Fundamentals of Astrophysics: Questions and Exercises

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1 Introduction

Quick Questions

1 Discuss how we can estimate the temperature of warm or hot object, e.g. a stove, without touching it.

Solution A warm or hot object will radiate its heat in the infrared (IR), which we cannot see, but can feel. □

2 Discuss the ways we estimate distances and sizes in our everyday world.

Solution For nearby objects, we infer distance through our stereoscopic vision, through the parallax effect. For more distant objects, if we intuitively know the size, then the ratio of this size to the observed angular size gives us a measure of distance. □

3 Discuss what sets our perception limits on the smallest intervals of time. How might this differ in creatures of different size, e.g. a fly vs. a human?

Solution Our detection of a signal, e.g. by our eyes or our hands, must then be transmitted by our nerves to our brain. The associated time delay is given by the distance divided the speed of nerve transmission, which for humans is of order 0.01-0.1 s. For smaller animals, like a bird or even a fly, the smaller size implies generally much shorter reaction times. □

4 For a typical car highway speed of 100 km/hr, about how long does it take to travel from coast to coast? How does this compare to how long it would it take drive to the moon? To the Sun? To alpha centauri?

Solution Coast-to-coast distance is $d \approx 5000\,\text{km}$, so $t = d/v = 50\,\text{hr} \approx 2\,\text{d}$.
Moon distance is $d \approx 400,000\,\text{km}$, so $t = d/v = 4000\,\text{hr} \approx 160\,\text{d} \approx 0.5\,\text{yr}$.
Sun distance is $d \approx 1.5 \times 10^8\,\text{km}$, so $t = d/v = 1.5 \times 10^6\,\text{hr} \approx 5.4 \times 10^9\,\text{s} \approx 170\,\text{yr}$.
Alpha centauri distance is $d \approx 4.2\ \text{ly} \approx 4 \times 10^{13}\,\text{km}$, so $t = d/v = 4 \times 10^{11}\,\text{hr} \approx 1.5 \times 10^{15}\,\text{s} \approx 50\,\text{Myr}$. □

5 Why might astronomical observations be useful in measuring the speed of light?

Solution By timing the eclipses of the Jupiter moon Io, Ole Rmer estimated that light would take about 22 minutes to travel a distance equal to the diameter of Earth's orbit around the Sun. $\square$

6 How does the speed of sound on Earth compare to the speed of light?

Solution Speed of sound $c_s \approx 1500\,\mathrm{km/hr} \approx 400\,\mathrm{m/s}$, vs. speed of light $c = 3^8\,\mathrm{m/s}$. So $c/c_s \approx 7.5 \times 10^5$, i.e.almost factor million. $\square$

7 About how old is the oldest living thing on Earth? What about the oldest animal?

Solution Oldest trees are about 5000 yr. Oldest animal is a shark, about 400 years old. $\square$

2 Astronomical Distances

Quick Questions

1 Betelgeuse has a diameter of about $1800\,R_\odot$, and a distance of $d \approx 220$ pc. What is its angular diameter in milli-arcsec?

Solution .
$\alpha = (1800 \times R_\odot/220\,\text{pc})\,rad \approx 9\,\text{au}/(220\,\text{pc})\,rad = 0.074\,\text{arcsec} = 74$ milli-arcsec. □

2 What is the distance (in pc)to a star with a parallax of 0.1 arcsec?

Solution $d = \text{arcsec}/p\,\text{pc} = 1/0.1\text{pc} = 10\text{pc}$. □

3 The average separation between human eyes is $s \approx 60$ mm. Derive then a general formula for the distance d (in m) to an object with visual parallax angle α.

Solution $d = s/\alpha = 0.06/(\alpha/rad)$ m. □

4 If we lived on Mars instead of Earth, what would be the length of a parsec (in km).

Solution .
Radius of Mars' orbit $s_{mars} \approx 2.3 \times 10^8$ km, so $pc_{mars} = s(rad/arcsec) = 4.7 \times 10^{13}\,\text{km} = 1.5\,\text{pc}_{earth}$. □

5 Over a period of several years, two stars appear to go around each other with a fixed angular separation of 1 arcsec. What is the physical separation, in au, between the stars if they have a distance $d = 10$ pc from Earth?

Solution $\alpha = 1\,\text{arcsec}$, $s/\text{au} = \alpha/\text{arcsec}\,d/\text{pc} = 10$, so $s = 10\,\text{au}$. □

6 What angle α would the Earth-Sun separation subtend if viewed from a distance of $d = 1$ pc? Give your answer in both radian and arcsec. How about from a distance of $d = 1$ kpc?

