CQF Exam One
Risk Techniques and Regulation

June 2021 Cohort

Instructions

Answers to all questions **are required**. Requested mathematical and all computational workings must be provided. Portal, upload and logistical questions to Orinta.Juknaite@fitchlearning.com. Clarifying questions are welcome to Richard.Diamond@fitchlearning.com. Tutor is unable to re-explain calculation or confirm correct numerical answers/reqs sections. Please make a good use of lecture exercises/solutions.

- Computation can be implemented on Excel spreadsheets or coded in Python/other languages. All code/Excel to be uploaded as ONE zip file named LASTNAME_CODE.zip
- Report for this exam can be simple: summary table and plots for Q1, text answers for Q2, table with numerical answers for Q3, typed or handwritten working for Q4.
  Copy final values and plots into Word/LaTeX. Merge all answers/pages into ONE pdf file, named LASTNAME_REPORT_E1.
- Submissions in Excel files only will receive a deduction in marks. Submissions in Python notebook only will receive a deduction in marks, particularly where there is unnecessary output. Absence of summary tables will receive a deduction. Submission in multiple pdf/image files will result in a delay and/or fail. Complex submissions will be graded with/after Extensions.

Questions

Question 1. (42 marks) As a market risk analyst, each day you calculate VaR from the available prior data. Then, you wait ten days to compare your prediction VaR_{t-10} to the realised return and check if the prediction about the worst loss was breached. You are given a dataset with *Closing Prices* S&P500.

- Implement VaR backtesting by computing 99%/10day Value at Risk using the rolling window of 21 daily returns to compute σ. The rolling window technique will give you the time series of $\text{VaR}_{10D,t}$.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

Report (a) the percentage of VaR breaches and (b) the number of consecutive breaches.
(c) Provide a plot which clearly identifies breaches.

Repeat backtesting for the sample of 42 daily returns to compute σ.

Provide numbers (a), (b) for both periods (21 returns, 42 returns) in one summary table.
Further Instructions are offered to help with this question (end of paper), including computational detail and consecutive breaches count.

Question 2. (16 marks) In this question you will have to recite a regulation and/or identify the relevant articles by a number. The EU Capital Requirements Regulation 2 (CRR2), can be found at `https://eba.europa.eu/regulation-and-policy/single-rulebook/interactive-single-rulebook/100427` in interactive form. Recite article(s) that specifically prescribe requirements for the following:

(a) Stable funding: article(s) titles only (not text) and a brief definition of Net Stable Funding Ratio.

(b) Internal risk model for 'correlation trading': article(s) titles only and a list of main risks to be captured.

Question 3. (16 marks) VaR and ES sensitivities are computed *with regard to* each asset i, individually. For a given allocated portfolio below, provide a summary table with computed $\frac{\partial \text{VaR}(w)}{\partial w_i}$ and $\frac{\partial \text{ES}(w)}{\partial w_i}$.

Asset	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	$\boldsymbol{w}$
1	0	0.30	50%
2	0	0.20	20%
3	0	0.15	30%

$$\boldsymbol{Corr} = \begin{pmatrix} 1 & 0.8 & 0.5 \\ 0.8 & 1 & 0.3 \\ 0.5 & 0.3 & 1 \end{pmatrix}$$

$$\frac{\partial \text{VaR}(w)}{\partial w_i} = \mu_i + \text{Factor} \times \frac{(\boldsymbol{\Sigma w})_i}{\sqrt{\boldsymbol{w^T \Sigma w}}} \quad \text{and} \quad \frac{\partial \text{ES}(w)}{\partial w_i} = \mu_i - \frac{\phi(\text{Factor})}{1-c} \times \frac{(\boldsymbol{\Sigma w})_i}{\sqrt{\boldsymbol{w^T \Sigma w}}}$$

where $()_i$ refers to i-th element, confidence $c = 99\%$, and $\phi()$ is simply Normal *pdf*. Factor $= \Phi^{-1}(1-c)$ computed with *Normal icdf*. Signs in formulae are for the left tail – lower percentile of 0.01.

Question 4. (26 marks) Answer this question with step-by-step mathematical derivation: each next line must mathematically and clearly follow from the previous line, making no omitted transformations. Begin with the definition of Expected Shortfall as a conditional expectation (which implies integration),

$$\text{ES}_c(X) \quad = \quad \mathbb{E}[\, X \,|\, X \leq \text{VaR}_c(X)]$$

and obtain its computation formula for the case of Normal Distribution, eg $\text{VaR}(X) = \mu + \Phi^{-1}(1-c) \times \sigma$.

You can provide a handwritten working but not a rough work: use ample and clear spacing between lines, no crossings/corrections. Submitting copied in formulae from any other source will immediately score less than 25% of marks.

END OF EXAM (Version D1)

Further Instructions

Please make good use of lecture exercises and tutorial/lab sessions. The tutor is unable to confirm numerical answers and methods of calculation, eg can't check spreadsheets/code before submission.

For VaR backtesting, you need to compute the 99%/10day Value at Risk on the rolling basis – you will have a column of VaR_t values.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Practical VaR calculation drops μ_{10D} for two reasons. First, 21-day sample average return (or alike) is not a robust quantity. Second, for a diversified portfolio/market index the quantity is negligible.
- Appropriate Factor value to be used (Standard Normal Percentile), the tutor will not confirm the numerical value. It is also your task to identify the eligible number of observations for which VaR is available and can be backtested: N_{obs} will not be confirmed.
- Compute a column of rolling standard deviation over DAILY log-returns for periods Day 1 - Day 21, Day 2 - Day 22, ... Compute VaR for each day t, after the initial period (Day 1 - Day 21).
- **Regardless** of how many observations are in a sample (21 or 42.), variance is *an average of squared daily differences* $\frac{\sum(r_t-\mu)^2}{(N-1)}$. Timescale remains 'daily' and 10-day VaR projection made by $\sigma_t \times \sqrt{10}$.
- VaR is fixed at time t and compared to future-realised return $\ln(S_{t+10}/S_t)$. You need 11 dates (rows) to capture 10-period return.
  A breach occurs when that 'future realised' return is below the VaR_t quantity (negative scale).

  $$r_{10D,t+10} < \text{VaR}_{10D,t} \qquad \text{means breach, given both numbers are negative.}$$

  In Excel, you will have a column for VaR_t series, a column of $r_{10D,t+10}$ series, and indicator column $\{0,1\}$ for a breach using $IF()$ function.
- Computation of 10-day return and VaR with the overlap of final day of the sample is acceptable, but best not to overlap.
- To obtain the conditional probability of breach $N_{conseq}/N_{breaches}$, identify consecutive breaches. For example, the sequence $1,1,1$ means two consecutive breaches occurred.

Extra Tasks

These tasks are not part of the exam and not graded. However, they remind a couple of useful techniques for a risk manager.

1. For comparison to naive sample std dev, backtesting is done with EWMA (but not GARCH which tends to the long-term average variance), with the same rolling window of 21 or 42 observations. EWMA estimated for each next period as follows:

$$\sigma^2_{t+1\,|\,t} = \lambda\,\sigma^2_{t\,|\,t-1} + (1-\lambda)\,r_t^2$$

we recommend $\lambda = 0.72$, which is smaller than the original RiskMetrics method, but minimises out of sample forecasting errors.

2. VaR practitioners/risk managers also test the issue of independence of breaches in VaR quantitatively. The applicable statistical test is: Christoffersen's exceedances independence.