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Kepler's Harmony of the World: A Realization for the Ear: Three and a half centuries after their conception, Kepler's data plotting the harmonic movement of the planets have been realized in sound with the help of modern astronomical knowledge and a computer-sound synthesizer
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John Rodgers
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Kepler's Harmony of the World: A Realization for the Ear

Three and a half centuries after their conception, Kepler's data plotting the harmonic movement of the planets have been realized in sound with the help of modern astronomical knowledge and a computer–sound synthesizer

The heavenly motions are nothing but a continuous song for several voices, to be perceived by the intellect, not by the ear; a music which, through discordant tensions, through syncopations and cadenzas as it were, progresses toward certain predesigned six-voiced cadences, and thereby sets landmarks in the immeasurable flow of time.

Johannes Kepler, *Harmonices Mundi*, Book V, Chapter 7

There is no news in the association of music with the heavens and the heavenly bodies. It is one of man's oldest metaphysical ideas: we have evidence of this mythical linkage from scriptures, poets, philosophers, and scientists, not only among peoples who write their histories but among oral cultures who similarly imagine that moving bodies in the sky thump, roar, and sing.

It is new, however, when a scientist of the heavens lives a life that inspires a major figure in the musical world to celebrate that life in music. As the subject for his last and longest opera, Paul Hindemith chose the life of Johannes Kepler (Fig. 1), the first figure in the whole history of science whose life has been so treated. Hindemith (Fig. 2) named his opera *Die Harmonie der Welt* from the title of one of Kepler's major works, and in it he chronicles musically the somber and even desperate life Kepler lived in the bizarre circumstances of Germany before and during the Thirty Years War.

Clearly Hindemith was attracted to Kepler's story in good part because he saw deep parallels between the social and political climate of their native Germany in Kepler's time and in his own, parallels that extend to their respective exiles. Hindemith was exiled because of the disfavor he incurred in the Third Reich, first for his "decadent non-Aryan music," and second for his insistence on making music with the best musicians he could find, whatever their "race" (he played the viola in a famous string trio with Szymon Goldberg, violinist, and Emmanuel Feuermann, cellist). Kepler was exiled because he was a Protestant caught in the conflict of the Thirty Years War.

But Hindemith was also attracted by the motive forces behind Kepler's life and his extraordinary accomplishment: a strong inner conviction that planetary behavior must be rational, governed by simple analogies and laws; an iron scientific discipline that made Kepler search for decades for ways to explain, and not merely explain away, the uncomfortable discrepancies between accepted astronomical ideas and the actual astronomical data; and an intense, indeed religious, devotion to these investigations because they would lead him to understand how (and perhaps why) God had put the universe together in exactly the form we observe.

Ruff first became aware of Kepler's ideas in Hindemith's class in the history of music theory at Yale in 1952. Later, he and Rodgers explored both the musical and scientific implications of Kepler's data and found that they are inseparable and now, at long last, realizable. Kepler had thrown a challenge to the musicians of his own day—to set to a sacred text the musical march of the heavens, whose harmonies he could demonstrate in numbers—but there is no evidence that the challenge was ever met. The authors were led to pick up this challenge three and a half centuries later, because the arrival of the computer age at last made possible the realization for the ear of the "song for several voices" that Kepler could hear only in the intellect.

Early Greek views

In the context of western European culture, the concept that the world has a harmony, that the planets make music, goes back to the early Greek philosophers, to Pythagoras in the sixth century B.C., or at least to his followers in the next century and a half. He, or they, discovered the extraordinary fact that musical concords, like those we now call the octave, the fifth, or the major third, represent simple but exact numerical ratios like 2:1, 3:2, or 4:3. From this and other phenomena, they deduced that all things are fundamentally numerical, to be understood by, or as, number, and above all that the majestic and regular cycles of the heavenly bodies embody number and therefore make perfect music, far purer than that produced by our im-

John Rodgers and Willie Ruff are both members of the faculty of Yale University. Rodgers is Silliman Professor of Geology and an enthusiastic pianist. A member of the Mitchell-Ruff Duo, Ruff plays the French horn and bass and is Associate Professor of Music. They came to Kepler and his "harmony of the world" through Hindemith and his opera Die Harmonie der Welt. *Studying the score led Ruff to look into Kepler's work and to discover the musical notation he had written down for the music of the planets. The computer program by which the astronomical data were converted into sounds was written by Mark Rosenberg, using the Music 4BF program developed at Bell Laboratories and the IBM 360/91 computer at the Princeton University Computer Center. A long-playing record of the results has been prepared by the authors; inquiries should be sent to Professor Ruff, School of Music, Yale University, New Haven, CT 06520.*

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perfect strings, reeds, and horns here on earth. (Of course some of these ideas go back beyond the Greeks; certainly the contrast between the heavens and the earth, the perfect and the imperfect, the seat of God and the seat of man, is also Judaic, and it may well be older.)

