CSCI 445 –Final Report

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Multirobot system

Localization

For the localization of the robot at the beginning of the program, we used the particle filtering method that we had created in labs 8 and 9. The basic process behind this was to set a cloud of particles randomly placed throughout the scene configuration, and through automated movements, use an estimation, based on the orientation and position of the particles, to find, or localize, where the actual robot was.

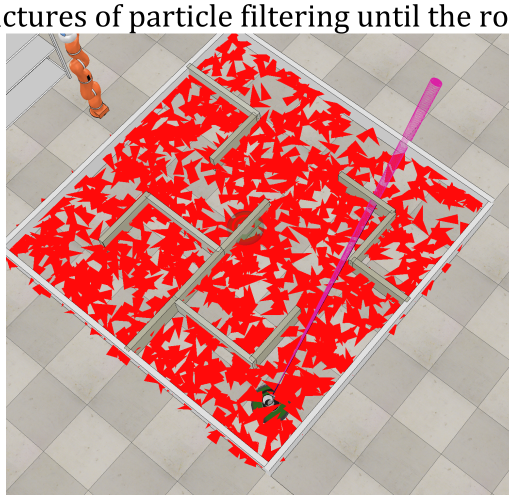
To increase the accuracy of our particle filtering we increased the number of particles used to 5000, giving us plenty of points to take and get a more accurate positioning. From there, we did our automatic particle filtering. Because we did not know the layout of the environment ahead of time, we decided to set a function that would continuously rotate the robot math.pi/5 degrees until the localization standards were met. This part of the process was done in the self.positioning() function in our finalproject.py file.

Every time the robot's position was estimated through the particle filtering, the function isLocalized() was called to test whether or not a good estimation of the robots position had been found. Here we used preset variables min_dist_to_localize and min_theta_to_localize to compare to the current estimated position and angle and test whether or not we had gotten to the location of the robot. The position to the goal of the current particles was found through the Euclidean distance of all of the particles in our particle filter. Similarly the theta was based on the get_estimate() function defined in particle_filter.py and the angles of all of the particles.

Once the distance and theta values were within our specified bounds (which we modified based on trial and error in the different environments), we scaled the values to the needed size to fit the scaling of the .png images

For the second environment that we received we set the min_dist_to_localize to 2.1 and min_theta_to_localize to math.pi/4. We found that these values localized the robot the fastest in the new environment. Theta localization value remained the same with both configurations we were provided but the distance localization value changed due to the configuration change of the environment.

Pictures of particle filtering until the robot had been localized:

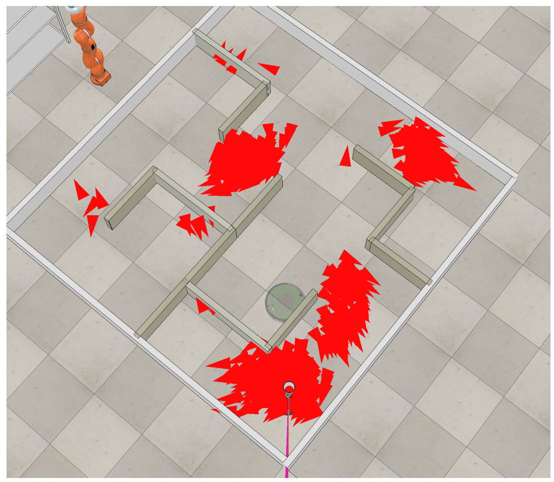
1. 

2. 

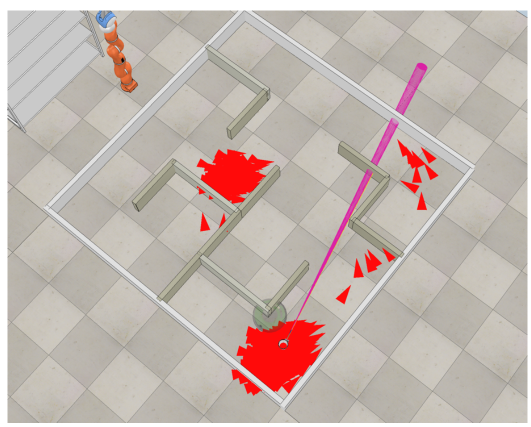
3. 

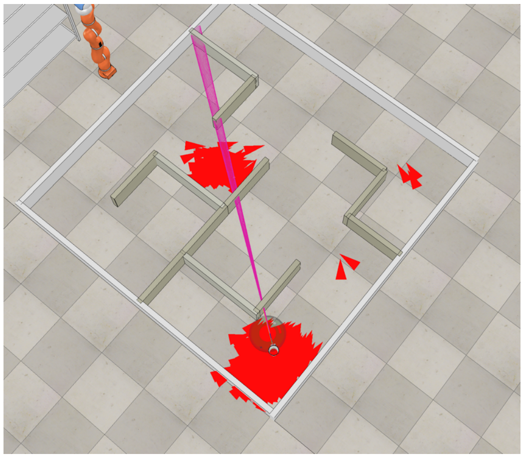
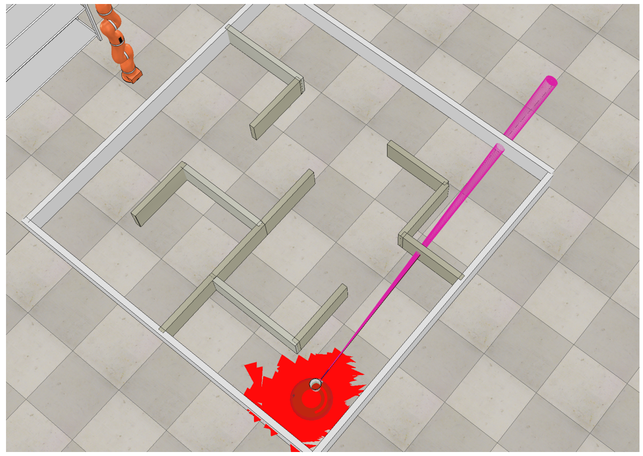
4. 

5. 

6. 

7. 

8. 

9.  10. 

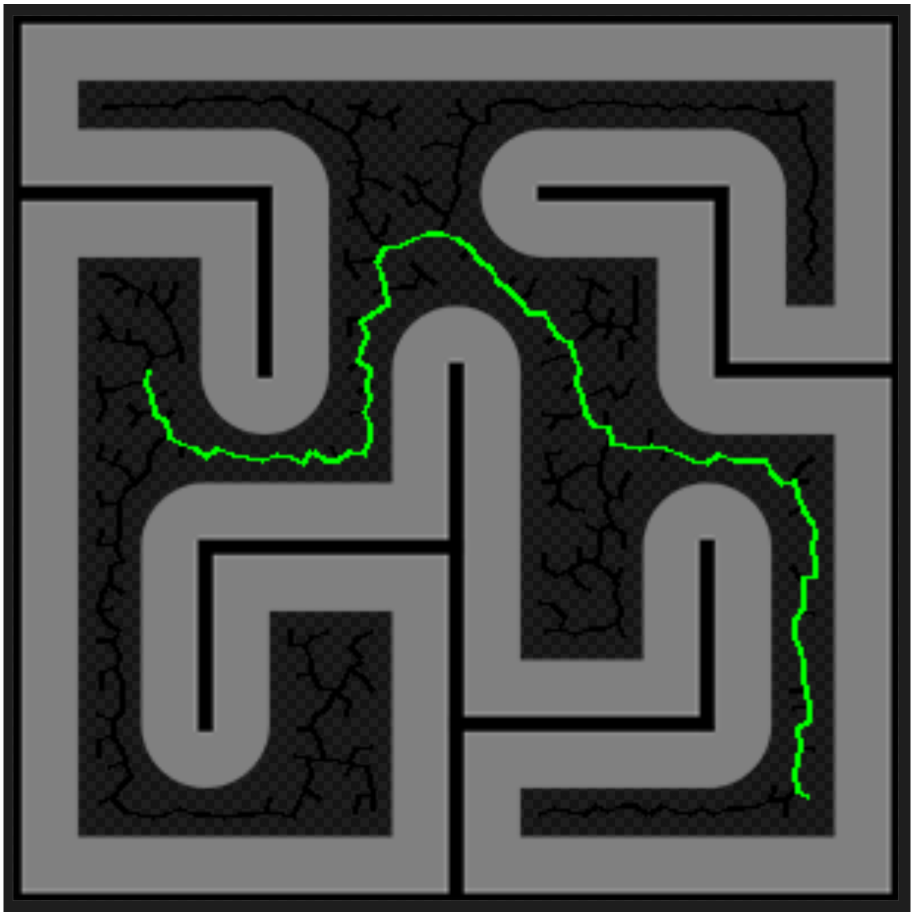
Path Planning

Once the robot had been localized, we set the starting point of the robot to the points found through particle filtering. From here we decided to plan our path by using RRT, which we had done in lab 10. We had many challenges with the image configuration and background. Some of these issues included, incorrect scaling, not registering the background color of the image, odd orientation problems with the image, and the path building from the end location as if it was building from the start.