Solution $\alpha = (s/\text{au})/(d/\text{pc})\,\text{arcsec}$, so for $d = 1\,\text{pc}$, $\alpha = 1\text{arcsec}$.
For $d = 1\,\text{kpc}$, $\alpha = 0.001\text{arcsec}$. □

3 Stellar Luminosity

Quick Questions

1 Compute $L_\odot$ given $F_\odot = 1.4\text{kW/m}^2$ and the known value of an au.

Solution $L_\odot = F_\odot\, 4\pi\, \text{au}^2 = 1.4{\times}10^3\text{W/m}^2\, 4\pi(1.5{\times}10^{11})^2 = 3.9{\times}10^{26}\,\text{W}$ $\square$

2 Recalling the relationship between an au and a parsec from equation (2.7), use eqns. (3.8) and (3.9) to compute the apparent magnitude of the Sun. What then is the Sun's distance modulus?

Solution
By (3.9), $M = 5 - 2.5\log(L/L_\odot) = 5$. From (3.8), we get $m_\odot = M + 5\log(d/10\text{pc}) = 5{+}5\log(1\,\text{au}/10\text{pc}) = 5{+}5\log((1/2{\times}5\text{pc})/10\text{pc}) = \boxed{-26.5 = m_\odot}$.
$m_\odot - M_\odot = 5 - 26.5 = \boxed{-21.5 = m_\odot - M_\odot}$ $\square$

3 Two stars have apparent magnitude $m_1 = +1$ and $m_2 = -1$. What the ratio of their fluxes f_1/f_2.

Solution $f_1/f_2 = 10^{(m_2-m_1)/2.5} = 10^{(-1-1)/2.5} = 0.16.$ $\square$

4 Two stars have apparent magnitude $m_1 = +1$ and $m_2 = +6$. What the ratio of their fluxes f_1/f_2.

Solution $f_1/f_2 = 10^{(m_2-m_1)/2.5} = 10^{(6-1)/2.5} = 100.$ $\square$

4 Surface Temperature from a Star's Color

Quick Questions

1 Derive equation (4.6) from equation (4.4) using the definition (4.5).

2 Using $B_\nu d\nu = B_\lambda d\lambda$ and the relationship between frequency ν and wavelength λ, derive equation (4.4) from equation (4.3).

3 As discussed in chapter 32, the Cosmic MIcrowave Background (CMB) has a temperature of 2.7 K. What is its peak wavelength, both in nm and mm?

Solution $\lambda_{max} = 500\,\mathrm{nm}(6000\,\mathrm{K}/2.7\,\mathrm{K}) = 1.1 \times 10^6\,\mathrm{nm} = 1.1\,\mathrm{mm}$ □

4 Two photons have wavelength ratio $\lambda_2/\lambda_1 = 2$.

a) What is the ratio of their period P_2/P_1?

Solution $\frac{P_2}{P_1} = \frac{\lambda_2/c}{\lambda_1/c} = \frac{\lambda_2}{\lambda_1} = \boxed{2}$ □

b) What is the ratio of their frequency ν_2/ν_1?

Solution $\frac{\nu_2}{\nu_1} = \frac{c/\lambda_2}{c/\lambda_1} = \frac{\lambda_1}{\lambda_2} = \boxed{\frac{1}{2}}$ □

c) What is the ratio of their energy E_2/E_1?

Solution $\frac{E_2}{E_1} = \frac{h\nu_2}{h\nu_1} = \frac{\nu_2}{\nu_1} = \boxed{\frac{1}{2}}$ □

5 The star αCenA has a luminosity $L = 1.5L_\odot$, and lies at a distance of 4.37 ly. What are its absolute magnitude M and apparent magnitude m? What then is its distance modulus?

Solution $M = 4.8 - 2.5\log(L/L_\odot) = 4.8 - \log(1.5) = 4.63$.
$m = M + 5\log(d/10pc) = 4.63 + \log(4.37/3.6/10) = 3.76$.
$m - M = -0.87$. □

6 Two stars have the same luminosity, but star 2 is 10 times as far as star 1. Give the ratio of their fluxes f_2/f_1 and the difference in the apparent magnitude $m_2 - m_1$?

