Kepler's Celestial Music

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KEPLER'S CELESTIAL MUSIC

By D. P. Walker

In the long tradition of the music of the spheres[1] Kepler's celestial harmonies are unique in several respects. First, they are real but soundless, whereas the Greek and medieval music of the spheres is either metaphorical, becoming eventually a literary topos, or is audible music, which, for various reasons, only very exceptional people, like Pythagoras,[2] can hear. Secondly, they are polyphonic (that is, are harmonies in the modern sense of the word), whereas earlier ones, from Plato to Zarlino,[3] consist only of scales. Thirdly, they are in just intonation, that is, having consonant thirds and sixths, whereas all earlier systems use Pythagorean intonation, in which the smallest consonance is the fourth.[4] Fourthly, these consonances are geometrically determined, by the regular polygons inscribable in a circle, whereas earlier theorists derive musical intervals arithmetically from simple numerical ratios. Finally,

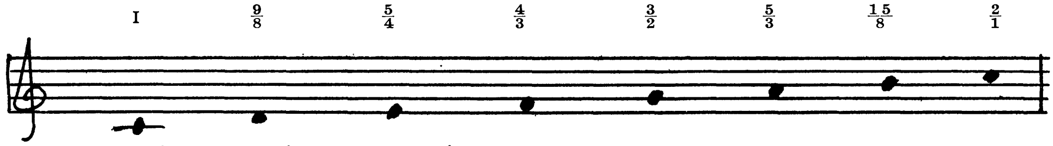
[1] There is quite a full bibliography in the article 'Harmonie' in *Die Musik in Geschichte und Gegenwart* (hereafter *Musik in G. & G.*), v, Kassel 1949–, cols. 1594–99; cf. also James Hutton, 'Some English Poems in Praise of Music', in *English Miscellany*, ed. M. Praz, ii, Rome 1951, pp. 1–63.

[2] See D. P. Walker, *Spiritual and Demonic Magic*, London 1958, p. 37, and John Hollander, *The Untuning of the Sky*, Princeton 1961, p. 29.

[3] Zarlino, in his *Istitutioni Harmoniche*, pt. i, c. vi (*Tutte l'Opere*, i, Venice 1589, pp. 16–21, cf. pp. 123–6), gives a good account of ancient opinions on the subject.

[4] Just intonation is based on a scale so divided that it contains the maximum number of perfectly just consonances, which are: octave $\frac{2}{1}$, fifth $\frac{3}{2}$, fourth $\frac{4}{3}$, major third $\frac{5}{4}$, minor third $\frac{6}{5}$, major sixth $\frac{5}{3}$, minor sixth $\frac{8}{5}$ (these ratios are of frequencies; to obtain the ratios of length of string, which Kepler and his contemporaries used, one has merely to invert the fractions). From Zarlino's time onwards the just scale is usually:

Intervals from lowest note	I	tone $\frac{9}{8}$	major third $\frac{5}{4}$	fourth $\frac{4}{3}$	fifth $\frac{3}{2}$	major sixth $\frac{5}{3}$	major seventh $\frac{15}{8}$	octave $\frac{2}{1}$



Intervals between notes: major tone, minor tone, semitone $\frac{9}{8}$ $\frac{10}{9}$ $\frac{16}{15}$ $\frac{9}{8}$ $\frac{10}{9}$ $\frac{9}{8}$ $\frac{16}{15}$

All the consonances here are just, except for the fifth D–A, and the minor third D–F; to make these just would necessitate having two D's.

Pythagorean intonation is based on a scale so divided that all the fifths are just and all the tones equal:

Intervals from lowest note	I	$\frac{9}{8}$	$\frac{81}{64}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{27}{16}$	$\frac{243}{128}$	$\frac{2}{1}$

Intervals between notes: $\frac{9}{8}$ $\frac{9}{8}$ $\frac{256}{243}$ $\frac{9}{8}$ $\frac{9}{8}$ $\frac{9}{8}$ $\frac{256}{243}$

Here all the fifths and fourths are just, but all the thirds and sixths are dissonant.

Equal temperament is based on a scale so divided that all its semitones, and therefore also all its tones, are equal. The ratio of the semitone is $\sqrt[12]{2}$. Here all the consonances, except for the octave, are false. Cf. J. M. Barbour, *Tuning and Temperament*, East Lansing 1953, and *Musik in G. & G.*, article 'Intervall'.

Kepler's *musica mundana* is centred on, and perceived from, the sun. These peculiarities, all of which are interconnected, are the subject of this article.

First, there is the basic problem of the objective validity of Kepler's celestial harmonies. Kepler, when finding the ratios of musical consonances in the extreme angular speeds of the planets as seen from the sun,[5] allowed himself some margin of error. This was of course quite in accordance with his metaphysics—one would not expect to find an exact copy of a geometrical archetype in the natural world. But it is evident that, given a wide enough margin of error, one could find musical ratios in any old set of numbers. Was Kepler, as Athanasius Kircher suggested in his *Musurgia* (1650)[6], playing a game with such lax rules that he was bound to win? Or did he in fact discover a pattern, a regularity which really does exist? I think the answer is that he was not playing too easy a game and that he had every reason to suppose that he had made a genuine discovery—for the following reason. He had been trying, ever since the *Mysterium Cosmographicum* (1596), to find these ratios in the heavens, in the distances between the orbits of the planets, and then in their orbital speeds,[7] and he did not find them. It is clear, therefore, that he was not willing to stretch his margin of error so that he could find them wherever he looked for them. He found them only when he placed himself in the sun and looked at the angular speeds of the planets from there.

Kepler's insistence that his celestial harmonies should be real, should be empirically confirmed by astronomical observation, is typical of all his thinking, and in particular of his thinking about music. While searching for, and discovering, purely metaphysical or aesthetic causes for things being as they are and not otherwise—beautiful, simple patterns, mathematically determined and logically interconnected, he always gave absolute priority to empirical evidence;[8] if the theoretical pattern, however beautiful, did not fit the facts, it was discarded. Though Kepler was resolved that his celestial music should not be merely analogical, he by no means despised analogies; and, as we shall see, it is not always clear whether his musical and geometric analogies are only metaphorical or whether they express a real connexion between the two terms of the analogy.

The two novelties in Kepler's celestial music of polyphony and just intonation are closely connected; if thirds and sixths are not admitted as consonances, there can be no polyphony. Although in the sixteenth and seventeenth centuries the question was debatable, Kepler believed, with the majority of competent scholars, that ancient music, though perhaps not

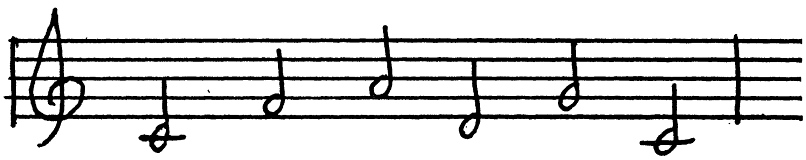
[5] It is the extreme speeds that give the basic 'scales' of each planet; other speeds within these limits are also used to make the harmonies; cf. *infra*, pp. 247–8.

[6] Athanasius Kircher, *Musurgia Universalis*, ii, Rome 1650, p. 379: 'Ludere autem in sola proportione, nullius ingenij est, cùm vix ullę numeris subiectae res sint, quae non aliquas ex musicis proportionibus denominationes habeant'; he has just (*ibid.*, pp. 377–8) criticized the largeness of Kepler's margin of error.

[7] See Kepler, *Harmonices Mundi Libri V* (hereafter *Harm. Mund.*), Lib. v, c. iv (*Gesammelte Werke,* ed. Max Caspar, vi, Munich 1940, pp. 306–12), and Max Caspar's *Nachbericht* (*ibid.*, pp. 470ff.).

[8] Cf. E. A. Burtt, *The Metaphysical Foundations of Modern Physical Science*, London 1949, pp. 50–51.

strictly monodic, was not polyphonic in any way resembling modern music,[9] and that this difference was reflected in the prevailing system of intonation: Pythagorean (in which the thirds and sixths are dissonant) for the ancients, and just (in which they are consonant) for the moderns.[10] This last opinion was also debatable. The acrimonious controversy between Zarlino and Vincenzo Galilei,[11] both of whom Kepler had read,[12] was about the question whether contemporary *a capella* singing was in just intonation or in some kind of tempered scale, Zarlino asserting the former and Galilei the latter; but both agreed that ancient vocal music was monodic and used the Pythagorean scale. In fact, Galilei was almost certainly nearer the truth than Zarlino and Kepler, for the following reasons. First, all systems of intonation, including equal temperament, are mathematical ideals,[13] to which actual musical practice can approximate only very roughly. Secondly, even an approximation is much more difficult to achieve in just intonation than in Pythagorean or equal temperament, because, unless some of the consonances are tempered, just intonation is hopelessly unstable even in the simplest diatonic music: if all the intervals in the following example are sung justly, the singer will end a comma ($\frac{81}{80}$) flatter than he began:[14]



$\frac{4}{3}$ $\frac{5}{4}$ $\frac{2}{3}$ $\frac{4}{3}$ $\frac{2}{3}$ $\frac{80}{81}$ instead of 1.