To fix the scaling we always scaled the y value as 300-y*100, to fit the images defined. We fixed the background image color by using a modified image that another team provided us, which is the image included in our file. To fix the orientation problems we had to modify a few things when following the path, which I will describe later. Finally for building the path from the start, we supplied the arm location, which we found in vrep and scaled, as the starting location to our rrt.build function, and set the goal as our starting location. This consistently solved the path problem in both the original and final configurations that we were given.

The final step to our path planning, was adding a buffer to the obstacle space in the _extend() function from rrt.py. This was essential because particle filtering was never exactly perfect, and some inconsistencies in robot location could cause it to run into the wall. With the buffer we eliminated the changes of that happening and felt more confident with our implementation.

An example of one of the paths found is included in the image below. (Found in file PATH_TO_FOLLOW.png)



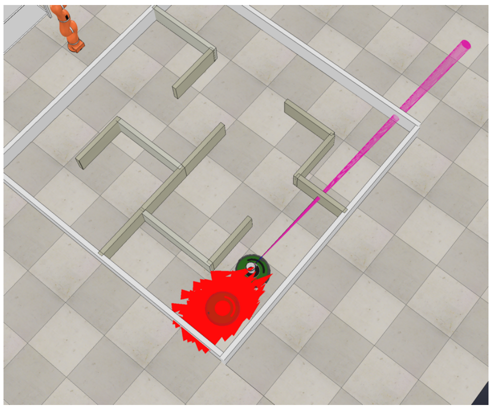
Path following

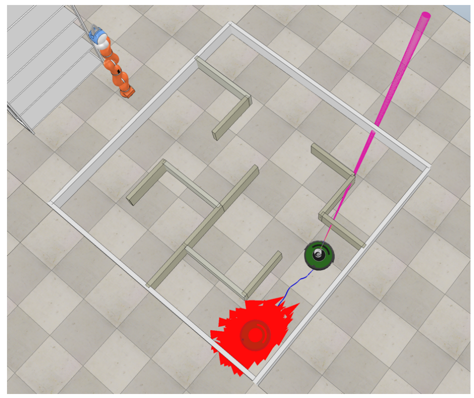
Next, we got to the path following process. This is where a lot of challenges occurred when we transitioned to the final project configuration. With the original configuration, it was easy for us to set the x odometry as the starting x position, and the y odometry as the starting y position. This was not the case for the final configuration. While a correct path was consistently being planned, when the robot went to follow it, it never would follow the correct path. Per Shihan's guidance we flipped the x and y odometry so that the y odometry would be the starting x position and the x odometry the starting y position. Now the robot seemed to go in the correct order but would always do the exact reflection of the path found. So, we then multiplied the y odometry value by -1 and the path was correct.

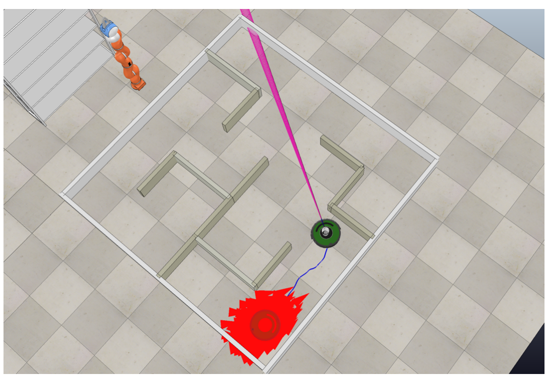
The next step in our path following was removing the first few locations found in the path planning. Due to the odometry being slightly off, this would cause the robot to do some loops in the scene, and by removing them, we set the first point in the path to a plausible location for the robot to easily get to. Then, because we reversed the starting and goal points, we reversed the path found, and that is when we began following it.
Like the switching of the x and y odometry, we did the same thing with our x and y goal points for each point in the path. This made sure that the robot always continued on the correct path.

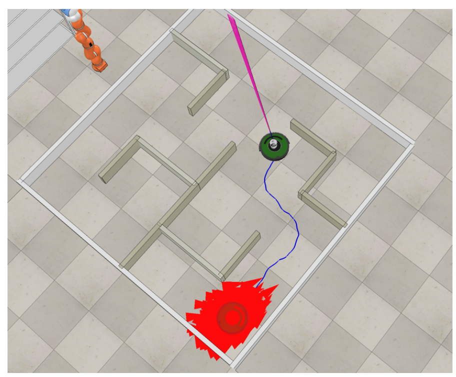
Another change we made was implementing a pidDistance calculation. Due to the odometry we found that without giving the robot time to adjust to the proper locations, we had more occurrences of the robot hitting the wall. While this decreased speed, it maintained accuracy and minimized drift.

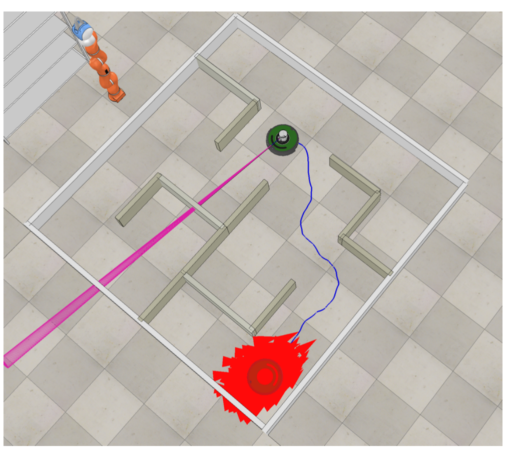
From here, it was a simple while loop, looping through all the points we found in our path. This is reflected in the images of our simulation below.

1. 

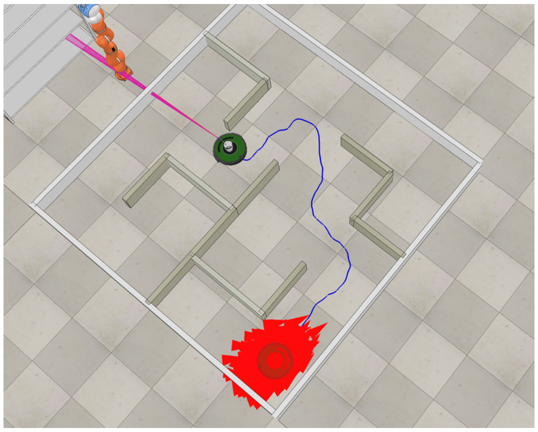
2. 

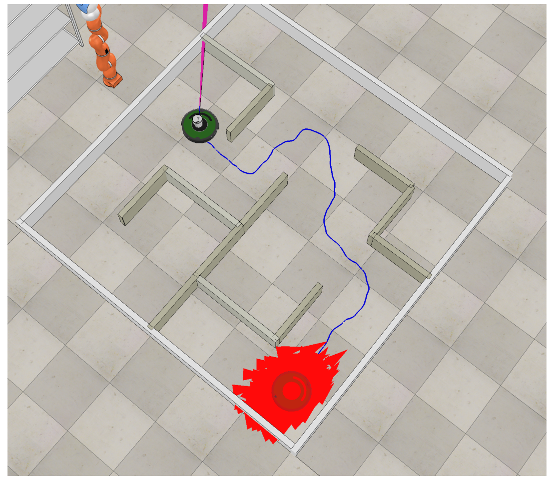
3. 