Solution $f_2/f_1 = (L_2/L_1)/(d_2/d_1)^2 = 1/(10)^2 = 10^{-2}$
$m_2 - m_1 = 2.5\log(f_1/f_2) = 5$ □

7 From figure 4.3, estimate the temperatures T (in K) of stars with colors B-V=1 and B-V=0.

Solution $B - V = 0$ gives $T \approx 11,000\,\mathrm{K}$; $B - V = 1$ gives $T \approx 5000\,\mathrm{K}$. □

8 Assuming the Earth has an average temperature equal to that of typical spring day, i.e. 50°F, compute the peak wavelength of Earth's blackbody radiation. What part of the EM spectrum does this lie in?

Solution

$T = 50^\circ\,\mathrm{F} = (50^\circ\,\mathrm{F} - 32^\circ\,\mathrm{F})\frac{5}{9}^\circ\,\mathrm{C} = 10^\circ\,\mathrm{C} = 283\,\mathrm{K} \approx 300\,\mathrm{K}$

$\lambda_{max} = 500\,\mathrm{nm}\frac{6000\,\mathrm{K}}{300\,\mathrm{K}} = 10000\,\mathrm{nm} = \boxed{10\,\mu\mathrm{m}}$.

This is in the InfraRed (IR). □

5 Stellar Radius from Luminosity and Temperature

Quick Questions

1 What is the luminosity (in $L_\odot$) of a star with 10 times the solar temperature and ten times the solar radius?

Solution $L/L_\odot = (T/T_\odot)^4\,(R/R_\odot)^2 = 10^4\,10^2 = 10^6$. □

2 What is the radius (in $R_\odot$) of a star that is both twice as luminous and twice as hot as the Sun?

Solution $R/R_\odot = (L/L_\odot)^{1/2}\,(T/T_\odot)^{-2} = 2^{1/2}\,2^{-2} = 2^{-3/2}$. □

3 Suppose star 2 is both twice as hot and twice as far star 1, but they have the same apparent magnitude. What is the ratio of their stellar radii, R_2/R_1?

Solution $m_1 = m_2$ implies $F_2 = F_1$, so $F_2/F_1 = 1 = (L_2/L_1)/(d_2/d_1)^2$, giving $L_2/L_1 = 4$. $R_2/R_1 = (L_2/L_1)^{1/2}(T_1/T_2)^2 = 2 * (1/2)^2 = 1/2$. □

4 A red giant star has a temperature of 3000 K and luminosity $L = 1000L_\odot$. About what is its radius, R (in $R_\odot$)?

Solution $T = 3000\,\mathrm{K} = T_\odot/2$, so $R/R_\odot = (L/L_\odot)^{1/2}\,(T/T_\odot)^{-2} = 1000^{1/2}(1/2)^2 = 2.5\sqrt{10} \approx 8$. □

6 Composition and Ionization from Stellar Spectra

Quick Questions

1 On the H-R diagram, where do we find stars that are:

a) Hot and luminous? $\boxed{\text{Upper left}}$

b) Cool and luminous? $\boxed{\text{Upper right}}$

c) Cool and Dim? $\boxed{\text{Lower right}}$

d) Hot and Dim? $\boxed{\text{Lower left}}$

2 Of the above, which of these are known as:

a) White Dwarfs? $\boxed{\text{d}}$

b) Red Giants? $\boxed{\text{b}}$

c) Blue supergiants? $\boxed{\text{a}}$

d) Red dwarfs? $\boxed{\text{c}}$

3 For the `Gaia` H-R diagram in figure 6.5, about what is the luminosity ratio between a white-dwarf star (at, say, color index 1), and a main sequence star of the same color? What is the implied ratio in the stellar radii, $R_{\rm wd}/R_{\rm ms}$?

Solution From figure 6.5, the absolute magnitude of a white dwarf is $M_{wd} \approx +14$, while for mass sequence star of same color index 1 it is $M_{ms} \approx 3$, giving a luminosity ratio $L_{ms}/L_{wd} = 10^{(M_{wd}-M_{ms})/2.5} = 10^{11/2.5} = 10^{4.4}$.
Since same color implies same temperature, we have $R_{\rm wd}/R_{\rm ms} = (L_{\rm wd}/L_{\rm ms})^{1/2} = 10^{-2.2} = 6.3 \times 10^{-3}$. □

4 Referring to figure 6.3, what is the typical factor difference between the abundance fractions of odd vs. even atomic numbers? Discuss possible reasons for this difference? (Hint: Which can be made directly by fusion of Helium?)

