SFU NatLangLab

CMPT 413 / 825: Natural Language Processing

Sequence Models

Fall 2020
2020-10-14

Adapted from slides from Danqi Chen and Karthik Narasimhan

Announcements

- HW2 programming due Wednesday 10/14
- HW2 conceptual questions due Thursday 10/15
- Project abstract / title due Friday 10/16
    - Not graded
    - If turned in by Friday 10/16, you will get some feedback on if your project seems reasonable.
    - If turned in by Sunday 10/18, we will do our best to give you feedback.
- Project Proposal due Friday 10/23 (5%)

Project Proposal

(due 10/23 - 5%)

- What problem are you addressing? Why is it interesting?
- What specific aspects will your project be on?
  - Re-implement paper? Compare different methods?
- What data do you plan to use?
- What is your method?
  - What do you plan to implement? Are there existing codebases you will use?
- How do you plan to evaluate? What metrics?
- What computational resources will you need?

Overview

- What is sequence modeling?
- Hidden markov models (HMM)
- Decoding algorithms: Greedy, Viterbi, Beam
- Maximum entropy markov models (MEMM)

Sequence Tagging

Input: sequence of words; Output: sequence of labels

Input	British	left	waffles	on	Falkland	Islands
Output1	N	N	V	P	N	N
Output2	N	V	N	P	N	N
⋮						

N	Noun, e.g. islands
V	Verb, e.g. leave, left
P	Preposition, e.g. on

What are POS tags?

- Word classes or syntactic categories
  - Reveal useful information about a word (and its neighbors!)

The/DT cat/NN sat/VBD on/IN the/DT mat/NN

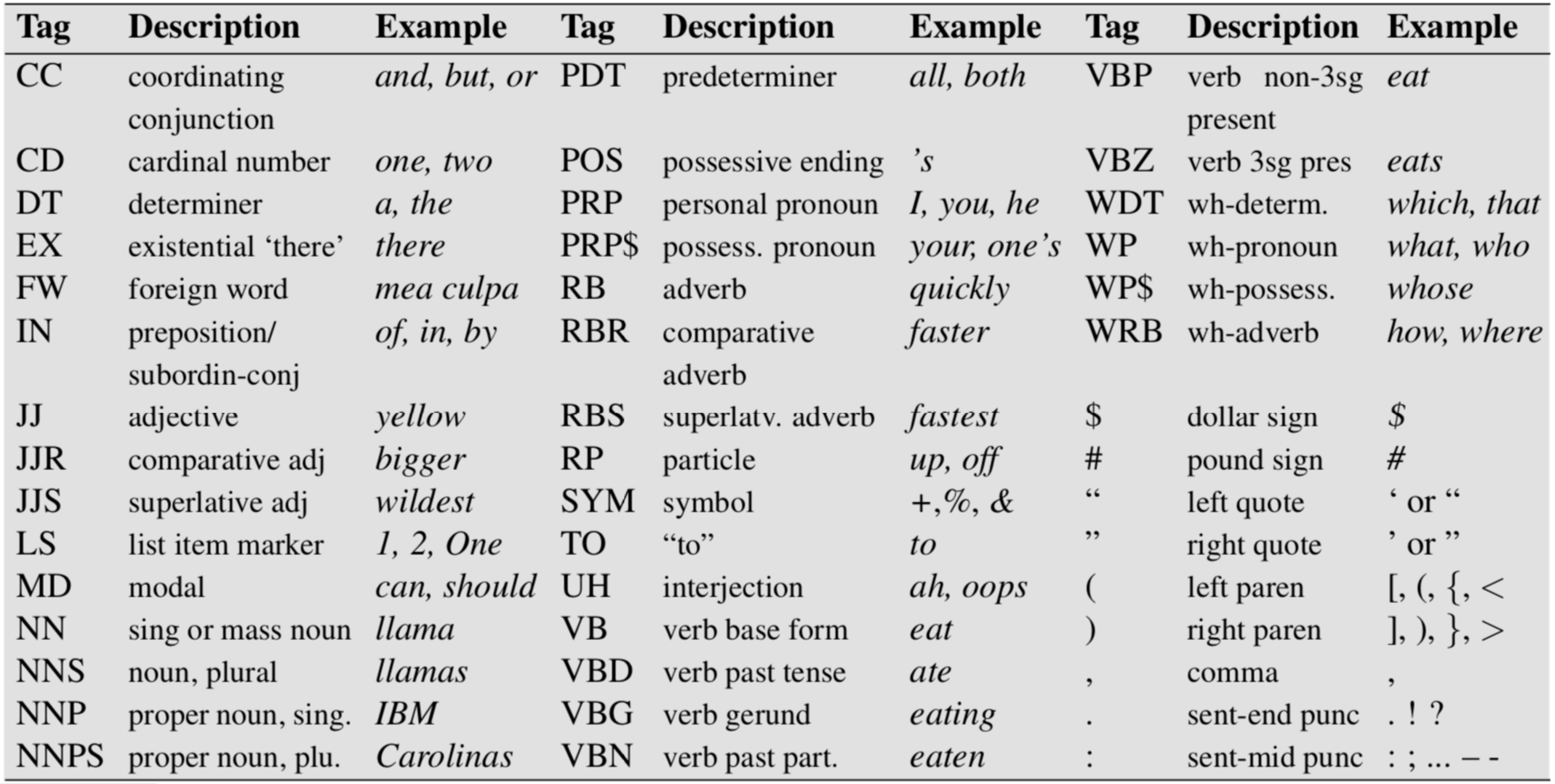
British/NNP left/NN waffles/NNS on/IN Falkland/NNP Islands/NNP

The/DT old/NN man/VB the/DT boat/NN

Parts of Speech

- Different words have different functions
- Closed class: fixed membership, **function words**
    - e.g. prepositions (*in, on, of*), determiners (*the, a*)
- Open class: New words get added frequently
    - e.g. nouns (Twitter, Facebook), verbs (google), adjectives, adverbs

Penn Tree Bank tagset



Tag	Description	Example	Tag	Description	Example	Tag	Description	Example
CC	coordinating conjunction	*and, but, or*	PDT	predeterminer	*all, both*	VBP	verb non-3sg present	*eat*
CD	cardinal number	*one, two*	POS	possessive ending	*'s*	VBZ	verb 3sg pres	*eats*
DT	determiner	*a, the*	PRP	personal pronoun	*I, you, he*	WDT	wh-determ.	*which, that*
EX	existential 'there'	*there*	PRP$	possess. pronoun	*your, one's*	WP	wh-pronoun	*what, who*
FW	foreign word	*mea culpa*	RB	adverb	*quickly*	WP$	wh-possess.	*whose*
IN	preposition/ subordin-conj	*of, in, by*	RBR	comparative adverb	*faster*	WRB	wh-adverb	*how, where*
JJ	adjective	*yellow*	RBS	superlatv. adverb	*fastest*	$	dollar sign	*$*
JJR	comparative adj	*bigger*	RP	particle	*up, off*	#	pound sign	*#*
JJS	superlative adj	*wildest*	SYM	symbol	+,%, &	“	left quote	‘ or “
LS	list item marker	*1, 2, One*	TO	“to”	*to*	”	right quote	’ or ”
MD	modal	*can, should*	UH	interjection	*ah, oops*	(	left paren	[, (, {, <
NN	sing or mass noun	*llama*	VB	verb base form	*eat*	)	right paren	],), }, >
NNS	noun, plural	*llamas*	VBD	verb past tense	*ate*	,	comma	,
NNP	proper noun, sing.	*IBM*	VBG	verb gerund	*eating*	.	sent-end punc	. ! ?
NNPS	proper noun, plu.	*Carolinas*	VBN	verb past part.	*eaten*	:	sent-mid punc	: ; ... – -

Figure 8.1 Penn Treebank part-of-speech tags (including punctuation).

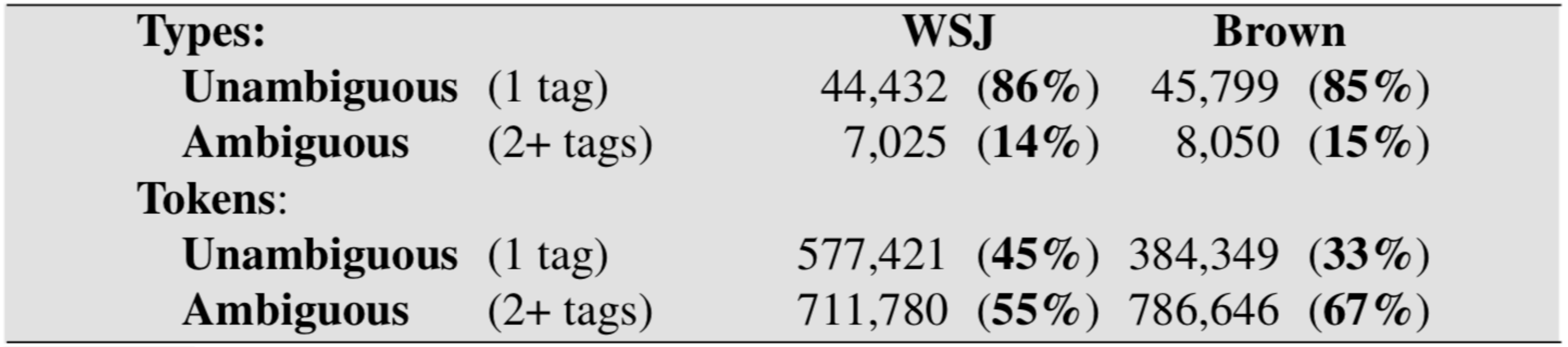
[45 tags]

(Marcus et al., 1993)

Other corpora: Brown, WSJ, Switchboard

Part of Speech Tagging

- Disambiguation task: each word might have different senses/functions
  - The/DT man/NN bought/VBD a/DT boat/NN
  - The/DT old/NN man/VB the/DT boat/NN

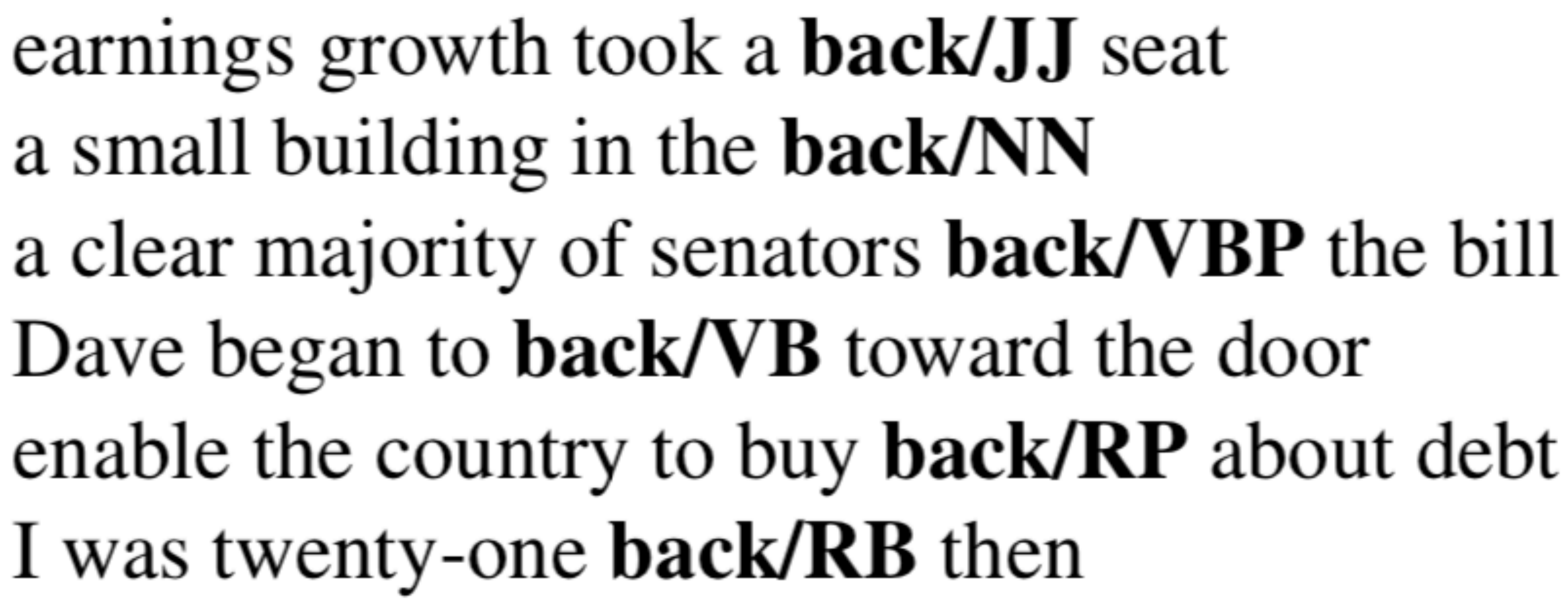


Types:		**WSJ**	**Brown**
Unambiguous	(1 tag)	44,432 (**86%**)	45,799 (**85%**)
Ambiguous	(2+ tags)	7,025 (**14%**)	8,050 (**15%**)
Tokens:			
Unambiguous	(1 tag)	577,421 (**45%**)	384,349 (**33%**)
Ambiguous	(2+ tags)	711,780 (**55%**)	786,646 (**67%**)