[9] See D. P. Walker, 'Musical Humanism in the 16th and early 17th centuries' (hereafter 'Musical Humanism'), *The Music Review,* 1941–42, sections ix–xi, and *infra*, p. 231.

[10] Kepler does not explicitly state this connexion in the *Harmonice Mundi*, but I think only because it is so obvious; he is a very elliptical writer. It is a commonplace in the 16th century musical treatises he used (*vide infra*, note 12). In a letter of 1599 to Herwart von Hohenburg (Kepler, *Ges. Werke*, xiv, p. 72) Kepler does make this connexion quite clearly: after explaining how the Greeks, owing to their Pythagorean intonation, failed to use the imperfect consonances, he goes on: 'Since this is the case, I am extremely surprised . . . that Ursus should think the music of the ancients much nobler than ours. I believe that one voice singing to the lyre had its own grace, and this for the sake of pleasure is being revived everywhere nowadays; but I shall never believe that monody is more delightful than four voices preserving unity in variety . . .' ('Quae cum ita sint, vehementer miror Ursum (et antea quoque mirabar, quam haec scirem), qui veterum Musicam putat longè nobiliorem fuisse nostrâ. Credo gratiam habuisse suam, vocis humanae unius accommodationem ad lyram, quae hodie voluptatis causa passim revocatur: sed unius simplicis vocis modulationem suaviorem esse quatuor vocibus in varietate identitatem tuentibus, numquam credidero. At nuspiam legimus cecinisse illos diversis vocibus in unum.') Reimarus Ursus was Imperial Mathematician and an enemy of Tycho Brahe; I have not been able to find anything about music in his published works.

[11] See Walker, 'Musical Humanism', end of sect. iv.

[12] Caspar (Kepler, *Ges. Werke*, vi, p. 477) gives a list of musical treatises which Kepler had read; this does not include Zarlino, but Kepler cites him in the *Harm. Mund.* (Kepler, *ibid.*, p. 139). The list wrongly ascribes to J. T. Freigius a work by F. Beurhusius (*Erotematum Musicae Libri duo*, Noribergae 1580), for which he wrote the preface.

[13] Just intonation, and to a lesser degree Pythagorean, have a physical basis: the series of overtones, and combination tones; but these were not yet discovered in Kepler's day. Cf. *infra*, p. 241.

[14] Cf. J. M. Barbour, *op. cit.*, pp. 196–9, and Wilhelm Dupont, *Geschichte der musikalischen Temperatur*, Kassel 1935, pp. 11–12.

Nevertheless, though none of these systems is ever exactly put into practice, it is not a matter of indifference for practical music which of them prevails as a theoretical ideal, because musicians will attempt to attain it and produce different results, which are easily distinguishable by ear.[15] For music which is monodic, or in which the interest is concentrated on melody, Pythagorean intonation is more suitable than just, since all the fifths and fourths can be untempered, and the very narrow semitones give greater sharpness to the shape of the melody.[16] For polyphonic music such as that of the sixteenth to the nineteenth centuries, in which the major triad occupies a dominating and central position, just intonation has the advantage of making this chord as sweet as possible and in general of making all chords, both major and minor, more consonant, though it has the disadvantage of much greater instability of pitch, of unequal tones, and of much wider semitones ($\frac{16}{15}$ as compared with $\frac{256}{243}$, differing by a comma, $\frac{81}{80}$). These remarks are borne out by the history of Western music. Music in the ancient world was monodic, or at least dominated by melody, and the standard intonation was Pythagorean; although Ptolemy, and before him Didymus, gave the ratios of just intonation, they did not accept thirds and sixths as consonances.[17] Pythagorean intonation, transmitted mainly through Boethius and Macrobius,[18] was the only system known to medieval musical theorists; but with the full development of polyphony in the later Middle Ages theorists begin to accept thirds and sixths as 'imperfect consonances', though still giving the Pythagorean ratios ($\frac{81}{64}$, $\frac{32}{27}$, $\frac{27}{16}$, $\frac{128}{81}$).[19] If medieval musicians were aiming at Pythagorean intonation, their major triads would be no more consonant than their minor triads; and in fact it is not until the later sixteenth century that harmony begins to be dominated by the major triad, as opposed to the minor, and that major and minor tonality begins to replace the modes. This brings us to the period of Zarlino, the first widely read and influential theorist to advocate

[15] To convince himself that he can hear the difference between just and Pythagorean thirds and sixths, the reader who owns a violin or 'cello may make the following simple experiment. Having tuned the instrument as accurately as possible, play E on the D-string with the open G-string; then, taking care not to move your finger, play the E with the open A-string. If the major sixth has been made as sweet as possible, it will be found that the finger has to be leaned considerably forward to produce a perfect fourth. The difference between the two E's is a comma ($\frac{81}{80}$). Then try the experiment the other way round.

[16] Present-day violinists who believe that they are playing in 'natural' or 'true' intonation, as opposed to equal temperament, make very narrow semitones by sharpening upward leading-notes and flattening downward ones. In consequence, e.g. G sharp followed by A is sharper than A flat followed by G; and in consequence of this their double-stopped thirds and sixths are very harsh. In other words they are attempting to play in Pythagorean intonation, as befits a melody instrument. They are of course also pushed towards this kind of intonation by the fact that their instrument is tuned in fifths (cf. J. M. Barbour, *op. cit.*, p. 200).

[17] Ptolemy, *Harm.*, lib. i, c. xv, and *Musik in G. & G.* (hereafter *MGG*), articles 'Intervall', 'Didymos'.

[18] Boethius, *De Institutione Musica*, lib. i, c. vii and *passim*; Macrobius, *Commentariorum in Somnium Scipionis Libri II*, lib. ii, c. i (it is interesting that, although Macrobius transmits the Pythagorean ratios correctly, he did not understand that they were ratios; having stated, rightly, that the tone cannot be exactly divided into two halves, he gives as a reason that 9 cannot be divided into two equal integers—the true reason being, of course, that there is no rational square root of $\frac{9}{8}$).

[19] *MGG*, article 'Intervall', cols. 1344–5.

the use of just intonation,[20] in which the major triad is much sweeter than the minor. The great growth of instrumental music and the development of harmony towards greater freedom of modulation from the sixteenth to the eighteenth centuries are reflected in the eventual triumph of equal temperament as the ideal intonation. All instruments of fixed intonation must have some kind of temperament, and in the sixteenth century an attempt at equal temperament was used for fretted instruments (lutes and viols), and usually meantone temperament,[21] which provides just thirds and sixths, for keyboard instruments. For instrumental music and for any music which modulates freely equal temperament had enormous advantages: as compared with just intonation or meantone temperament, all its fifths and fourths are very nearly true and its semitones narrower; as compared with Pythagorean, its thirds and sixths are less dissonant; as compared with any other system, all keys are equally in tune, and there is no instability of pitch.

We may say then that Kepler was right in accepting a real connexion between the growth of polyphony and the prevalence of just intonation as an ideal, though his reasons for this acceptance are not of course the same as those I have just given. For Kepler just intonation and polyphony had finally prevailed because they were natural, that is, they corresponded to the archetypes in the mind of God,[22] on which the created world was modelled, and which are also in the mind of man, the image of God. Modern music and intonation are thus justified and welcomed by Kepler in two ways: first, empirically, because an unprejudiced observer gifted with a good ear can realize that thirds and sixths are consonant—they will please and satisfy him because they correspond to the archetypes in his mind, and, by using a monochord, he can discover that their ratios are 5:4, 6:5, etc.; secondly, the investigator of nature can find these consonances in God's creation, in Kepler's case in the harmony of the spheres, and thus confirm his own empirical knowledge and the instinctively natural, polyphonic practice of modern musicians. The final step is to show by reasons drawn from geometry, that supreme set of archetypes which is coeternal with God, why these ratios and no others produce musical consonances.

Kepler is always most emphatic in affirming that polyphony is a modern invention and therefore quite unknown to the ancients, though for historical evidence he merely refers the reader to Galilei's *Dialogo della Musica antica et moderna* (1581);[23] and, unlike most of his contemporaries he sees this as the

[20] Ramis de Pareia (*Musica Practica*, Bologna 1482) and Foligno (*Musica Theorica*, Venice 1529) had both given the ratios of just consonances.

[21] So called because the major third is divided into two equal tones (ratio $\frac{\sqrt{5}}{2}$); see Percy C. Buck, *Acoustics for Musicians*, Oxford 1918, pp. 95–99.

[22] For Kepler's archetypes see Pauli's contribution to C. G. Jung and W. Pauli, *The Interpretation of Nature and the Psyche*, London 1955.