Solution According to figure 6.3, even atomic number elements have nearly a 100 times the abundance of next heigher odd element.
Such even number elements can be synthesized by multiple addition of a Helium nucleus, whereas odd element require more complex synthesis. □

7 Surface Gravity and Escape/Orbital Speed

Quick Questions

1 What is the ratio of the *energy* needed to escape the Earth vs. that needed to reach LEO? What is the ratio of the associated *speeds*?

Solution $E_{esc} = GM/R$, $E_{leo} = GM/2R$ so $E_{esc} = 2E_{leo}$
$V_{esc} = \sqrt{2GM/R} = \sqrt{2}V_{leo}$, since $V_{leo} = \sqrt{GM/R}$. □

2 *Weight change on different bodies* Suppose a man weighs 200 lbs. on Earth. What is his weight on:

a) The Sun.
Solution $g_\odot/g_e = (M_\odot/M_e)/(R_\odot/R_e)^2 \approx 27$.
So $W_\odot = 27 \times 200 = 5400$ lbs. □

b) A red giant with $M = 1M_\odot$ and $R = 100R_\odot$.
Solution $W = 1/100^2\, W_\odot = 0.54$ lbs. □

c) A white dwarf with $M = 1M_\odot$ and $R = 0.01R_\odot$.
Solution $W = 1/(0.01)^2\, W_\odot = 5.4 \times 10^7$ lbs. □

d) A neutron star with $M = 1M_\odot$ and $R = 10$ km.
Solution $W = 1 \times (7 \times 10^5/10)^2\, W_\odot = 2.7 \times 10^{12}$ lbs. □

3 In CGS units, the Sun has $\log g_\odot \approx 4.44$. Compute the $\log g$ for stars with:

Solve for $\log g_{ast}$ $\quad g = \frac{GM}{R^2} \implies g_*/g_\odot = \frac{GM/R^2}{GM_\odot/R_\odot^2} = \frac{M}{M_\odot}(\frac{R^2}{R_\odot^2})^{-1} \implies$
$\log g_* = \log(\frac{M}{M_\odot}(\frac{R}{R_\odot})^{-2}) + \log g_\odot = 4.44 + \log(\frac{M}{M_\odot}) - 2\log(\frac{R}{R_\odot})$ □

a) $M = 10M_\odot$ and $R = 10R_\odot$
Solution $\log g_* = 4.44 + \log(\frac{10M_\odot}{M_\odot}) - 2\log(\frac{10R_\odot}{R_\odot}) = \boxed{3.44}$ □

b) $M = 1M_\odot$ and $R = 100R_\odot$
Solution $\log g_* = 4.44 + \log(\frac{1M_\odot}{M_\odot}) - 2\log(\frac{100R_\odot}{R_\odot}) = \boxed{0.44}$ □

c) $M = 1M_\odot$ and $R = 0.01R_\odot$
Solution $\log g_* = 4.44 + \log(\frac{1M_\odot}{M_\odot}) - 2\log(\frac{0.01R_\odot}{R_\odot}) = \boxed{8.44}$ □

d) The Sun has an escape speed of $V_{e\odot} = 618$ km/s. Compute the escape speed V_e of the stars in the above.
Reformulate into solar units $V_e = \sqrt{\frac{2GM}{R}} \implies V_e = 618\,\text{km/s}\sqrt{\frac{M/M_\odot}{R/R_\odot}}$ □

a.

Solution $V_e = 618\,\mathrm{km/s}\sqrt{\frac{10M_\odot/M_\odot}{10R_\odot/R_\odot}} = \boxed{618\,\mathrm{km/s}}$ □

b.

Solution $V_e = 618\,\mathrm{km/s}\sqrt{\frac{1M_\odot/M_\odot}{100R_\odot/R_\odot}} = \boxed{61.8\,\mathrm{km/s}}$ □

c.

Solution $V_e = 618\,\mathrm{km/s}\sqrt{\frac{1M_\odot/M_\odot}{0.01R_\odot/R_\odot}} = \boxed{6180\,\mathrm{km/s}}$ □

4 The Earth orbits the Sun with an average speed of $V_{\mathrm{orb}} = 2\pi\mathrm{au/yr} = 30\,\mathrm{km/s}$. Compute the orbital speed V_{orb} (in km/s) of a body at the following distances from the stars with the quoted masses:

Solution $V_{orb} = \sqrt{\frac{GM}{r}} \implies V_{orb}/V_e = \sqrt{\frac{GM/r}{GM_\odot/r_e}} = \sqrt{\frac{M/M_\odot}{r/r_e}} \implies V_{orb} = 30\,\mathrm{km/s}\sqrt{\frac{M/M_\odot}{r/1\,\mathrm{au}}}$ □

a) $M = 10M_\odot$ and $d = 10\,\mathrm{au}$.

