ECON535 **Department of Economics**
Econometric Analysis II **UIUC**

Problem Set V
Due: March 7th, 2017 (Tuesday)

1. To explore the small-sample properties of the distribution of the t-statistic under the null hypothesis for various true distributions of the error term in the linear regression model, perform the following Monte Carlo study. The design variables are:

$$Y_i = \alpha + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i \quad (i = 1, \ldots, n)$$

The vector $X_i = (X_{i1}, X_{i2})'$ is $N(0, I)$, where I is a (2×2) identity matrix. Let $\alpha = 1.0$ for all experiments. Run experiments with the following values for $\beta = (\beta_1, \beta_2)'$.

β_1	β_2
0.0	1.0

for $n = 50$ and 100 and three distributions of ε_i. The three distributions for ε_i are:

(1) ε_i is i.i.d. $N(0, 1)$.

(2) ε_i is i.i.d. standard Cauchy.

(3) $\varepsilon_i = \eta_i - 1$, where η_i is i.i.d. standard exponential.

Perform 300 replications per experiment and hold the draws of the X_i fixed both across and within experiments. For each replication, compute the t-statistic for the following hypothesis test:

$$H_0 : a'\beta = 0 \qquad \text{versus} \qquad H_a : a'\beta \neq 0 \ ,$$

where $a' = (1, 0)$. Tabulate the 10%, 5%, and 1% quantiles of the empirical distribution of the t-statistic for each of the three distributions for ε_i for $n = 50$ and 100. Compare these empirical quantiles to the 10%, 5%, and 1% quantiles for the t-distribution appropriate for this hypothesis test assuming that ε_i is $N(0, 1)$.

Let

$$\frac{1}{300} \sum_{s=1}^{300} I\{|T(s)| > c_{\alpha/2, n-K}\} = \mathrm{P}(|T| > c_{\alpha/2, n-K}) \ ,$$

where $I\{(|T(s)| > c_{\alpha/2, n-K}\}$ is the event that $|T(s)|$ is greater than the $\alpha/2$ quantile of t-distribution with $n - K$ degrees of freedom, where $T(s)$ is the realized value of the t-statistic for replication s and K is the number of regressors. Using the binomial approximation, test the null hypothesis that $\mathrm{P}(|T| > c_{\alpha/2, n-K})$ is equal to α for $\alpha = 0.10, 0.05,$ and 0.01. Use these hypothesis testing result to assess the quality of the approximation provided by the asymptotic distribution for each true distribution of ε_i.

2. To explore the implication of testing multiple hypothesis with the same regression perform the following Monte Carlo experiment. The design variables are as follows:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_{10} X_{i10} + \varepsilon_i \quad (i = 1, \ldots, n)$$

The vector $X_i = (X_{i1}, X_{i2}, \ldots, X_{i10})'$ is $N(0, \Sigma)$, where $\Sigma = I_{10} + (\iota\iota' - I_{10})\rho$, and ι is a 10×1 vector of 1's, I_{10} is a (10×10) identity matrix and $|\rho| < 1$. Let $\varepsilon_i = N(0, 1)$.

Let $\beta_0 = 1.0$ for all experiments. Run experiments with the following values for $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)'$.

β_1	β_2	β_3	β_4
0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0
0.0	1.0	0.0	0.0
0.0	0.0	1.0	0.0
0.5	0.0	0.0	1.0

for values of $\rho \in (0.0, 0.9)$, and for $n = 200$, for a total of 10 experiments. Set $\beta_k = 0$ for $k = 5, 6, \ldots, 10$ for all 10 experiments. For each experiment you should do 200 replications. The sample of X_i drawn for each value of ρ should be fixed both within and across experiments. For each replication, compute t-statistics for the following ten hypothesis tests:

$$H_{0k} : \beta_k = 0 \quad \text{versus} \quad H_{ak} : \beta_k \neq 0 \qquad \text{for} \;\; k = 1, \ldots, 10 \; .$$

a) For each experiment, tabulate the empirical power function for each of the 10 tests assuming an $\alpha = 0.05$ size for each individual test, i.e., what fraction of the 200 replications does the absolute value f the t-statistic exceed 1.96. Compute the fraction of the 200 replications that at least one of the individual null hypotheses is rejected using this critical value for each individual test. How does this rejection frequency compare to the $\alpha = 0.05$?

b) For each replication except the ones for the first row of the above table and each value of ρ, compare the frequency of rejection of the null hypothesis for the remaining elements β that are equal to zero. The relationship between the value of ρ and the difference between these two rejection frequencies increasing or decreasing? Try with both n=50 and n=200 and compare the results.

c) For each experiment, tabulate the empirical power function for each of the 10 tests using the Bonferroni bound to adjust the size of the individual hypothesis tests, i.e., use 2.81 as the critical value for the individual hypothesis tests. Compute the fraction of the 200 replications that at least one of the individual null hypotheses is rejected. How does this rejection frequency compare to $\alpha = 0.05$?