4. 

5. 

6. 

7. 

8. 

Sample output of system (yellow highlight is localization, blue is path planning, green is path following, pink is arm control)

```
PS C:\Users\Lauren Hampson\OneDrive\ROBOTICS\fp\final_configuration> python run.py --sim
finalproject
theta: 0.5788803805691348 33.16740264953833
self.odometry.x 0.5 self.odometry.y 0.5 self.odometry.theta 0.5788803805691348
self.pf.x 1.4413007946574377 self.pf.y 1.4631348358440532 self.pf.theta 3.17071617733052
EUCLIDEAN_DISTANCE  1.3467278485421896
DISTANCE_theta  2.5918357967613854
theta: 1.1577607611382705 66.33480529907672
self.odometry.x 0.5 self.odometry.y 0.5 self.odometry.theta 1.1577607611382705
self.pf.x 1.2941123601378473 self.pf.y 1.3226953827715957 self.pf.theta 3.10254487491885
EUCLIDEAN_DISTANCE  1.1434343590068492
DISTANCE_theta  1.9447841137805792
theta: 1.736641141707405 99.50220794861504
self.odometry.x 0.5000041877133693 self.odometry.y 0.5000003746573494 self.odometry.theta
1.736641141707405
self.pf.x 1.3775513571037081 self.pf.y 1.2218214671562346 self.pf.theta 3.014696662755686
EUCLIDEAN_DISTANCE  1.1362722931064015
DISTANCE_theta  1.2780555210482811
theta: 2.3155215222765375 132.66961059815324
self.odometry.x 0.500007648653908 self.odometry.y 0.5000027598003425 self.odometry.theta
2.3155215222765375
self.pf.x 1.4095672049457848 self.pf.y 1.1926310256202246 self.pf.theta 2.526349231694386
EUCLIDEAN_DISTANCE  1.1432552213108556
DISTANCE_theta  0.21082770941784856
theta: 2.8944019028456696 165.8370132476914
self.odometry.x 0.5000118490724795 self.odometry.y 0.5000089929363845 self.odometry.theta
2.8944019028456696
self.pf.x 2.149282335074971 self.pf.y 0.8611432307885898 self.pf.theta 3.7053766790730496
EUCLIDEAN_DISTANCE  1.68834566180862
DISTANCE_theta  0.81097477622738
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self.odometry.x 0.5000118490724795 self.odometry.y 0.5000089929363845 self.odometry.theta
3.473282283414803
self.pf.x 2.343029249837611 self.pf.y 0.7018542491955163 self.pf.theta 4.479382575255751
EUCLIDEAN_DISTANCE  1.8540373909383208
DISTANCE_theta  1.0061002918409478
theta: 4.052162663983935 232.17181854676787
self.odometry.x 0.500011210218444 self.odometry.y 0.5000095403407041 self.odometry.theta
4.052162663983935
self.pf.x 2.381284606468413 self.pf.y 0.6117663137784264 self.pf.theta 5.096068125173551
EUCLIDEAN_DISTANCE  1.884589920339993
DISTANCE_theta  1.043905461189616
theta: 4.631043044553062 265.3392211963057
self.odometry.x 0.500018871796282 self.odometry.y 0.5000062362570619 self.odometry.theta
4.631043044553062
```

```
self.pf.x 2.3926077453356163 self.pf.y 0.5798447677100268 self.pf.theta 5.416261607419503
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DISTANCE_theta  0.785218562866441
theta: 5.20992342512219 298.50662384584365
self.odometry.x 0.5000343948402015 self.odometry.y 0.5000086396173645 self.odometry.theta
5.20992342512219
self.pf.x 2.4718024511825103 self.pf.y 0.5478743698086065 self.pf.theta 2.9859534884309604
EUCLIDEAN_DISTANCE  1.9723489539477204
DISTANCE_theta  2.2239699366912293
theta: 5.788803805691319 331.67402649538167
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EUCLIDEAN_DISTANCE  2.095371458954238
DISTANCE_theta  4.936273732488338
theta: 0.08449887908086479 4.841429144919833
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DISTANCE_theta  1.2742109253510325
theta: 0.6633792596499996 38.00883179445816
self.odometry.x 0.500056886582257 self.odometry.y 0.500075658616611 self.odometry.theta
0.6633792596499996
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EUCLIDEAN_DISTANCE  2.206927800131557
DISTANCE_theta  1.059286250010706
theta: 1.2422596402191348 71.17623444399652
self.odometry.x 0.5000445369947606 self.odometry.y 0.5000854676065691 self.odometry.theta
1.2422596402191348
self.pf.x 2.671834414427581 self.pf.y 0.4374566637742314 self.pf.theta 2.3315968119227364
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DISTANCE_theta  1.0893371717036016
theta: 1.8211400207882693 104.34363709353484
self.odometry.x 0.5000486781462936 self.odometry.y 0.5000861943633185 self.odometry.theta
1.8211400207882693
self.pf.x 2.6298159948706235 self.pf.y 0.5171895007691821 self.pf.theta 2.7700702596389033
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DISTANCE_theta  0.948930238850634
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EUCLIDEAN_DISTANCE  2.119880954383056
DISTANCE_theta  0.9657615083604569
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DISTANCE_theta  1.0373188638840682
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theta: 5.29631407015264 303.45644319550127
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```
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DISTANCE_theta  5.3780230440073895
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DISTANCE_theta  0.8142249119392402
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DISTANCE_theta  0.7949137136405287
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DISTANCE_theta  0.9088544011863042
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DISTANCE_theta  0.8420385744793997
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DISTANCE_theta  0.9398133296364857
theta: 4.224943954044828 242.07145724608267
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theta: 4.803824334613958 275.23885989562075
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self.pf.x 2.6623600465593023 self.pf.y 0.37223668664470916 self.pf.theta 5.710010676913093
EUCLIDEAN_DISTANCE  2.166056654333079
DISTANCE_theta  0.9061863422991348
theta: 5.384596481132663 308.51465274989596
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self.pf.x 2.705393184625387 self.pf.y 0.3710035136626831 self.pf.theta 0.011749119771716764
EUCLIDEAN_DISTANCE  2.209195624661236
DISTANCE_theta  5.372847361360946
```

```
theta: 5.9634768617017935 341.6820553994341
self.odometry.x 0.4999260380442419 self.odometry.y 0.500235756639732 self.odometry.theta
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EUCLIDEAN_DISTANCE  2.2372738334966007
DISTANCE_theta  5.448553805207733
theta: 0.25917193509133896 14.84945804897224
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0.25917193509133896
self.pf.x 2.711969471761089 self.pf.y 0.36704805729994755 self.pf.theta 1.0091635529468332
EUCLIDEAN_DISTANCE  2.2160467660325285
DISTANCE_theta  0.7499916178554943
start_x
271.1969471761089
start_y
36.70480572999475
Localized
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<rrt.Vertex object at 0x0A409150>, <rrt.Vertex object at 0x0A409510>, <rrt.Vertex object at
0x0A409D10>, <rrt.Vertex object at 0x0A409870>, <rrt.Vertex object at 0x0A409130>, <rrt.Vertex
object at 0x0A409650>, <rrt.Vertex object at 0x0A409C50>, <rrt.Vertex object at 0x0A4091D0>,
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0x0A3ACC70>, <rrt.Vertex object at 0x0A3AC670>, <rrt.Vertex object at 0x0A3ACA10>, <rrt.Vertex
object at 0x0A3AC5D0>, <rrt.Vertex object at 0x0A3ACC90>, <rrt.Vertex object at 0x0A3AC590>,
