

cpt_s 350

Homework 10

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1. $A \leq_m B$ and $B \leq_m C$ implies $A \leq_m C$

Since $A \leq_m B$, we have a poly-time computable function f such that

$\forall x, x \in A \text{ iff } f(x) \in B.$

Since $B \leq_m C$, we have a poly-time computable function g such that

$\forall y, y \in B \text{ iff } g(y) \in C.$

To show $A \leq_m C$, we need to find a poly-time computable function h such that

$\forall x, x \in A \text{ iff } h(x) \in C.$ Here we take $h = g \circ f.$

2. Show the following problem is in NP.

Given: a directed graph G ,

Q: is there a walk on G passing every node of G exactly once?

In this problem, we need to guess the question is true and check it.

Guess: there is a sequence of nodes(walk) w such that the length of walk $|w| \leq k$

where k is the number of nodes (bound the size of the walk).

Check: 1. w is indeed a walk on G in determine poly-time.

2. w covers every node in G exactly once in determine poly-time. (Since k is limited, we can check it by hash table, 2D-array, etc.)

3. If both 1 and 2 are true, return true; else, crash.

3. Show that the following problem is in NP.

Given: a directed graph G ,

Q: is there a walk passing all nodes of G ?

Guess: there is a walk w runs the following algorithm in T steps where $T \geq k$ (the number of nodes in G) to bound the size of the walk or running time.

Check: 1. w is indeed a walk on G in determine poly-time T .

2. w covers every node in G in determine poly-time. (Since k is limited, we can check it by hash table, 2D-array, etc.)

3. If both 1 and 2 are true, return true; else, crash.

4. Boolean circuit.