Cpt S 450 Homework #3

Please print your name!

No late homework!

1. (easy) Write an algorithm that selects both the maximal element and the minimal element from an array A of n elements, using only $1.5 \cdot n$ comparisons.

2. (not so easy) The algorithm $S(A, n, i)$ selects all the j-th smallest elements (with $j \leq i$) from an array A of n elements, by using linearselect to select each of the j-th smallest elements (with $j \leq i$). Clearly, one could also implement S alternatively as $T(A, n, i)$, which first sort A (on average-case and on worst-case, the sorting takes time $O(n \log n)$ using mergesort) and then select the first i elements. Please compare the average-case complexities of the two algorithms; i.e., For the average-case complexities, under what conditions (on the choices for i), S is better than T or vice versa.

3. (hard) In class, we have demonstrated the worst case complexity analysis for linearselect where each group has $k = 5$ numbers. Please show the worst case complexities for $k = 3$ and $k = 7$.

4. (hard) Let ilselect(A, n, i) be an algorithm that selects the i-smallest from an array A with n integers. It works as follows:

```
ilselect(A, n, i){
    r=partition(A, 1, n);
      //test if A[r] is the element to be selected
   if i == r, return A[r];
      //test if quickselect from the low-part
   if i < r, return quickselect(A, 1, r - 1, i);
      //test if linearselect from the high-part
   if i > r, return linearselect(A, r + 1, n, i - r);
}
```

That is, the algorithm runs quickselect on the low-part or runs linear select on the high-part. Show the worst-case complexity and the average complexity of the algorithm.

5. We use 5com to denote an operation that sorts 5 numbers. Show the minimal number of 5com operations that one need to sort n distinct numbers.