Figure 8.2 Tag ambiguity for word types in Brown and WSJ, using Treebank-3 (45-tag) tagging. Punctuation were treated as words, and words were kept in their original case.

Part of Speech Tagging

- Disambiguation task: each word might have different senses/functions
  - The/DT man/NN bought/VBD a/DT boat/NN
  - The/DT old/NN man/VB the/DT boat/NN



earnings growth took a **back/JJ** seat
a small building in the **back/NN**
a clear majority of senators **back/VBP** the bill
Dave began to **back/VB** toward the door
enable the country to buy **back/RP** about debt
I was twenty-one **back/RB** then

Some words have many functions!

A simple baseline

- Many words might be easy to disambiguate
- **Most frequent class:** Assign each token (word) to the class it occurred most in the training set. (e.g. man/NN)
- Accurately tags **92.34%** of word tokens on Wall Street Journal (WSJ)!
- State of the art ~97-98%
- Average English sentence ~ 14 words
  - Sentence level accuracies: 0.92^{14} = **31%** vs 0.97^{14} = **65%** vs 0.98^{14} = **75%**

The/DT old/JJ man/NN the/DT boat/NN

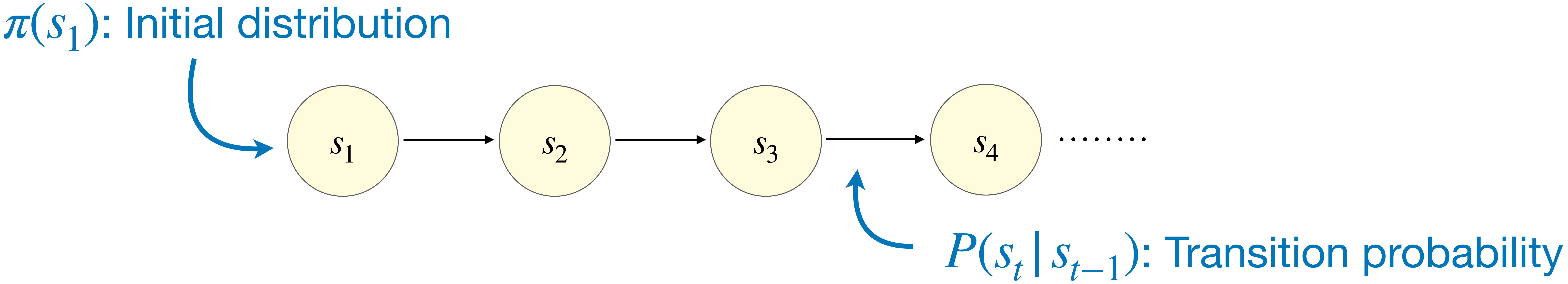
- POS tagging not solved yet!

Hidden Markov Models

Some observations

- The function (or POS) of a word depends on its context
  - The/DT old/NN man/VB the/DT boat/NN
  - The/DT old/JJ man/NN bought/VBD the/DT boat/NN
- Certain POS combinations are extremely unlikely
  - *<JJ, DT>* or *<DT, IN>*
- Better to make decisions on entire sequences instead of individual words (Sequence modeling!)

Markov chains



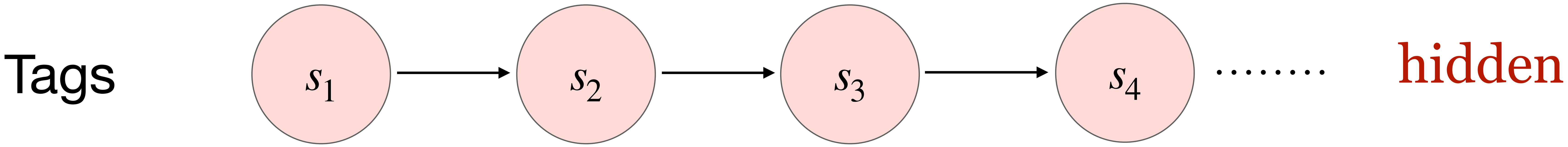
$\pi(s_1)$: Initial distribution

$P(s_t \mid s_{t-1})$: Transition probability

- Model probabilities of sequences of variables
- Each state can take one of K values ({1, 2, ..., K} for simplicity)
- Markov assumption: $P(s_t \mid s_{<t}) \approx P(s_t \mid s_{t-1})$

Where have we seen this before?

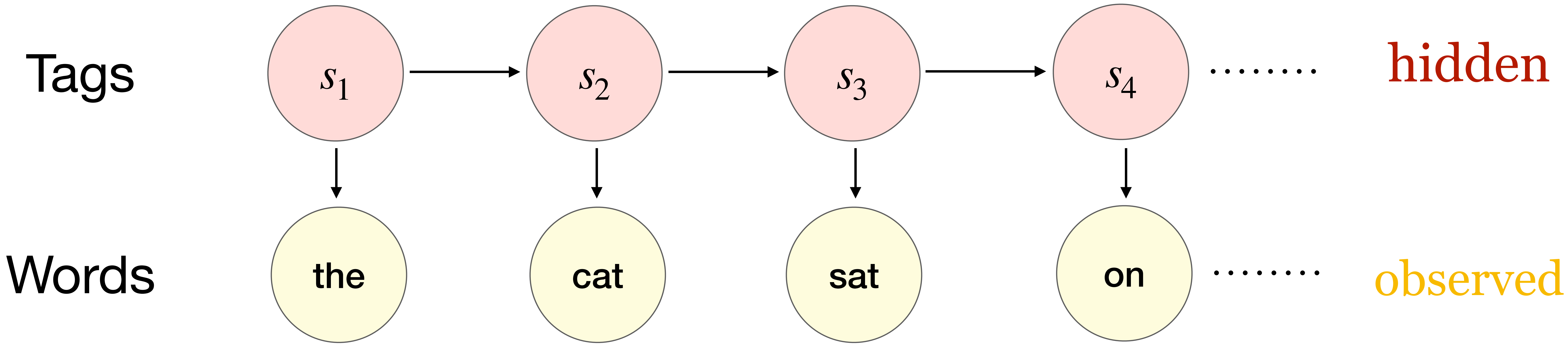
Markov chains



The/?? cat/?? sat/?? on/?? the/?? mat/??

- We don’t observe POS tags at test time

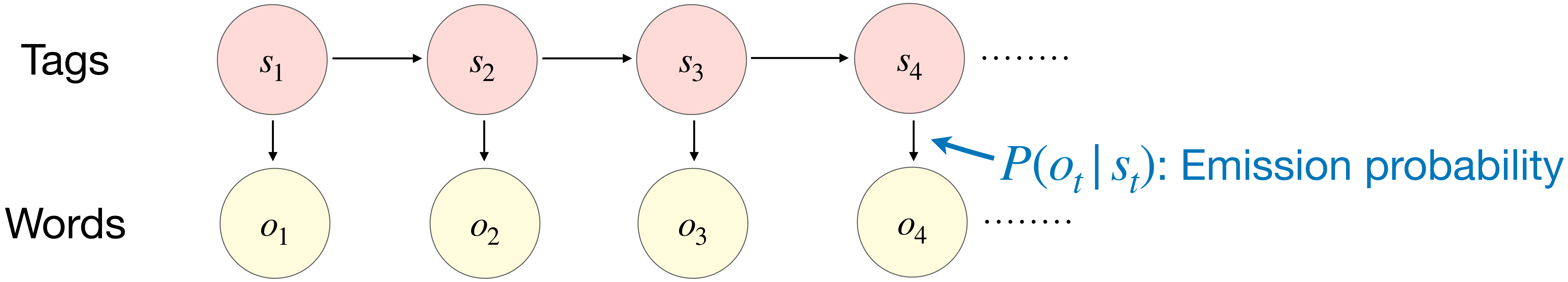
Hidden Markov Model (HMM)



The/?? cat/?? sat/?? on/?? the/?? mat/??

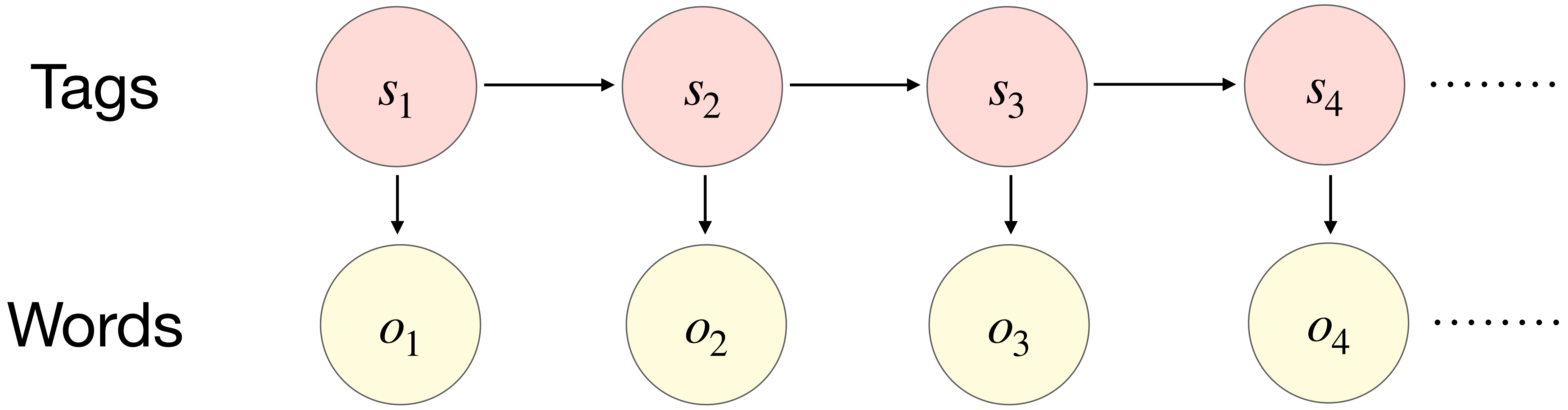
- We don’t observe POS tags at test time
- But we do observe the words!
- HMM allows us to *jointly reason* over both hidden and observed events.