[23] In the *Harmonice Mundi* Kepler does not mention that Galilei, under the influence of Girolamo Mei (see C. V. Palisca, *Girolamo Mei (1519–1594) Letters on Ancient and Modern Music to Vincenzo Galilei and Giovanni Bardi . . .*, n.p. (American Institute of Musicology), 1960), was writing against modern polyphony and just intonation, and in favour of a revival of ancient monody and Pythagorean intonation. But in a letter of 1618 to Matthäus Wacker von Wackenfels, Kepler tells how in October 1617, when setting off from Linz to Regensburg, he foresaw a slow journey and therefore took with him Galilei's dialogue, which, though he found the Italian difficult, he read with the greatest pleasure; in it he

extraordinary and unique advance that it was,[24] an advance that for him is paralleled by the new astronomy and his own discovery of the celestial polyphony. In the Fifth Book of the *Harmonice Mundi*, after he has gone through the 'scales' played by each planet, which are like 'simple song or monody, the only kind known to the ancients',[25] he begins his chapter on the chords made by all six planets,[26] and by five and by four of them, thus:[27]

> Now, Urania, a more majestic sound is needed, while through the harmonic ladder of celestial movements I ascend yet higher, where the true Archetype of the world's structure lies hidden. Follow me, modern musicians, and express your opinion on this matter by means of your arts,[28] unknown to antiquity; Nature, always generous with her gifts, has at last, having carried you two thousand years in her womb, brought you forth in these last centuries, you, the first true likenesses of the universe; by your symphonies of various voices, and whispering through your ears, she has revealed her very self, as she exists in her deepest recesses, to the Mind of man, the most belovèd daughter of God the Creator.

Though modern music reveals the archetypical structures of the heavens, it is not an imitation of the celestial music, nor derived from it; but both are

found valuable information about the ancients, and, although he often disagreed with the author's opinions, he enjoyed the virtuosity with which Galilei expounded views opposite to his own, by extolling ancient music and denigrating modern (Kepler, *Ges. Werke*, xvii, p. 254: 'inveni enim thesaurum antiquitatis egregium, et quamvis in re ipsa crebro ab ipso dissentiam, delectatus tamen sum artificio disputantis in contrarium, et in re Mathematica oratorem agentis, praesertim ubi veterem Musicam extollit, novam deprimit'). Kepler cites Galilei much more frequently than any other modern writer on music.

[24] Cf. Kepler's letter quoted above, p. 230, n. 10, and *infra*, pp. 234–5.

[25] Kepler, *Ges. Werke*, vi, p. 316: '. . . quae proportio est Cantus simplicis seu Monodiae, quam Choralem Musicam dicimus, et quae sola Veteribus fuit cognita, ad cantum plurium vocum, Figuratum dictum, inventum proximorum saeculorum: eadem est proportio Harmoniarum, quas singuli designant Planetae, ad Harmonias junctorum.'

[26] The moon is excluded because 'Luna seorsim suam Monodiam cantillat, Terris ut cunis assidens' (Kepler, *ibid.*, p. 323). In the translation of the Fifth Book of the *Harm. Mund.* by Charles Glenn Wallis (in Ptolemy, *The Almagest* . . ., (Great Books of the Western World, no. 16), 1952, p. 1040), the last part of this sentence is rendered: 'like a dog sitting on the earth', 'canis' presumably being read for 'cunis'—not a happy emendation; the translation as a whole is very poor. The German translation by Caspar of the whole of the *Harm. Mund.* (Kepler, *Welt-Harmonik*, Munich-Berlin 1939) is of course excellent.

[27] Kepler, *ibid.*, p. 323: 'Nunc opus, Uranie, sonitu majore: dum per scalam Harmonicam coelestium motuum, ad altiora conscendo; quâ genuinus Archetypus fabricae Mundanae reconditus asservatur. Sequimini Musici moderni, remque vestris artibus, antiquitati non cognitis, censete: vos his saeculis ultimis, prima universitatis exempla genuina, bis millium annorum incubatu, tandem produxit sui nunquam non prodiga Natura: vestris illa vocum variarum concentibus, perque vestras aures, sese ipsam, qualis existat penitissimo sinu, Menti humanae, Dei Creatoris filiae dilectissimae insusurravit.'

[28] I.e. 'prove me right by using justly intoned polyphony'. In a side-note Kepler suggests that modern composers should write six-part motets on one of the Psalms, or some other scriptural text, in return for this eulogy he has given them. Kepler will see that they are published, and says that: 'He who most nearly expresses the celestial music described in this book; to him Clio promises a garland, Urania promises Venus as a wife' ('Qui propius Musicam coelestem exprimet hoc opere descriptam; huic Clio sertum, Urania Venerem sponsam spondent.')

likenesses of the same archetypes, the geometric beauties coeternal with the Creator; and modern music, as we shall see, thereby even allows us to experience something of God's satisfaction in His own handiwork.

Kepler's whole-hearted and joyous acceptance of polyphony as a step forward, comparable in importance with the Copernican revolution, is in marked contrast to the attitude of his contemporaries, even of those who also believed that ancient music was monodic. Zarlino and his followers, such as the composers of *musique mesurée à l'antique* or the Florentine *Camerata* of Bardi, concede that modern music has acquired additional sweetness and variety through the use of polyphony, but they also believe that, with regard to rhythm and the treatment of text, we still have much to learn from the ancients;[29] there is no feeling that music has acquired another dimension, but merely that one aspect of the art has been elaborated, while another equally, or even more important aspect has degenerated.[30] Another, more subtle contrast with Kepler is provided by Sethus Calvisius, with whom Kepler had a long correspondence on music and on chronology,[31] and whose musical treatises he recommends, rather lukewarmly, in the *Harmonice Mundi*.[32] Calvisius, in his essay *De Initio et Progressu Musices, aliisque rebus eo spectantibus* (1600),[33] gives a competent, if brief history of musical theory and practice from the Flood to the present day, and from it Kepler could have gathered all the elements necessary to produce a realization of musical progress. The ancients rejected thirds and sixths; their music was monodic or nearly so; at some time in the Middle Ages polyphony was invented, and soon became decadently over-complicated;[34] at the time of the Reformation, 'the repurging of celestial doctrine, together with other good arts and languages', an improvement in musical style began, especially in the treatment of text, which has reached its culmination with Orlando di Lasso and other more recent composers.[35] Music has now attained such heights that no further progress seems possible; all we can do now is to use it to thank God that in this last age of the world He has advanced this art, 'among the other liberal arts, to its highest perfection', as a prelude to the music of the Church Triumphant in heaven, soon to begin and never to cease.[36] Unlike Kepler, Calvisius however, though he knows and states that the ancients had no polyphony, never singles

[29] See Walker, 'Musical Humanism', sect. ix.

[30] G. M. Artusi, another author whom Kepler cites (*ibid.*, pp. 181, 182, 185), is almost as severe on modern polyphony as Galilei and Mei. In his *L'Artusi overo delle Imperfettioni della Moderna Musica Ragionamenti due*, Venice 1600, polyphony is condemned as positively pernicious, because, by its mixture and confusion of rhythms, modes and genera, it prevents the production of the 'effects'.

[31] The letters concerning music are: Kepler, *Ges. Werke*, xv, pp. 469ff.; xvi, pp. 47ff., 55ff., 216ff.; xviii, pp. 455ff.

[32] Kepler, *ibid.*, vi, p. 185.

[33] Calvisius, *Exercitationes Musicae Duae. Quarum Prior est, de Modis musicis, quos vulgò Tonos vocant, rectè cognoscendis, & dijudicandis. Posterior, de Initio* . . ., Lipsiae 1600.

[34] Calvisius, *op. cit.*, pp. 91–94, 124–8. Calvisius is referring mainly to the complexities of medieval rhythmical notation.

[35] Calvisius, *ibid.*, pp. 133–5, 'usque ad coelestis doctrinae, una cum bonis artibus & linguis, repurgationem . . .'.

[36] Calvisius, *ibid.*, p. 138 (last page), '. . . quod hoc ultimo mundi articulo, inter alias liberales artes, hanc etiam ad summam perfectionem deducere, & quasi προαύλιον praelusionem fieri voluit [sc. Deus], perfectissimae illius Musicae in vita coelesti, ab universo triumphantis Ecclesiae & beatorum Angelorum choro, propediem inchoandae, & per omnem aeternitatem continuandae'.

out this fact as an example of progress, and he sees the present good state of music on a par with that of the other liberal arts, which have been revived after the long medieval darkness.

The main reason, according to Kepler, why this musical revelation and revolution was so long delayed was that the ancients did not stay close enough to empirically established facts, to the judgements of the ear. In the preface to the Third Book of the *Harmonice Mundi*, which deals with practical music, Kepler gives a brief history of intonation. The Pythagoreans discovered by ear the perfect consonances of the octave, fifth and fourth, and their ratios (2:1, 3:2, 4:3); but then turned away too soon from the evidence of their ears and towards speculation in numbers—a double error, first in that musical theory must not only start from observation but also be constantly checked by it, and secondly, in that the grounds of consonance must be sought not in numbers, but in geometry:[37]

> The Pythagoreans were so addicted to this kind of philosophizing in numbers, that they failed to keep to the judgment of their ears, though it was by means of this that they had initially been brought to this philosophy; they defined solely by their numbers what is a melodic interval and what is not, what is consonant and what dissonant, thus doing violence to the natural instinctive judgment of the ear.

