snow-TWCR: Model equations and workflow definitions (v1.0.0)

Theodorou, 2026

1. Overview

The snow-TWCR workflow produces gridded maps of **snow depth** (cm) and **snow water equivalent (SWE)** (mm) on a DEM grid for specified survey dates. The workflow combines (i) point snow observations, (ii) terrain and coastal predictors, and (iii) storm-integrated meteorological forcing from automatic weather stations (AWS) and uses a **two-stage occurrence × magnitude model**.

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2. Spatial grid and predictors

All predictions are generated on the **DEM raster grid** (one prediction per DEM cell).

For any grid cell x:

- $z(x)$: elevation (m) from DEM
- $d(x)$: distance to sea (km)
- $P_{\text{snow}}(x)$: storm-cumulative snow-favorable precipitation (mm)

I use transformed predictors:

$$D(x) = \log(1 + d(x)), Q(x) = \log(1 + P_{\text{snow}}(x))$$

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3. Storm-integrated snow-favorable precipitation $\boldsymbol{P}_{\text{snow}}(\boldsymbol{x})$

Storm integration period:

- Start: **12 Jan 2025 00:00 (local time)**
- End: **survey day 12:00 (local time)**
  Time resolution: **10 minutes**

Let $i = 1, \ldots, N$index AWS stations with valid metadata (latitude, longitude, elevation).
Let $P_i(t)$ be the observed 10-minute precipitation increment (mm) at station i and time step t.

Let $T_{w,i}(t)$ be station wet-bulb temperature (°C), computed from air temperature and relative humidity.

3.1 Distance-weighted station blending (continuous, not nearest-station)

Define distances (km) from grid cell $x$to station i: $d_i(x)$.
Define normalized inverse-distance weights:

$$w_i(x) = \frac{d_i(x)^{-p}}{\sum_{j=1}^{N} d_j\,(x)^{-p}}$$

where p is the IDW power (here $p = 2$).

Distance-weighted precipitation at grid cell $x$and time t:

$$P(x,t) = \sum_{i=1}^{N} w_i\,(x)\,P_i(t)$$

3.2 Lapse-rate adjustment of wet-bulb temperature to each grid cell

I use a constant lapse rate:

Γ
$= -6.5$

Convert station wet-bulb temperature to a "sea-level equivalent":

$$T_{w0,i}(t) = T_{w,i}(t) - \Gamma \frac{z_i}{1000}$$

where z_i is the station elevation (m).

Blend T_{w0} to the grid cell and adjust to local elevation:

$$T_w(x,t) = \left(\sum_{i=1}^{N} w_i\,(x)\,T_{w0,i}(t)\right) + \Gamma \frac{z(x)}{1000}$$

3.3 Snow fraction from wet-bulb temperature

Precipitation phase is represented as a snow fraction $f_s(x,t) \in [0,1]$ using a linear transition band centered near the rain–snow transition:

- All snow for $T_w(x,t) \leq -0.5°\text{C}$: $f_s = 1$

- All rain for $T_w(x,t) \geq 1.5°\text{C}$: $f_s = 0$
- Linear transition in between:

$$f_s(x,t) = 1 - \frac{T_w(x,t) - (-0.5)}{1.5 - (-0.5)} \text{ for } -0.5 < T_w(x,t) < 1.5$$

3.4 Storm-cumulative snow-favorable precipitation

At each time step, snow-favorable precipitation is:

$$P_{\text{snow}}(x,t) = P(x,t)\, f_s(x,t)$$

Storm-cumulative snow-favorable precipitation:

$$P_{\text{snow}}(x) = \sum_t P_{\text{snow}}(x,t) = \sum_t P(x,t)\; f_s(x,t)$$

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4. SWE computation at observation points

At observation points, SWE is computed using snow depth (cm) and density (g cm $^{-3}$):

$$SWE\ (mm) = depth(cm)\ \times \rho(g\ cm^{-3}) \times 10$$

If density is measured at a point, the measured value is used. If density is missing, it is estimated from an elevation-based rule:

- For each survey day, measured densities $\rho$are linked to DEM elevation and constrained to decrease with elevation (monotonic).
- Density for missing locations is interpolated between nearby density measurements in elevation.
- Outside the observed density elevation range, density is clamped to the nearest endpoint (lowest-elevation density for below range; highest-elevation density for above range).

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5. Two-stage mapping model (occurrence × magnitude)

Let $y(x)$be the target variable: either **Depth (cm)** or **SWE (mm)**.
Define snow presence:

$$S(x) = \mathbb{I}[y(x) > 0]$$

5.1 Stage 1: occurrence model (logistic regression)

I fit a logistic regression for snow presence:

$$S(x) \sim \text{Bernoulli}(p(x))$$

$$\text{logit}(p(x)) = \alpha_0 + \alpha_1 z(x) + \alpha_2 z(x)^2 + \alpha_3 \log(1 + d(x)) + \alpha_4 \log(1 + P_{\text{snow}}(x))$$

5.2 Stage 2: magnitude model (log-linear regression on snow-present points)

For points with $y > 0$, I fit:

$$\log(1 + y(x)) \\ = \beta_0 + \beta_1 z(x) + \beta_2 z(x)^2 + \beta_3 \log(1 + d(x)) + \beta_4 \log(1 + P_{\text{snow}}(x)) + \varepsilon(x)$$

Back-transformation to original units:

$$\widehat{m}(x) = \exp\left(\widehat{\log(1 + y(x))}\right) - 1$$

5.3 Soft coupling (final prediction)

The final prediction is:

$$\hat{y}(x) = \hat{p}(x) \times \widehat{m}(x)$$

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6. Post-processing rules

6.1 Snowline proxy and low-elevation probability cutoff

A data-driven snowline proxy is defined as the 10th percentile of observed snow-present elevations:

$$E_{\text{snowline}} = \text{quantile}_{0.10}(z \mid S = 1)$$

For grid cells below this snowline proxy, predictions are set to zero when predicted probability is small:

$$\hat{y}(x) = 0 \text{if } z(x) < E_{\text{snowline}} \text{ and } \hat{p}(x) < 0.06$$

6.2 High-elevation probability floor

To avoid unrealistically low snow probability at high elevations, for elevations above the snowline proxy plus a margin:

$$\hat{p}(x) \leftarrow \max\left(\hat{p}(x), 0.85\right) \text{for } z(x) \geq E_{\text{snowline}} + 10\text{ m}$$

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7. Hard conditioning at measurement grid cells

To ensure consistency with observed values, grid cells coincident with observation points are overwritten with the observed value (mean if multiple observations fall into the same DEM cell).

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8. Physical caps applied to outputs

Depth caps are applied as fixed event constraints:

- 14 Jan 2025: Depth $\leq$ 80 c m
- 15 Jan 2025: Depth $\leq$ 150 c m

SWE caps are derived from the fixed depth cap and a robust density estimate for the day:

$$\text{SWE}_{\text{cap}} = \text{Depth}_{\text{cap}} \times \rho_{0.95} \times 10$$

where $\rho_{0.95}$is the 95th percentile of measured densities for that day (fallback to global mean density if sparse).