T-tests

Data Science Immersive

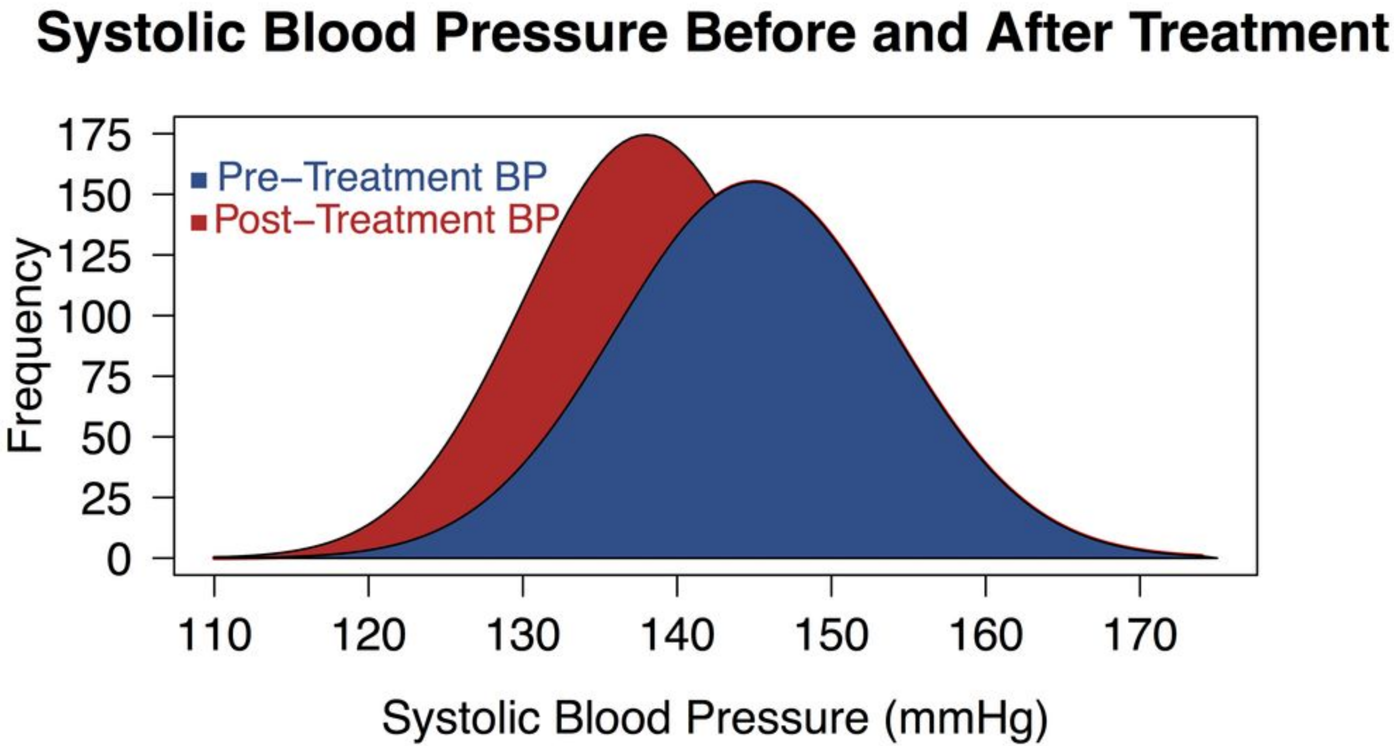
FLATIRON SCHOOL

ESTD 1759
GUINNESS
GUINNESS

Examples

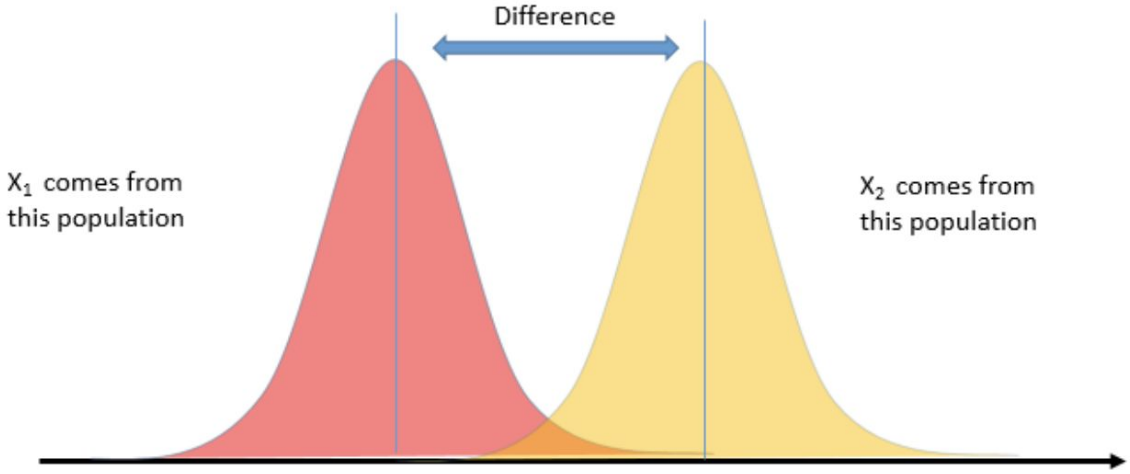
- Chemistry - do inputs from two different barley fields produce different yields?
- Astrophysics - do star systems with near-orbiting gas giants have hotter stars?
- Economics - demography, surveys, etc.
- Medicine - BMI vs. Hypertension, etc.
- Business - which ad is more effective given engagement?