Thus misled, the Pythagoreans, and following them Plato,[38] restricted consonances to ratios made out of their tetractys (1.2.3.4.), and therefore failed to include thirds and sixths, without which there can be no polyphony. They wrongly accepted as a melodic interval the Pythagorean semitone, or Platonic limma, $\frac{256}{243}$ (the difference between two major tones and a fourth: $\frac{4}{3} \div (\frac{9}{8} \times \frac{9}{8})$), and wrongly excluded the minor tone, $\frac{10}{9}$ (the difference between a just major third and a major tone: $\frac{5}{4} \div \frac{9}{8}$). This 'harmonic tyranny' continued until the time of Ptolemy, who, maintaining the judgement of the ear against Pythagorean philosophy, admitted as melodic intervals the minor tone ($\frac{10}{9}$) and just semitone ($\frac{16}{15}$), and gave the ratios of just thirds and sixths ($\frac{5}{4}$, $\frac{6}{5}$, $\frac{5}{3}$, $\frac{8}{5}$). But Ptolemy, though he had thus emended the Pythagoreans' system and rightly trusted his ear, was still misled by their preoccupation with 'abstract numbers', and in consequence both wrongly excluded thirds and sixths from the consonances, which 'all well-eared musicians of today' accept, and wrongly included among the melodic intervals a division of the fourth into $\frac{7}{6}$ and $\frac{8}{7}$, which is 'most abhorrent to the ears of all men'.[39]

Kepler has several reasons for insisting that the causes of consonance must be sought not in numbers but in geometrical figures. First, one cannot find any sufficient reason why God should have chosen the numbers 1.2.3.4.5.6. as those out of which consonances should be generated, and have excluded

[37] Kepler, *Ges. Werke*, vi, p. 99: 'Huic enim philosophandi formae per Numeros, tantopere fuerunt dediti Pythagoraei; ut jam ne aurium quidem judicio starent, quarum tamen indicijs ad Philosophiam hanc initio perventum erat: sed quid concinnum esset, quid inconcinnum; quid consonum, quid dissonum, ex solis suis Numeris definirent, vim facientes instinctui naturali auditus.'

[38] Kepler, *Ges. Werke*, vi, pp. 94–95, 100; cf. his earlier criticism of Plato in a letter of 1599, xiv, pp. 71–72.

[39] Kepler, *Ges. Werke*, vi, p. 99; Ptolemy, *Harm.*, lib. i, c. xv. The appendix to the *Harm. Mund.* contains a critique of Ptolemy's musical analogies (Kepler, *ibid.*, pp. 369ff.).

7.11.13. etc.[40] The reason given in the *Timaeus*, namely the two families of squares and cubes generated by the triad 1.2.3, itself the principle of all things:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & & & 9 & \\ 8 & & & & & & 27 \end{array}$$

is no good, because it excludes the number 5, 'which will not allow itself to be robbed of its right of citizenship among the sources of consonances',[41] that is, of the thirds and sixths, all of which in their ratios have the number 5. Secondly, numbers are not suitable as causes of musical intervals, because the terms of musical ratios are continuous, not discrete quantities, and therefore these causes must be sought in geometrical figures.[42] By the terms (*termini*) of the intervals Kepler must mean musical sounds of different pitches, and presumably believes that these have no natural units by which they can be counted. This is odd, since, when explaining (correctly) sympathetic vibration,[43] Kepler is evidently considering musical sound as made up of a series of pulses (*ictus*) caused by a vibrating string, and these would provide a natural unit for counting, as we now count frequencies. But Kepler must have been in a bit of a muddle about the nature of musical sound, since he apparently believed that the pitch of a string falls as the amplitude of its vibration decreases.[44] Finally, numbers are metaphysically and epistemologically inferior to geometrical figures and proportions. Numbers do not exist in physical things, but only 'dispersed units' so exist; numbers are thus abstract, in the sense that an Aristotelian *tabula rasa* mind could develop them by abstraction from the repetitive sense-experience of any kind of unit—they are 'of second, even of third or fourth intention'.[45] But this is not true of geometrical figures and proportions; these do exist, as imperfect copies, in physical things; and the mind or soul recognizes and classes them by comparing them with the God-implanted archetypes within itself.[46] In the *Harmonice Mundi* Kepler quotes a long passage from Proclus's commentary on Euclid, which is a defence of the Platonic doctrine, that all mathematical ideas exist

[40] Kepler, *ibid.*, p. 100; cf. *infra*, p. 241 on Kepler's rejection of harmonic proportion.

[41] Kepler, *ibid.*: 'Nam causa illa de Ternario principiorum, et familiâ quadratorum et cuborum inde deductâ, causa est nulla; cùm quinarius ab illa exulet, qui sibi inter Musicorum intervallorum Ortum jus civitatis eripi non patitur'; cf. Kepler, *ibid.*, pp. 94–95; Plato, *Timaeus*, 35 B.

[42] Kepler, *ibid.*, p. 100: 'Cùm enim intervallorum Consonorum termini, sint quantitates continuae: causas quoque quae illa segregant à Dissonis, oportet ex familia peti continuarum quantitatum, non ex Numeris abstractis, ut quantitate discretâ . . .'

[43] Kepler, *ibid.*, pp. 105–6.

[44] Kepler, *Ges. Werke*, vi, p. 144.

[45] Kepler, *ibid.*, p. 431 (*Apologia* against Fludd): 'Omnis numerus, ut sit numerus, menti inesse debet, ut docet Aristoteles; in sensibus inque materia numerus non est, sed unitates dispersae'; p. 212, 'Numerus definitur esse multitudo ex unitatibus conflata . . .'; p. 222, 'sunt enim illi [sc. numeri] secundae quodammodò intentionis, imò et tertiae, et quartae, et cujus non est dicere terminum: nec habent in se quicquam, quod non vel à quantitatibus, vel ab alijs veris et realibus entibus, vel etiam à varijs Mentis intentionibus acceperint'.

[46] Kepler, *ibid.*, pp. 215–16; the mind recognizes these proportions intellectually, the soul instinctively.

innate in the soul, against the Aristotelian epistemology of their being universals abstracted from multiple sense-experience. He then concedes that Aristotle was right as far as numbers are concerned, and was right to refute Pythagorean number-philosophy, and Kepler himself rejects Plato's numerology in the *Republic*; but 'with regard to continuous quantities I am entirely in agreement with Proclus'.[47]

This inferiority of numbers to geometric figures and ratios is important for Kepler's attitude to analogies or symbols. Analogies based purely on numbers correspond to no archetype in the soul of man or mind of God, whereas geometric analogies do so correspond, and, in many cases, are therefore more than analogies: they display the reasons why God created things as they are and not otherwise, or why we are pleased or displeased with certain experiences. Not only in the *Apologia* against Fludd, but elsewhere in the *Harmonice Mundi*, Kepler takes care explicitly to reject any number-symbols which might suggest themselves to the reader.[48]

Harmony, musical or of any other kind, consists in the mind's recognizing and classing certain proportions between two or more continuous quantities by means of comparing them with archetypical geometrical figures. Now we know by experience that there are seven musical consonances, which have these ratios: $\frac{2}{1}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, $\frac{5}{3}$, $\frac{8}{5}$, and which can be multiplied indefinitely by doubling their ratios, i.e. by inserting octaves ($\frac{3}{2}$, e.g., a fifth, when doubled becomes $\frac{3}{1}$, a twelfth). What class of geometric figures will yield these ratios, and no others? As early as 1599 Kepler was looking for the answer in the arcs of a circle cut off by regular, geometrically constructable, inscribed polygons.[49]

There are two main reasons why he should have looked here. First, he had already had at least partial success in using the five regular Platonic solids to account for the number of planets and the size of their orbits;[50] and in the *Mysterium Cosmographicum* he had, as he later wrote,[51]

> wrongly attempted to deduce the number and ratios [of the consonances] from the five regular solid bodies, whereas the truth is rather that both the five regular solid figures and the musical harmonies and divisions of the monochord have a common origin in the regular plane figures,

that is, the number of the regular solids is determined by their surfaces, which

[47] Kepler, *ibid.*, pp. 218–22, 'De numeris quidem haud contenderim; quin Aristoteles rectè refutaverit Pythagoricos. . . . At quod attinet quantitates continuas, omninò adsentior Proclo.'

[48] E.g. Kepler, *ibid.*, p. 123. (There are six possible consonant triads; this fact is *not* to be explained by the six days of creation and the Trinity.)

[49] Kepler, *Ges. Werke*, xiii, pp. 349–50, letter to Herwart von Hohenburg, dated 30 May 1599 (consonances from arcs of a circle); xiv, pp. 29–37, letter to the same, 6 August 1599 (consonances from regular inscribed figures).

[50] See A. Koyré, *La Révolution astronomique*, Paris 1961, pp. 143ff.