Solution $V_{orb} = 30\,\mathrm{km/s}\sqrt{\frac{10M_\odot/M_\odot}{10\,\mathrm{au}/1\,\mathrm{au}}} = \boxed{30\,\mathrm{km/s}}$ □

b) $M = 1M_\odot$ and $d = 100\,\mathrm{au}$.

Solution $V_{orb} = 30\,\mathrm{km/s}\sqrt{\frac{1M_\odot/M_\odot}{100\,\mathrm{au}/1\,\mathrm{au}}} = \boxed{3\,\mathrm{km/s}}$ □

c) $M = 1M_\odot$ and $d = 0.01\,\mathrm{au}$.

Solution $V_{orb} = 30\,\mathrm{km/s}\sqrt{\frac{1M_\odot/M_\odot}{0.01\,\mathrm{au}/1\,\mathrm{au}}} = \boxed{300\,\mathrm{km/s}}$ □

5 Rank the following in order of increasing energy change ΔE required:

a) Escaping the solar system from 1 au.
b) Escaping the Earth from its surface.
c) Escaping the Earth from low-earth-orbit.
d) Escape the solar system from Earth-Sun orbit.
e) Reducing Earth-Sun orbit to an orbit that would impact the Sun.

Solution c, b, d, e, a □

8 Stellar Ages and Lifetimes

Quick Questions

1 What are the luminosities (in $L_\odot$) and the expected main sequence lifetimes (in Myr) of stars with masses:

a) $10\,M_\odot$?

Solution $L/L_\odot \propto (M/M_\odot)^3$

$t_{ms} = 10000\text{Myr}(\frac{M_\odot}{M})^2$

$t_{ms} = 10000\,\text{Myr}(\frac{M_\odot}{10M_\odot})^2 = \boxed{100\,\text{Myr} = t_{ms,a}}$

$L/L_\odot = (10M_\odot/M_\odot)^3 \implies L = \boxed{10^3 L_\odot = L_a}$ □

b) $0.1\,M_\odot$?

Solution $t_{ms} = 10000\text{Myr}(\frac{M_\odot}{0.1M_\odot})^2 = \boxed{10^6\text{Myr} = t_{ms,b}}$

$L/L_\odot = (0.1M_\odot/M_\odot)^3 \implies L = \boxed{10^{-3}L_\odot = L_b}$ □

c) $100\,M_\odot$?

Solution $t_{ms} = 10000\text{Myr}(\frac{M_\odot}{100M_\odot})^2 = \boxed{1\text{Myr} = t_{ms,c}}$

$L/L_\odot = (100M_\odot/M_\odot)^3 \implies L = \boxed{10^6 L_\odot = L_c}$ □

2 Confirm the integration result in equation (8.3).

3 What is the age (in Myr) of a cluster with a MS turnoff luminosity at:

a) $L = 1L_\odot$

b) $L = 10L_\odot$

c) $L = 100L_\odot$

d) $L = 1000L_\odot$

e) $L = 10^5 L_\odot$

f) $L = 10^6 L_\odot$

Solution $t_{cluster} = 10,000\,\text{Myr}(\frac{L}{L_\odot})^{-2/3}$

a. $t_{cluster} = 10,000\,\text{Myr}$

b. $t_{cluster} = 2200\,\text{Myr}$

c. $t_{cluster} = 460\,\text{Myr}$

d. $t_{cluster} = 100\,\text{Myr}$

e. $t_{cluster} = 22\,\text{Myr}$

e. $t_{cluster} = 4.6\,\text{Myr}$

e. $t_{cluster} = 1\,\text{Myr}$ □

9 Stellar Space Velocities

Quick Questions

1 What's the Doppler shift ratio $\Delta\lambda/\lambda_o$ for a star moving away by a speed $V_r = 150\,\text{km/s}$?

Solution $\Delta\lambda/\lambda_o = V_r/c = 150/(3 \times 10^5) = 5 \times 10^{-4}$. $\square$

2 A line with rest wavelength $\lambda_o = 1000\,\text{nm}$ is observed in a star to have a wavelength $\lambda = 999\,\text{nm}$. What is the star's radial speed V_r (in km/s) and is it moving toward or away from us?