Components of an HMM



1. Set of states S = {1, 2, ..., K} and observations O
2. Initial state probability distribution $\pi(s_1)$
3. Transition probabilities $P(s_{t+1} \mid s_t)$
4. Emission probabilities $P(o_t \mid s_t)$

Assumptions



1. Markov assumption:

$$P(s_{t+1} \mid s_1, \ldots, s_t) = P(s_{t+1} \mid s_t)$$

Transition Probabilities

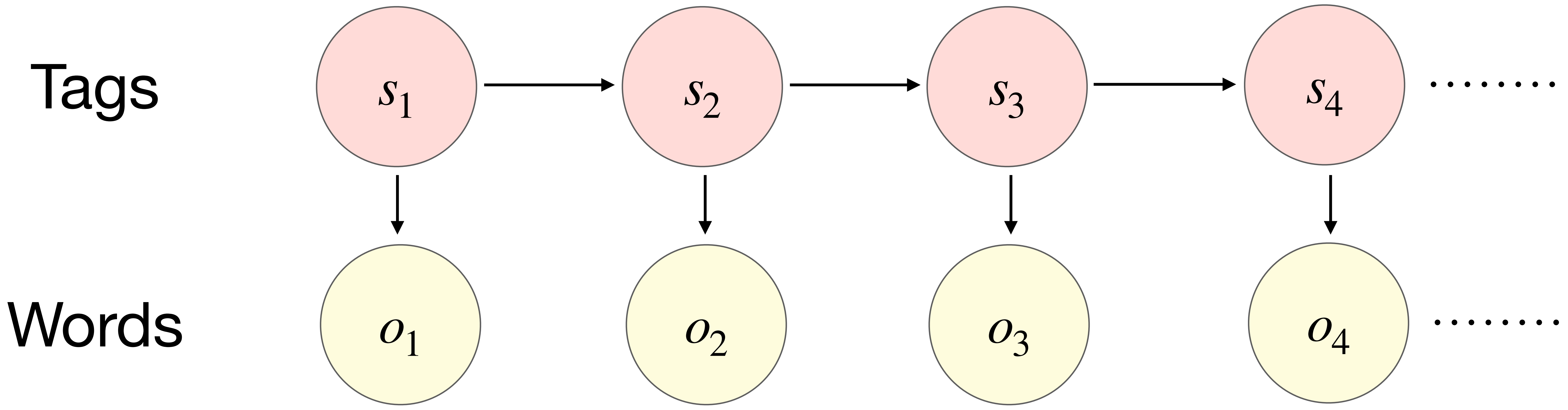
2. Output independence:

$$P(o_t \mid s_1, \ldots, s_t) = P(o_t \mid s_t)$$

Emission Probabilities

Which is a stronger assumption?

Sequence likelihood



$$
\begin{aligned}
P(S, O) &= P(s_1, s_2, \ldots, s_n, o_1, o_2, \ldots, o_n) \\
&= \pi(s_1) P(o_1|s_1) \prod_{t=2}^{n} P(s_t, o_t|s_{t-1}) \\
&= \pi(s_1) P(o_1|s_1) \prod_{t=2}^{n} P(o_t|s_t) P(s_t, |s_{t-1})
\end{aligned}
$$

Example: POS tagging

the/DT cat/NN sat/VBD on/IN the/DT mat/NN

$\pi(DT) = 0.8$

s_{t+1}

s_t

	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01

…

P(the|DT, cat|NN, sat|VBD, on|IN, the|DT, mat|NN) = ??

Example: POS tagging

$\boxed{\pi(DT) = 0.8}$

the/DT cat/NN sat/VBD on/IN the/DT mat/NN

s_{t+1}

s_t

	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01

Where did these numbers come from?

Learned from data!

P(the|DT, cat|NN, sat|VBD, on|IN, the|DT, mat|NN) =

$$= \overset{0.8}{\pi(\mathrm{DT})}\overset{0.5}{P(\mathrm{the}|\mathrm{DT})}\ \overset{0.8}{P(\mathrm{NN}|\mathrm{DT})}\overset{0.2}{P(\mathrm{cat}|\mathrm{NN})}\ \overset{0.6}{P(\mathrm{VBD}|\mathrm{NN})}\overset{0.1}{P(\mathrm{sat}|\mathrm{VBD})}\ \ldots$$

$$= 1.84 * 10^{-5}$$

Learning from fully observed data

Fully labeled! All tags are known during training

Training set:

1 Pierre/NNP Vinken/NNP ,/, 61/CD years/NNS old/JJ ,/, will/MD join/VB the/DT board/NN as/IN a/DT nonexecutive/JJ director/NN Nov./NNP 29/CD ./.

2 Mr./NNP Vinken/NNP is/VBZ chairman/NN of/IN Elsevier/NNP N.V./NNP ,/, the/DT Dutch/NNP publishing/VBG group/NN ./.

3 Rudolph/NNP Agnew/NNP ,/, 55/CD years/NNS old/JJ and/CC chairman/NN of/IN Consolidated/NNP Gold/NNP Fields/NNP PLC/NNP ,/, was/VBD named/VBN a/DT nonexecutive/JJ director/NN of/IN this/DT British/JJ industrial/JJ conglomerate/NN ./.

...

38,219 It/PRP is/VBZ also/RB pulling/VBG 20/CD people/NNS out/IN of/IN Puerto/NNP Rico/NNP ,/, who/WP were/VBD helping/VBG Huricane/NNP Hugo/NNP victims/NNS ,/, and/CC sending/VBG them/PRP to/TO San/NNP Francisco/NNP instead/RB ./.

Learning from fully observed data

Training set:

1 Pierre/NNP Vinken/NNP ,/, 61/CD year
join/VB the/DT board/NN as/IN a/DT no
Nov./NNP 29/CD ./.
2 Mr./NNP Vinken/NNP is/VBZ chairman
N.V./NNP ,/, the/DT Dutch/NNP publish
3 Rudolph/NNP Agnew/NNP ,/, 55/CD ye
chairman/NN of/IN Consolidated/NNP Go
,/, was/VBD named/VBN a/DT nonexecut
this/DT British/JJ industrial/JJ conglomer
. . .
38,219 It/PRP is/VBZ also/RB pulling/VE
of/IN Puerto/NNP Rico/NNP ,/, who/WP
Huricane/NNP Hugo/NNP victims/NNS ,/
them/PRP to/TO San/NNP Francisco/NN

Easy!

Maximum likelihood estimate:

$$P(s_i \mid s_j) = \frac{C(s_j, s_i)}{C(s_j)}$$

$$P(o \mid s) = \frac{C(s, o)}{C(s)}$$

Estimating probabilities

$$\pi(DT) = \frac{2}{2}$$

s_{t+1}

s_t

	DT	NN	IN	VBD
DT				
NN				
IN				
VBD				

o_t

	the	cat	sat	on	mat	a	cleaned	chair	man
DT									
NN									
IN									
VBD									

Training corpus:

a/DT man/NN cleaned/VBD the/DT mat/NN

the/DT cat/NN sat/VBD on/IN the/DT chair/NN

Estimating probabilities

$$\pi(DT) = \frac{2}{2}$$

s_{t+1}

s_t

	DT	NN	IN	VBD
DT				
NN				
IN				
VBD				

o_t

	the	cat	sat	on	mat	a	cleaned	chair	man
DT	3/4					1/4			
NN									
IN									
VBD									

Training corpus:

a/DT man/NN cleaned/VBD the/DT mat/NN
the/DT cat/NN sat/VBD on/IN the/DT chair/NN

Estimation probabilities

$$\pi(DT) = \frac{2}{2}$$

s_{t+1}

s_t

	DT	NN	IN	VBD
DT				
NN				
IN				
VBD				

o_t

	the	cat	sat	on	mat	a	cleaned	chair	man
DT	3/4					1/4			
NN		1/4			1/4			1/4	1/4
IN				1/1					
VBD			1/2				1/2		

Training corpus:

a/DT man/NN cleaned/VBD the/DT mat/NN

the/DT cat/NN sat/VBD on/IN the/DT chair/NN

Estimation probabilities

$$\pi(DT) = \frac{2}{2}$$

s_{t+1}

s_t

	DT	NN	IN	VBD
DT				
NN				
IN				
VBD	1/2		1/2	

o_t

	the	cat	sat	on	mat	a	cleaned	chair	man
DT	3/4					1/4			
NN		1/4			1/4			1/4	1/4
IN				1/1					
VBD			1/2				1/2		

Training corpus:

a/DT man/NN cleaned/VBD the/DT mat/NN

the/DT cat/NN sat/VBD on/IN the/DT chair/NN

Estimation probabilities

$$\pi(DT) = \frac{2}{2}$$

s_{t+1}

s_t

	DT	NN	IN	VBD	EOS
DT		4/4			
NN				2/4	2/4
IN	1/1				
VBD	1/2		1/2		

o_t

	the	cat	sat	on	mat	a	cleaned	chair	man
DT	3/4					1/4			
NN		1/4			1/4			1/4	1/4
IN				1/1					
VBD			1/2				1/2		

Training corpus:

a/DT man/NN cleaned/VBD the/DT mat/NN

the/DT cat/NN sat/VBD on/IN the/DT chair/NN

Learning from partially observable data (unsupervised learning)

No labels (or partial labels).

Still want to estimate parameters to maximize likelihood of training data.

Parameters: $\theta = \{P(s_i \mid s_j), P(o \mid s)\}$

Guaranteed to iteratively improve likelihood
$L(\theta_t) \geq L(\theta_{t-1})$

EM: Expectation-Maximization algorithm

Initialize parameters to some random values

E-Step: Compute **expected** counts $\bar{C}$ using current parameters

M-Step: Take expected counts use it to re-estimate parameters that **maximizes** the likelihood

$$P(s_i \mid s_j) = \frac{\bar{C}(s_j, s_i)}{\bar{C}(s_j)}, \quad P(o \mid s) = \frac{\bar{C}(s, o)}{\bar{C}(s)}$$

Iterate until convergence.

Example: POS tagging

the/?? cat/?? sat/?? on/?? the/?? mat/??

$\pi(DT) = 0.8$

s_{t+1}

s_t

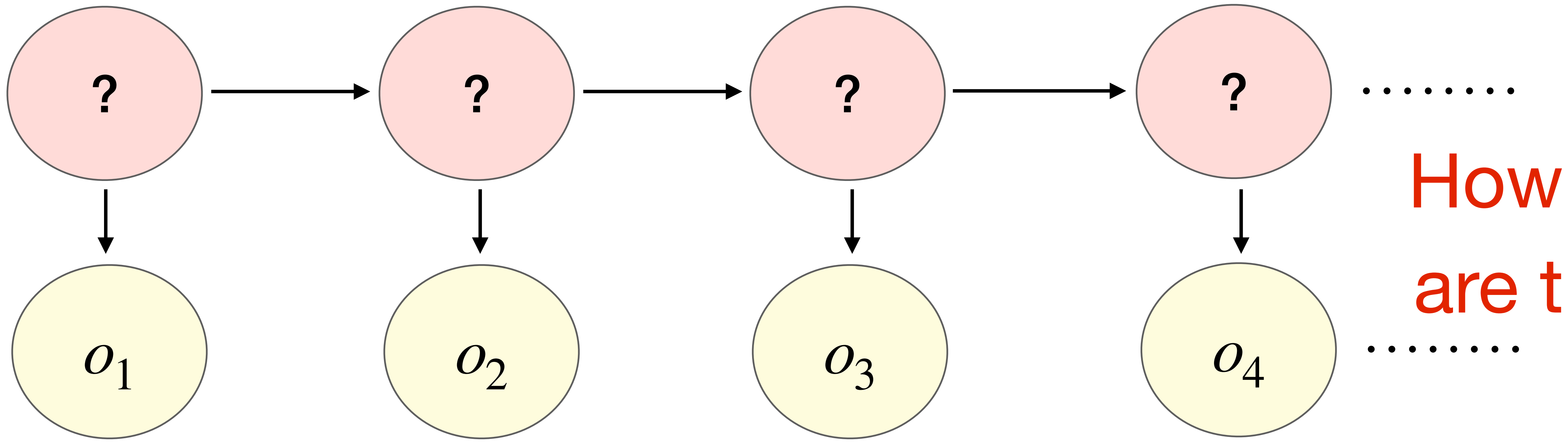
	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01

How to find best sequence?