[51] Kepler, *Ges. Werke*, vi, p. 119, 'Legat curiosus lector, quae de his sectionibus ante annos 22 scripsi in *Mysterio Cosmographico*, Capite XII et perpendat, quomodo fuerim illo loco hallucinatus super causis sectionum et Harmoniarum; perperam nisus earum numerum et rationes deducere ex numero quinque corporum Regularium solidorum: cùm verum sit hoc potius, tam quinque figuras solidas, quàm Harmonias Musicas et chordae sectiones, communem habere originem ex figuris Regularibus planis.' *Myst. Cosm.* (1596) on consonances in *Ges. Werke*, i, pp. 40–43.

must be regular polygons, and the three basic regular polygons, triangle, square and pentagon, can generate only five solids. Secondly, Kepler had archetypical reasons for using divisions of a circle rather than of any other figure. One of his favourite analogies, which is certainly more than a metaphor, is that of the sphere representing the Trinity; the centre is the Father, the surface the Son, and the intervening space the Holy Ghost.[52] In the *Harmonice Mundi*, when explaining his use of the circle as the cause of consonances, he recalls this 'symbolisatio' and extends it. A section through the centre of the sphere produces the plane figure of a circle, which represents the soul of man; this section is made by rotating a straight line, representing corporeal form, which extends from the centre of the sphere to any point on its surface; thus the soul is to the body as a curve to a straight line, that is, 'incommunicable and incommensurable'; and the soul is to God as a circle to a sphere, that is, partaking of the divine three-dimensional sphericity, but joined to, and shaping, the plane generated by the bodily line. 'Which cause', continues Kepler, 'established the Circle as the subject and source of terms for harmonic proportions'.[53]

Using only a rule and compasses, one can divide the circumference of a circle into equal parts in only four basic ways (with one exception, the pentecaidecagon, which will be dealt with later), namely, by inscribing in it its diameter, an equilateral triangle, a square,[54] and a pentagon; by continuously doubling the number of sides of these figures an infinite number of further divisions is possible. Figures, such as the heptagon, which cannot be so constructed, are not demonstrable, and are thus 'unknowable', even to God;[55] they are therefore excluded from the archetypes. The arcs cut off by these basic demonstrable figures provide the following ratios by comparing the arc subtended by one side with the whole circumference, and the arc subtended by the remaining sides with the whole:

	One side to whole	*Residue to whole*
diameter	$\frac{1}{2}$:1. Octave	$\frac{1}{2}$:1
triangle	1:3. Twelfth	2:3. Fifth
square	1:4. Double octave	3:4. Fourth
pentagon	1:5. Double octave plus major third.	4:5. Major third.

This gives us all but three of the seven basic consonances: minor third and sixth, and major sixth. The last can be obtained by dividing the pentagon into 2 and 3; which yields 2:5, a tenth, and 3:5, a major sixth. Just as an infinite number of consonances can be generated by doubling the ratios, so there are an infinite number of regular polygons obtainable by doubling the number of sides. By using two of these polygons, hexagon and octagon, we can get the missing consonances: 5:6, minor third, and 5:8, minor sixth.[56]

The salient feature of this method of explaining the ratios of consonances is that it does not work very well, and it does not work well because of the

[52] Kepler, *ibid.*, i, pp. 9, 23–24.

[53] Kepler, *ibid.*, vi, p. 224.

[54] The square is really a doubling of the diameter.

[55] Kepler, *ibid.*, vi, pp. 47ff.

[56] Kepler, *Ges. Werke*, vi, pp. 101–18. For minor third, cf. *infra*, p. 243, n. 75.

thirds and sixths. This is the main point I want to make here: if Kepler had accepted the still current, very ancient Pythagorean and Platonic system of intonation, involving only the consonances 1:2, 2:3, 3:4, he would have had no difficulties at all; but he did not accept it, and that on purely empirical grounds—because, before he set out on his investigation into causes, he had already established by ear that just thirds and sixths are consonant. The Pythagorean system would have fitted Kepler's geometrical explanation so well because he could have defined the admissible polygons as those whose sides are either directly commensurable with the diameter of the circle (the diameter itself) or commensurable in square (the triangle and the square),[57] and he need have used no other figures. We may, I think, take Kepler's word for it that he did originally adopt just intonation solely on the judgement of his ear. He emphatically states this in the *Harmonice Mundi*, and gives as evidence the fact that he already used this system of consonances in the *Mysterium Cosmographicum*, that is, at a time when he was still far from finding any satisfactory theoretical justification of it:[58]

> The evidence of my book the *Mysterium Cosmographicum* alone will be enough to protect the sense of hearing against the objections of sophists who will dare to deny that the ear can be trusted in such minute divisions [of the monochord] and such very subtle distinctions of consonances. For the reader will see that there I relied on the judgment of the ear in establishing the number of divisions [i.e. consonances], at a time when I was still struggling to find causes, and that I did not then do what the Ancients did. They, having advanced a little way by the judgment of the ear, soon despised their guide and finished the rest of their journey following mistaken Reason, having, as it were forcibly led their ears astray and ordered them to be deaf.

Moreover, it is clear from his correspondence that he was in the habit of using a monochord, and he gives advice on how to achieve more accurate results by checking the consonance one is investigating with its residue; for a major third, e.g., check $\frac{4}{5}$ with $\frac{1}{5}$, i.e. $\frac{1}{4}$, a double octave.[59] He also gives, in the *Harmonice Mundi*, an ingenious method of making audible the slight error in a rough and ready kind of equal temperament used on lutes and described by Galilei.[60]

In Kepler's correspondence of the year 1599, when he began his harmonic investigations with the regular polygons, it is always the thirds and sixths that give trouble. The pentagon, necessary for the major third and sixth, is

[57] (Side of triangle)^2 = 3(radius)^2; (side of square)^2 = 2(radius)^2. The hexagon might have given trouble; but I am sure Kepler would have found a way round it.

[58] Kepler, *ibid.*, pp. 119–20, 'Igitur vel solo allegato mei Mysterij Cosmographici testimonio, satis est munitus auditus, contra Sophistarum obtrectationes, fidem auribus derogare ausuros circa divisiones adeò minutas, et dijudicationem concordantiarum subtilissimam: quippe cùm videat lector me fidem aurium illo tempore secutum esse, in constituendo sectionum numero, cùm adhuc de causis laborarem; nec idem hîc fecisse, quod fecêre Veteres; qui aurium judicio progressi aliquatenus, mox contemptis ducibus, reliquum itineris, Rationem erroneam secuti, perfecerunt; auribus vi quasi pertractis, et planè obsurdescere jussis'.

[59] Kepler, *Ges. Werke*, xvi, p. 159; cf. xv, p. 450.

[60] Kepler, *ibid.*, vi, pp. 143–5.

indeed constructable, but its sides are incommensurable with the diameter even in square.[61] The octagon, necessary for the minor third and sixth, also has irrational sides even in square; but in any case by what rule do we allow it to divide the circle into 3 and 5 parts, but exclude the division into 1 and 7, which would produce dissonant intervals? With regard to the pentagon, the answer was to be that its irrationality involves the 'divine' proportion of the golden section, to which we shall return. For the octagon Kepler tried out various solutions, using regular solids, stars, comparing the arcs not only with the circle but also with the semicircle, etc.;[62] and finally arrived at the rule he uses in the *Harmonice Mundi*, namely, that the harmonic section of a circle must be such that the two parts compared both with the whole and also with each other produce ratios that do not involve numbers such as 7.9.11.13., which are the number of sides of undemonstrable figures (heptagon etc.).[63] Thus the octagon may divide the circle into 3 and 5 parts, since $\frac{3}{5}$, $\frac{5}{8}$, $\frac{3}{8}$ involve no 'ungeometric' numbers, but not into 1 and 7 parts. By means of this rule, which comes dangerously close to being an arithmetic rather than geometric explanation of consonance, Kepler already in 1599 gives the neat table of consonant ratios that appears in the *Harmonice Mundi*:[64]

- $\frac{1}{1}$
  - $\frac{1}{2}$
    - $\frac{1}{3}$
      - $\frac{1}{4}$
        - $\frac{1}{5}$
          - $\frac{1}{6}$ 7
          - $\frac{5}{6}$ 11
        - $\frac{4}{5}$ 9
      - $\frac{3}{4}$ 7
    - $\frac{2}{3}$
      - $\frac{2}{5}$ 7
      - $\frac{3}{5}$
        - $\frac{3}{8}$ 11
        - $\frac{5}{8}$ 13
  - $\frac{1}{2}$

These fractions are generated by adding numerator and denominator to form a new denominator, which has as numerators both numbers of the previous fraction, the generation being blocked by the appearance of an 'ungeometric' number.

These sections of a circle or a string are not of course harmonic in the usual mathematical and musical sense of the term. Kepler does define ordinary harmonic proportion and give the formula for finding an harmonic mean.[65] Three numbers are in harmonic proportion, if the greatest is to the least as the difference between the greatest and the middle is to the difference between the middle and the least $\left(\frac{a}{c} = \frac{(a-b)}{(b-c)}\right.$; the formula for the harmonic mean is therefore $\left. b = \frac{2ac}{(a+c)}\right)$. Thus in terms of string lengths, the harmonic division of an octave, $1:\frac{1}{2}$, gives us 1, $\frac{2}{3}$, $\frac{1}{2}$, i.e. a fifth $(1:\frac{2}{3})$ and a fourth

[61] (Side of pentagon)^2 = ($\frac{1}{2}$ radius)^2 $(10-2\sqrt{5})$. Kepler, in the First Book of the *Harm. Mund.*, elaborates a detailed system, based on Euclid Book X, for grading the irrationality of the sides of polygons. For trouble about the pentagon in the letters of 1599, see Kepler, *ibid.*, xiv, pp. 30–32, 46–48, 65–66.

[62] Kepler, *ibid.*, xvi, pp. 31–38, 46–48.

[63] *Ibid.*, pp. 48, 66.

[64] Kepler, *ibid.*, vi, p. 118.

[65] Kepler, *Ges. Werke*, vi, pp. 120–1.