Problem



Systolic Blood Pressure Before and After Treatment
Pre–Treatment BP
Post–Treatment BP
Frequency
175
150
125
100
75
50
25
0
110
120
130
140
150
160
170
Systolic Blood Pressure (mmHg)

Problem



Difference
X_1 comes from
this population
X_2 comes from
this population

Hypothesis

- Null Hypothesis / Alternative Hypothesis Structure

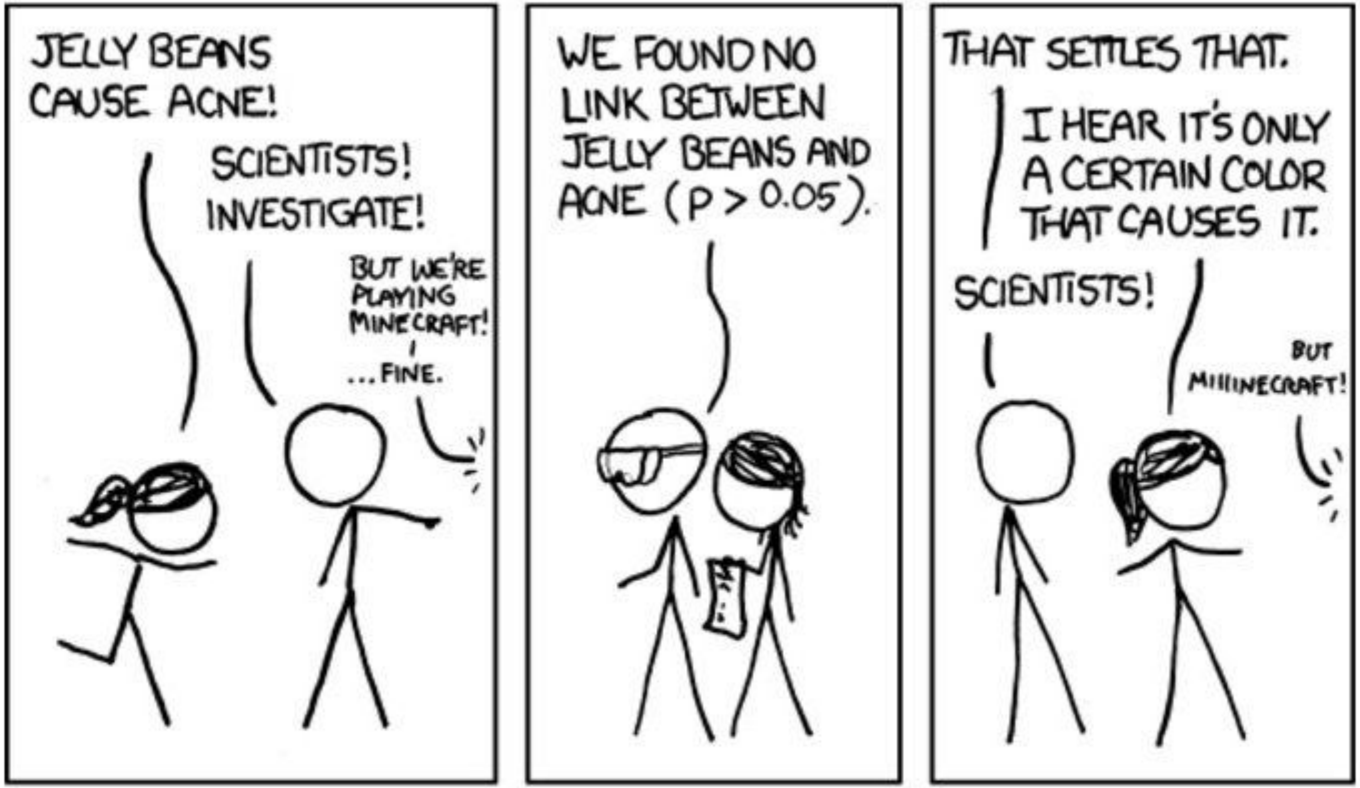
- Error
  - I: False positive rate
  - II: False negative rate