Decoding with HMMs



How many sequence of states are there for $s_t \in \{1,2,...,K\}$?

K^n

- **Task:** Find the most probable sequence of states $\langle s_1, s_2, \ldots, s_n \rangle$ given the observations $\langle o_1, o_2, \ldots, o_n \rangle$

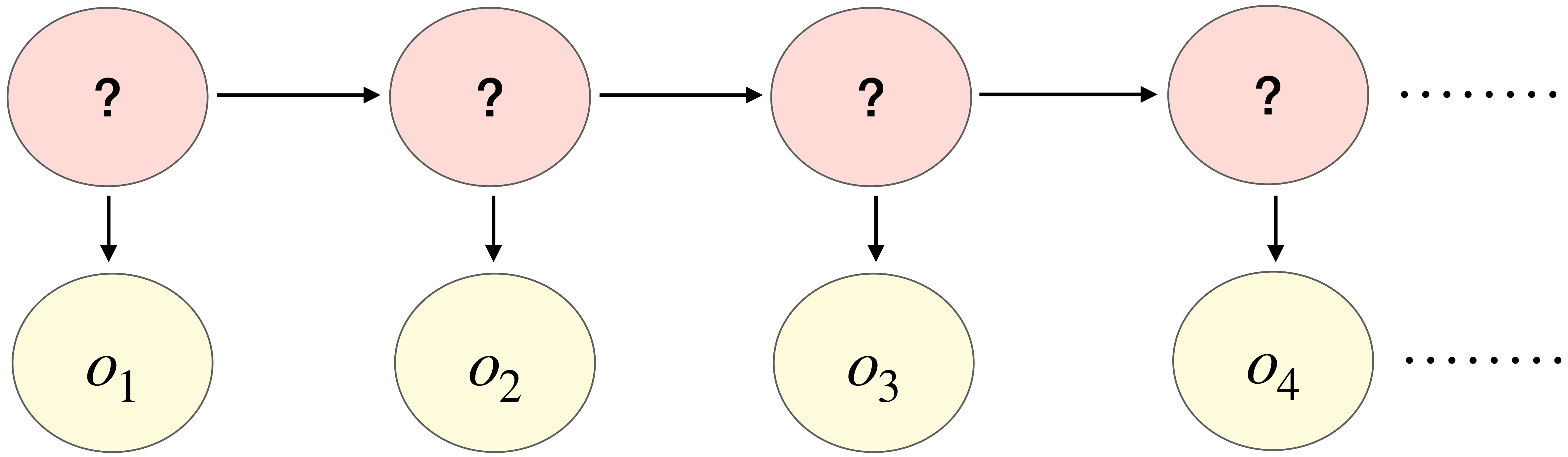
What is the best sequence of tags for the observed sequence: the cat sat on the mat

$$\hat{S} = \arg\max_S P(S|O) = \arg\max_S \frac{P(S)P(O|S)}{P(O)}$$

Bayes Rule

$$\hat{S} = \arg\max_S P(S)P(O|S)$$

Decoding with HMMs



- **Task:** Find the most probable sequence of states $\langle s_1, s_2, \ldots, s_n \rangle$ given the observations $\langle o_1, o_2, \ldots, o_n \rangle$

$$\pi(< \text{SOS} >) = 1$$

$$P(s_1|s_0) = P(s_1| < \text{SOS} >)$$

or

$$P(s_1|s_0) = \pi(s_1)$$

$$\hat{S} = \arg\max_{S} P(S)P(O|S)$$

$$= \arg\max_{S} \prod_{t=1}^{n} P(o_t|s_t)P(s_t, |s_{t-1})$$

Emission Probabilities

Transition Probabilities

Greedy decoding

s

	DT	NN	IN	VBD
π	0.8	0.1	0.05	0.05

s_{t+1}

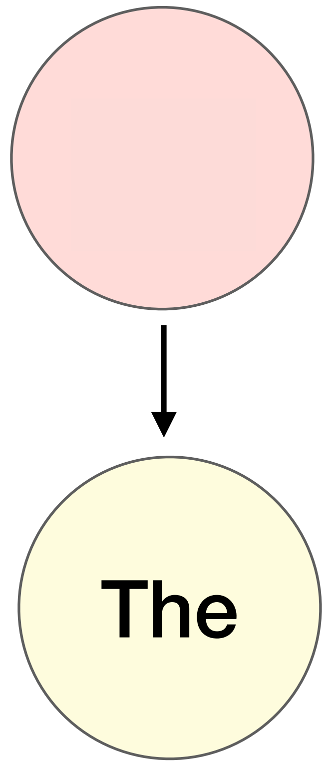
s_t

	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

s_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01



$$\arg\max_s P(\text{The}|s)\pi(s_1 = s) = \text{DT}$$

$$\hat{S} = \arg\max_S P(S)P(O|S)$$

$$= \arg\max_S \prod_{t=1}^{n} P(o_t|s_t)P(s_t,|s_{t-1})$$

Emission Transition

Greedy decoding

s

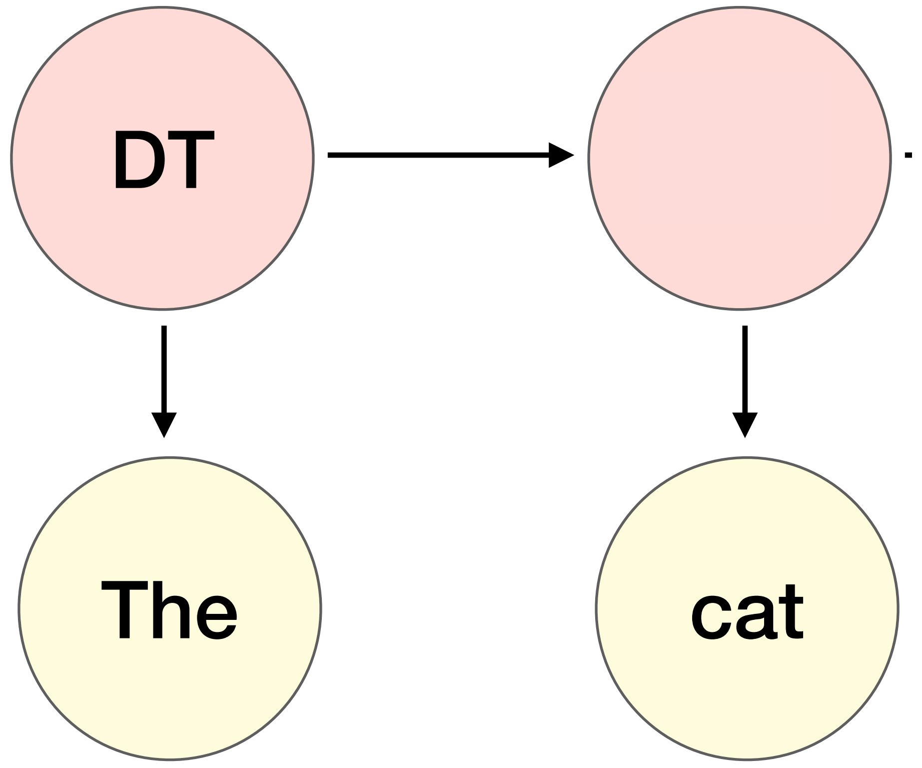
	DT	NN	IN	VBD
π	0.8	0.1	0.05	0.05

s_{t+1}

s_t	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

s_t	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01



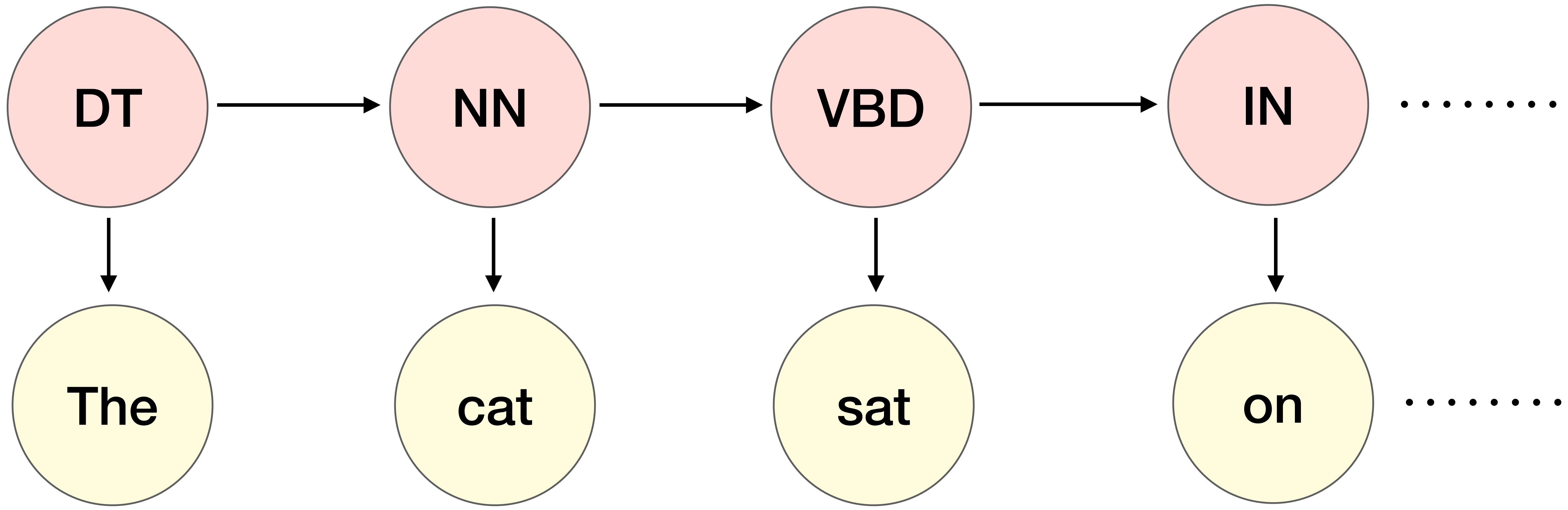
$$\arg\max_s P(\text{cat}|s)P(s_2 = s|DT) = \text{NN}$$