Furthermore the early Greeks reasoned that, because the heavens are perfect, the heavenly revolutions must exhibit the perfect form, the circle, and the motions must be steady, at a constant velocity. It follows that each heavenly body—Mercury, Venus, Earth, Mars, Jupiter, Saturn, and the sun, seven in all or, if the "fixed" stars are included, eight—sings a single unchanging note, the combination of these notes being the heavenly harmony.

The word *harmony* comes to us from the Greeks, but we must remember that what they meant by it was somewhat different from what we mean today. At first (in Homer) it meant a proper joining, a good job of morticing timber or a valid contract; as later applied to music, it should be translated *tuning*. A well-tuned lyre has harmony, an ill-tuned not; naturally the heavens are the best tuned of all. It seems probable that the Pythagoreans thought in terms of tuning in the numerical concords they had discovered, but this is not certain.

Because the *average* motions of Venus and Mercury, as seen from the earth, are the same as that of the sun, perhaps these three would all sing the same note; hence only five or six notes would be needed. This and other difficulties show how imperfectly we understand much of the Greek theory. Moreover, the phrase "harmony of the spheres," or "music of the spheres," refers only to a rather late form of the theory, when the planets were thought to be attached to transparent, rotating spheres; before that they were sometimes supposed to be carried by material circles, perhaps like hollow, fire-filled hoops with one hole through which the light shone out. Just as it is not certain that the Pythagoreans' notion of tuning was in terms of their numerical concords, so too are we uncertain whether the highest note was that of the stars and the lowest that of the moon, which moves the slowest if the earth is absolutely fixed, or vice versa, if the earth in fact rotates and it is the stars

Figure 1. Johannes Kepler (1571–1630), the founder of modern astronomy, believed that each planet produced its own distinctive musical note and that the regular movement of the planets created a "celestial music," or harmony, reflecting God's perfection in ordering the universe. (From Kepler, *Collected Works*.)

Figure 2. Paul Hindemith (1895–1963), who based his longest opera—*Die Harmonie der Welt*—on the life of Kepler, wrote: "The science of music deals with the proportions objects assume in their quantitative and spatial, but also in their biological and spiritual, relations. Kepler's three basic laws of planetary motion . . . could perhaps not have been discovered without a serious backing in music theory. It may well be that the last word concerning the interdependence of music and the exact sciences has not been spoken." (Hindemith 1952).

that are fixed (the idea that the earth rotates is probably later than the Pythagoreans).

As seen from our imperfect earth, however, the planets certainly do not move with steady velocities or in unchanging circles relative to the fixed stars. The sun comes the closest, marking out the circle called the ecliptic, but the moon's motion appears quite irregular, and the other planets even stop dead in the sky and retrace their steps from time to time. The timing of these irregularities is itself quite regular, however, and since it was clear to the Pythagoreans and their followers that God's heavenly bodies must move perfectly—that is, in circles at constant velocities—much ingenuity was expended in calculating what combinations of circular motions would best "save the appearances."

Having the earth rotate daily helped somewhat, and Aristarchus's hypothesis (third century B.C.) that the sun and not the earth is in or near the center of the circles (except that of the moon) helped a great deal. But even so, the "appearances" still required rather complicated combinations of circles, and the last great astronomers of the ancient world, Hipparchus (second century B.C.) and Ptolemy (second century A.D.), rejected both these hypotheses as contrary to the plain evidence of our senses that the earth is stationary. When Copernicus in the fifteenth century revived the sun-centered hypothesis, he did so precisely because that made it easier for him to believe that the planets move in circles.

Kepler, who was born on 27 December 1571 and died 15 November 1630 (in the Julian calendar), was the heir to all these ideas. He too believed that the planets move in circles—around the sun, for he was a convinced Copernican—and, in possession of by far the best planetary observations made up to that time (those of his predecessor as Imperial Astronomer, Tycho Brahe), he set out to prove it. But the more he struggled with the data and the calculations, the more difficult it became to fit them to the circular hypothesis. The idea of constant velocity had to go first, and finally he was forced against all his convictions to the idea of some sort of oval. Finally he tried the ellipse, and at once all the data fell into place. For some reason God, in creating the world, had chosen to make the planets move not in circles but in ellipses, at variable, although regularly variable and readily calculable, velocities.