<rrt.Vertex object at 0x0A3ACAD0>, <rrt.Vertex object at 0x0A419430>, <rrt.Vertex object at
0x0A4190D0>, <rrt.Vertex object at 0x0A419090>, <rrt.Vertex object at 0x0A4195D0>, <rrt.Vertex
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0x0ADCC890>, <rrt.Vertex object at 0x0ADCC690>, <rrt.Vertex object at 0x0ADCCB90>, <rrt.Vertex
object at 0x0ADCCC90>, <rrt.Vertex object at 0x0ADCC7F0>, <rrt.Vertex object at 0x0ADCC770>,
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0x0ADCC2B0>, <rrt.Vertex object at 0x00C04B70>, <rrt.Vertex object at 0x00C04710>, <rrt.Vertex
object at 0x00C04D30>, <rrt.Vertex object at 0x00C04470>, <rrt.Vertex object at 0x00C045D0>]
0.5700221181812415 -2.6547448520022527
0.6035471060368041 -2.6918403394540222
0.6533982254634734 -2.695695970648515
0.7016350480425038 -2.6825350836161324
0.7510314001767537 -2.6747890913730963
0.8008814147298877 -2.6786589814299733
0.8475659725666218 -2.660755013724779
0.8936269452238288 -2.641302846805075
0.9435612886905775 -2.6438643585744774
0.9855823989770687 -2.6709609688844504
1.0351974074088561 -2.6771537823253397
1.0847828047352441 -2.670728179475471
1.0958169616861184 -2.7194954572381618
1.1458168944395735 -2.7195774612013515
1.1930044093139187 -2.7030445110718846
```

```
1.2409228295549888 -2.7173212404452465
1.2892752182019265 -2.704591491345775
1.329323509296031 -2.674655987668004
1.376800305358921 -2.6589730722629286
1.4259174123154863 -2.649618404753213
1.4131323834033729 -2.6012806034408196
1.4507905612270546 -2.5683889381895746
1.4920747692574095 -2.5401823047976504
1.499186903320939 -2.490690714107288
1.496069997560461 -2.4407879596895175
1.5137604303489445 -2.394022068403708
1.4802944983154454 -2.3568732946267823
1.4913410608740316 -2.3081088254341493
1.5166842156353857 -2.2650074970701413
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1.5422427789726243 -2.169656144215075
1.5340011258269732 -2.1203400702287287
1.5402559689462196 -2.0707328435621064
1.5530576431421041 -2.022399447883938
1.6020921802843473 -2.0126211981813587
1.61451915942859 -1.9641901111674522
1.6564389301240703 -1.9369369861202521
1.705637564963414 -1.928021026836757 5
1.7521810413252237 -1.9097534378289638
1.8021613606404017 -1.911156178415404
1.8478655645760127 -1.8908799425070288
1.8978495593292144 -1.8896149253087708
1.9109095653394876 -1.8413506917293216
1.955005352690959 -1.8177803387864018
1.9982373650263074 -1.79266076256712
1.9994818488407406 -1.7426762523661026
2.0392716912309097 -1.7123980680981268
2.073959585923386 -1.6763876534684163
2.1028536738326826 -1.6355816283754248
2.148550187273627 -1.6152880663583316
2.176021297548539 -1.5735108225494872
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2.241120473823685 -1.5000374860082963
2.2627651542359675 -1.4549652350122546
2.262698697374474 -1.4049652791774185
2.2442852760958485 -1.358479303692162
2.2274768188654517 -1.3113892222998569
2.2103363477880182 -1.2644189737243634
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1.9160686705887169 -1.2019592287558423
1.8697610780104683 -1.1831016873040268
1.8204304419013433 -1.1749476489751316
1.7941033074570623 -1.2174550835243108
1.7451898705064368 -1.2070880354547213
1.6968753503117568 -1.2199607639284316
1.655745220690771 -1.1915299278984994
1.6101515448634551 -1.2120534930019031
1.5604008070506818 -1.2170398831876676
1.5145203642241458 -1.1971656544972016
1.5221857719907788 -1.1477567329974234
1.4937326831610322 -1.1066419943256702
1.4920217153676374 -1.0566712770083255
1.521526968372493 -1.0163049538525133
1.4838828485727347 -0.9833972002904506
1.509081747613179 -0.940211375218449 3
1.5103397601792241 -0.8902272036800146
1.4976830150110467 -0.8418556541726047
1.509208718149569 -0.793202204478234
1.5176016543091313 -0.7439116514013867
1.5333279419410903 -0.6964492043793789
1.5076658016652233 -0.6535370317885889
```

```
1.538289407962492 -0.6140124225261321
1.5530278750548143 -0.5662340007463313
1.5735878046727083 -0.5206567115102738
1.6233063300886146 -0.5153587656377802
1.6561327306460447 -0.47764368342332447
1.7051012364093407 -0.4675399492962778
1.7501694637436678 -0.4458868921231805
1.8 -0.45
odometry
1.7504656159974374
-0.4538039106297495
position
0.48008817434310913
1.9017589092254639
particle_filter
2.7100276026027137
0.3686664307211017
differences
-0.88
-0.2999999999999998
Coordinates
-0.88
-0.2999999999999998
Initial params
0.93
-0.32855317055145083
-18.824710018240058
rotate base
inverse kinematics
Go to [-0.613,0.12], IK: [72.12684109125145 deg, 71.31395901276565 deg]
goto GRIPPER
close GRIPPER
Go to 72.12684109125145,71.31395901276565 deg
Go to 71.58516349056856,69.31395901276565 deg
Go to 71.04348588988567,67.31395901276565 deg
Go to 70.50180828920278,65.31395901276565 deg
Go to 69.9601306885199,63.31395901276565 deg
Go to 69.41845308783701,61.31395901276565 deg
Go to 68.87677548715412,59.31395901276565 deg
Go to 68.33509788647123,57.31395901276565 deg
Go to 67.79342028578834,55.31395901276565 deg
Go to 67.25174268510546,53.31395901276565 deg
Go to 66.71006508442257,51.31395901276565 deg
Go to 66.16838748373968,49.31395901276565 deg
Go to 65.62670988305679,47.31395901276565 deg
Go to 65.0850322823739,45.31395901276565 deg
Go to 65,45 deg
Move sphere and gripper to 0,-30 deg
Move sphere and gripper to 0.0,-28.0 deg
Move sphere and gripper to 0.0,-26.0 deg
Move sphere and gripper to 0.0,-24.0 deg
Move sphere and gripper to 0.0,-22.0 deg
Go to 65,45 deg
Go to 63.888888888888886,43.0 deg
Go to 62.77777777777777,41.0 deg
Go to 61.66666666666666,39.0 deg
Go to 60.55555555555554,37.0 deg
Go to 59.44444444444443,35.0 deg
Go to 58.333333333333314,33.0 deg
Go to 57.2222222222222,31.0 deg
Go to 56.111111111111086,29.0 deg
Go to 54.99999999999997,27.0 deg
Go to 53.88888888888886,25.0 deg
Go to 52.77777777777774,23.0 deg
Go to 51.66666666666663,21.0 deg
Go to 50.555555555555515,19.0 deg
Go to 49.4444444444444,17.0 deg
Go to 48.333333333333286,15.0 deg
Go to 47.222222222222217,13.0 deg
Go to 46.11111111111106,11.0 deg
Go to 44.99999999999994,9.0 deg
```

```
Go to 43.88888888888883,7.0 deg
Go to 42.777777777777715,5.0 deg
Go to 41.6666666666666,3.0 deg
Go to 40.555555555555486,1.0 deg
Go to 40,0 deg
Move sphere and gripper to 0,-20 deg
Move sphere and gripper to 0.0,-18.0 deg
Move sphere and gripper to 0.0,-16.0 deg
Move sphere and gripper to 0.0,-14.0 deg
Move sphere and gripper to 0.0,-12.0 deg
Move sphere and gripper to 0.0,-10.0 deg
Move sphere and gripper to 0.0,-8.0 deg
Move sphere and gripper to 0.0,-6.0 deg
Move sphere and gripper to 0.0,-4.0 deg
Move sphere and gripper to 0.0,-2.0 deg
Move sphere and gripper to 0.0,0.0 deg
Move sphere and gripper to 0.0,2.0 deg
Move sphere and gripper to 0.0,4.0 deg
Move sphere and gripper to 0.0,6.0 deg
Move sphere and gripper to 0.0,8.0 deg
Move sphere and gripper to 0.0,10.0 deg
```

Robotic Arm

Overall our solution for the Kuka robot is very consistent and robust. The custom Kuka API or functionality it is in an independent class we called MyArmRobot, which makes it modular and reusable in the same way we import our PyCreate2 library.