Solution $V_o = c(\lambda - \lambda_o)/\lambda_o = c(-1/1000) = -300\,\text{km/s}$.
Since $V_o < 0$, it's moving away. $\square$

3 A star with parallax $p = 0.05\,\text{arcsec}$ has a proper motion $\mu = 1\,\text{arcsec/yr}$. What is its tangential speed V_t, in both km/s and au/yr?

Solution $V_t = 4.7\mu/p\,\text{km/s} = 4.71/0.05 = 94\,\text{km/s} = 20\text{au/yr}$. $\square$

4 What is the total space velocity V (in km/s) of a star that has both the properties listed in the previous two questions?

Solution $V = \sqrt{V_r^2 + V_t^2} = 314\,\text{km/s}$. $\square$

10 Using Binary Systems to Determine Masses and Radii

Quick Questions

1 An asteroid of mass $m \ll M_\odot$ has a circular orbit at a distance $a = 4\,\text{au}$ from the Sun.

a) What is the asteroid's orbital period P, in yrs?
Solution $P/yr = (a/\text{au})^{3/2} = 4^{3/2} = 8$, so $P = 8\,\text{yr}$. □

b) How would this change for an asteroid with orbital eccentricity $\epsilon = 0.5$ and a semi-major axis $a = 4\,\text{au}$?
Solution Kepler's 3rd law does not depend on eccentricity, so it would not change, with period still $P = 8\,\text{yr}$. □

c) How would this change if asteroid mass is doubled?
Solution For asteroid mass $m \ll M_\odot$, Kepler's 3rd law also not depend on the mass m, so doubling the mass would not change the period, which again would be $P = 8\,\text{yr}$. □

d) What is the period (in yr) for a similar asteroid with $a = 4\,\text{au}$ orbiting a star with mass $M = 4M_\odot$.
Solution .
For star of mass M, Kepler's 3rd law becomes $P/yr = (M/M_\odot)^{1/2}\,(a/\text{au})^{2/3}$, so now $P = 4^{1/2} \times 8 = 16\,\text{yr}$. □

2 In eclipsing binaries, note that the net area of stellar surface eclipsed is the same whether the smaller or bigger star is in front. So why then is one of the eclipses deeper than the other? What quantity determines which of the eclipses will be deeper?

Solution The eclipse surface is the same, but the surface brightness of one star can be greater than the other.
The surface brightness depends on temperature, so the eclipse of the hotter star will be deeper. □

3 About what are the luminosities L (in $L_\odot$) of main-sequence stars with the following masses:

a) $M = 2M_\odot$
b) $M = 7M_\odot$
c) $M = 10M_\odot$

d) $M = 50M_\odot$
e) $M = 100M_\odot$

Solution $L/L_\odot = (M/M_\odot)^3$, so
a. $L = 8\,L_\odot$.
b. $L = 343\,L_\odot$.
c. $L = 1000L_\odot$.
d. $L = 1.25 \times 10^5\,L_\odot$.
e. $L = 10^6\,L_\odot$. $\square$

11 Stellar Rotation

Quick Questions

1 A line with rest wavelength $\lambda_o = 500\,\text{nm}$ is rotational broadened to a full width of 0.5 nm. Compute the value of $V \sin i$, in km/s.

Solution
$\frac{\Delta\lambda_{full}}{\lambda_o} = \frac{2V \sin i}{c} \implies V \sin i = \frac{\Delta\lambda_{full}}{2\lambda_o} c = \frac{0.5\,\text{nm}}{2 \cdot 500\,\text{nm}} (c) = \frac{1}{2 \cdot 10^3} (c) = \frac{300\,\text{km/s}}{2} =$
$\boxed{150\,\text{km/s}}$ $\square$

2 Derive equation (11.3) from the definitions of rotational Doppler width $\Delta\lambda_{\text{rot}}$ (11.1) and equivalent width W_λ (11.2), using the wavelength scaling given in footnote 11.1.

3 Write a formula for the equatorial rotation speed V_{rot} (in km/s) in terms of a star's rotation period (in days) and radius R (in $R_\odot$).

Solution $V_{\text{rot}} = 2\pi R/P =$
$(R/R_\odot)/(P/day)(2\pi \times 7 \times 10^5/(24 \times 3600) = 51(R/R_\odot)/(P/day)\,\text{km/s}.$ $\square$

4 If the equatorial rotation speed is V_{rot}, what is the associated speed at any latitude ℓ? Does make any difference if this is in the northern ($\ell > 0$) or southern ($\ell < 0$) hemisphere?

Solution $V(\theta) = V_{\text{rot}} \cos \ell$, and since cos is even function, speed is same for a given latitude in north or south. $\square$