The shock of this idea at that time

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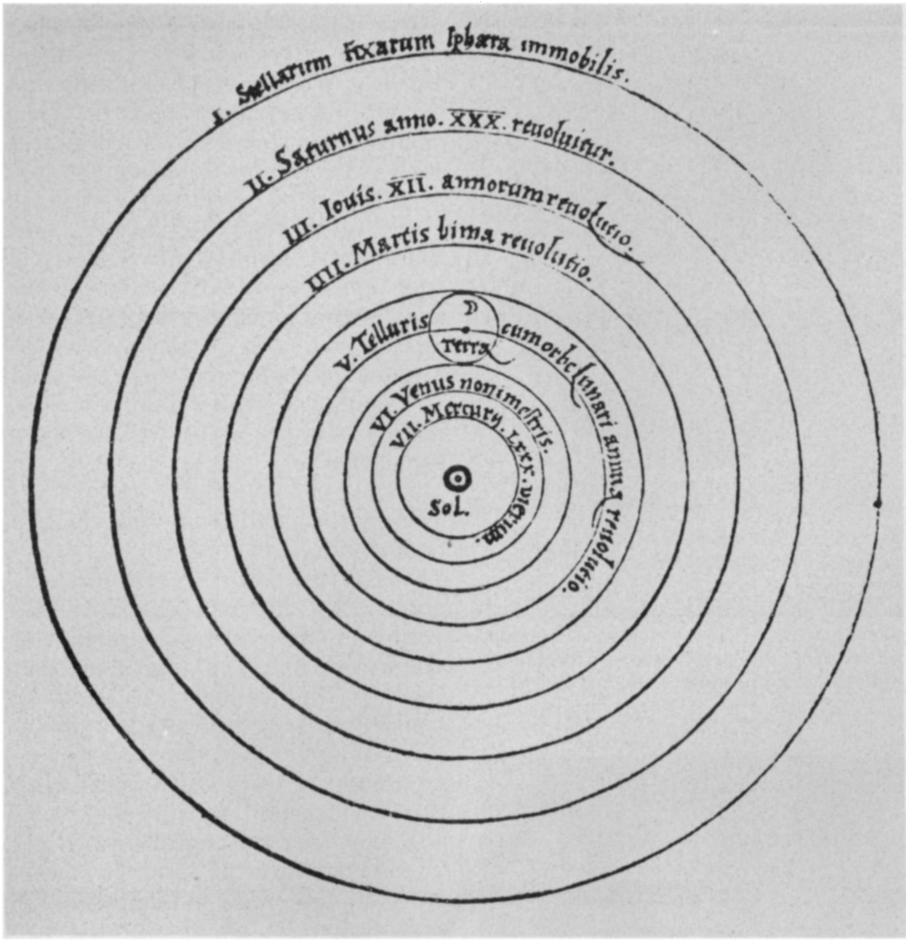

Figure 3. Kepler's ideas about the movements of the planets were based on the sun-centered planetary system of Copernicus, shown here in a drawing from *On Revolutions*, Book 6, published by Copernicus in 1543, the year of his death. In this schematic figure, Copernicus showed the orbits as circular; Kepler discovered that they were actually elliptical. (Courtesy of The Beinecke Rare Book and Manuscript Library, Yale University.)

can hardly be imagined today. Why should God deliberately have chosen eccentric and variable motion when he could have ordered the planets so much more simply and "perfectly"? Kepler was sure there must be some good reason, and he set himself to find it. And here the ancient idea of a harmony produced by the planets came to his assistance. A musician himself, steeped in the music of his time, which was the high point of the great polyphonic vocal tradition exemplified by Palestrina and Vittoria, he hit on the idea that, as the velocities of the planets vary, so the notes they sing must vary too, and perhaps God chose to have them vary in the particular way they do precisely in order to create a celestial six-part harmony. (The six parts are the earth and the five visible planets; as the moon revolves around the earth, it is not part of the main scheme, and Kepler knew that the stars are "fixed"; see Fig. 3).

Feverishly he sought the clue; he tried ordinary linear velocity and some other variables and got only discord. But when he tried angular velocity, the rate at which the planets would appear to move through the heavens if viewed from the sun, it seemed to him that he had found the harmony God had created, that he had, as he himself put it, "seen into the mind of the Creator." If the notes the planets sing are proportional to their constantly but regularly changing singular velocities, then the "harmony of the world" is the sounding together of these six changing voices, a sublime celestial motet of which our earthbound music is only a pale reflection, just as to the Greeks the heavens were the perfectly tuned lyre.