To use the arm robot, you just need to call the standard constructor in Python and send two arguments: the instance of the robotic arm from the PyCreate2 suite and a time helper from the same suite as it can be appreciated in the following line:

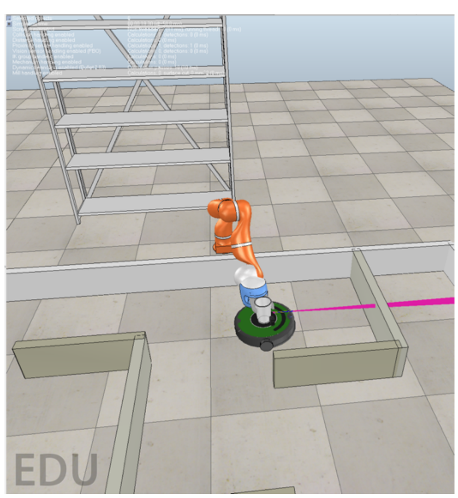
```
self.my_arm_robot = MyArmRobot(factory.create_kuka_lbr4p(), factory.create_time_helper())
```

After we have an instance of the robotic arm we need to pass the coordinates where the robot is located so it can calculate the joint angles to grab the cup using Inverse Kinematics, which can be done by calling the `set_create_coordinates` function as in the following line:

```
# Configuration for first map
my_arm_x_pos = 1.6
my_arm_y_pos = 3.4
cup_y_diff = 0.005
self.my_arm_robot.set_create_coordinates(my_arm_x_pos - float(create2_x_pos),
                                         my_arm_y_pos - float(create2_y_pos) + cup_y_diff)
# where the first parameter is coordinate in X-axis and the second in the Y-axis
respectively taking the arm as a point of reference.

# Configuration for final map
my_arm_x_pos = -0.4
my_arm_y_pos = 1.6
cup_y_diff = 0.005
self.my_arm_robot.set_create_coordinates(my_arm_x_pos - float(create2_x_pos),
                                         my_arm_y_pos - float(create2_y_pos) + cup_y_diff)
```

This functionality calculates the angle we need to rotate the base swoop which will compensate the difference between the iRobot Create2 goal position, and the position calculated using odometry and the particle filter specially in the Y-axis as it can be appreciated in the following image:



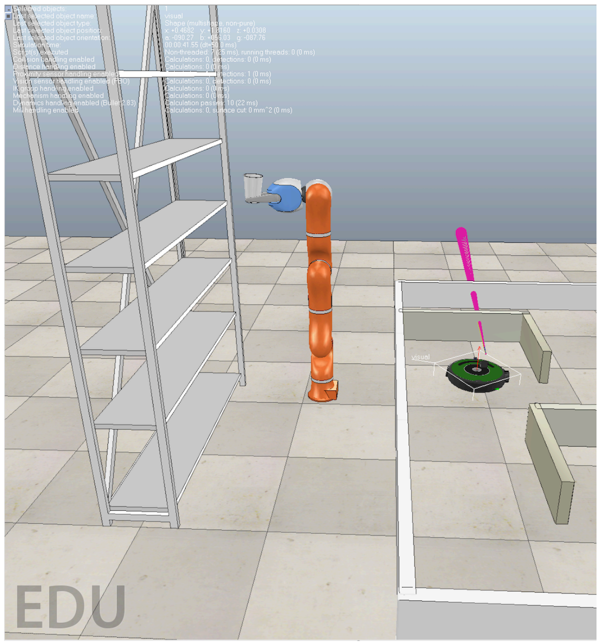
iCreate Robot located at (0.45, 1.8) and Kuka robot at (-0.4, 1.6)

Interface

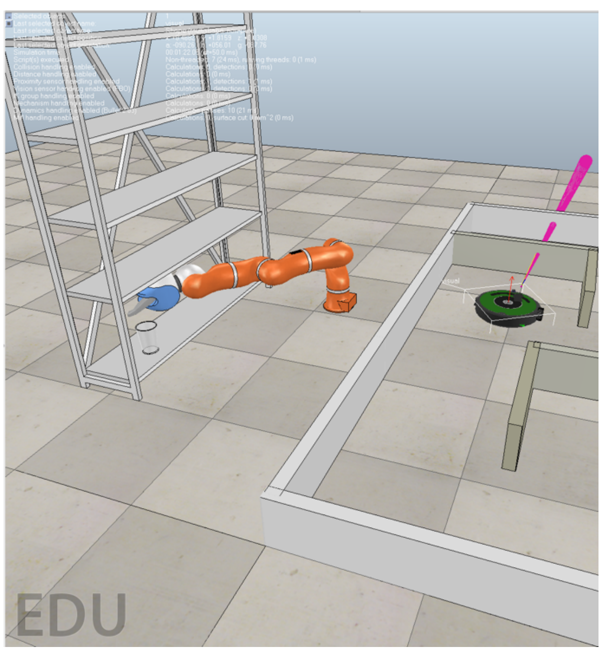
These are the functions that can be called after the robotic arm has been initialized and the coordinates set up:

```
grab_cup_and_go_to_initial_position() # Grabs cup and moves it in front of the shelf

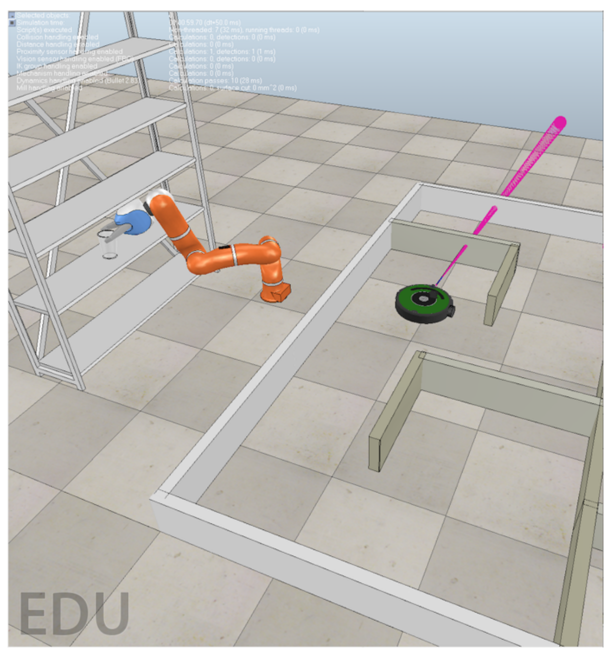
# After the cup is in front of the shelf we can call one of the following functions:
place_cup_in_shelf_zero(), place_cup_in_shelf_one(), place_cup_in_shelf_two()
place_cup_in_shelf_three() and place_cup_in_shelf_four()
```



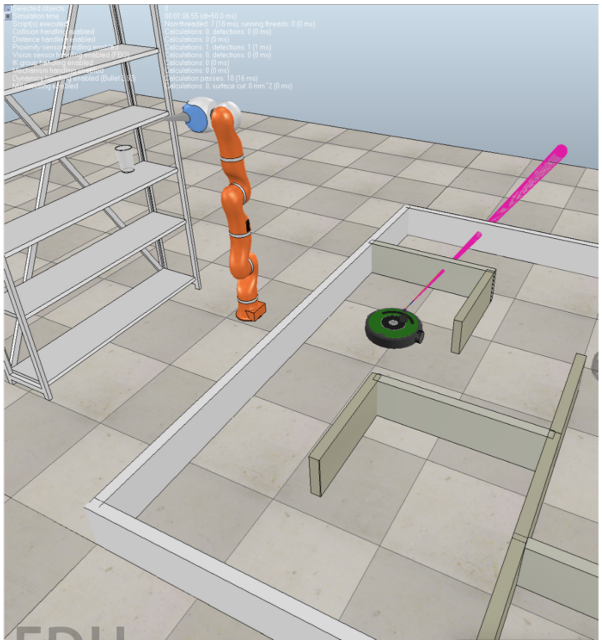
Calling grab_cup_and_go_to_initial_position()