$$\hat{S} = \arg\max_S P(S)P(O|S)$$

$$= \arg\max_S \prod_{t=1}^{n} P(o_t|s_t)P(s_t,|s_{t-1})$$

Emission Transition

Greedy decoding



$$\forall t, \hat{s}_t = \arg\max_s P(o_t|s_t)P(s|\hat{s}_{t-1})$$

- Not guaranteed to be optimal!
  - Local decisions
- Fast! $O(K \times n)$

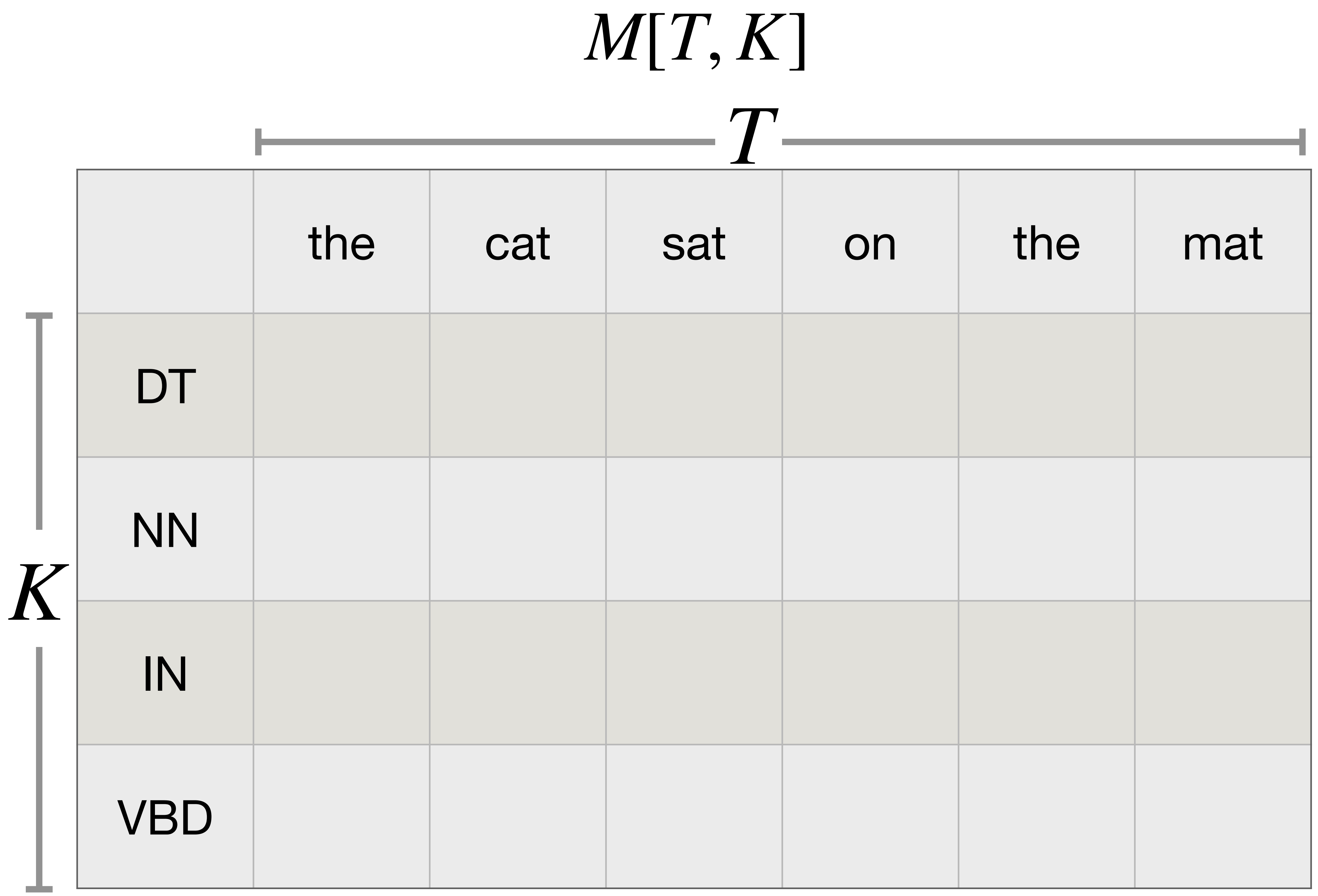
Viterbi decoding

- Use **dynamic programming**!

- Probability lattice, $M[T, K]$

  - T : Number of time steps

  - K : Number of states



	the	cat	sat	on	the	mat
DT						
NN						
IN						
VBD						

- $M[i, j]$: Most probable sequence of states ending with state **j** at time **i**