($\frac{2}{3}:\frac{1}{2} = 1:\frac{3}{4}$). Similarly, the harmonic division of a fifth yields a major and a minor third; and that of a major third yields a major and minor tone. Kepler rejects this method of generating musical consonances on the grounds that there is an indefinite number of harmonic proportions which yield consonant ratios between the extreme terms but not between the middle and the extremes, e.g. 5, $\frac{20}{7}$, 2. It was very unfortunate that Kepler's dislike of numbers should have led him to reject harmonic proportion, the standard explanation, from Zarlino onwards, of the consonances. Harmonic proportion and the harmonic series 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ . . . , together with the phenomenon of sympathetic vibration, point directly to the physical basis of consonance, namely, that a vibrating string does in fact divide itself up into parts a $\frac{1}{2}$, $\frac{1}{3}$, etc. of the total length, which also vibrate at frequences 2, 3, etc. times that of the whole string, thus producing the overtones:

String lengths: 1 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$ . . .

Fundamental. Overtones

Frequencies: 1 2 3 4 5 6 7 8 . . .

Nor is it anachronistic to make this comment. Descartes, in his *Compendium Musicae*, written at the same time as the *Harmonice Mundi*, was on the brink of discovering overtones,[66] and this, a little later, Mersenne in his *Harmonie Universelle* (1636) achieved.[67] Moreover, this is the kind of physical explanation that would have delighted Kepler; but it would also have worried him. The harmonic divisions of a string continue indefinitely, and thus produce the 'ungeometric' division into $\frac{1}{7}$ and an interval (a very flat minor seventh) not accepted as consonant.[68] He would therefore have had great difficulty in combining his regular polygons with the series of overtones.

Another difficulty with the regular polygons, which Kepler had cleared up, to his own satisfaction, by 1607, was that raised by the pentecaidecagon. This figure is constructable and cannot be excluded on the grounds that its sides are irrational even in square, since this would entail excluding also the pentagon and octagon. But he was determined to exclude somehow, and did so on the grounds that it does not have its own independent construction, but can be constructed only by combining a triangle with a pentagon.[69]

[66] Descartes, *Oeuvres*, ed. Adam and Tannery, xi, Paris 1908, pp. 97, 99, 103.

[67] See Hellmut Ludwig, *Marin Mersenne und seine Musiklehre*, Berlin 1935, pp. 40ff.

[68] Mersenne (*Harmonie Universelle*, Paris 1636, Livre Premier des Consonances, pp. 87, 89) suggests that long custom might lead us to accept 7-ratios as consonant. The series of overtones also contains, of course, an indefinite number of other dissonant ratios.

[69] Kepler, *Ges. Werke*, vi, pp. 46–47; cf. xv, p. 391.

At first sight it is not clear why Kepler should be so anxious to reject this polygon, since a circle can, on his own principles, be divided by it into 12 and 3 parts and thus produce consonant ratios. But in the letter to Hewart von Hohenburg of January 1607, which gives a summary of the projected *Harmonice Mundi*, we find the reason, expressed in a typically enigmatic way.[70]

> And so this fifteen-angled figure is sent back among the five foolish virgins. For it comes too late after all the doors have been shut by the numbers 7.9.11.13.

That is to say: to include the pentecaidecagon as a consonance-generating polygon would spoil the neatness and elegance of the table of ratios given above.

The acceptance of the pentagon, although its irrationality is greater than that of the triangle or square, is justified, as I have mentioned, by that irrationality involving the golden section.[71] This is the proportion between three quantities which fulfil the two following conditions:

(1) That they are in geometric proportion, i.e. that the greatest term is to the middle term as the middle to the least $\left(\frac{a}{b}=\frac{b}{c} \text{ or } b^2=ac\right)$.

(2) That the greatest term is the sum of the two lesser $\Big(a=b+c$; so that the formula is: $\frac{a}{b}=\frac{b}{(a-b)}$ or $b^2=a(a-b)$; therefore $b=\frac{a}{2}(\sqrt{5}-1)\Big)$.

In other words, a line is divided into two parts in this proportion, if the whole is to the greater part as the greater part to the less. The side of an inscribed decagon is to the radius of the circle as the greater part to the whole in the golden section (decagon side $=\frac{1}{2}$ radius $(\sqrt{5}-1)$). The square on the side of a pentagon is equal to the square on the side of the decagon inscribed in the same circle plus the square on the radius ((pentagon side)^2 $= r^2+\frac{r^2}{4}(6-2\sqrt{5})$; therefore pentagon side $=\frac{r}{2}\sqrt{10-2\sqrt{5}}$). Also the side of a pentagon is to the line joining two of its vertices as the greater part to the whole in the golden section. Finally, and most importantly for Kepler, this proportion has the property of generating itself indefinitely: by adding the greater part to the whole one obtains a new whole, and the old whole becomes the new greater part $\left(\frac{(a+b)}{a}=\frac{a}{b}\right)$.

Since the pentagon contains the divine proportion within itself, as that between a side and the line joining two vertices, and not only, like the decagon, in relation to the radius of the circle in which it is inscribed,[72] Kepler is able to regard the pentagon as the archetypical figure of this proportion and hence of generation in general. In the *Harmonice Mundi* he reinforces the belief that the golden section is the archetype of generation by the following consideration.[73] An approximation to this proportion can be obtained by this sequence (Fibonacci numbers):

[70] Kepler, *ibid.*, xv, pp. 395–6, 'Itaque haec figura quindecangulum refertur inter quinque fatuas virgines. Venit enim serò postquam jam januae omnes per numeros 7.9.11.13 occlusae sunt.'

[71] Kepler, *ibid.*, vi, pp. 42–45, 63–64, 175ff.

[72] Kepler, *Ges. Werke*, vi, pp. 63–64.

[73] *Ibid.*, p. 175.

c or $a-b$	b	a	
1	1	2	
1	2	3	
2	3	5	
3	5	8	
5	8	13	
8	13	21	etc.

These sets of numbers satisfy the second of the above two conditions ($c = a-b$); they fail to satisfy the first condition ($ac = b^2$) in such a way that ac alternately exceeds or falls short of b^2 by unity, so that as the sequence is carried on b^2 approaches indefinitely nearer in value to ac or $a(a-b)$:

	$a(a-b)$	b^2
masc.	2	1
fem.	3	4
m.	10	9
f.	24	25
m.	65	64
f.	168	169

Where $a(a-b)$ exceeds b^2, the number is, Kepler says, masculine, where it falls short feminine. He then continues:[74]

> Since such is the nature of this [golden] section, which is used for the demonstration of the pentagon, and since God the Creator has fitted the laws of generation to that [proportion]—to the genuine and by itself perfect proportion of ineffable terms [has fitted] the propagation of plants which each have their seed within themselves; and [to] the paired proportions of numbers (of which the one falling short by unity is compensated by the other exceeding [by unity]) [has fitted] the conjunction of male and female—what wonder then, if the progeny of the pentagon, the major third or 4:5 and minor third, 5:6,[75] move our souls, images of God, to emotions comparable to the business of generation.

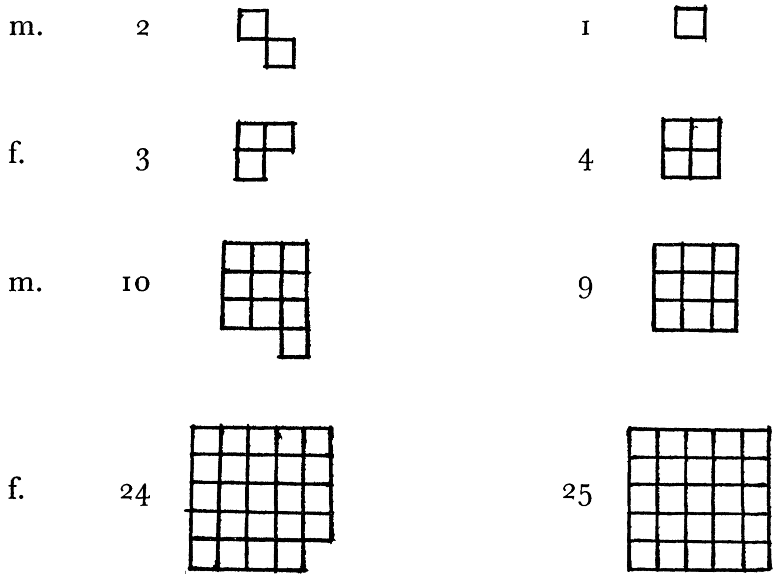
Kepler is so fond of sexual, male-female, analogies[76] that we become inclined to accept them even when, as in this case, it is not obvious why he

[74] *Ibid.*, pp. 175–6, 'Haec cùm sit natura hujus sectionis, quae ad quinquanguli demonstrationem concurrit; cùmque Creator Deus ad illam conformaverit leges generationis; ad genuinam quidem et seipsâ solâ perfectam proportionem ineffabilium terminorum, rationes plantarum seminarias, quae semen suum in semetipsis habere jussae sunt singulae: adjunctas verò binas Numerorum proportiones (quarum unius deficiens unitas alterius excedente compensetur) conjunctionem maris et foeminae: quid mirum igitur, si etiam soboles quinquanguli Tertia dura seu 4.5. et mollis 5.6. moveat animos, Dei imagines, ad affectus, generationis negocio comparandos?'

