*A.1) Show that A and B are independent (p(A,B) = p(A) * p(B)?):*

$p(A,B,C,D) = p(A)p(B)p(C|B,A)p(D|C)$

$p(A,B) = \int_C \int_D p(A)p(B)p(C|B,A)p(D|C)$

$p(A,B) = p(A)p(B)$ → A and B are independent

A.2) If we observe D, are A and B still independent? (p(A,B|D) = p(A|D)p(B|D)?):

$p(A,B|D) = p(A,B,D) / p(D)$

$p(A,B|D) = \int_c p(A,B,C,D) / p(D)$

$p(A,B|D) = \int_c p(A)p(B)p(C|B,A)p(D|C)/p(D)$

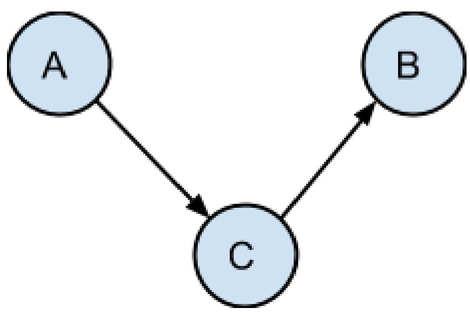
$p(A,B|D) = p(A)p(B)\int_c p(D|C) / p(D) \neq p(A|D)p(B|D)$ → A and B are dependent

B.1) Given joint probabilities, show that A and B are marginally dependent and independent when conditioned on C?

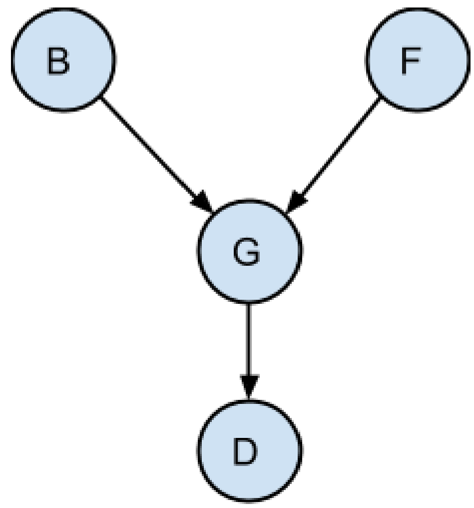
B.2) Evaluate p(A), p(B|C), p(C|A) to show that p(A,B,C) = p(A)p(C|A)p(B|C).

See .m file

B.3) Draw the corresponding DAG:



C) p(B,F,G,D) = p(B)p(F)p(G|B,F)p(D|G)



C.1) Given D = 0, what is the probability that the tank is empty (p(F=0|D=0)?

p(F|D) = p(F,D) / p(D) = $\sum_{B}\sum_{G}$p(B,F,G,D) / $\sum_{B}\sum_{F}\sum_{G}$p(B,F,G,D)

p(F=0|D=0) = .21

(see .m file)

C.2) Given D = 0 and B = 0, what is the probability that the tank is empty (p(F=0|D=0,B=0))?

p(F|D,B) = p(F,D,B) / p(D,B) = $\sum_{G}$p(B,F,G,D) / $\sum_{F}\sum_{G}$p(B,F,G,D)

p(F=0|D=0,B=0) = .11

(see .m file)

C.3) Explain why the two probabilities are equal/unequal:

They are unequal because knowing the state of the battery adds additional information to our probabilistic model. The additional knowledge about B will affect the probability of the gauge reading 0 or 1, which in turn will reflect the driver's estimate. Because it is highly unlikely that the battery will be flat and the tank will be empty, the numerator in the case when the battery state is known is much smaller, reducing the posterior probability.