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 \neq 0$$

- Choosing the right error rate
  - Alpha, α
  - Sigma, σ
  - Depends on field of study, $0.2 \leq \alpha \leq 0.00001$

P-Values



JELLY BEANS CAUSE ACNE!
SCIENTISTS! INVESTIGATE!
BUT WE'RE PLAYING MINECRAFT!
...FINE.
WE FOUND NO LINK BETWEEN JELLY BEANS AND ACNE (P > 0.05).
THAT SETTLES THAT.
I HEAR IT'S ONLY A CERTAIN COLOR THAT CAUSES IT.
SCIENTISTS!
BUT MIIIINECRAFT!

What’s a t-test

What’s different?

- Sometimes the population standard deviation is irrelevant, and sometimes it’s unknown. (we’ll get to the different types of t-test later)
- Sometimes a sample is too small to be confident that it’s an accurate representation of reality

A t-test is like a modified z-test:

- Penalize for small sample size - “degrees of freedom”
- Use sample std. dev. ***s*** to estimate population $\boldsymbol{\sigma}$

Assumptions

- Normally distributed (and similarly distributed) samples
- Controlled for selection bias

What’s a t-test

The z-distribution is skewed to fit the level of uncertainty of a t-test.

t curve approaches z curve as df -> ∞

t curve and *z curve*

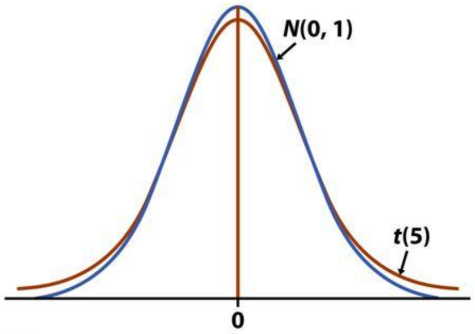


Figure 7-1
Introduction to the Practice of Statistics, Fifth Edition
© 2005 W.H. Freeman and Company

Both the standard normal curve *N(0,1)* (the *z* distribution), and all *t(k)* distributions are density curves, symmetric about a mean of 0, but *t* distributions have more probability in the tails.

As the sample size increases, this decreases and the *t* distribution more closely approximates the *z* distribution. ***By n = 1000 they are virtually indistinguishable from one another.***

What's going on?

Z statistic

$$\boxed{z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}}$$

Unknown σ

t statistic

$$\boxed{t = \frac{\bar{X} - \mu}{s / \sqrt{n}}}$$

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Low *v*

v approaches infinity

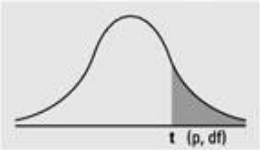
$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

What this really comes down to

$$\lim_{n\to\infty} \frac{n}{n-1} = 1$$

Tables

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150

3 types of t-test

- Do we have a good sample?
  - **Single sample**
  - The One Sample *t* Test determines whether the sample mean is statistically different from a known or hypothesized population mean. (sample vs. population)
  - Note this is the only one where we need the population mean

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- Has our sample changed?
  - **Paired data**
  - Two samples of related data points (same person, same wheat field, etc.)

