

1. Convert English statements to FOL:

- a. $Pit(3,1)$
- b. $\exists x_1, y_1, x_2, y_2 \text{ Breeze}(x_1, y_1) \Leftrightarrow Pit(x_2, y_2) \wedge (\neg(x_1 = x_2) \vee \neg(y_1 = y_2))$
- c. $\forall x, y \text{ Agent}(x, y) \wedge \text{Wumpus}(x, y) \wedge \text{Alive}(\text{Wumpus}) \Rightarrow \neg \text{Alive}(\text{Agent})$
- d. $\exists x_1, y_1, x_2, y_2 \text{ Stench}(x_1, y_1) \wedge \text{Stench}(x_2, y_2) \wedge (\neg(x_1 = x_2) \vee \neg(y_1 = y_2))$
- e. $\text{Breeze}(2,2) \wedge Pit(2,3)$

2. Convert FOL sentences to CNF:

1. $\text{Wumpus}(2,3)$
2. $Pit(S_1, S_2)$
3. $\text{Action}(\text{Shoot}, t) \vee \text{Arrow}(A, t)$
4. $(\neg \text{Agent}(x, y, t_1) \vee \neg \text{Orientation}(\text{Right}, t_1) \vee \neg \text{Action}(\text{TurnLeft}, t_1) \vee \text{Agent}(x, y, t_2))$
 $\wedge (\neg \text{Agent}(x, y, t_1) \vee \neg \text{Orientation}(\text{Right}, t_1) \vee \neg \text{Action}(\text{TurnLeft}, t_1) \vee \text{Orientation}(\text{Up}, t_2))$

3. Resolution proof:

a. Convert to CNF:

- i. $\neg \text{Orientation}(\text{Right}, t_1) \vee \neg \text{Action}(\text{TurnLeft}, t_1) \vee \text{Inc}(t_1, t_2) \vee \text{Orientation}(\text{Up}, t_2)$
- ii. $\neg \text{Alive}(\text{Agent}, t_1) \vee \text{Action}(\text{GoForward}, t_1) \vee \neg \text{Inc}(t_1, t_2) \vee \text{Alive}(\text{Agent}, t_2)$
- iii. $\neg \text{Action}(\text{TurnLeft}, t) \vee \neg \text{Action}(\text{GoForward}, t)$

iv. $Alive(Agent, 1)$

v. $Orientation(Right, 1)$

vi. $Action(TurnLeft, 1)$

vii. $Inc(1, 2)$

viii. $Inc(2, 3)$

ix. Query: $Alive(Agent, T) \wedge Orientation(Up, T)$

b. Proof:

i.

Assume the query is false: $\neg Alive(Agent, T) \vee \neg Orientation(Up, T)$

$\{t_2/T\}$

ii.

$\neg Alive(Agent, t_1) \vee Action(GoForward, t_1) \vee \neg Inc(t_1, T) \vee \neg Orientation(Right, t_1) \vee Action(TurnLeft, t_1)$

$\{t_1/1\}$

iii.

$\neg Alice(Agent, 1) \vee Action(GoForward, 1) \vee \neg Inc(1, T) \vee \neg Orientation(Right, t_1) \vee \neg Action(TurnLeft, 1)$

iv.

$Action(GoForward, 1) \vee \neg Inc(1, T)$

$\{T/2\}$

v.

$Action(GoForward, 1)$

vi.

$\neg Action(TurnLeft, 1)$ which is not correct.

4. Vampire theorem:

1. Input:

fof(a1, axiom,

! [T1, T2] : ((orientation(Right,T1) & action(TurnLeft,T1) & inc(T1,T2)) => orientation(Up, T2))).

fof(a2, axiom,

! [T1, T2] : ((alive(Agent,T1) & ~action(GoForward,T1) & inc(T1,T2)) => alive(Agent, T2))).

fof(a3, axiom,

! [T] : (action(TurnLeft,1) => ~action(GoForward,T))).

fof(a4, axiom, alive(Agent,1)).

fof(a5, axiom, orientation(Right,1)).

fof(a6, axiom, action(TurnLeft,1)).

fof(a7, axiom, inc(1,2)).

fof(a8, axiom, inc(2,3)).

fof(c1, conjecture,

? [T] : (alive(Agent,T) & orientation(Up,T))).

2. Output:

% Refutation found. Thanks to Tanya!

% SZS status ContradictoryAxioms for input

% SZS output start Proof for input

3. ! [X7] : (action(X3,1) => ~action(X6,X7)) [input]

6. action(X3,1) [input]

17. ! [X0] : (action(X1,1) => ~action(X2,X0)) [rectify 3]

18. ! [X2,X1] : ! [X0] : (action(X1,1) => ~action(X2,X0)) [closure 17]

19. ! [X2,X1,X0] : (action(X1,1) => ~action(X2,X0)) [flattening 18]

24. action(X0,1) [rectify 6]

25. ! [X0] : action(X0,1) [closure 24]

31. ! [X2,X1,X0] : (\sim action(X2,X0) | \sim action(X1,1)) [ennf transformation 19]

35. ! [X0,X1,X2] : (\sim action(X0,X2) | \sim action(X1,1)) [rectify 31]

40. \sim action(X0,X2) | \sim action(X1,1) [cnf transformation 35]

43. action(X0,1) [cnf transformation 25]

47. \$false [subsumption resolution 43,40]

% SZS output end Proof for input

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% Version: Vampire 4.5.1 (commit unknown)

% Termination reason: Refutation

% Memory used [KB]: 383

% Time elapsed: 0.003 s

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