Theory of Signal Detection Homework

Introduction

In this homework, you will develop simulations that explore the Theory of Signal Detection.

Step 1

We will be considering a Yes-No experiment. For concreteness, let's suppose this is an experiment in which the observer has to say whether a patch of dots is moving left or right, like the experiment that you did.

Conceptually, each trial results in an internal response representing dot motion, on an axis we can think of as having 0 represent no motion, negative numbers representing leftward motion, and positive numbers representing rightward motion. To make things concrete, we'll assume that this representation corresponds to the dot correlations used to describe the stimuli. On each trial, the mean response of a stimulus is its correlation, but there is trial-to-trial noise so that on any trial, the response is given by a draw from a normal distribution with mean `dotCorr` and standard deviation `dotSd`. Note that because the noise is normal, the internal response can sometimes be less than -1 and sometimes greater than 1, even though the dot correlations themselves are bounded below and above by these values.

TSD says that the subject compares the internal response on each trial to a criterion, `rightCrit`, and says right if the response exceeds the criterion.

Consider a simple experiment where on each trial the dots are either going right with correlation `dotCorr`, or left with correlation `–dotCorr`. (In this homework, we'll be considering this symmetric case so that we'll always think of `dotCorr` as a positive number.) The subject indicates right or left on the basis of the value of `rightCrit`. We'll define a hit as a response of right when the dots are going right and a false alarm as a response of right when the dots are going left.

Write a program that simulates out and plots the ROC curve for any choice of `dotCorr` and `dotSd`. To generate the curve, you'll simulate out the hit and false alarm rates for many values of `rightCrit`. On your plot, show what happens when you increase `dotCorr` with `dotSd` fixed and what happens when you increase `dotSd` with `dotCorr` fixed. Call this plot Figure 1.

Step 1A

Write a function that takes `dotCorr`, `dotSd`, and `rightCrit` as arguments and uses the cumulative normal distribution (`normcdf` function in Matlab) to compute and return the theoretical hit and false alarm rates corresponding to the arguments. Use this function to add a smooth theoretical curve through the ROC curves in Figure 1.

Step 2

Define four payoffs Vh, Vfa, Vm, and Vcr using the convention given in class. Define pRight as the probability that a rightward stimulus is presented on any trial. Given values for dotCorr, dotSd, Vh, Vfa, Vm, Vcr, pRight, write a function that finds the expected payoff for any choice of rightCrit. You'll want to use the function you write in Step 1A to get the necessary hit and false alarm rates. Hold dotCorr and dotSd fixed at some interesting values and make a plot of how the expected payoff varies as a function of rightCrit for a two different choices of the payoffs and probability rightward. Call this Figure 2. Do the graphs you get make sense? Explain why?

Step 3

From your work in Step 1A, you know how to compute the hit and false alarm rates for any triplet of dotCorr, dotSd, and rightCrit. You can also compute dPrime from dotCorr and dotSd, using the definition of d prime for the equal variance normal case.

Write a function that computes d-prime from an observed hit and false alarm rate, using the formula (difference of z-scores) from class for the equal variance normal case. Call this dPrimeData. Compute the hit and false alarm rates for a large number of choices of dotCorr, dotSd, and rightCrit and from these hit and false alarm rates compute dPrimeData. Make scatter plots that compare dPrimeData with dPrime. Do they agree? Why or why not? Call this Figure 3.

Step 4

Suppose the subject adopts a neutral criterion rightCrit = 0 and that pRight = 0.5. Choose a fixed reasonable value for dotSd and compute the probability that a subject gets a Y/N trial right as a function of the value of dotCorr. Make a plot of this probability correct as a function of the value of dotCorr. (For each positive value of dotCorr, simulate out a series of trials where half are going left (-dotCorr), and half are going right (dotCorr). Figure out what fraction of these trials the subject gets right. That's the probability you want to plot for each value of dotCorr).

Now consider a TAFC version of the experiment. On each trial, the subject sees two patches of moving dots and has to decide which one is moving right. The subject uses the following decision rule. Each dot patch leads to an internal response, which as in all of the above is given by a draw from a normal distribution with a mean equal to the dot correlation (+/- dotCorr) and standard deviation dotSd. The subject chooses the stimulus on each trial that has the larger response. Simulate out the percent correct as a function of dotCorr for this experiment and add it to your plot for the Y/N experiment. Call this Figure 4. Comment on why it makes sense that the two curves differ.

Extra Credit 1. It is a theorem that for any value of dotCorr, the area under the ROC curve obtained for the Y/N experiment (for various choices of rightCrit) is equal to the percent correct obtained in the TAFC experiment. So for each value of dotCorr, you can compute the ROC curve and integrate it numerically to find the area underneath it. Do this, and add the result to your plot. Does it overlay the TAFC curve your obtained?

Extra Credit 2. To get some intuition for the relation between d-prime and percent correct in a TAFC task, make plot of one against the other.

Turn In

Turn in your four figures and any comments you'd like to make about them. Also include Matlab code that generates the plots. I'll only look at the code if your figures don't make sense, but I'd like to have it if I need it.

You should do this assignment on your own, without a lab partner.