$$t = \frac{m}{s/\sqrt{n}}$$

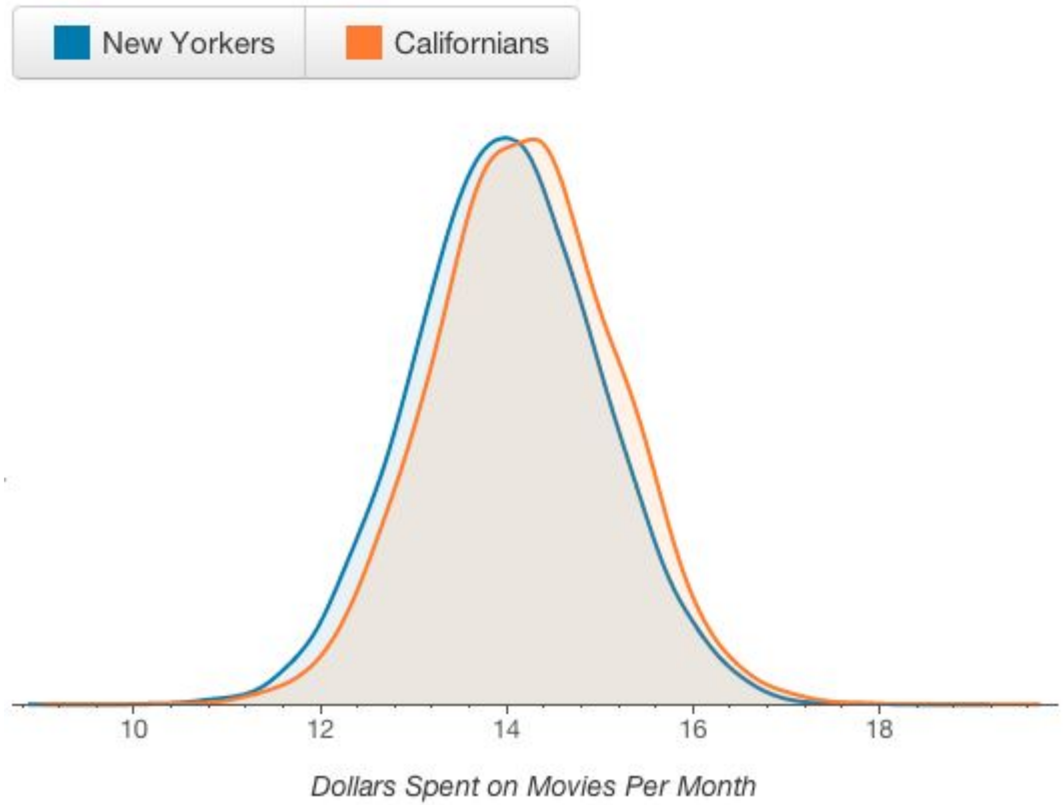
- Are two different samples different or alike?
  - **Independent samples**
  - Two samples of unrelated data points (not before/after, etc.) (different people, different wheat fields)