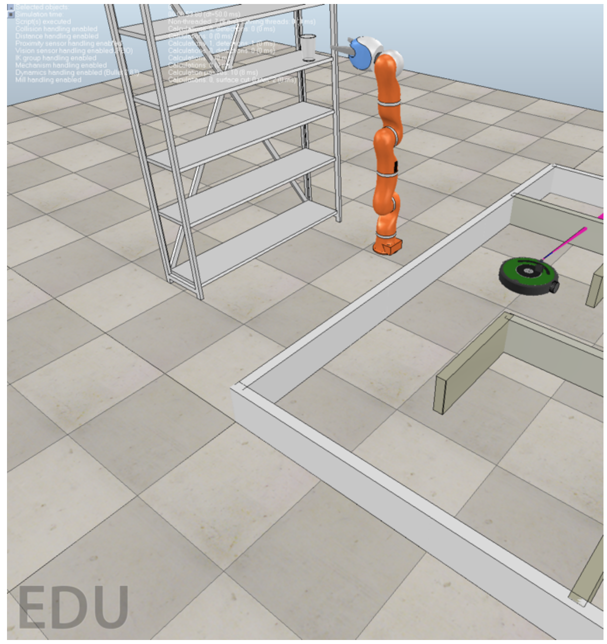
Calling place_cup_in_shelf_zero()



Calling place_cup_in_shelf_one()



Calling place_cup_in_shelf_two()



Calling place_cup_in_shelf_three()

Other parameters that can be modified are the distance x and y to the shelves if these are located in different space by changing the global variables:

```
ADJACENT_SIDE_TO_SHELVES = 0.5  # X DISTANCE TO CENTER OF SHELVES
OPPOSITE_SIDE_TO_SHELVES = 0.6  # Y DISTANCE TO CENTER OF SHELVES
```

We can also set the speed the robot arm moves between angles or the gripper head size:

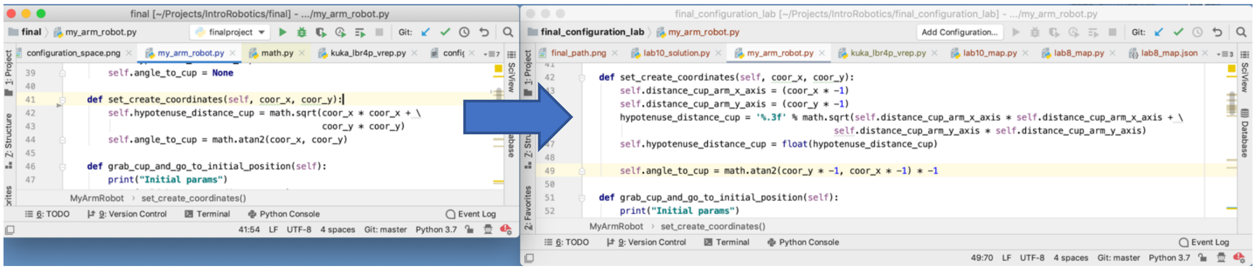
```
SPEED_FACTOR = 2.0
GRIPPER_HEAD_SIZE = 0.317
```

Or the height where the cup is located by changing the parameter:

```
self.cup_z_coordinate  = 0.12
```

Implementation

Our solution for the robotic arm has a deliberative architecture since the reachable workspace, and the distance (x,y,z) between the robotic arm and the iCreate must be defined before the arm starts moving to reach its goal. We used the modules from Programming Assignment 2: inverse kinematics to grab the cup, forward kinematics to move the cup in front of the shelf and inverse kinematics again to place the cup in one of the shelves. For the final map, it required almost no modification since our solution works in environments with different coordinate systems. For this scenario, it can be done by translating or negating coordinates x and y and the parameter `angle_to_cup` where the iCreate is located.



Changes done between initial and final map

In order to guarantee that the robotic arm places the cup successfully in every iteration, it was necessary to make a wrapper function for `go_to, forward_kinematics`, and `inverse_kinematics` methods from the create2 suite. We called the private function `__move(to_theta_1, to_theta_2)`, which takes into account the angles we want to move the robot. Then it analyzes the direction we want the arm to go and gives a proportionality factor to each angle per joint based in the most significant difference of these angles in relation to its current angles (`MyArmRobot` stores information about the last moved angle of each joint). With this information, we can smoothly move our robotic arm to a certain angle using the time helper and a proportional difference with the current angle in a for loop instead of abruptly moving the arm to a particular position or angle in one "command" or "action".

Error handling

It is worth mentioning that our solution handles the case where inverse kinematics cannot resolve a specific position because either there is no solution or there is a math error. To solve this issue, we created a function `print_arm_reachability`, along with a modification of the `inverse_kinematics` module, detects the reachable workspace points for x and z. If it does not return a solution, then it can try with another pair of points in the (x,z) coordinate system within a range of $\pm$ 0.05 difference, which does not affect its correctness. This function also helped us to determine the best position where to place the cup on each shelf using it along inverse kinematics.

Example of log for reachable workspace:

```
Angels-MacBook-Pro:final_configuration_lab angelricardonietogarcia$ python3 run.py --sim
finalproject.py
print_arm_reachability
opposite = 0.000, x = 0.000:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095: z 1.100:
opposite = 0.004, x = 0.005:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095: z 1.100:
opposite = 0.008, x = 0.010:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095: z 1.100:
```

```
opposite = 0.012, x = 0.015:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095: z 1.100:
opposite = 0.015, x = 0.020:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095: z 1.100:
opposite = 0.019, x = 0.025:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095: z 1.100:
opposite = 0.023, x = 0.030:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.027, x = 0.035:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.031, x = 0.040:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.035, x = 0.045:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.038, x = 0.050:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.042, x = 0.055:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.046, x = 0.060:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.050, x = 0.065:z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z
0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955:
z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z
1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.054, x = 0.070:z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z
0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950:
z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z
1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.058, x = 0.075:z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z
0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950:
z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z
1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.061, x = 0.080:z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z
0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950:
z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z
1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
```

```
opposite = 0.065, x = 0.085:z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z
0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950:
z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z
1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.069, x = 0.090:z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z
0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950:
z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z
1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090: z 1.095:
opposite = 0.073, x = 0.095:z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z
0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950:
z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z
1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090:
opposite = 0.077, x = 0.100:z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z
0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950:
z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z
1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090:
opposite = 0.081, x = 0.105:z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z
0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945:
z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z
1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050:
z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090:
opposite = 0.085, x = 0.110:z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z
0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945:
z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z
1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050:
z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090:
opposite = 0.088, x = 0.115:z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z
0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945:
z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z
1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050:
z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090:
opposite = 0.092, x = 0.120:z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z
0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945:
z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z
1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050:
z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090:
opposite = 0.096, x = 0.125:z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z
0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940:
z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z
0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045:
z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085: z 1.090:
opposite = 0.100, x = 0.130:z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z
0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940:
z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z
0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045:
z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085:
opposite = 0.104, x = 0.135:z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z
0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940:
z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z
0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045:
z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085:
opposite = 0.108, x = 0.140:z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z
0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940:
z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z
0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045:
z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085:
opposite = 0.111, x = 0.145:z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z
0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940:
z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z
0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045:
z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085:
opposite = 0.115, x = 0.150:z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z
0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935:
z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z
0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040:
z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085:
```