Kepler's laws

Kepler's astronomical insights are summed up in his three great laws (although he himself did not call them that, nor did he think them as important as his musical discovery). The first law states that the orbit of each planet around the sun is an ellipse, and that the sun is not in the center but at one focus of the ellipse. An ellipse and its two foci can be easily understood by the simple construction shown in Figure 4.

If we take a pencil and a piece of string of length $2a$, hold the two ends of the string together at a single point O, place the point of the pencil P inside the loop of the string, and pull the string taut with the pencil point, then by moving the pencil over paper we will trace a circle whose radius (O–P, or a) is half the length of the string (the diameter A–A', the longest straight line that can be drawn inside the circle, is equal to $2a$). But if we hold the two ends of the string at two *different* points, F and F', and trace out a curve with the point of the pencil P (the string always being kept taut), the curve will be an ellipse, of which the two points F and F' are called the foci. The line passing through the two foci and extending out to the ellipse itself (A–A') is the major axis, the longest straight line that can be drawn inside the ellipse (and, as with the circle, equal to the length of the string, $2a$); its midpoint O is the center of the ellipse; and the line through O perpendicular to the major axis (B–B') is the minor axis, the smallest diameter of the ellipse. The length A–O is a (half the major axis), and symmetry and the original construction demand that B–F and B–F' also be equal to a. The length B–O is defined as b (half the minor axis), and the length F–O is defined as f (the focal distance).

By the theorem still called Pythagorean, $a^2 = b^2 + f^2$. Obviously the degree of flattening of the ellipse $(a - b)/a$, how much it differs from a circle, depends on f or rather on its ratio to a. In order to compare the shapes of large and small ellipses (ellipses with different a), it is convenient to define the ratio f/a as e (the eccentricity). Clearly if f and hence e are zero, F and F' coincide with O, and the "ellipse" is a circle; whereas if e should equal one ($f = a$), the ellipse would be flattened to a straight line, $2a$ in length. (If any two of the four quantities, a, b, e, and f, are known, the two others can be calculated.)

Thus from Kepler's first law it follows

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that: (1) a planet's mean distance from the sun (which is at F) is a (when the planet is at B or B'), (2) its minimum distance (perihelion) is $a - f = a\ (1-e)$ (when the planet is at A), (3) its maximum distance (aphelion) is $a + f = a\ (1+e)$ (when the planet is at A'). The mean distance here is the arithmetic mean between the extremes. Furthermore, if one defines t as the time elapsed since the planet was last at perihelion, expressed as 360ths of the planet's period of revolution around the sun, p (in years), then

$$d = \frac{a(1 + e \cos t)(1 - e \cos t)^2}{(1 - e^2)}$$

(t as the argument of the cosine should be taken in degrees).

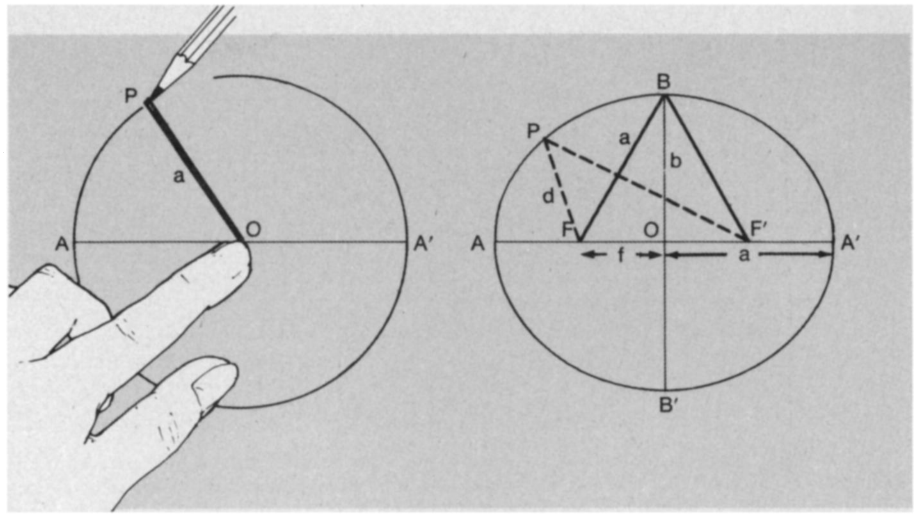

Figure 4. Both the circle (*left*) and the ellipse (*right*) were constructed with a piece of string of length $2a$. The ellipse has an eccentricity of one-half, which is twice that of the orbit of any planet; ellipses with eccentricities of less than one-quarter are flattened only 3% or less and are difficult for the eye to distinguish from circles.

