Basic Integration Problems

I. Find the following integrals.

1. $\int (5x^2 - 8x + 5)dx$

2. $\int (-6x^3 + 9x^2 + 4x - 3)dx$

3. $\int (x^{\frac{3}{2}} + 2x + 3)dx$

4. $\int \left(\frac{8}{x} - \frac{5}{x^2} + \frac{6}{x^3} \right)dx$

5. $\int (\sqrt{x} + \frac{1}{3\sqrt{x}})dx$

6. $\int (12x^{\frac{3}{4}} - 9x^{\frac{5}{3}})dx$

7. $\int \frac{x^2 + 4}{x^2}dx$

8. $\int \frac{1}{x\sqrt{x}}dx$

9. $\int (1 + 3t)t^2\ dt$

10. $\int (2t^2 - 1)^2\ dt$

11. $\int y^2 \sqrt[3]{y}\ dy$

12. $\int d\theta$

13. $\int 7\sin(x)dx$

14. $\int 5\cos(\theta)d\theta$

15. $\int 9\sin(3x)dx$

16. $\int 12\cos(4\theta)d\theta$

17. $\int 7\cos(5x)dx$

18. $\int 4\sin\left(\frac{x}{3}\right)dx$

19. $\int 4e^{-7x}dx$

20. $\int 9e^{\frac{x}{4}}dx$

21. $\int -5\cos \pi x\ dx$

22. $\int -13e^{6t}dt$

II.   Evaluate the following definite integrals.

1. $\int_1^4 (5x^2 - 8x + 5)dx$

2. $\int_1^9 (x^{\frac{3}{2}} + 2x + 3)dx$

3. $\int_4^9 (\sqrt{x} + \frac{1}{3\sqrt{x}})dx$

4. $\int_1^4 \frac{5}{x^3}dx$

5. $\int_{-1}^2 (1+3t)t^2\, dt$

6. $\int_{-2}^1 (2t^2 - 1)^2\, dt$

Solutions

I. Find the following integrals.

1. $\int(5x^2 - 8x + 5)dx = \boxed{\dfrac{5x^3}{3} - 4x^2 + 5x + C}$

2. $\int(-6x^3 + 9x^2 + 4x - 3)dx = \boxed{\dfrac{-3x^4}{2} + 3x^3 + 2x^2 - 3x + C}$

3. $\int(x^{\frac{3}{2}} + 2x + 3)dx = \boxed{\dfrac{2x^{\frac{5}{2}}}{5} + x^2 + 3x + C}$

4. $\int\left(\dfrac{8}{x} - \dfrac{5}{x^2} + \dfrac{6}{x^3}\right)dx = \int\left(\dfrac{8}{x} - 5x^{-2} + 6x^{-3}\right)dx$

$= 8Ln(x) - \dfrac{5x^{-1}}{-1} + \dfrac{6x^{-2}}{-2} = \boxed{8Ln(x) + \dfrac{5}{x} - \dfrac{3}{x^2} + C}$

5. $\int(\sqrt{x} + \dfrac{1}{3\sqrt{x}})dx = \int\left(x^{\frac{1}{2}} + \dfrac{1}{3}x^{-\frac{1}{2}}\right)dx$

$= \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{1}{3}\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} = \boxed{\dfrac{2}{3}x^{\frac{3}{2}} + \dfrac{2}{3}x^{\frac{1}{2}} + C}$

6. $\int(12x^{\frac{3}{4}} - 9x^{\frac{5}{3}})dx = \boxed{\dfrac{48x^{\frac{7}{4}}}{7} - \dfrac{27x^{\frac{8}{3}}}{8} + c}$

7. $\int\dfrac{x^2 + 4}{x^2}dx = \int 1 + 4x^{-2}\ dx = \boxed{x - \dfrac{4}{x} + C}$

8. $\int\dfrac{1}{x\sqrt{x}}dx = \int x^{-\frac{3}{2}}dx = \boxed{-\dfrac{2}{\sqrt{x}} + C}$

9. $\int(1 + 3t)t^2\ dt = \int t^2 + 3t^3\ dt = \boxed{\dfrac{t^3}{3} + \dfrac{3t^4}{4} + C}$

10. $\int(2t^2 - 1)^2\ dt = \int 4t^4 - 4t^2 + 1\ dt = \boxed{\dfrac{4t^5}{5} - \dfrac{4t^3}{3} + t + C}$

11. $\int y^2 \sqrt[3]{y}\, dy = \int y^{\frac{7}{3}} dy = \boxed{\dfrac{3y^{\frac{10}{3}}}{10} + C}$

12. $\int d\theta = \boxed{\theta + C}$

13. $\int 7\sin(x)dx = \boxed{-7\cos(x) + C}$

14. $\int 5\cos(\theta)d\theta = \boxed{5\sin(\theta) + C}$

15. $\int 9\sin(3x)dx = \boxed{-3\cos(3x) + C}$

16. $\int 12\cos(4\theta)d\theta = \boxed{3\sin\ 4\theta\ + C}$

17. $\int 7\cos(5x)dx = \boxed{\dfrac{7\sin(5x)}{5} + C}$

18. $\int 4\sin\left(\dfrac{x}{3}\right)dx = \boxed{-12\cos\left(\dfrac{x}{3}\right) + C}$

19. $\int 4e^{-7x}dx = \boxed{-\dfrac{4e^{-7x}}{7} + C}$

20. $\int 9e^{\frac{x}{4}}dx = \boxed{36e^{\frac{x}{4}} + C}$

21. $\int -5\cos\ \pi x\ dx = \boxed{-\dfrac{5\sin(\pi x)}{\pi} + C}$

22. $\int -13e^{6t}dt = \boxed{-\dfrac{13e^{6t}}{6} + C}$

II. Evaluate the following definite integrals.

1. $\int_1^4 (5x^2 - 8x + 5)dx = \left(\dfrac{5x^3}{3} - 4x^2 + 5x\right)\Big|_1^4 = \dfrac{188}{3} - \dfrac{8}{3} = \boxed{60}$

2. $\int_1^9 (x^{\frac{3}{2}} + 2x + 3)dx = \left(\dfrac{2x^{\frac{5}{2}}}{5} + x^2 + 3x\right)\Bigg|_1^9 = \dfrac{1026}{5} - \dfrac{22}{5} = \boxed{\dfrac{1001}{5}} = 200.2$

3. $\int_4^9 (\sqrt{x} + \dfrac{1}{3\sqrt{x}})dx = \left(\dfrac{2}{3}x^{\frac{3}{2}} + \dfrac{2}{3}x^{\frac{1}{2}}\right)\Big|_4^9 = 20 - \dfrac{20}{3} = \boxed{\dfrac{40}{3}} = 13.333$

4. $\int_1^4 \dfrac{5}{x^3}dx = -\dfrac{5}{2x^2}\Big|_1^4 = -\dfrac{5}{32} + \dfrac{5}{2} = \boxed{\dfrac{75}{32}} = 2.344$

5. $\int_{-1}^2 (1+3t)t^2\, dt = \left(\dfrac{t^3}{3} + \dfrac{3t^4}{4}\right)\Big|_{-1}^2 = \dfrac{44}{3} - \dfrac{5}{12} = \boxed{\dfrac{57}{4}} = 14.25$

6. $\int_{-2}^1 (2t^2 - 1)^2\, dt = \left(\dfrac{4t^5}{5} - \dfrac{4t^3}{3} + t\right)\Big|_{-2}^1 = \dfrac{7}{15} + \dfrac{254}{15} = \boxed{\dfrac{87}{5}} = 17.4$