[75] Kepler continues this passage by arguing that the minor third ($\frac{6}{5}$) derives primarily, not from the hexagon, but from the decagon, and therefore is of 'the class of five-angled figures'. He refers the reader back to ch. iii of this Book (Third). This must be a mistake for ch. ii, where, on pp. 115–16, one finds the relevant *Propositio* XIII. I do not find Kepler's argument convincing; but I am not sure that I have understood it fully. Where it suits him, Kepler derives the minor third from the hexagon (see next note).

[76] E.g., *ibid.*, pp. 135 (major thirds male, minor feminine, because former from the irrational pentagon, latter from the rational hexagon), 292 (cube and dodecahedron

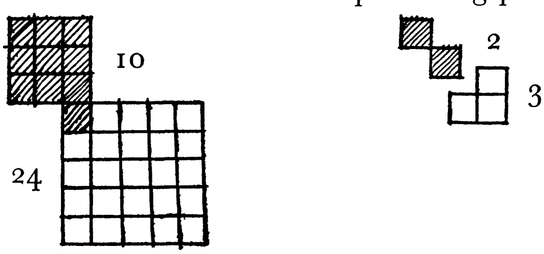
has chosen one term for male and the other for female. In a letter of 1608 to Joachim Tanckius we find a fuller treatment of the golden section series, though here he does not connect it with major and minor thirds. As in the *Harmonice Mundi*, Kepler represents the numbers thus:



but here he adds the explanation:

> Non puto me posse clarius et palpabilius rem explicare, quam si dicam te videre imagines illic mentulae, hic vulvae.

and moreover the numbers are shown in compromising positions:



masculine, octohedron and icosahedron feminine, because latter inscribable in former; tetrahedron androgynous, because inscribable in itself), 326–7 (Earth male, Venus female). On p. 176 is a truly remarkable analogy, which shows what a sensitive musician Kepler was. At a time when no other writer on music even remotely approaches the concept of a leading note, he clearly expresses the feeling of expectation produced by semitones. The major third, says Kepler, is masculine because, when singing e.g. C D E, one feels an urge to overcome the semitone and reach the fourth, F; this third, then, is 'active and full of efforts' ('actuosa et conatuum plena'), its generative power ('vis γόνιμος') is striving to reach the fourth, and when the semitone is sung and the fourth reached this is like an ejaculation ('quaerens finem suum, scilicet Diatessaron, cujus semitonium est ei [sc. tertiae durae] quasi ἔκχυσις, toto conatu quaesita'). The minor third is feminine because, when singing e.g. D E F, one feels a tendency to sink back a semitone on to E; this third therefore is passive, and is always sinking to the ground, like a hen ready to be mounted by a cock ('semper se, veluti gallina, sternit humi, promptam insessori gallo'). This is not of course identical with the modern concept of a leading-note, since Kepler is thinking here in purely melodic terms.

This letter,[77] as Caspar points out, is important for an understanding of Kepler's attitude to analogies or symbols. He had been sent by Tanckius a work on the monochord by Andreas Reinhard (*Monochordum*, Leipzig 1604),[78] which contained some sexual analogies. Having commented playfully on these, Kepler goes on to say that 'by this titillation' Reinhard has excited him to vie with him in finding symbols of male and female; and then, after the long passage on the golden section, 'which the lecherous feelings roused by Reinhard's speculations had forced out of him',[79] he states:[80]

> I too play with symbols, and have planned a little work, Geometric Cabala, which is about the Ideas of natural things in geometry; but I play in such a way that I do not forget that I am playing. For nothing is proved by symbols, nothing hidden is discovered in natural philosophy through geometric symbols; things already known are merely fitted [to them]; unless by sure reasons it can be demonstrated that they are not merely symbolic but are descriptions of the ways in which the two things [i.e. the two terms of the analogy] are connected and of the causes of this connexion.

He gives as an example of a geometric analogy which is also a causal explanation his theory of the weather. Bad weather accompanies certain planetary aspects because there is a Soul of the Earth or *Archeus Subterraneus* which is capable of perceiving geometric relationships, in this case, the angles formed by planetary rays meeting on the Earth, and is thereby excited to expel 'subterranean humours'. Thus 'the geometry of the aspects becomes an objective cause',[81] whereas it would be useless to rely on such 'symbolisations' as that Saturn brings snow, Mars thunder, Jupiter rain, etc.

When in this letter Kepler is expounding the analogy between the golden section and generation, he may be only 'playing and not forgetting that he is playing'. But I think there is little doubt that by the time of the *Harmonice Mundi* he is convinced that this analogy shows also a real causal connexion. Such a deeply pious man would not write in jest that 'creator Deus' has fitted the modes of vegetable and animal generation to the archetypical figure of the pentagon. Polyphonic music, with its thirds and sixths, excites and moves us deeply as does sexual intercourse because God has modelled both on the same geometric archetype. There are also, as we have seen and shall see, other archetypical causes of the emotive power of music: the connexions between our music and the celestial harmonies.

I shall not here give a general description of Kepler's celestial music,

[77] Kepler, *Ges. Werke*, xvi, pp. 154ff.

[78] I have not been able to see this work, as the British Museum copy has been destroyed.

[79] Kepler, *ibid.*, p. 158, 'Atque hic excursus esto, quem mihi extorsit prurigo a Reinhardi speculationibus concitata'.

[80] *Ibid.*, 'Ludo quippe et ego Symbolis, et opusculum institui, Cabalam Geometricam, quae est de Ideis rerum Naturalium in Geometria: sed ita ludo, ut me ludere non obliviscar. Nihil enim probatur Symbolis, nihil abstrusi eruitur in Naturali philosophia, per Symbolas geometricas, tantum ante nota accommodantur: nisi certis rationibus evincatur, non tantum esse Symbolica sed esse descriptos connexionis rei utriusque modos et causa.'

[81] *Ibid.*, 'geometria aspectuum fit causa objectiva'.

since this has already been done by several modern scholars.[82] I wish merely to discuss a few problems raised by it and aspects of it which have not, I think, been dealt with before.

There are some difficulties in Kepler's planetary chords that are due, at least in part, to his use of the musical terms *durus* and *mollis*. In Book III he describes the two genera, *molle* and *durum*, in such a way that they seem to be the same as our minor and major modes or tonalities. We are therefore disconcerted when we come to the planetary harmonies in Book V to find that the two chords of all six planets 'generis duri' consist of an E minor $\frac{6}{3}$ and a C major $\frac{6}{4}$, and those 'generis mollis' of an E flat major $\frac{6}{3}$ and a C minor $\frac{6}{4}$; the two chords, *durum* and *molle*, for five planets (Venus omitted), on the other hand, are what one would expect: a G major $\frac{5}{3}$ and a G minor $\frac{5}{3}$:[83]

Harmoniae Planetarum Omnium | *Harmoniae Planetarum Quinque*

Generis Duri | *Generis Mollis* | *Generis Duri* | *Generis Mollis*

I think that here Kepler is using the terms in their original sense simply to mean respectively any scale or chord which contains B sharp (*durum*) or which contains B flat (*molle*); this is not surprising, since the theoretical distinction of our major and minor modes was only just beginning to emerge at the time he was writing. The inclusion of a $\frac{6}{4}$ chord is odder, since in practice this

[82] W. Harburger, *Johannes Keplers Kosmische Harmonie*, Leipzig 1925; A. Koyré, *La Révolution astronomique* (hereafter *Rév. astr.*), Paris 1961, pp. 328–45; Caspar's *Nachbericht* to the *Harm. Mund.* (Kepler, *Ges. Werke*, vi, pp. 461ff.

[83] Kepler, *Ges. Werke*, vi, pp. 325–7.

chord was treated as a dissonance and was banned by most theorists—but not by all. Zarlino, though of course aware of current practice, gives a defence of the fourth as a consonance,[84] and Kepler had read Zarlino.[85] In his earlier attempts to find harmonies in the orbital speeds of planets Kepler also gives $\frac{6}{4}$ chords. In one letter he notes that modern musicians may object to the fourth instead of the fifth being at the bottom of the chord, and says that he has answers to this objection; but he does not unfortunately give them.[86] Since he relied so much on the judgement of his ear, he may well just have observed that $\frac{6}{4}$ chords are nearly as sweet as $\frac{5}{3}$ ones, and considerably sweeter than $\frac{6}{3}$ ones. It is indeed a still not fully explained mystery of musical history how the $\frac{6}{4}$ came to be treated as a dissonance.

In any case, we are not justified in expecting the celestial harmonies to be exactly the same as our music. As I have already mentioned, Kepler makes it clear that our music is not an imitation of celestial music; their relationship is that of two independent products of the same geometric archetypes. Since Kepler does not stress the differences between the two—quite naturally, since he is interested in their similarities—it may make their relationship clearer, if I point out two of the most important of these differences.