Kepler's second law is a little more abstract (Fig. 5). If a planet P is moving around the sun, which by the first law occupies one focus F of the planet's elliptical orbit, we can imagine the line that connects the planet with the sun (e.g. P–F) sweeping out an area of the ellipse. The second law states that the areas swept out by this line in any two equal periods of time are equal (for example, the two shaded areas in Fig. 5). For this to be true, the planet must move faster when it is nearer the sun (i.e. in the half of the ellipse toward A) than when it is farther away (in the half toward A'). Thus, although the velocity of the planet is variable, it varies according to a strict rule. As Kepler knew, there is more than one kind of velocity, but all are governed by this law. The linear velocity v (in kilometers per second, for example) varies proportionally as follows:

$$v_{\text{max}}\ (\text{at } A) \sim \frac{1}{\sqrt{a}(1-e)}$$

$$v_{\text{mean}}\ (\text{at } B \text{ or } B') \sim \frac{1}{\sqrt{a}}$$

$$v_{\text{min}}\ (\text{at } A') \sim \frac{1}{\sqrt{a}(1+e)}$$

The mean is now the *harmonic* mean between the extremes.

The *angular* velocity ω, measured from the sun at F, varies as follows:

$$\omega_{\text{max}}\ (\text{at } A) \sim \frac{1}{a^{3/2}(1-e)^2}$$

$$\omega_{\text{mean}}\ (\text{at } B \text{ or } B') \sim \frac{1}{a^{3/2}}$$

$$\omega_{\text{min}}\ (\text{at } A') \sim \frac{1}{a^{3/2}(1+e)^2}$$

In terms of t,

$$\omega \sim \frac{1}{a^{3/2}}\left(\frac{(1-e^2)}{(1 + e \cos t)(1 - e \cos t)^2}\right)^2$$

Kepler's third law, sometimes called the harmonic law, states that the square of a planet's period of revolution around the sun, p, is exactly proportional to the cube of its mean distance from the sun, a. Thus

$$p^2 \sim a^3, \text{ or } p \sim a^{3/2}$$

We can therefore substitute p for $a^{3/2}$ in the expressions for ω given above, which then become equations if p is expressed in years and ω in revolutions per year or in degrees per 1/360 year, as follows:

$$\omega_{\text{max}}\ (\text{at } A) = \frac{1}{p(1-e)^2}$$

$$\omega_{\text{mean}}\ (\text{at } B \text{ or } B') = \frac{1}{p}$$

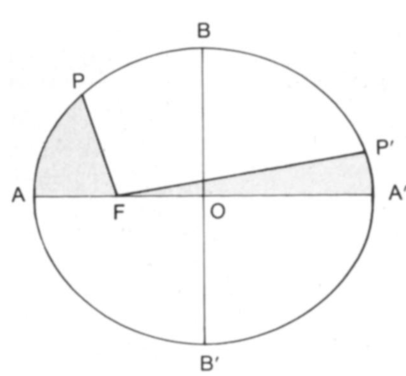

Figure 5. This ellipse illustrates Kepler's second law—that the area FAP equals the area $FA'P'$; hence, because FA is less than FA', PA is greater than $P'A'$.

$$\omega_{\text{min}}\ (\text{at } A') = \frac{1}{p(1+e)^2}$$

The mean is the square of the harmonic mean between the square roots of the extremes. In terms of t,

$$\omega = \frac{1}{p}\left(\frac{(1-e^2)}{(1 + e \cos t)(1 - e \cos t)^2}\right)^2$$

Reconstructing Kepler's "harmony of the world"

If now we assume with Kepler that the planets obey his laws and that the notes they sing are proportional to their angular velocities around the sun, then all we need to reconstruct Kepler's harmony of the world is, for each planet, p, the period of revolution around the sun; e, the eccentricity of its orbit; and t', some measure of how much of its revolution the planet, at some particular moment in time, has covered since it was last at its perihelion (when it was singing its highest note), so that its song can be correlated to real time and to the songs of other planets. Data for p and e for the planets (and also for maximum, mean, and minimum a and v) are available today in published tables to a degree of accuracy undreamed of in Kepler's day (see Table 1). For the particular starting point in time, we chose 27 December 1571 (Julian), the day Kepler was born.