Viterbi decoding

s

	DT	NN	IN	VBD
π	0.8	0.1	0.05	0.05

s_{t+1}

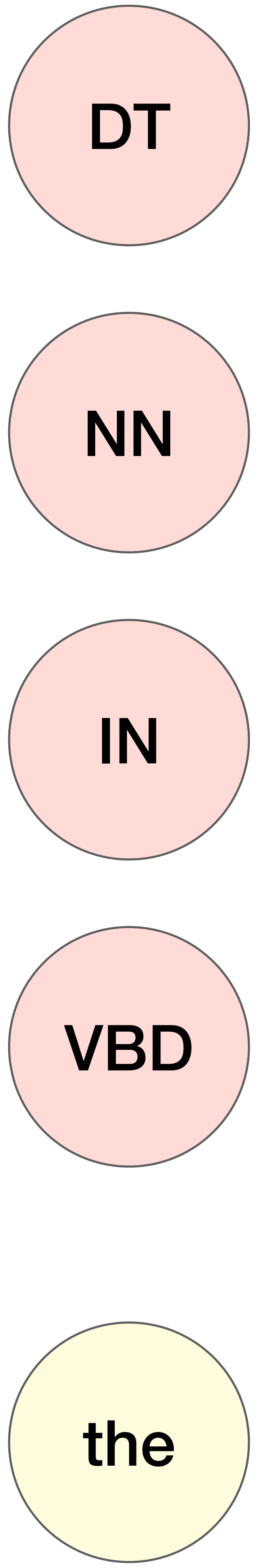
s_t

	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

s_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01



$$M[1, DT] = \pi(DT)\ P(\text{the} \mid DT) = 0.8 \times 0.5 = 0.4$$

$$M[1, NN] = \pi(NN)\ P(\text{the} \mid NN) = 0.1 \times 0.01 = 0.001$$

$$M[1, IN] = \pi(IN)\ P(\text{the} \mid IN) = 0.05 \times 0 = 0$$

$$M[1, VBD] = \pi(VBD)\ P(\text{the} \mid VBD) = 0.05 \times 0 = 0$$

Forward

$M[T, K]$

	the	cat	sat	on	the	mat
DT	0.4					
NN	0.001					
IN	0					
VBD	0					

Viterbi decoding

s_{t+1}

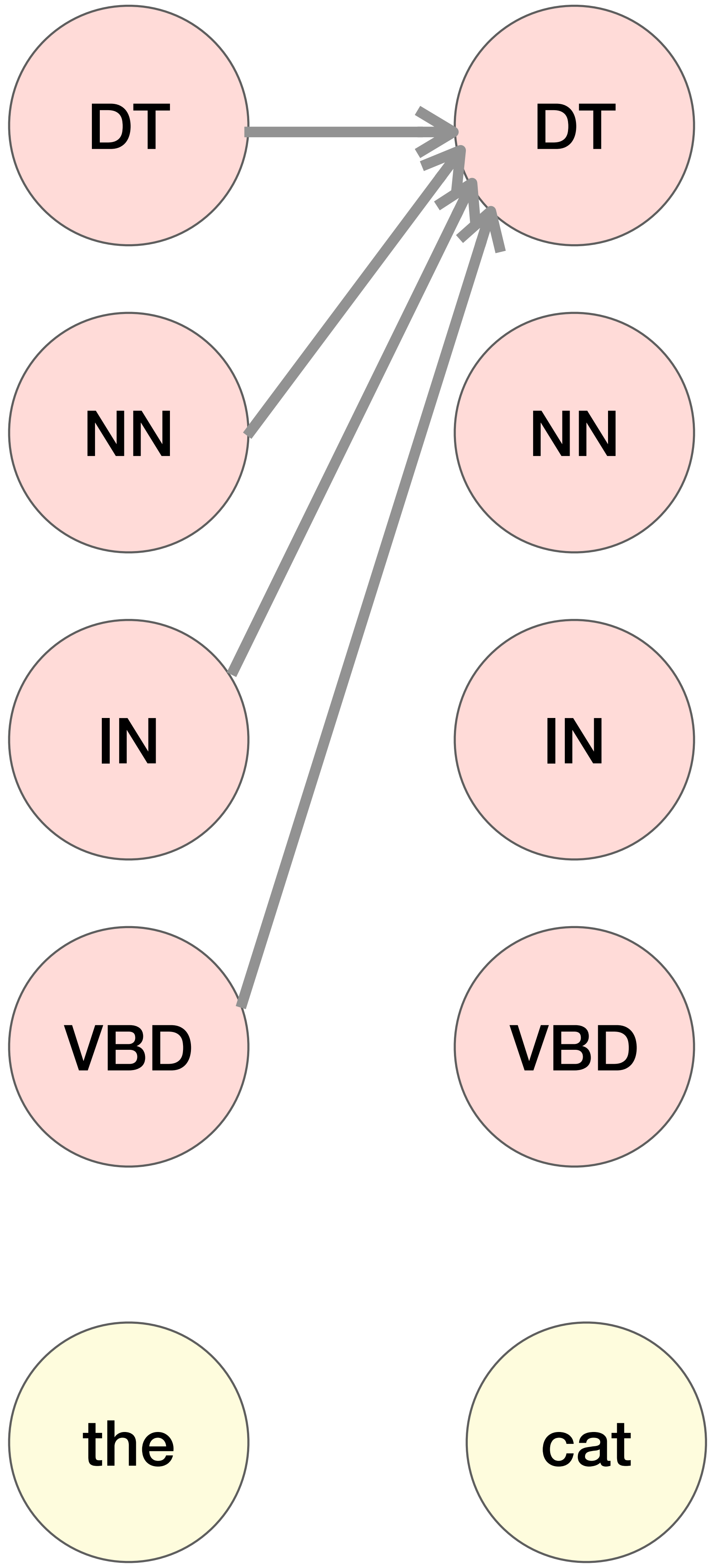
s_t

	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

s_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01



$$M[2,DT] = \max_{k} M[1,k]\ P(DT\,|\,k)\ P(\text{cat}\,|\,DT) = 0$$

	the	cat	sat	on	the	mat
DT	0.4	0				
NN	0.001					
IN	0					
VBD	0					

$M[1,K]$

Forward

Viterbi decoding

s_{t+1}

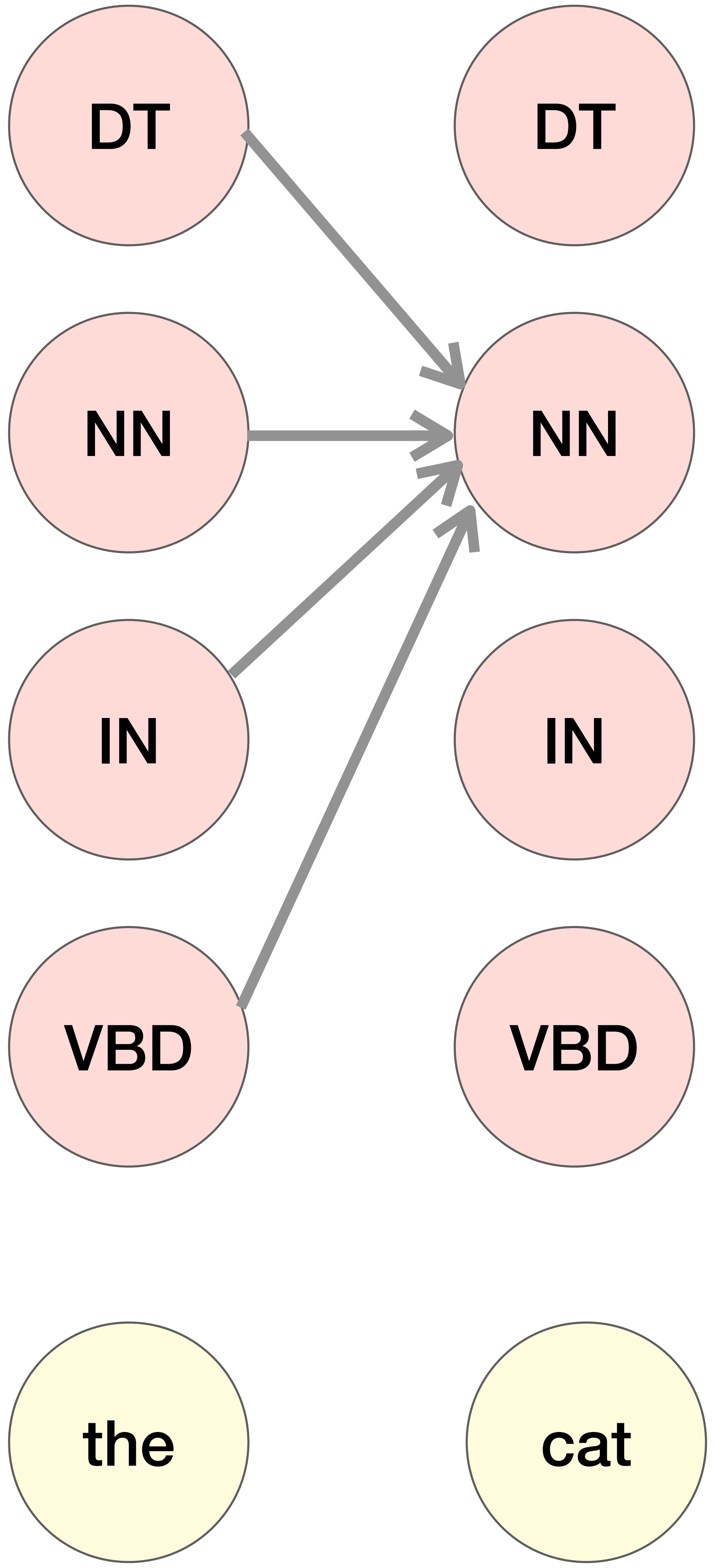
s_t

	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

s_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01



$$M[2,NN] = \max_{k} M[1,k]\ P(NN\,|\,k)\ P(\text{cat}\,|\,NN) = 0.064$$

	the	cat	sat	on	the	mat
DT	0.4	0				
NN	0.001	0.064				
IN	0					
VBD	0					

$M[1,K]$

Forward

Viterbi decoding

s_{t+1}

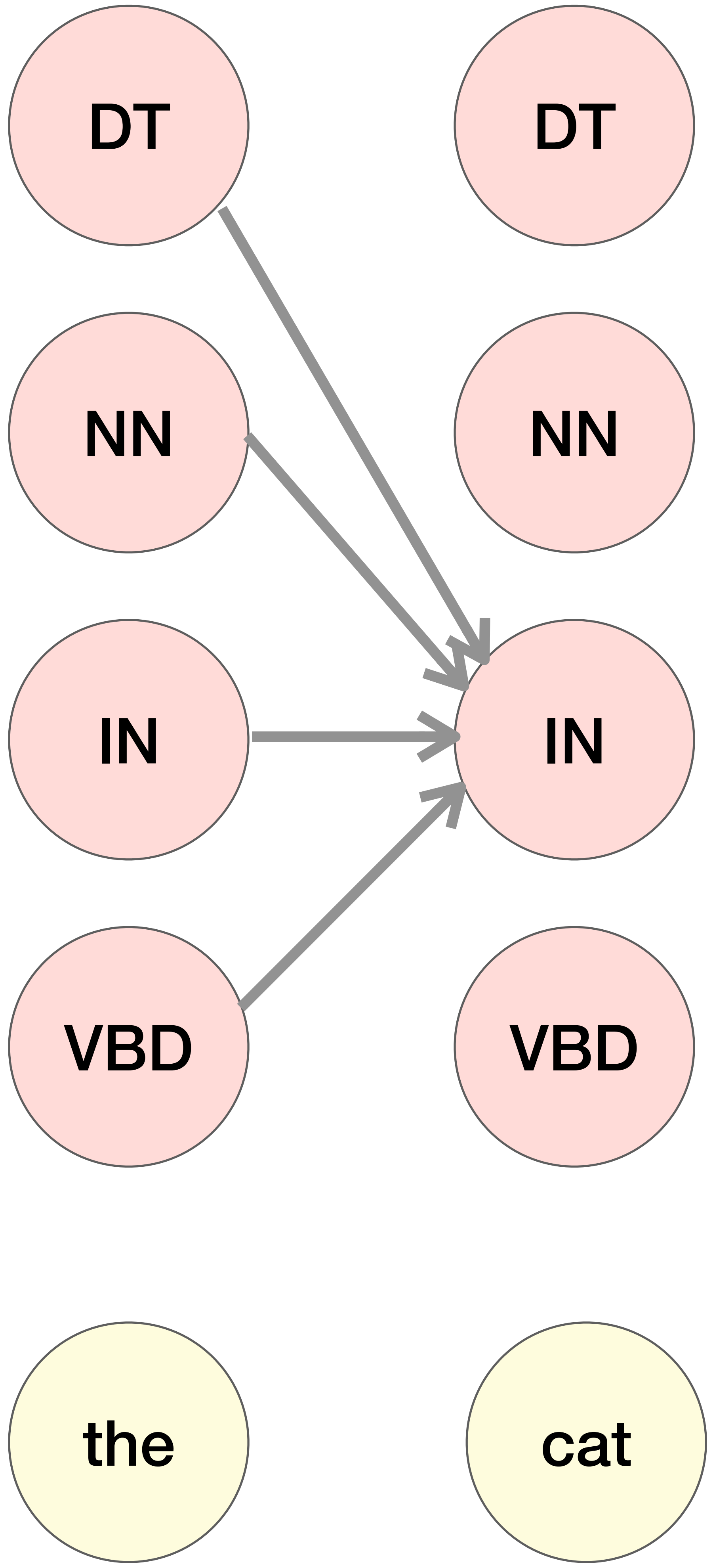
s_t

	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

s_t

	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01



$$M[2,IN] = \max_{k} M[1,k]\ P(IN\,|\,k)\ P(\text{cat}\,|\,IN) = 0$$

	the	cat	sat	on	the	mat
DT	0.4	0				
NN	0.001	0.064				
IN	0	0				
VBD	0					

$M[1,K]$

Forward

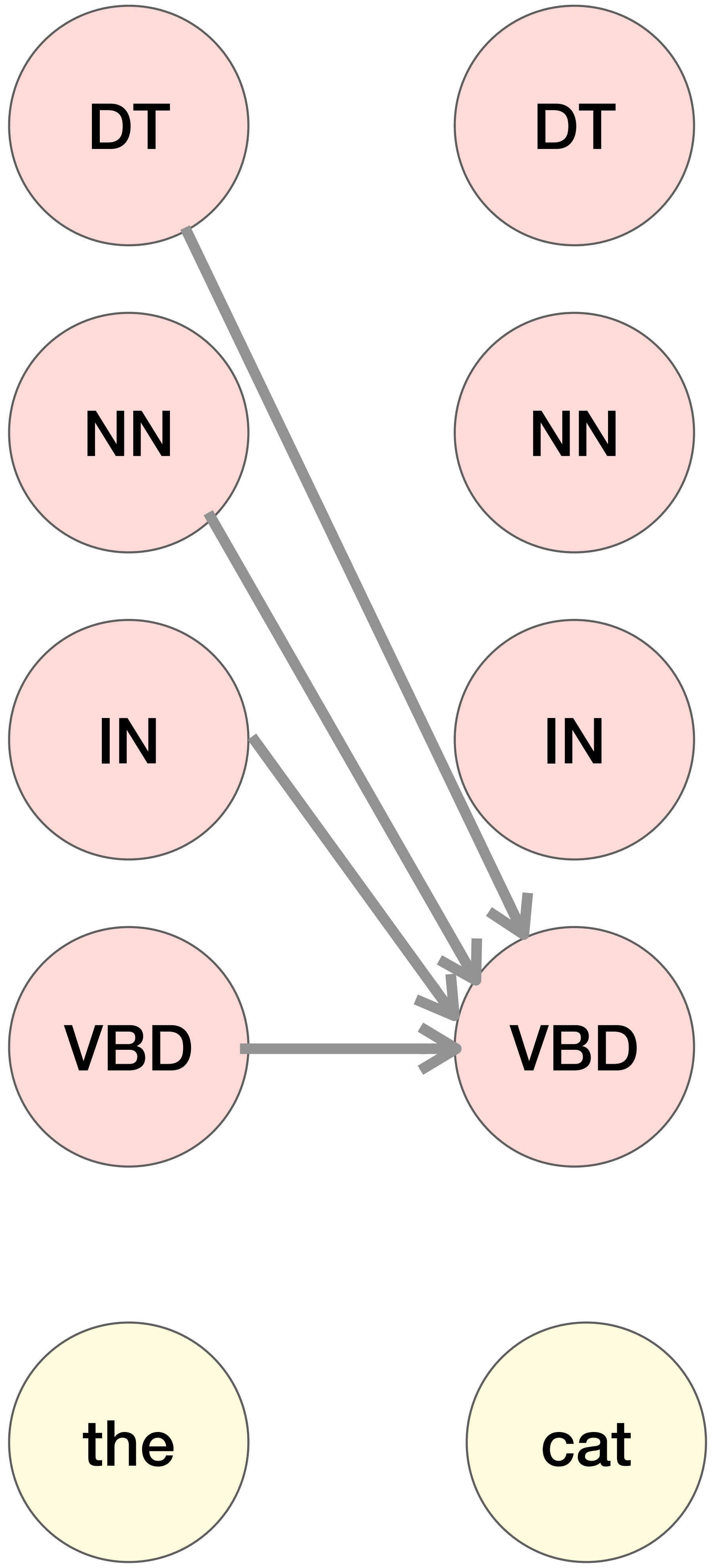
Viterbi decoding

s_{t+1}

s_t	DT	NN	IN	VBD
DT	0.05	0.8	0.05	0.1
NN	0.05	0.2	0.15	0.6
IN	0.5	0.2	0.05	0.25
VBD	0.3	0.3	0.3	0.1

o_t

s_t	the	cat	sat	on	mat
DT	0.5	0	0	0	0
NN	0.01	0.2	0.01	0.01	0.2
IN	0	0	0	0.4	0
VBD	0	0.01	0.1	0.01	0.01



