Cpts 540 AI hw 8

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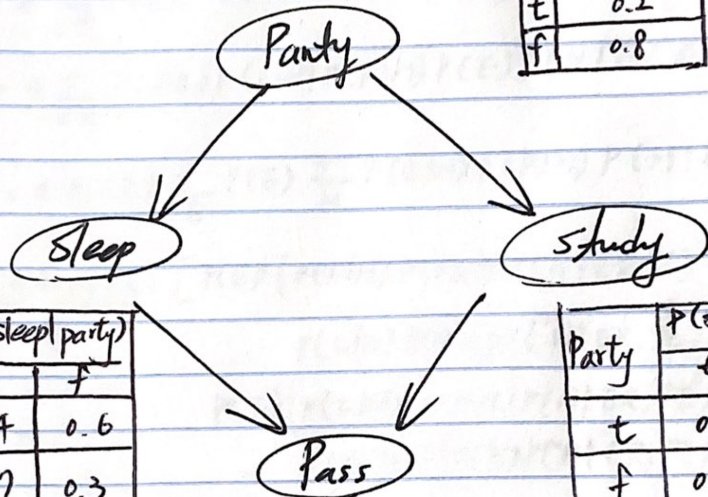
1. Construct a Bayesian network.

	P(party)
t	0.2
f	0.8



Party	P(sleep|party)	
	t	f
t	0.4	0.6
f	0.7	0.3

Party	P(study | party)	
	t	f
t	0.3	0.7
f	0.6	0.4

Sleep	Study	P(pass|sleep, study)	
		t	f
t	t	0.9	0.1
t	f	0.4	0.6
f	t	0.6	0.4
f	f	0.1	0.9

2. a. $P(ER=t, E=t, H=t, L=t, P=t)$

$= (0.3)(0.2)(0.9)(0.8)(0.7)$

$= 0.03024$

b. $P(H=t \mid E=f) = 0.26$

c. $P(L=t \mid ER=t, E=t)$

$= 0.8(0.9) + 0.4(0.1) = 0.72 + 0.04 = 0.76$

2.d. $P(ER=t \mid L=t, P=t)$

$$= \frac{P(ER=t, L=t, P=t)}{P(L=t, P=t)}$$

$$= \alpha P(ER=t, L=t, P=t)$$

$$= \alpha \sum_{EH} P(ER=t, L=t, P=t, E, H)$$

$$= \alpha \sum_{EH} P(ER) P(L \mid H) P(P \mid H) P(E) P(H \mid ER, E)$$

$$= \alpha P(ER) \sum_{E} P(E) \sum_{H} P(L \mid H) P(P \mid H) P(H \mid ER, E)$$

$$= \alpha P(ER) \Big[P(E) \big(P(L \mid H) P(P \mid H) P(H \mid ER, E) + P(L \mid \neg H) P(P \mid \neg H) P(\neg H \mid ER, \neg E) \big) + P(\neg E) \big(P(L \mid H) P(P \mid H) P(H \mid ER, \neg E) + P(L \mid \neg H) P(P \mid \neg H) P(\neg H \mid ER, \neg E) \big) \Big]$$

$$= \alpha (0.3) \Big[(0.2) \big((0.8)(0.7)(0.9) + (0.4)(0.5)(0.1) \big) + (0.8) \big((0.8)(0.7)(0.4) + (0.4)(0.5)(0.6) \big) \Big]$$

$$= 0.11252 \alpha$$

$$P(\neg ER \mid L, P) = 0.20552 \alpha$$

$$\alpha = \frac{1}{0.11252 + 0.20552} = 3.1403$$

$$P(ER \mid L, P) = 0.11252 \alpha = 0.3546$$

$$P(\neg ER \mid L, P) = 0.20552 \alpha = 0.6453$$

2.e. $\mathbf{P}(P \mid ER = f, E = f)$

$$P(P = t \mid ER = f, E = f)$$

$$= \frac{P(P \wedge \neg ER \wedge \neg E)}{P(\neg ER)P(\neg E)}$$

$$= \frac{\sum_{H,L} P(P \wedge \neg ER \wedge \neg E \wedge H \wedge L)}{P(\neg ER)P(\neg E)}$$

$$= \frac{\sum_{H,L} P(\neg ER)P(\neg E)P(P|H)P(H|\neg ER, \neg E)P(L|H)}{P(\neg ER)P(\neg E)}$$

$$= \sum_{H,L} P(P|H)P(H|\neg ER, \neg E)P(L|H)$$

$$= \sum_{H} P(P|H)P(H|\neg ER, \neg E)\sum_{L} P(L|H)$$

$$= [P(P|H)P(H|\neg ER, \neg E)(P(L|H)+P(\neg L|H)) +$$
$$P(P|\neg H)P(\neg H|\neg ER, \neg E)(P(L|\neg H)+P(\neg L|\neg H))]$$

$$= [(0.7)(0.2)(0.8+0.2) +$$
$$(0.5)(0.8)(0.4+0.6)]$$

$$= 0.54$$

$$P(P = f \mid ER = f, E = f) = 0.46$$

$$\Rightarrow \mathbf{P}(P \mid ER = f, E = f) = \langle 0.54, 0.46 \rangle$$

3. The sample's probability.

1. Get $ER = f$ because $P(\neg ER) = 0.7$ (max)

2. Get $E = f$ because $P(\neg E) = 0.8$ (max)

3. Get $H = f$ based on $ER = f$ and $E = f$

   and [redacted] $= 0.8$ (max)
   $P(H \mid \neg ER, \neg E)$

   [redacted]

4. Get $L = f$ based on $H = f$

   $P(\neg L \mid \neg H) = 0.6$ (max)

5. Get P based on $H = f$

   $P(P \mid \neg H) = P(\neg P \mid H)$, so both $P = t$ and $P = f$ works

   Choose $P = t$.

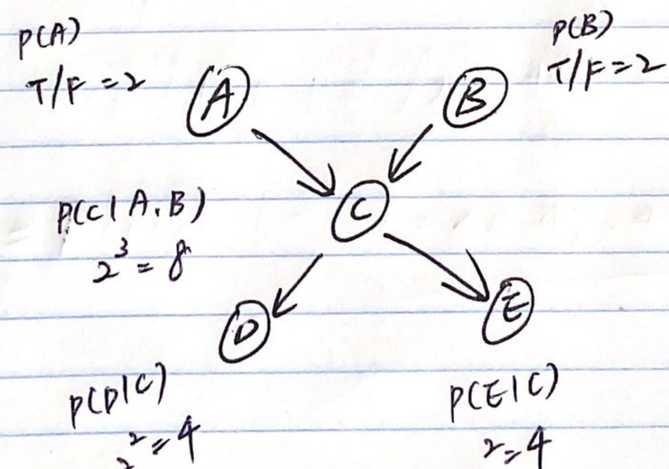
6. Caculate the probability:

   $P(ER = f, E = f, H = f, L = f, P = t)$

   $= P(\neg ER)\, P(\neg E)\, P(H \mid \neg ER, \neg E)\, P(\neg L \mid \neg H)\, P(P \mid \neg H)$

   $= (0.7)(0.8)(0.8)(0.6)(0.5) = 0.1344$

4. The minimum # of probabilities.



$2 + 2 + 8 + 4 + 4$

$= 20$