In order to convert these mathematical data into sounds whose changes the human ear can follow and so to bring the immensity of astronomical space and time down to a human

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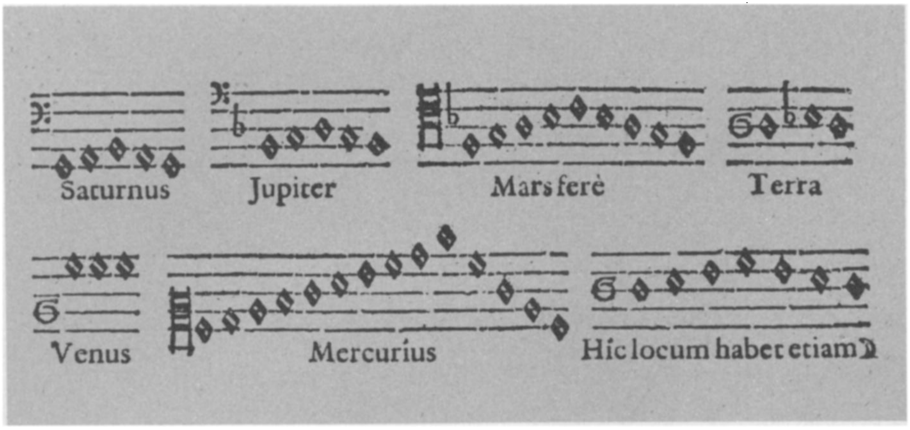

Figure 6. These notations for the pitches of the planets were described by Kepler as "the single movements [of the planets] in the familiar terms of notes. They do not form articulately the intermediate positions, which you see here filled by notes, as they do the extremes, because they struggle from one extreme to the opposite not by leaps and intervals but by a continuum of tunings and actually traverse all the means (which are potentially infinite)—which cannot be expressed by me in any other way than by a continuous series of intermediate notes." (From Johannes Kepler, *Harmonices Mundi.*)

level, we had to make several decisions, some of them rather arbitrary. For pitch, expressing angular velocity, Kepler had already made the conversion (Fig. 6): he had carefully chosen a very low G (G_2) for the lowest note of Saturn (the G next below the bottom of our present piano keyboard). The three laws then fix all the other pitches; the highest note of Mercury becomes the E (e''''') next above the top of the present keyboard, and the total range of the visible planets, including the earth, is nearly 8 octaves. Indeed, the range is approaching the limit of human hearing at either end.

The formulas presented above, operating upon the data in the table, give the variations of angular velocity through time in terms of revolutions per year; as the earth's period is exactly one year, its mean angular velocity is exactly one revolution (360°) per year. To simplify the calculations, we chose 800 vibrations per second to represent that angular velocity. What low G is in vibrations per second then depends only on how the musical scale is tuned; with Earth's note so fixed, Saturn's low note would be exactly the G Kepler chose for it if tuning A (a′) is 439.74 vibrations per second and the tuning is harmonic (the ratio of A to G is 9:8), or if tuning A is 438.75 vibrations per second and the tuning is well tempered (the ratio of A to G is $2^{1/6}$:1). (Today, tuning A is generally 440 vibrations per second, but in Kepler's time it was at least half a tone lower, perhaps around 413.)

We had also to alter the time scale. It would be pointless for us to listen to the songs of the planets in real time, for even Mercury takes 88 days to go from high pitch to low pitch and back again, and Saturn takes 29 years. We chose to let 5 seconds simulate one real year (a sidereal year, which contains 31,558,150 seconds), and thus the ratio is about 1 to 6 million. When the chosen ratios for time and pitch are combined, it turns out that in our realization each vibration representing Earth at mean velocity lasts 1/4,000 of a simulated year—about 2 simulated hours—and that each vibration for Mercury at maximum velocity lasts about 20 simulated minutes, and for Saturn at minimum velocity about 3 simulated days.

We had to calculate a function to govern the rate of change of pitch from high to low and back again as each planet swings around in its elliptical orbit. What is called "simple harmonic motion," governed by a cosine function, would give a reasonable first approximation, quite adequate for the least eccentric (most nearly circular) planetary orbits (such as Venus and Earth), but for the more eccentric ones (such as Mercury, Mars, Jupiter, and Saturn) the error in pitch is considerable at the middle of the rapid descent or ascent between the extreme pitches, reaching almost a major third for Mercury. For these planets we used the more complicated formulas, given above, that relate the angular velocity ω to t, the time since last perihelion; then the pitch remains close to its highest point for rather less time, and close to its lowest point for rather more, than it would in "simple harmonic motion."