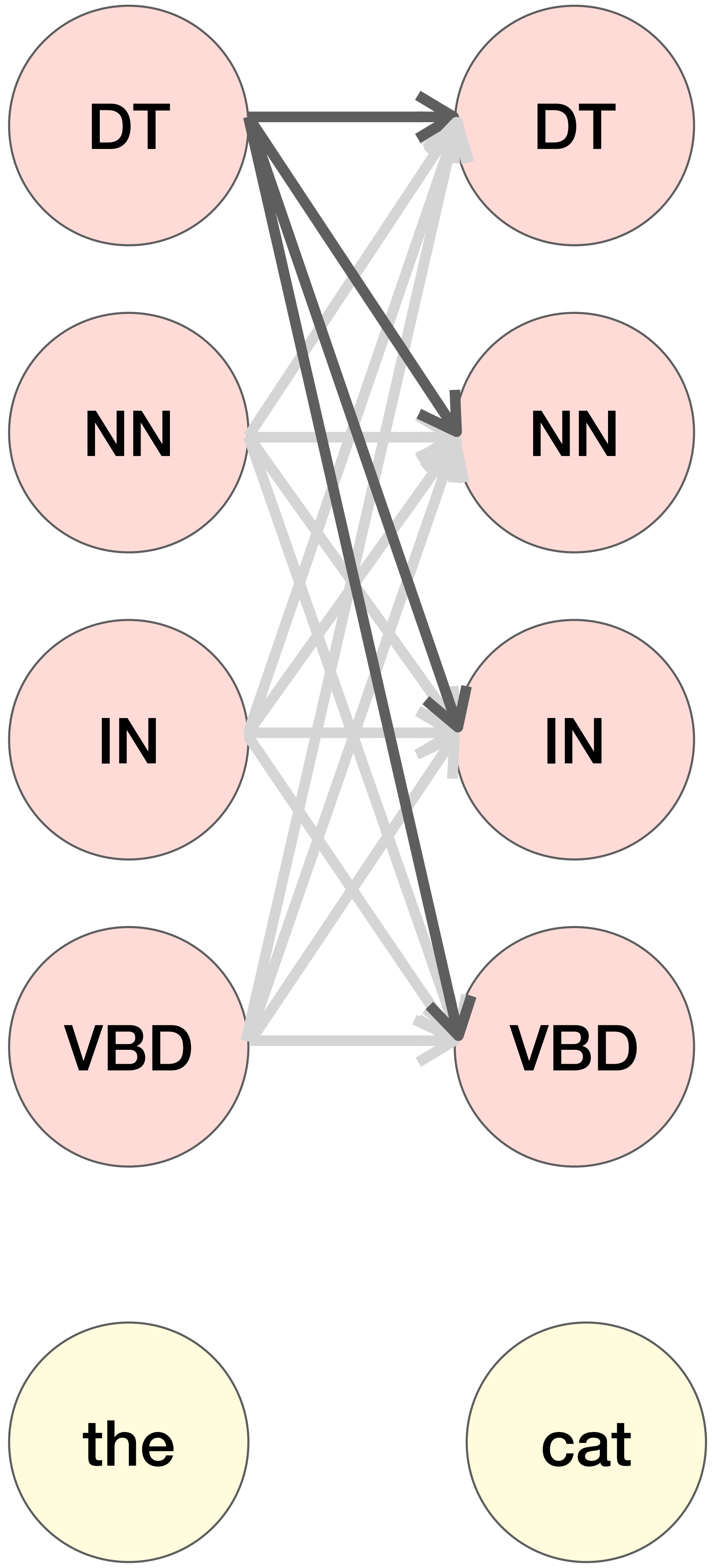
	the	cat	sat	on	the	mat
DT	0.4	0				
NN	0.001	0.064				
IN	0	0				
VBD	0	0.0004				

$M[1,K]$

$$M[2,VBD] = \max_k M[1,k]\ P(VBD\,|\,k)\ P(\text{cat}\,|\,VBD) = 0.0004$$

Forward

Viterbi decoding



$$M[2,DT] = \max_k M[1,k]\ P(DT\,|\,k)\ P(\text{cat}\,|\,DT) = 0$$

$$M[2,NN] = \max_k M[1,k]\ P(NN\,|\,k)\ P(\text{cat}\,|\,NN) = 0.064$$

$$M[2,IN] = \max_k M[1,k]\ P(IN\,|\,k)\ P(\text{cat}\,|\,IN) = 0$$

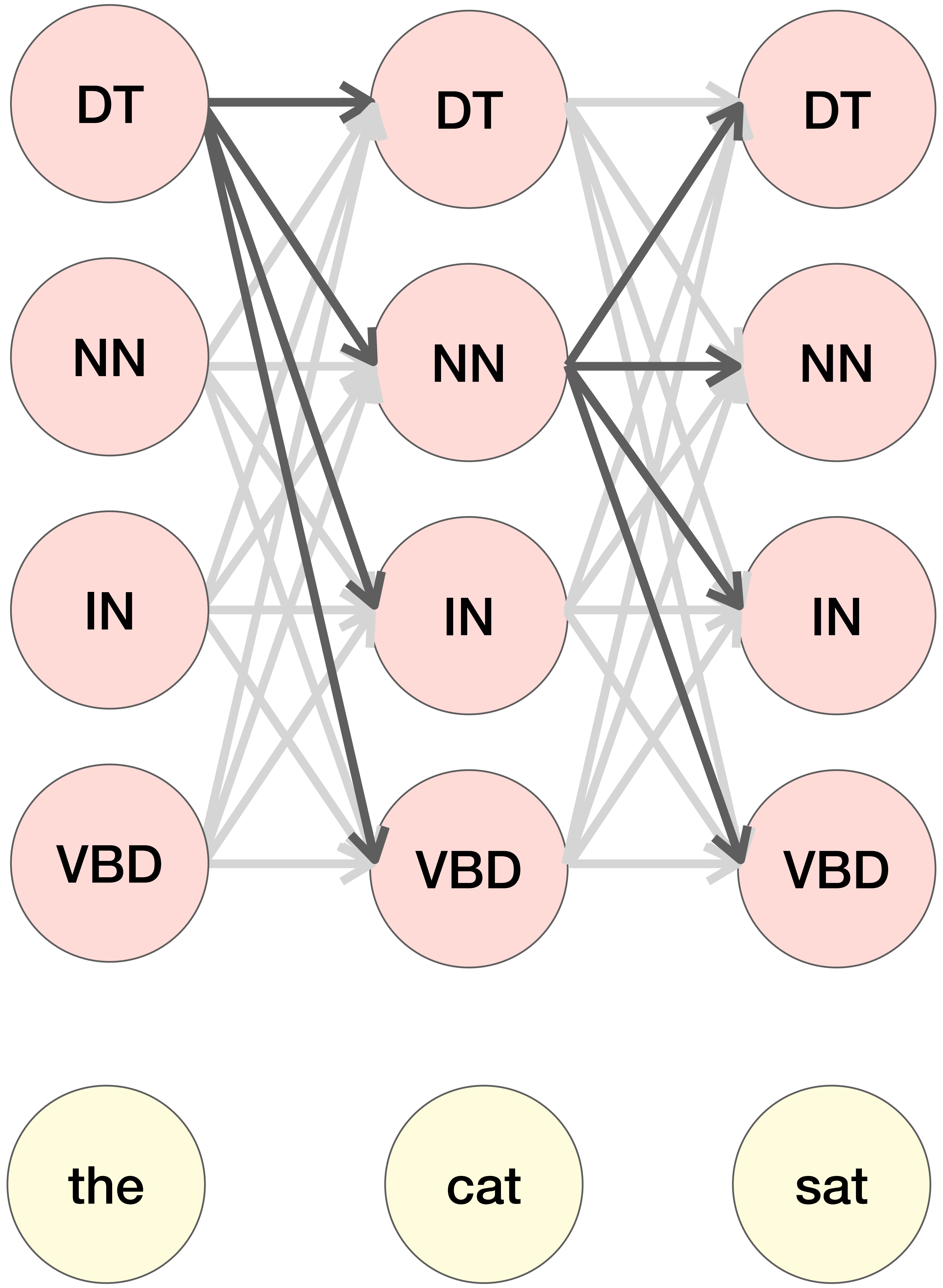
$$M[2,VBD] = \max_k M[1,k]\ P(VBD\,|\,k)\ P(\text{cat}\,|\,VBD) = 0.004$$

Forward

$M[T, K]$

	the	cat	sat	on	the	mat
DT	0.4	0				
NN	0.001	0.064				
IN	0	0				
VBD	0	0.004				

Viterbi decoding



$$M[3,DT] = \max_k M[2,k]\ P(DT \mid k)\ P(\text{sat} \mid DT) = 0$$

$$M[3,NN] = \max_k M[2,k]\ P(NN \mid k)\ P(\text{sat} \mid NN) = 0.000128$$

$$M[3,IN] = \max_k M[2,k]\ P(IN \mid k)\ P(\text{sat} \mid IN) = 0$$

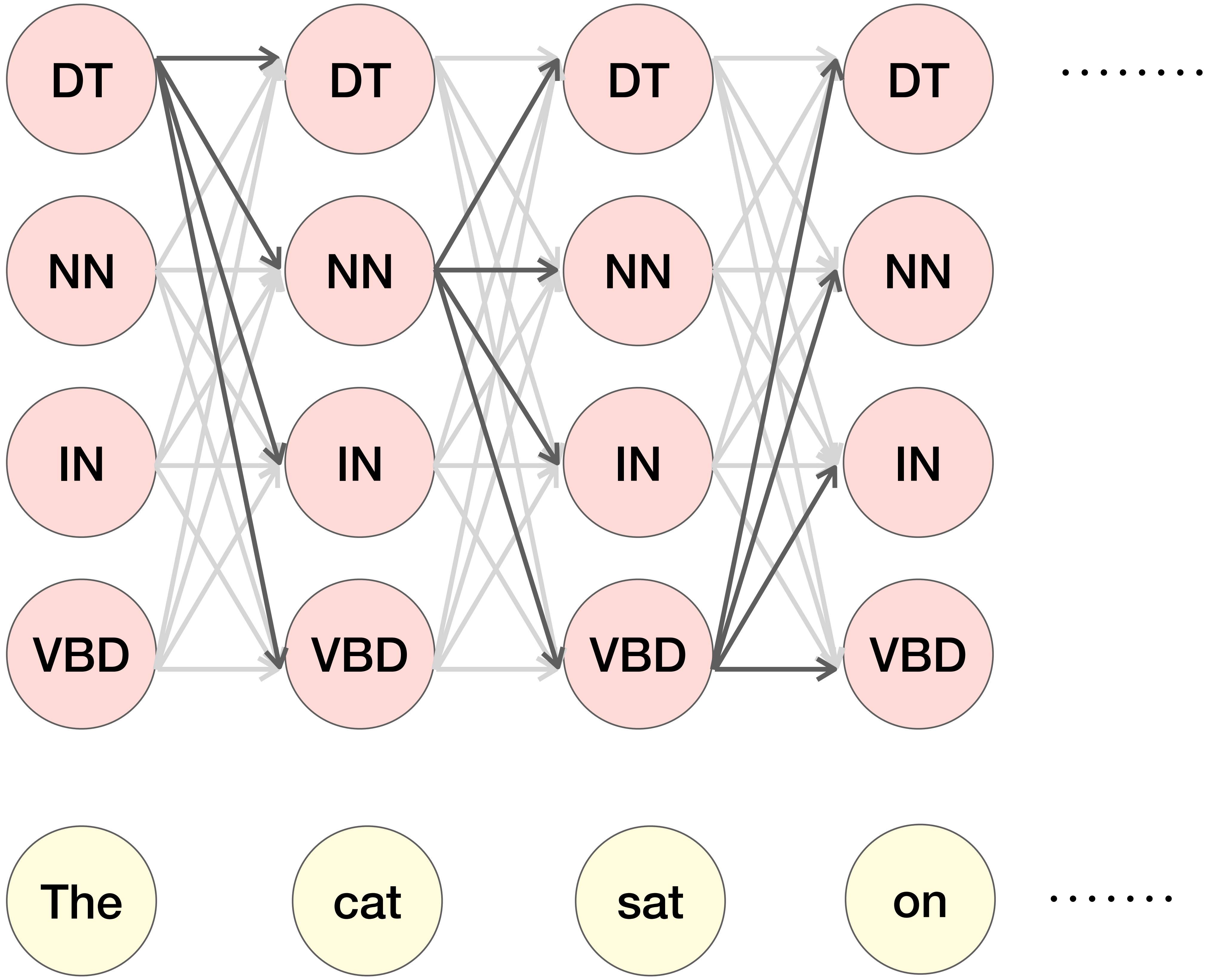
$$M[3,VBD] = \max_k M[2,k]\ P(VBD \mid k)\ P(\text{sat} \mid VBD) = 0.00384$$

Forward

$M[T, K]$

	the	cat	sat	on	the	mat
DT	0.4	0	0			
NN	0.001	0.064	0.000128			
IN	0	0	0			
VBD	0	0.008	0.00384			

