward period needs to be specified in the **Forward Period** field. Having made these specifications, PRIME interpolates in forward rate terms if the curve is asked for a forward rate. For example: Suppose there are two benchmark points in the curve, b1 and b2. Suppose further they have benchmark dates of 1 year and 2 years respectively (call them d1 and d2). Suppose that **Interpol Rate Type** is set to **Forward Rate** and the **Forward Period** is 6m. Consider the calculation of the 6 month forward rate with a reset start date in 15 months and reset end date in 21 months. Call this forward rate f*. With these settings, PRIME calculates the forward rate f* as follows: 1. Since the forward rate end date falls in between d1 and d2, PRIME will use these dates for interpolation. 2. Consider two more dates: d1', which is 6 months prior to d1 and d2', which is 6 months prior to d2. When calculating the exact dates for d1' and d2' from the 6M period, the curve's Day Method field is used (recommended to be set to same Day Method as defined in the main rate index related to the forward curve). 3. Given these 4 dates, PRIME calculates 4 spot rates (which will use spot rate interpolation where needed), s1', s1, s2', and s2: s1' corresponds to d1', s1 to d1, s2' to d2' and s2 to d2. 4. Next, two forward rates are calculated, f1 and f2. f1 is the forward rate over the period (d1', d1) and f2 is the forward rate over the period (d2', d2). Each forward rate is calculated as a function of the pair of spot rates. 5. Finally, given these two forward rates f1 and f2 that have end dates at each of the two benchmark dates that were picked, The desired forward rate, f*, is then calculated from f1 and f2 by linear interpolation. Note Only linear interpolation is supported in forward rate terms, that is, the **Interpolation** setting in the Yield Curve Definition window does not affect interpolation in forward rate terms. However, the same setting does, even if **Interpol Rate Type** is set to **Forward Rate**, still affect any interpolation in spot rate terms. Such interpolation could, for example, be needed in point 3 above. Note **Interpol Rate Type Forward Rate** is only intended to be used for curves that are used for estimating forward rates (that is, curves that are not used as discount curves).

Help file to Front Arena users.

7. References

CME, 2021. Eurodollar futures contract specs, https://www.cmegroup.com/trading/interest-rates/stir/eurodollar_contract_specifications.html

IBM, 2013. Financial Models in RiskWatch Masterbook. IBM AlgoOne, Version 4, published in 2013

Wiki, 2021. https://en.wikipedia.org/wiki/Eurodollar

8. Acknowledgement

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