Since Kepler's day, three additional planets have been discovered in the solar system. All are subject to Kepler's laws, as indeed they must be because of Newton's law of gravitation, the discovery of which, half a century later, would have been impossible without the knowledge of Kepler's laws, as Newton freely acknowledged. (Newton also destroyed, once and for all, the belief that the heavens obey different laws from the earth and what is on it.) It is no coincidence, given Kepler's preference for angular rather than linear velocity and his choice of pitches to represent it, that the planets he did not see he also would not have heard as musical tones.

One could of course choose the pitch so as to bring the new planets into the tone range, but then the inner or "terrestrial" planets, as far as Mars, would become inaudible at the other end of the scale. Or one could switch from angular to linear velocity and thus bring all the planets within a range of 4 octaves, but the variations for the individual planets would then be much less, about half as much. For the less eccentric, the variation would be barely perceptible to the ear, and the resulting sounds would be much less concordant, as Kepler understood when he rejected linear velocity in the first place. Furthermore, the periods of the newly discovered planets are all much longer than Saturn's, which is already 2½ minutes in our realization. Pluto, whose period is 248 years, would take over 20 minutes, and, despite its large eccentricity, its changes of pitch would be too slow to hear. It seems best therefore to remain with Kepler's choices, in which case the new planets would vibrate only two to ten times per second; in other words, they are not part of the vocal polyphony but of the *rhythm* section.

A more serious question arises out of the actual perturbations in the planetary motions from those that would be predicted by the use of Kepler's laws alone. Those laws express the influence of the sun on the motions of the rest of the "world," or solar system, as Kepler indeed understood, and they enabled Newton to advance

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Table 1. Data used in realizing Kepler's harmony of the world

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Basic data									
p (years)	0.240899	0.615185	*1.0* ∗	1.88082	11.8613	29.4568	84.0081	164.784	248.35
e	0.2056	0.0068	0.0167	0.0934	0.0484	0.0543	0.0460	0.0082	0.2481
Angular velocities (calculated, in revolutions per year)									
ω_{max}	6.578	1.648	1.034	0.6469	0.09310	0.03796	0.1308	0.006169	0.007122
ω_{mean}	4.15112	1.62557	*1.0*	0.531683	0.0843078	0.0339480	0.0119036	0.00606855	0.0042658
ω_{min}	2.856	1.604	0.9674	0.4473	0.07670	0.03054	0.01088	0.005970	0.002585
Figures used in the realization									
Period (seconds)	1.2046	3.076	*5.0*	9.404	59.306	147.284 (= 4 min., 22.3 sec.)	420.04 (= 7 min.)	823.92 (= 13 min., 44 sec.)	1241.75 (= 20 min., 42 sec.)
Pitch (vibrations per second)									
High	5262	1318	827.4	517.5	74.48	30.37	10.463	4.935	5.698
Mean	3321	1300	*800*	425.3	67.49	27.16	9.523	4.855	3.221
Low	2285	1283	773.9	355.8	61.36	24.43	8.704	4.776	2.068
t' for 27 December 1571 (in 360ths of the period *p*)	189.76	166.92	9.23	116.17	357.53	123.38	110.6	45.6	39.3
t'' for 27 December 1571	244.29	292.83	105.47	152.39	358.83	218.82	285.25	90.4	328.0

∗ Figures in italics are assigned by definition or by choice; the rest are calculated.

("on the shoulders of giants," as he said) to the universal law of gravitation, of which Kepler's laws are in fact direct "consequences." But the law of gravitation predicts that not only the sun but each of the planets influences the motion of all the others. The total mass of the planets (of which two-thirds is in Jupiter and another fifth in Saturn) is not much more than one-tenth of one percent of the mass of the sun, so that they virtually do not influence it and it overwhelmingly controls them. But relative to each other these masses are not negligible.

The inner planets are too close to the sun and too far from the massive Jupiter and Saturn to be perturbed obviously over centuries, say, or millennia, although Mars is approaching the danger zone. (The asteroids, mainly in the gap between Mars and Jupiter, may represent the materials of a planet that was too near Jupiter to survive or, more likely perhaps, too near ever to consolidate into a single planet in the first place.) But Jupiter and Saturn influence each other very considerably, and Uranus and Neptune have some effect; indeed it was precisely the effect of Uranus on Saturn and of Neptune on Uranus that led to their discovery.