Each planet has its own scale, determined by its extreme speeds (at aphelion and perihelion).[87] Saturn and Mercury, for example, have respectively:[88]

4 *octaves higher*

But, as Kepler points out, their passage from the lowest to the highest note and back again is not really articulated into steps of tone and semitone, as in a musical scale, but represents a continuous acceleration and deceleration of the planet's speed, so that, if they actually emitted sounds (which they do

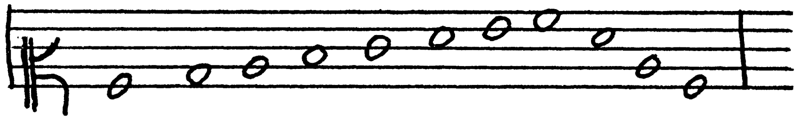
[84] Zarlino, *Istitutioni Harmoniche*, pt. iii, c. v, lx (*Tutte l'Opere*, Venice 1589, pp. 186–8, 302–4). Zarlino, in recommending the use of major $\frac{6}{4}$ chords, relies chiefly on the judgement of the ear. He cites an earlier work in defence of the fourth: Andreas Papinus, *De Consonantiis, seu pro Diatessaron Libri Duo*, Antwerp 1581. This is a long book; it ends with two musical examples to illustrate the author's views, the first of which must be unique in Western music in that its final chord is a major $\frac{6}{4}$.

[85] *Vide supra*, p. 230 n. 12.

[86] Kepler, *Ges. Werke*, xiv, p. 52; cf. *ibid.*, p. 27.

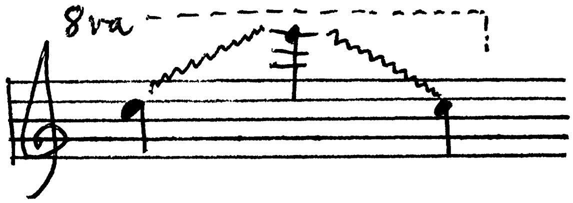
[87] Kepler, *Ges. Werke*, vi, p. 322.

[88] The original has for Mercury's scale:



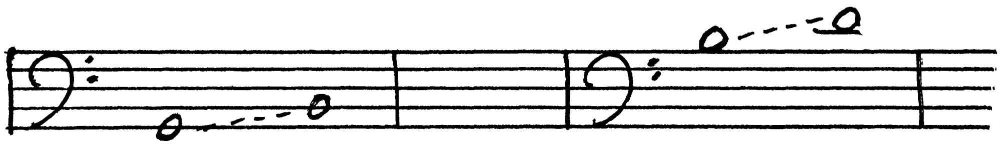
I am assuming that the C clef should be on the bottom line, as the Frisch edition gives it, and as Koyré transcribes it (*Rév. astr.*, p. 339). This emendation is necessary for Mercury's scale to fit into the planetary chords, and it fits Kepler's planetary speeds better, according to which the highest note of Mercury should be a major sixth and seven octaves above Saturn's lowest note (see Koyré, *ibid.*). On the other hand, Kepler states that Mercury's scale begins on A (Kepler, *Ges. Werke*, vi, pp. 321–2), and gives its compass as an octave and a minor third (*ibid.*, p. 312).

not), their 'scales' would sound like a siren giving an air-raid warning. Mercury's scale then would be like

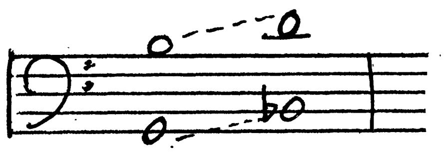


played with one finger on a violin.

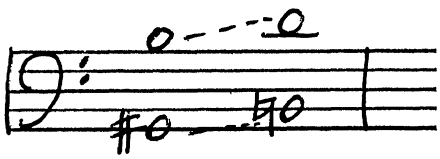
The chords which two or more planets can make are determined by the intervals comprised by their scales and the intervals between these scales.[89] Saturn and Jupiter, for example, have respectively:



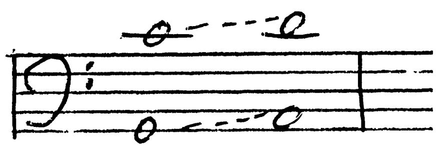
and can produce major tenths between



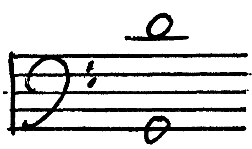
minor tenths between



elevenths between

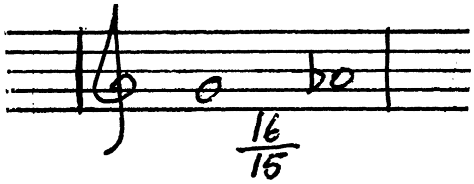


and one twelfth

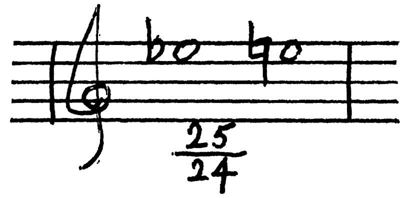


But with chords of more notes the possibilities become progressively more limited. Since the Earth and Venus have very narrow ranges:

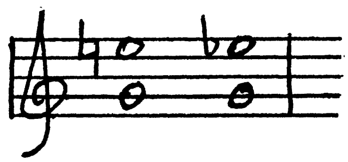
[89] Kepler, *ibid.*, pp. 314–16.



and



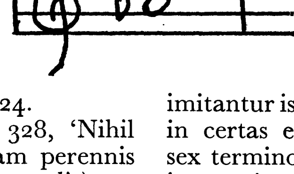
respectively, they can combine with other planets in only a very restricted number of ways. It is these two planets, then, which determine the four possible chords of all six planets, which must all contain the major or minor sixths:[90]



These six-planet chords, already given above, can evidently occur only at very long intervals of time. Kepler doubts whether any of them can yet have happened twice, and conjectures that perhaps it was only at the moment of creation that such a perfectly harmonious combination of all six planets occurred.[91] One may also, I think, suppose that Kepler believed that at the Last Day the heavens would, before their music ceased for ever, once again sound a perfect chord. In any case, here is another basic difference between earthly and heavenly music. In the latter, the unique piece of divine music begins with a concord, passes through an immense series of dissonances, which are only finally resolved (perhaps) on the final chord; whereas (at least in Kepler's time) human polyphony consists largely of concords, and dissonances are rapidly resolved. Kinds of human music which come near to Kepler's heavenly music would be: the *cadenza* of a classical concerto, which is a very long interpolation between a $\frac{6}{4}$ chord and its $\frac{5}{3}$ resolution, or a piece written entirely on a pedal-note (e.g. one of Bach's *Musettes*).

When, however, Kepler himself describes the likeness between earthly and heavenly music, he evidently assumes that there will be more than only one or two perfectly consonant chords in the whole course of the world's history:[92]

[90] I do not know why Kepler did not use the possibility:



[91] Kepler, *Ges. Werke*, vi, p. 324.

[92] Kepler, *Ges. Werke*, vi, p. 328, 'Nihil igitur sunt motus coelorum, quàm perennis quidam concentus (rationalis non vocalis) per dissonantes tensiones, veluti quasdam Syncopationes vel Cadentias (quibus homines imitantur istas dissonantias naturales) tendens in certas et praescriptas clausulas, singulas sex terminorum (veluti Vocum) ijsque Notis immensitatem Temporis insigniens et distinguens; ut mirum amplius non sit, tandem inventam esse ab Homine, Creatoris sui

> The motions of the heavens, therefore, are nothing else but a perennial concert (rational not vocal) tending, through dissonances, through as it were certain suspensions or cadential formulae[93] (by which men imitate those natural [i.e. celestial] dissonances), towards definite and prescribed cadences[93], each chord being of six terms (as of six voices), and by these marks [sc. the cadences] distinguishing and articulating the immensity of time; so that it is no longer a marvel that at last this way of singing in several parts, unknown to the ancients, should have been invented by Man, the Ape of his Creator; that, namely, he should, by the artificial symphony of several voices, play out, in a brief portion of an hour, the perpetuity of the whole duration of the world, and should to some degree taste of God the Creator's satisfaction in His own works, with a most intensely sweet pleasure gained from this Music that imitates God.

Once again, I wish to emphasize that this comparison is not only a metaphor; through the geometrical archetypes there is a real causal connexion between the two polyphonies, a connexion which accounts for their likeness. By this causal analogy between human music and planetary movements, and between music and sexual desire, Kepler gives to music a meaning and value that had not previously been attributed to it, a meaning which only polyphonic music, unknown to the ancients, could possibly have. The marvellous effects of music, emotional, moral and religious, are of course familiar enough; but in that tradition it was always music together with words that produced the 'effects', and it was always the words that bore the specific meaning, that determined the particular effect. That music alone could have a precise and profound meaning, was, I think, in Kepler's time an entirely novel idea. It is an idea that we have all come to accept, and, although we may find Kepler's explanation of it unconvincing, we cannot claim to have found a better one.

Simiâ, rationem canendi per concentum, ignotam veteribus; ut scilicet totius Temporis mundani perpetuitatem in brevi aliqua Horae parte, per artificiosam plurium vocum symphoniam luderet, Deique Opificis complacentiam in operibus suis, suavissimo sensu voluptatis, ex hac Dei imitatrice Musicâ perceptae, quadamtenus degustaret.'

[93] Kepler, like Calvisius, uses the term 'clausula' for cadence. The only time he uses the term 'cadentia' (*ibid.*, p. 182) is when discussing suspensions; he suggests that the word 'cadentia' derives from the fact that the dissonant suspended note *falls* to its resolution. In this passage, therefore, I believe that, in coupling 'cadentiae' with suspensions, Kepler was thinking of the regular $\frac{5}{4}$–$\frac{5}{3}$ suspensions at perfect cadences.