cpt_s 350

Homework 2

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Blum's theorem works.

Traditional method:

Alphabet = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

Input = a number with $2n$ digits.

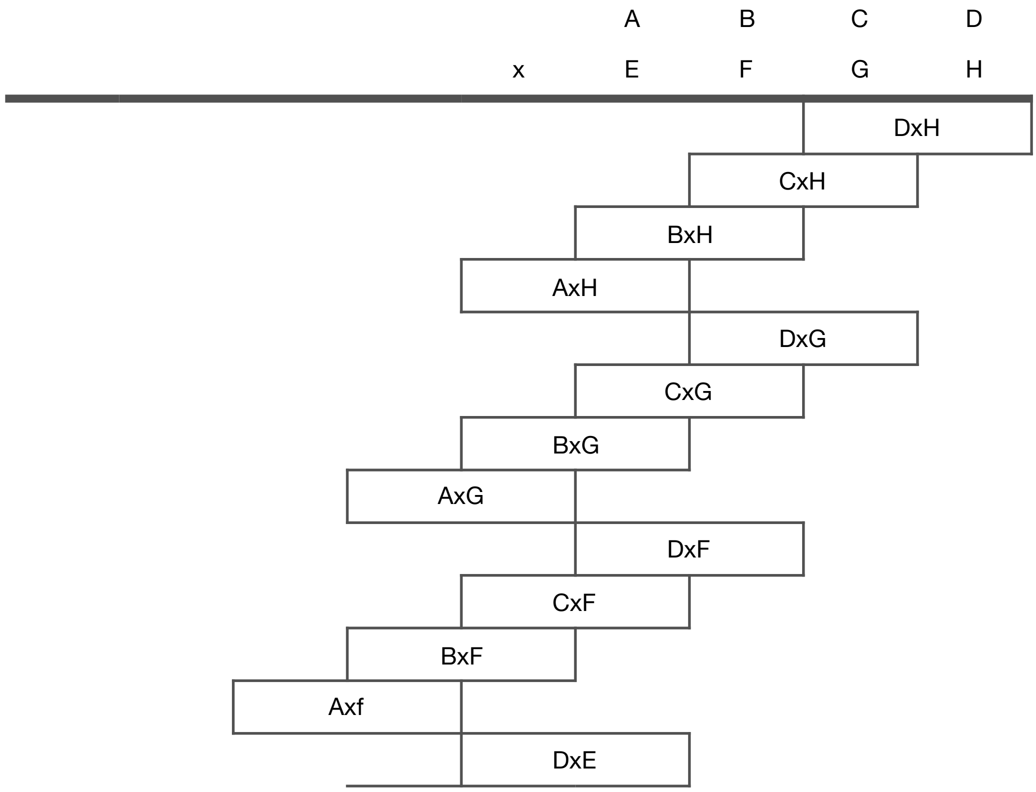
Speed-up method:

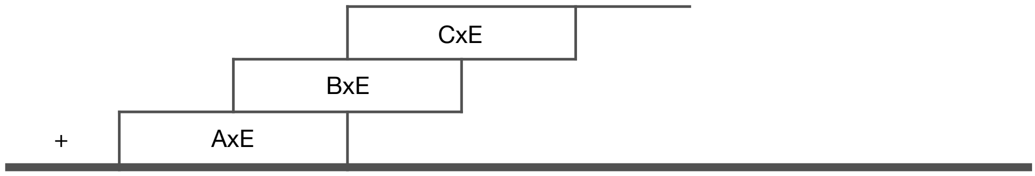
Alphabet = {0, 1, 2, 3, …, 9, 10, 11, …, 98, 99}.

Input = a number with n digits.

1. How to do $2n$ digits by $2n$ digits multiplication.

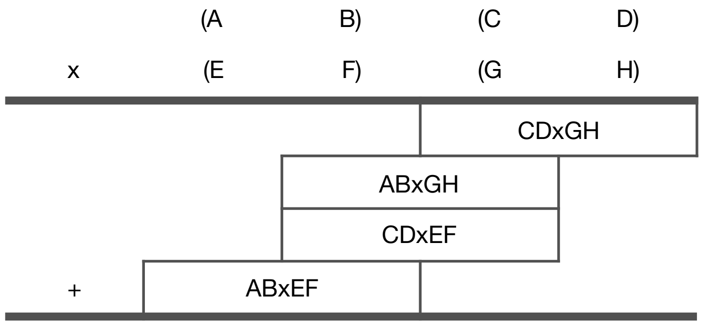
The traditional method is based on the multiplication table we learned before, we calculate multiplication digit by digit. The process is as follow:





In the process, We can find an interesting thing. Because we calculate digit by digit, we are going to spend $T_{multipy}((2n)^2)$ time to do the n digits multiplication. Also, they will spend $T_{sum}((2n)^2)$ time to sum the total numbers. Therefore, the total time will be $T_{traditional} = T_{multipy}((2n)^2) + T_{sum}((2n)^2) = T_{total}(2(2n)^2) = T_{total}(8n^2)$.

However, in the speed-up method, we have a better way to do multiplication. First of all, in order to calculate the answer in a flash, we need to remember the multiplication results within 100. For example, 10x10=100, 12x12=144, and 40x25=1000. Second, we group every two numbers from right to left, so each group has 2 digits. Next, we treat each group as a digit and do the same multiplication as the traditional way. The process shows as follow:



After done the process, we will find that the time we spend on the multiplication is $T_{multipy}(n^2)$, and so does the time of counting total number $T_{sum}(n^2)$. Therefore, the total time will be $T_{speed-up} = T_{multipy}(n^2) + T_{sum}(n^2) = T_{total}(2n^2)$.

2. Why the Speed-up method is at least two times faster than the traditional one?

Comparing to the two methods, we can get the difference between the two time complexity as follow:

$\lim_{n\to\infty} \frac{T_{traditional}(8n^2)}{T_{speed-up}(2n^2)}$ = 4 , which means the speed-up method at least four times faster

then the traditional one.