$$t = \frac{m_A - m_B}{\sqrt{\frac{S^2}{n_A} + \frac{S^2}{n_B}}}$$

$$S^2 = \frac{\sum (x - m_A)^2 + \sum (x - m_B)^2}{n_A + n_B - 2}$$

Practice

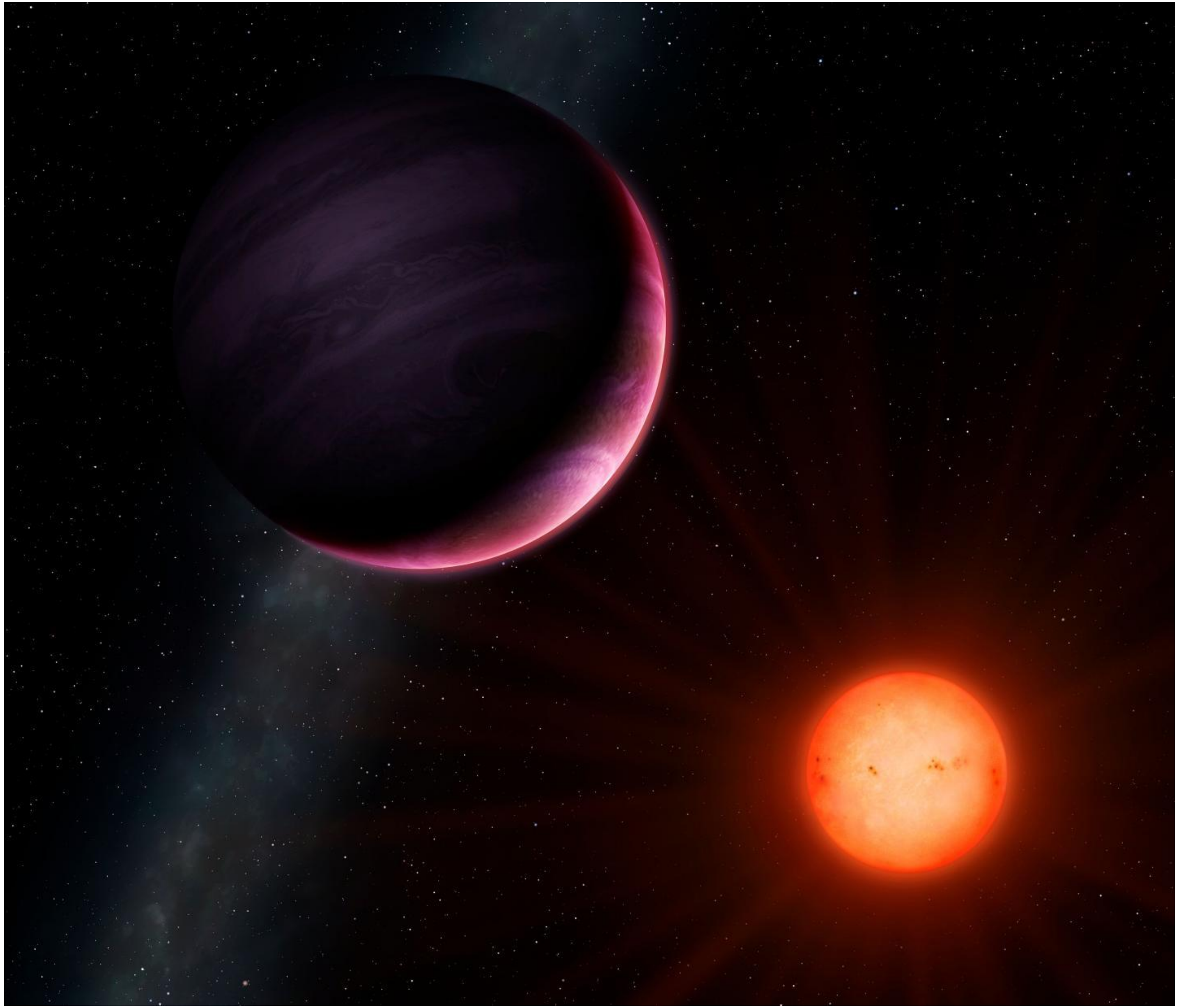
Effect Size



New Yorkers
Californians
10
12
14
16
18
Dollars Spent on Movies Per Month

Interpreting Results

- Reject or fail to reject null hypothesis
- Language is important!
- Don’t throw out failed experiments
  - This methodology, with this data, does not produce significant results
  - More data
  - More time
  - More details

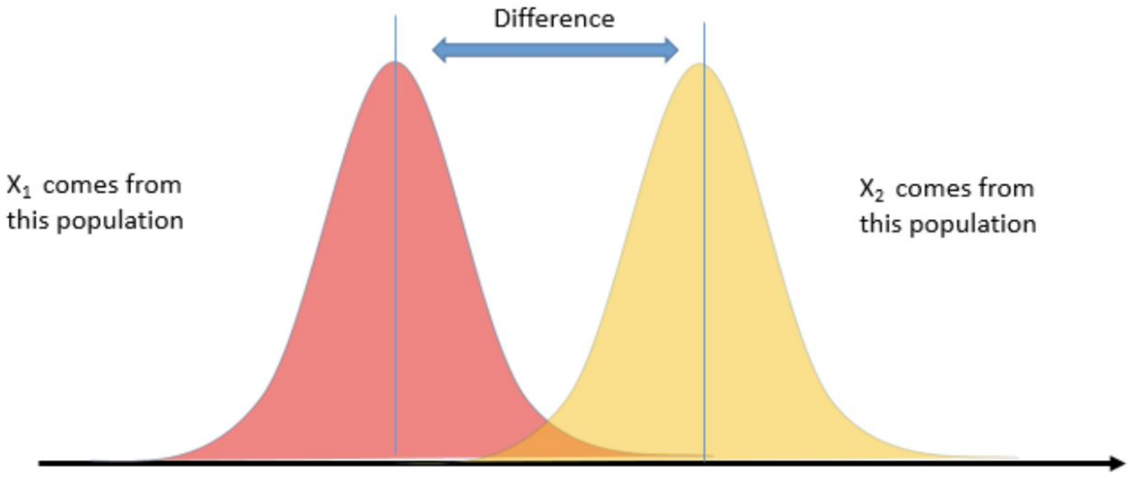


Hypotheses

- Null hypothesis / hypothesis structure & formulation

$$H_o: \mu_1 = \mu_2$$

- Why do w [illegible] ge!
- p-values



Quick Recap

T-tests compare means. Here's what we need:

- Mean **μ**
- Standard deviation **σ**
- Sample size **n**
- Distribution
- Z score

Error

- Choosing the appropriate error rate
  - Alpha, α
  - Sigma, σ
- Depends on field of study
  - $0.2 \leq \alpha \leq 0.00001$

Quick Maths

Normal distribution:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Z score:

$$z = \frac{(x - \mu)}{\sigma}$$

Substituting:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Integrating the normal distribution gets you something called the “error function” which is usually abbreviated as erf().

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

$$Y = \frac{1}{2\sigma} erf\left(\frac{z}{\sqrt{2}}\right)$$

Quick Maths

$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

$$\Gamma(n) = (n-1)!$$

$$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$$

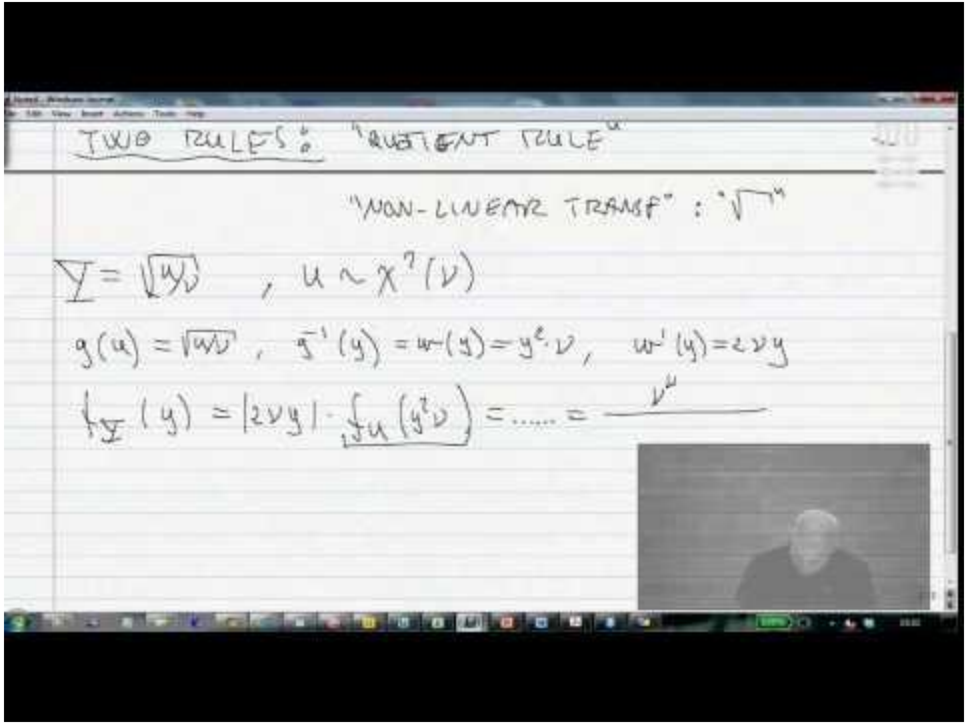
$$\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}\,dx$$

$$t = \frac{z}{\sqrt{u/v}}$$

$$\begin{aligned}
\operatorname{erf}(x) &= \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \\
&= \frac{2}{\sqrt{\pi}}\int_0^x (1-t^2+\frac{t^4}{2!}-\frac{t^6}{3!}+\cdots)dt \\
&= \frac{2}{\sqrt{\pi}}\left[t-\frac{t^3}{3}+\frac{t^5}{5\cdot 2!}-\frac{t^7}{7\cdot 3!}+\cdots\right]_0^x \\
&= \frac{2}{\sqrt{\pi}}\left(x-\frac{x^3}{3}+\frac{x^5}{5\cdot 2!}-\frac{x^7}{7\cdot 3!}+\cdots\right)
\end{aligned}$$

