cpt_s 350

Homework 10

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1. $A \leq_m B$ and $B \leq_m C$ implies $A \leq_m C$

   Since $A \leq_m B$, we have a poly-time computable function f such that

   $\forall x, x \in A$ iff $f(x) \in B$.

   Since $B \leq_m C$, we have a poly-time computable function g such that

   $\forall y, y \in B$ iff $g(y) \in C$.

   To show $A \leq_m C$, we need to find a poly-time computable function h such that

   $\forall x, x \in A$ iff $h(x) \in C$. Here we take $h = g \circ f$.

2. Show the following problem is in NP.

   Given: a directed graph G,

   Q: is there a walk on G passing every node of G exactly once?

   In this problem, we need to guess the question is true and check it.

   Guess: there is a sequence of nodes(walk) w such that the length of walk $|w| \leq k$

   where k is the number of nodes (bound the size of the walk).

   Check: 1. w is indeed a walk on G in determine poly-time.

   2. w covers every node in G exactly once in determine poly-time. (Since k is limited, we can check it by hash table, 2D-array, etc.)

   3. If both 1 and 2 are true, return true; else, crash.

3. Show that the following problem is in NP.

   Given: a directed graph G,

   Q: is there a walk passing all nodes of G?

   Guess: there is a walk w runs the following algorithm in T steps where $T \geq k$ (the number of nodes in G) to bound the size of the walk or running time.

   Check: 1. w is indeed a walk on G in determine poly-time T.

   2. w covers every node in G in determine poly-time. (Since k is limited, we can check it by hash table, 2D-array, etc.)

   3. If both 1 and 2 are true, return true; else, crash.

4. Boolean circuit.

   To compare two boolean circuit, we need to run all possible inputs. Since the input size is n and there is only 0 and 1 are possible for each input, so we will spend $O(2^n)$ to check if $C_1 = C_2$.