It follows that the real motions, the angular velocities, and hence the Keplerian pitches of these planets are controlled by more than Kepler's laws; the corrections are not large but they are appreciable. We had therefore to ask whether to try to reconstruct the Keplerian music of the planets as he predicted it would be or as it would actually be, if pitches did in fact inhere in angular velocity. As it is Kepler's harmony of the world that we are trying to realize, we have chosen to follow his predictions. But we feel we must point out that our realization is for an ideal motion of the planets and not for their actual motion.

We have used stereophonic sound to suggest the ceaseless swinging of the planets around the sun. The period for each planet is of course identical with the period of its pitch variation, but the point in space where it is closest to the sun, and hence sings its highest note, is in a different direction for each planet. For this, the function t'' had to be determined for the chosen starting point, giving the angle that the planet has swept out beyond some fixed point in the sky (as seen from the sun) to reach its position at the given moment. (The fixed point is the so-called "first point of Aries," the point where the sun appeared from the earth at the vernal equinox in 1571.) For this calculation, a simple cosine function was adequate for all planets.

The sound of the planets

Mercury, as the innermost planet, is the fastest and the highest pitched. It has a very eccentric orbit (as planets go), which it traverses in 88 days; its song is therefore a fast whistle, going from the E above the piano (e′′′′′) down more than an octave to about C# (c#′′′′) and back, in a little over a second.

Venus and Earth, in contrast, have nearly circular orbits. Venus's range is only about a quarter tone, near the E next above the treble staff (e′′′); Earth's is about a half tone, from G (g′′) to G# at the top of that staff. Together they drone a sixth, but the sixth is continuously changing from major to minor, or even down almost to a fifth, as the two planets go through their cycles—about 3 seconds for Venus, exactly 5 for Earth. Kepler compared Earth's sad minor second to the first minor second in the standard do-re-mi scale—mi-fa-mi—and for him it sang of Earth's unending misery-famine-misery.

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Next out from Earth is Mars, again with an eccentric orbit. Its song is distinctive, one of the easiest to pick out in the full "harmony." Alone in the alto, it ranges from the C above middle C (c″) down to about F# (f#′) and back, in nearly 10 seconds.

The distance from Mars to Jupiter is much greater than that between the inner planets (as mentioned above, the asteroids in this gap may represent a missing planet), and Jupiter's song is much deeper, in the baritone or bass, and much slower. It covers a minor third, from D to B (D to B_1) just below the bass staff. Still farther out and still lower is Saturn, only a little more than a deep growl, in which a good ear can sometimes hear the individual vibrations. Its range is a major third, from B to G (B_2 to G_2), the B at the top being just an octave below the B at the bottom of Jupiter's range. Thus the two planets together define a major triad, and it may well have been this concord—in the ratio 4:5:6, inevitable when angular velocities are equated with pitches—that made Kepler certain he had cracked the code and discovered the secret of the celestial harmony. Saturn's cycle is about 2½ times that of Jupiter (almost 2½ minutes vs. almost 1 minute), and their songs commonly strike the concordant ratios. This would be even more evident if the speed of the music were doubled so that the cycles were half as long and the pitches were all raised an octave: together the two planets then sing a majestic counterpoint in the key of G Major.

The outer, post-Kepler planets we have simulated not by musical tones at the given vibration frequency but by sharp rhythmic beats. Uranus is a rapid ticky-ticky-ticky, changing gradually from fewer than 9 to more than 10 beats per second and back, but over a period of 7 minutes, so that the change is not easy to detect. When the much steadier (because much less eccentric) Neptune is added, however, at nearly 5 beats per second, the changes in Uranus's rhythm become more obvious, because the ratio between the two planets shifts continually back and forth from less than to more than 2:1, and the resulting "beats" are readily discerned.

Finally, Pluto's base-drum beat is the foundation of the whole structure. Although its period is long—20 minutes and 42 seconds—its orbit is so eccentric that the gradual slowing down, and even more the speeding up, of the rhythm is manifest. At Pluto's slowest, the ratio with Neptune is almost 1:2½, yet near its fastest their rhythms are identical, and at the highest point Pluto is actually a little faster, because, in fact, at its closest point to the sun it is actually closer than Neptune ever comes (lending credence to the view that Pluto is only an escaped satellite of Neptune's). Thus the relation between the two rhythms is continually changing, and "beats" come and go as they come into a simple ratio and then recede from it again. The additional doubled rhythm of Uranus, ticking away above, adds greatly to the fascination of this cross-rhythm.

We cannot emphasize strongly enough that for Kepler the harmony of the world is to be heard not from the earth but from the sun. His ideas were rigorously Copernican and sun-centered, and he scolded severely those who slipped back into an earth-centered or earthbound point of view of the sort that permeates astrology.

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