As $n \to \infty$, $f(t) \to f(z)$, $t \to z$, $u \to v$

Full Proof


TWO RULES: "QUOTIENT RULE"
"NON-LINEAR TRANSF" : "√"
$Y = \sqrt{U/\nu}$, $U \sim \chi^2(\nu)$
$g(u) = \sqrt{u/\nu}$, $g^{-1}(y) = w(y) = y^2 \cdot \nu$, $w'(y) = 2\nu y$
$f_Y(y) = |2\nu y| \cdot f_U(y^2\nu) = \ldots\ldots =$

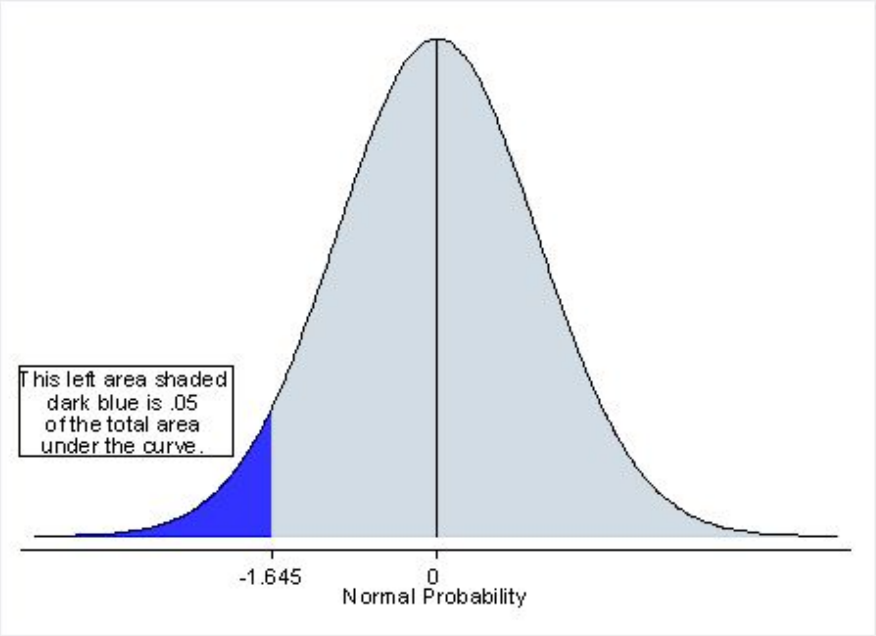
Practice with t tests

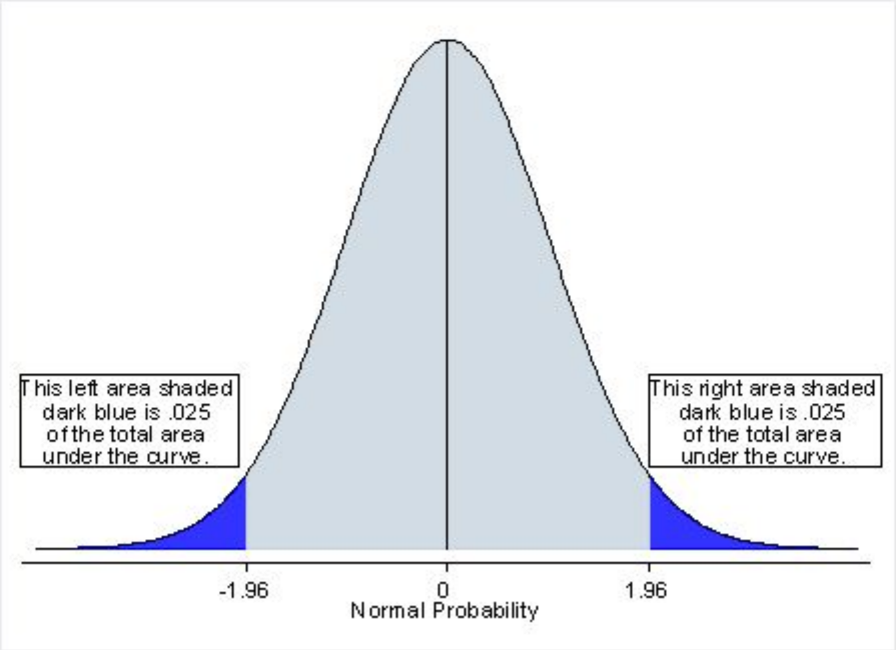
Do one together (one sided)

Then practice (one sided)

Then give example that requires two sided

One-Tail vs. Two Tail





Page Name

Code Examples