opposite = 0.119, x = 0.155:z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080: z 1.085:
opposite = 0.123, x = 0.160:z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080:
opposite = 0.127, x = 0.165:z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080:
opposite = 0.131, x = 0.170:z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080:
opposite = 0.134, x = 0.175:z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075: z 1.080:
opposite = 0.138, x = 0.180:z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075:
opposite = 0.142, x = 0.185:z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075:
opposite = 0.146, x = 0.190:z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075:
opposite = 0.150, x = 0.195:z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070: z 1.075:
opposite = 0.154, x = 0.200:z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070:
opposite = 0.157, x = 0.205:z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070:
opposite = 0.161, x = 0.210:z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070:
opposite = 0.165, x = 0.215:z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065: z 1.070:
opposite = 0.169, x = 0.220:z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065:

```
opposite = 0.173, x = 0.225:z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z
0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910:
z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z
0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015:
z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z 1.065:
opposite = 0.177, x = 0.230:z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z
0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905:
z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z
0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010:
z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060: z
1.065:
opposite = 0.181, x = 0.235:z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z
0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905:
z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z
0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010:
z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z 1.060:
opposite = 0.184, x = 0.240:z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z
0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900:
z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z
0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005:
z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z
1.060:
opposite = 0.188, x = 0.245:z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z
0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900:
z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z
0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005:
z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055: z
1.060:
opposite = 0.192, x = 0.250:z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z
0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900:
z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z
0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005:
z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z 1.055:
opposite = 0.196, x = 0.255:z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z
0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895:
z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z
0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000:
z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z 1.050: z
1.055:
opposite = 0.200, x = 0.260:z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995:
z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z
1.050: z 1.055:
opposite = 0.204, x = 0.265:z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995:
z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045: z
1.050:
opposite = 0.207, x = 0.270:z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z
0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885:
z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z
0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990:
z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z
1.045: z 1.050:
opposite = 0.211, x = 0.275:z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z
0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885:
z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z
0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990:
z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z
1.045: z 1.050:
opposite = 0.215, x = 0.280:z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z
0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880:
z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z
0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985:
z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z
1.040: z 1.045:
opposite = 0.219, x = 0.285:z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z
0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880:
```

z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045:
opposite = 0.223, x = 0.290:z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040: z 1.045:
opposite = 0.227, x = 0.295:z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040:
opposite = 0.230, x = 0.300:z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035: z 1.040:
opposite = 0.234, x = 0.305:z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035:
opposite = 0.238, x = 0.310:z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030: z 1.035:
opposite = 0.242, x = 0.315:z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030:
opposite = 0.246, x = 0.320:z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030:
opposite = 0.250, x = 0.325:z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025: z 1.030:
opposite = 0.254, x = 0.330:z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025:
opposite = 0.257, x = 0.335:z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020: z 1.025:
opposite = 0.261, x = 0.340:z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020:

opposite = 0.265, x = 0.345:z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015: z 1.020:
opposite = 0.269, x = 0.350:z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015:
opposite = 0.273, x = 0.355:z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010: z 1.015:
opposite = 0.277, x = 0.360:z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010:
opposite = 0.280, x = 0.365:z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005: z 1.010:
opposite = 0.284, x = 0.370:z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005:
opposite = 0.288, x = 0.375:z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000: z 1.005:
opposite = 0.292, x = 0.380:z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000:
opposite = 0.296, x = 0.385:z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995: z 1.000:
opposite = 0.300, x = 0.390:z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990: z 0.995:
opposite = 0.303, x = 0.395:z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z 0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990:
opposite = 0.307, x = 0.400:z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890: z 0.895:

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0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985: z 0.990:
opposite = 0.311, x = 0.405:z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z
0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790:
z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z
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0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985:
opposite = 0.315, x = 0.410:z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z
0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790:
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0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980: z 0.985:
opposite = 0.319, x = 0.415:z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z
0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790:
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0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975: z 0.980:
opposite = 0.323, x = 0.420:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
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0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975:
opposite = 0.326, x = 0.425:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
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0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970: z 0.975:
opposite = 0.330, x = 0.430:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
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0.945: z 0.950: z 0.955: z 0.960: z 0.965: z 0.970:
opposite = 0.334, x = 0.435:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
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z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950: z 0.955: z 0.960: z 0.965:
opposite = 0.338, x = 0.440:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950: z 0.955: z 0.960: z 0.965:
opposite = 0.342, x = 0.445:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950: z 0.955: z 0.960:
opposite = 0.346, x = 0.450:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950: z 0.955:
opposite = 0.350, x = 0.455:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950: z 0.955:
opposite = 0.353, x = 0.460:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
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0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945: z 0.950:
opposite = 0.357, x = 0.465:z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z
0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785:
z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z
0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885: z 0.890:
z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z 0.940: z
0.945:
opposite = 0.361, x = 0.470:z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z
0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780:
z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z
0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885:
z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z
0.940: z 0.945:
opposite = 0.365, x = 0.475:z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z
0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780:
z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z
0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885:
z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935: z
0.940:
opposite = 0.369, x = 0.480:z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z
0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780:
z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z
0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885:
z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930: z 0.935:
opposite = 0.373, x = 0.485:z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z
0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780:
z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z
0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880: z 0.885:
z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930:
opposite = 0.376, x = 0.490:z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z
0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775:
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0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880:
z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925: z 0.930:
opposite = 0.380, x = 0.495:z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z
0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775:
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0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880:
z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920: z 0.925:
opposite = 0.384, x = 0.500:z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z
0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775:
z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z
0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880:
z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915: z 0.920:
opposite = 0.388, x = 0.505:z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z
0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775:
z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z
0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875: z 0.880:
z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910: z 0.915:
opposite = 0.392, x = 0.510:z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z
0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770:
z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z
0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875:
z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905: z 0.910:
opposite = 0.396, x = 0.515:z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z
0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770:
z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z
0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875:
z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905:
opposite = 0.399, x = 0.520:z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z
0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770:
z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z
0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870: z 0.875:
z 0.880: z 0.885: z 0.890: z 0.895: z 0.900: z 0.905:
opposite = 0.403, x = 0.525:z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z
0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765:
z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z
0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870:
z 0.875: z 0.880: z 0.885: z 0.890: z 0.895: z 0.900:
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opposite = 0.407, x = 0.530:z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z
0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765:
z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z
0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870:
z 0.875: z 0.880: z 0.885: z 0.890: z 0.895:
opposite = 0.411, x = 0.535:z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z
0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765:
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0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865: z 0.870:
z 0.875: z 0.880: z 0.885: z 0.890:
opposite = 0.415, x = 0.540:z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z
0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760:
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0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865:
z 0.870: z 0.875: z 0.880: z 0.885:
opposite = 0.419, x = 0.545:z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z
0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760:
z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z
0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865:
z 0.870: z 0.875: z 0.880:
opposite = 0.423, x = 0.550:z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z
0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760:
z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z
0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860: z 0.865:
z 0.870: z 0.875:
opposite = 0.426, x = 0.555:z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z
0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755:
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0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860:
z 0.865: z 0.870:
opposite = 0.430, x = 0.560:z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z
0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755:
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0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855: z 0.860:
z 0.865:
opposite = 0.434, x = 0.565:z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z
0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750:
z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z
0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855:
z 0.860:
opposite = 0.438, x = 0.570:z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z
0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750:
z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800: z
0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850: z 0.855:
opposite = 0.442, x = 0.575:z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z
0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745:
z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z 0.795: z
0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845: z 0.850:
opposite = 0.446, x = 0.580:z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z
0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745:
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0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840: z 0.845:
opposite = 0.449, x = 0.585:z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z
0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740:
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0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835: z 0.840:
opposite = 0.453, x = 0.590:z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z
0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740:
z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z 0.790: z
0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830: z 0.835:
opposite = 0.457, x = 0.595:z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z
0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735:
z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z
0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820: z 0.825: z 0.830:
opposite = 0.461, x = 0.600:z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z
0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735:
z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z 0.785: z
0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815: z 0.820:
opposite = 0.465, x = 0.605:z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z
0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730:
```

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z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z 0.780: z
0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810: z 0.815:
opposite = 0.469, x = 0.610:z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z
0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725:
z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z
0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805: z 0.810:
opposite = 0.472, x = 0.615:z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z
0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725:
z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z 0.775: z
0.780: z 0.785: z 0.790: z 0.795: z 0.800: z 0.805:
opposite = 0.476, x = 0.620:z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z
0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720:
z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z 0.770: z
0.775: z 0.780: z 0.785: z 0.790: z 0.795: z 0.800:
opposite = 0.480, x = 0.625:z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z
0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715:
z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z
0.770: z 0.775: z 0.780: z 0.785: z 0.790:
opposite = 0.484, x = 0.630:z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z
0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715:
z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z 0.765: z
0.770: z 0.775: z 0.780: z 0.785:
opposite = 0.488, x = 0.635:z 0.625: z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z
0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710:
z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z 0.760: z
0.765: z 0.770: z 0.775: z 0.780:
opposite = 0.492, x = 0.640:z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z
0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705:
z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z 0.755: z
0.760: z 0.765: z 0.770:
opposite = 0.496, x = 0.645:z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z 0.645: z
0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700:
z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z 0.750: z
0.755: z 0.760: z 0.765:
opposite = 0.499, x = 0.650:z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z
0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695:
z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z
0.750: z 0.755:
opposite = 0.503, x = 0.655:z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z
0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695:
z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740: z 0.745: z
0.750:
opposite = 0.507, x = 0.660:z 0.605: z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z
0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690:
z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735: z 0.740:
opposite = 0.511, x = 0.665:z 0.600: z 0.605: z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z
0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685:
z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725: z 0.730: z 0.735:
opposite = 0.515, x = 0.670:z 0.595: z 0.600: z 0.605: z 0.610: z 0.615: z 0.620: z 0.625: z
0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675: z 0.680:
z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720: z 0.725:
opposite = 0.519, x = 0.675:z 0.590: z 0.595: z 0.600: z 0.605: z 0.610: z 0.615: z 0.620: z
0.625: z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675:
z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710: z 0.715: z 0.720:
opposite = 0.522, x = 0.680:z 0.585: z 0.590: z 0.595: z 0.600: z 0.605: z 0.610: z 0.615: z
0.620: z 0.625: z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665: z 0.670:
z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700: z 0.705: z 0.710:
opposite = 0.526, x = 0.685:z 0.580: z 0.585: z 0.590: z 0.595: z 0.600: z 0.605: z 0.610: z
0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660: z 0.665:
z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695: z 0.700:
opposite = 0.530, x = 0.690:z 0.575: z 0.580: z 0.585: z 0.590: z 0.595: z 0.600: z 0.605: z
0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655: z 0.660:
z 0.665: z 0.670: z 0.675: z 0.680: z 0.685: z 0.690: z 0.695:
opposite = 0.534, x = 0.695:z 0.570: z 0.575: z 0.580: z 0.585: z 0.590: z 0.595: z 0.600: z
0.605: z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z 0.645: z 0.650: z 0.655:
z 0.660: z 0.665: z 0.670: z 0.675: z 0.680: z 0.685:
opposite = 0.538, x = 0.700:z 0.560: z 0.565: z 0.570: z 0.575: z 0.580: z 0.585: z 0.590: z
0.595: z 0.600: z 0.605: z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640: z 0.645:
z 0.650: z 0.655: z 0.660: z 0.665: z 0.670: z 0.675:
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opposite = 0.542, x = 0.705:z 0.555: z 0.560: z 0.565: z 0.570: z 0.575: z 0.580: z 0.585: z
0.590: z 0.595: z 0.600: z 0.605: z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635: z 0.640:
z 0.645: z 0.650: z 0.655: z 0.660: z 0.665:
opposite = 0.545, x = 0.710:z 0.550: z 0.555: z 0.560: z 0.565: z 0.570: z 0.575: z 0.580: z
0.585: z 0.590: z 0.595: z 0.600: z 0.605: z 0.610: z 0.615: z 0.620: z 0.625: z 0.630: z 0.635:
z 0.640: z 0.645: z 0.650: z 0.655:
opposite = 0.549, x = 0.715:z 0.545: z 0.550: z 0.555: z 0.560: z 0.565: z 0.570: z 0.575: z
0.580: z 0.585: z 0.590: z 0.595: z 0.600: z 0.605: z 0.610: z 0.615: z 0.620: z 0.625: z 0.630:
z 0.635: z 0.640: z 0.645:
opposite = 0.553, x = 0.720:z 0.535: z 0.540: z 0.545: z 0.550: z 0.555: z 0.560: z 0.565: z
0.570: z 0.575: z 0.580: z 0.585: z 0.590: z 0.595: z 0.600: z 0.605: z 0.610: z 0.615: z 0.620:
z 0.625: z 0.630: z 0.635:
opposite = 0.557, x = 0.725:z 0.530: z 0.535: z 0.540: z 0.545: z 0.550: z 0.555: z 0.560: z
0.565: z 0.570: z 0.575: z 0.580: z 0.585: z 0.590: z 0.595: z 0.600: z 0.605: z 0.610: z 0.615:
z 0.620:
opposite = 0.561, x = 0.730:z 0.520: z 0.525: z 0.530: z 0.535: z 0.540: z 0.545: z 0.550: z
0.555: z 0.560: z 0.565: z 0.570: z 0.575: z 0.580: z 0.585: z 0.590: z 0.595: z 0.600: z 0.605:
z 0.610:
opposite = 0.565, x = 0.735:z 0.515: z 0.520: z 0.525: z 0.530: z 0.535: z 0.540: z 0.545: z
0.550: z 0.555: z 0.560: z 0.565: z 0.570: z 0.575: z 0.580: z 0.585: z 0.590: z 0.595: z 0.600:
opposite = 0.568, x = 0.740:z 0.505: z 0.510: z 0.515: z 0.520: z 0.525: z 0.530: z 0.535: z
0.540: z 0.545: z 0.550: z 0.555: z 0.560: z 0.565: z 0.570: z 0.575: z 0.580: z 0.585:
opposite = 0.572, x = 0.745:z 0.495: z 0.500: z 0.505: z 0.510: z 0.515: z 0.520: z 0.525: z
0.530: z 0.535: z 0.540: z 0.545: z 0.550: z 0.555: z 0.560: z 0.565: z 0.570:
opposite = 0.576, x = 0.750:z 0.485: z 0.490: z 0.495: z 0.500: z 0.505: z 0.510: z 0.515: z
0.520: z 0.525: z 0.530: z 0.535: z 0.540: z 0.545: z 0.550: z 0.555:
opposite = 0.580, x = 0.755:z 0.475: z 0.480: z 0.485: z 0.490: z 0.495: z 0.500: z 0.505: z
0.510: z 0.515: z 0.520: z 0.525: z 0.530: z 0.535: z 0.540:
opposite = 0.584, x = 0.760:z 0.465: z 0.470: z 0.475: z 0.480: z 0.485: z 0.490: z 0.495: z
0.500: z 0.505: z 0.510: z 0.515: z 0.520: z 0.525:
opposite = 0.588, x = 0.765:z 0.450: z 0.455: z 0.460: z 0.465: z 0.470: z 0.475: z 0.480: z
0.485: z 0.490: z 0.495: z 0.500: z 0.505:
opposite = 0.592, x = 0.770:z 0.435: z 0.440: z 0.445: z 0.450: z 0.455: z 0.460: z 0.465: z
0.470: z 0.475: z 0.480: z 0.485:
opposite = 0.595, x = 0.775:z 0.420: z 0.425: z 0.430: z 0.435: z 0.440: z 0.445: z 0.450: z
0.455: z 0.460:
opposite = 0.599, x = 0.780:z 0.400: z 0.405: z 0.410: z 0.415: z 0.420: z 0.425: z 0.430: z
0.435:
opposite = 0.603, x = 0.785:z 0.375: z 0.380: z 0.385: z 0.390: z 0.395:
opposite = 0.607, x = 0.790:
opposite = 0.611, x = 0.795:
opposite = 0.615, x = 0.800:
opposite = 0.618, x = 0.805:
opposite = 0.622, x = 0.810:
opposite = 0.626, x = 0.815:
opposite = 0.630, x = 0.820:
opposite = 0.634, x = 0.825:
opposite = 0.638, x = 0.830:
opposite = 0.641, x = 0.835:
opposite = 0.645, x = 0.840:
opposite = 0.649, x = 0.845:
opposite = 0.653, x = 0.850:
opposite = 0.657, x = 0.855:
opposite = 0.661, x = 0.860:
opposite = 0.665, x = 0.865:
opposite = 0.668, x = 0.870:
opposite = 0.672, x = 0.875:
opposite = 0.676, x = 0.880:
opposite = 0.680, x = 0.885:
opposite = 0.684, x = 0.890:
opposite = 0.688, x = 0.895:
opposite = 0.691, x = 0.900:
opposite = 0.695, x = 0.905:
opposite = 0.699, x = 0.910:
```