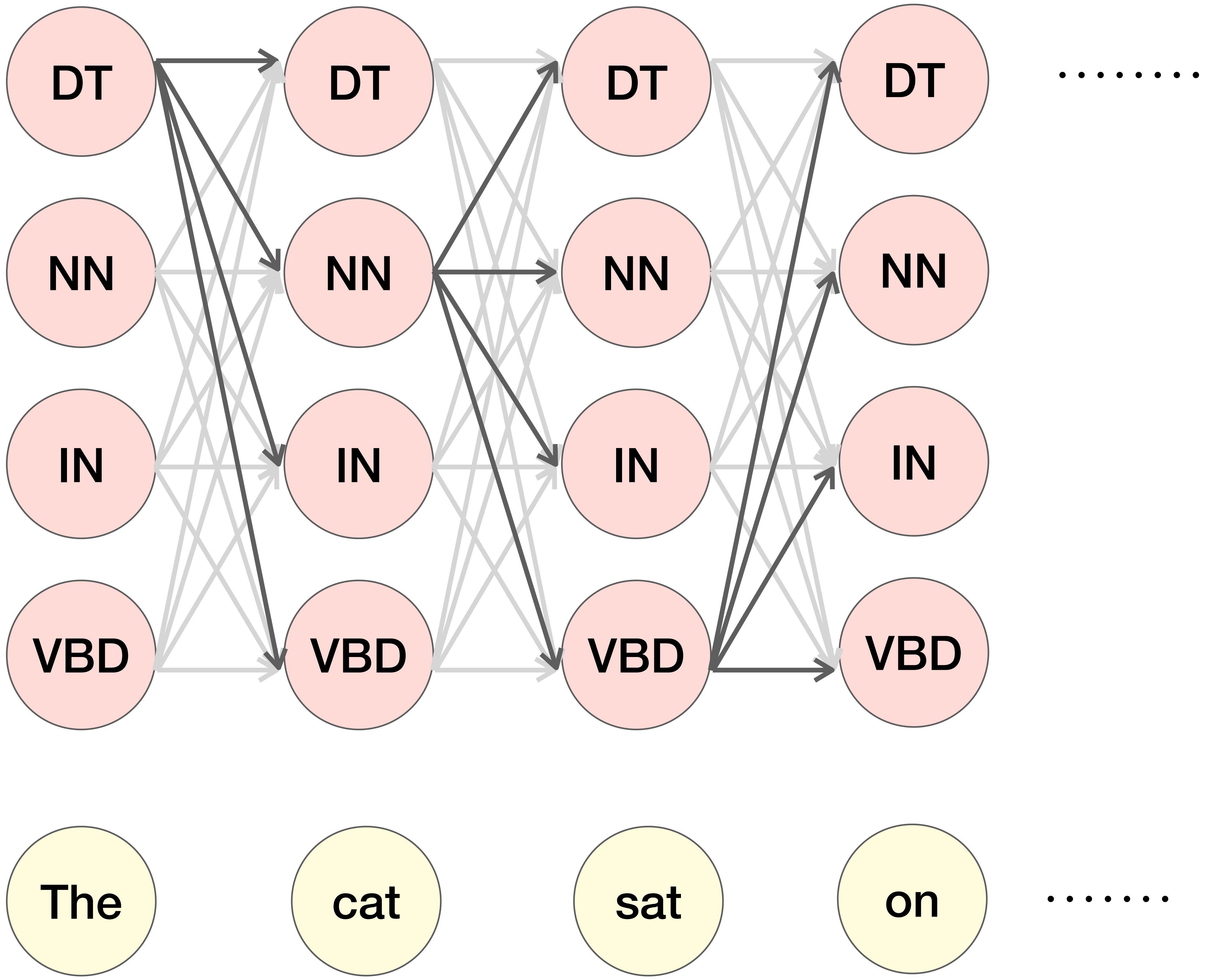
Viterbi decoding



$$M[i,j] = \max_k M[i-1,k]\ P(s_j \mid s_k)\ P(o_i \mid s_j) \quad 1 \le k \le K \quad 1 \le i \le n$$

Backward: Pick $\max_k M[n,k]$ and backtrack

Viterbi decoding



Time complexity?

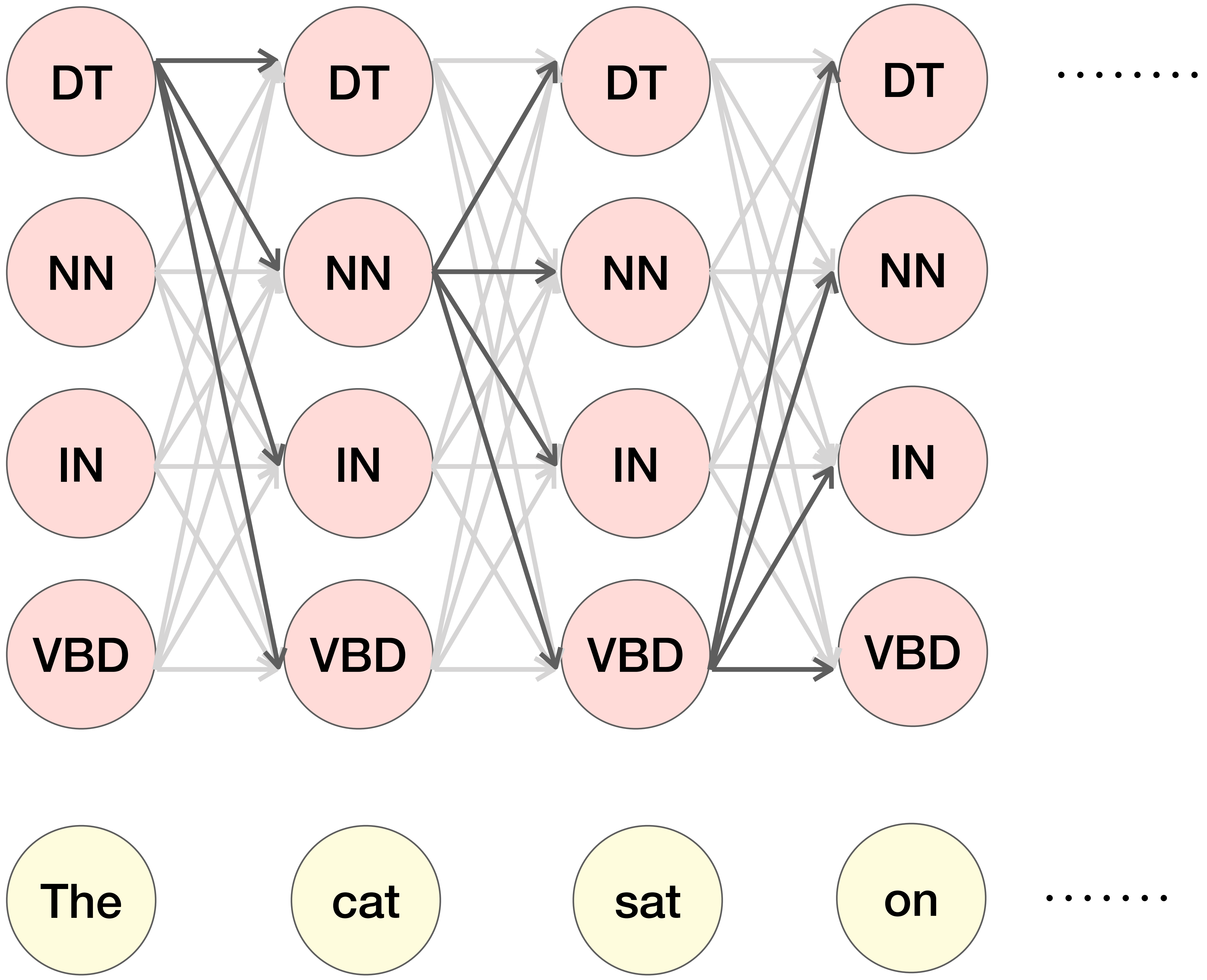
O(K x K x n)

$$M[i,j] = \max_k M[i-1,k]\ P(s_j \mid s_k)\ P(o_i \mid s_j) \quad 1 \leq k \leq K \quad 1 \leq i \leq n$$

Backward: Pick $\max_k M[n,k]$ and backtrack

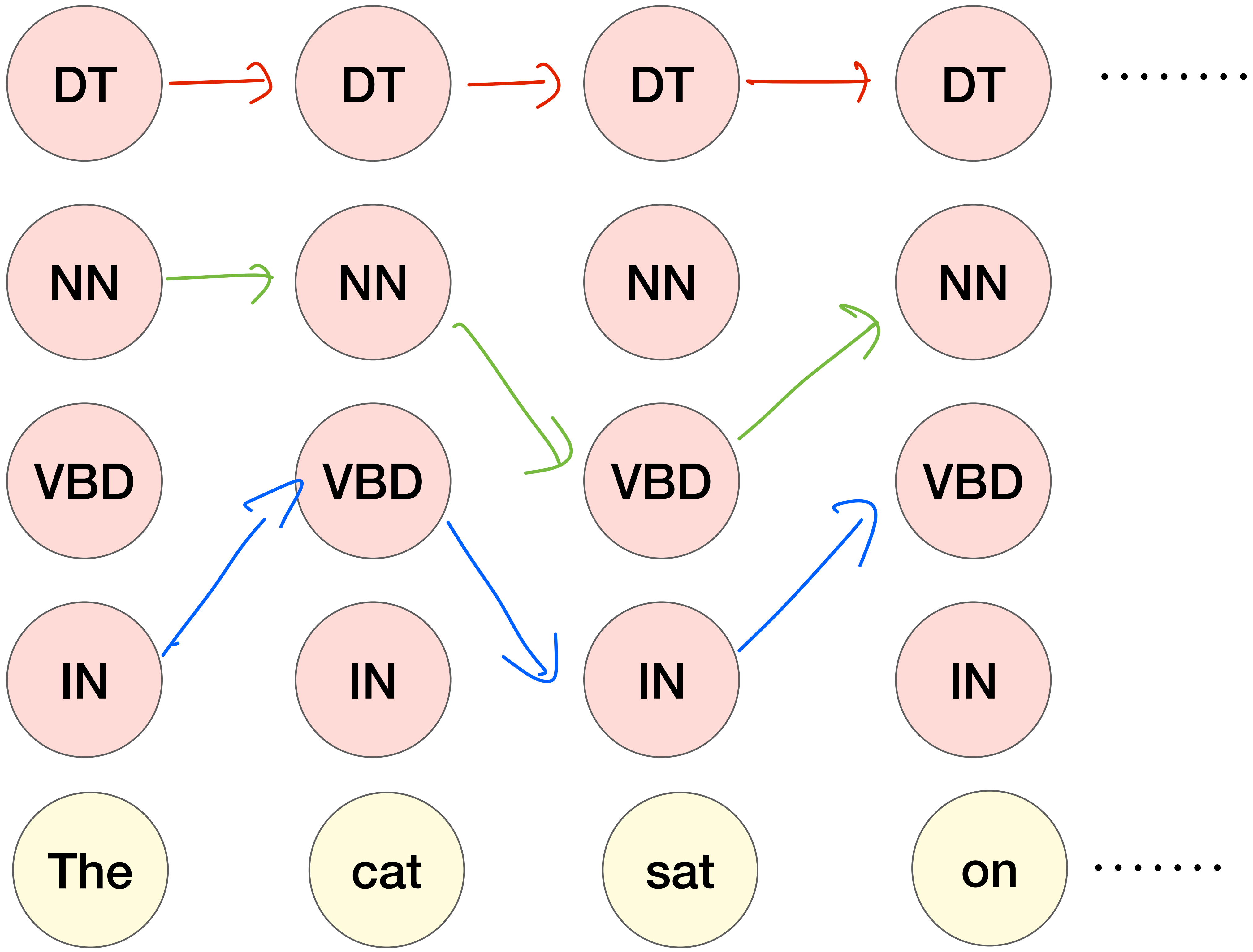
Beam Search

- If K (number of states) is too large, Viterbi is too expensive!



Beam Search

- If K (number of states) is too large, Viterbi is too expensive!



Many paths have very low likelihood!

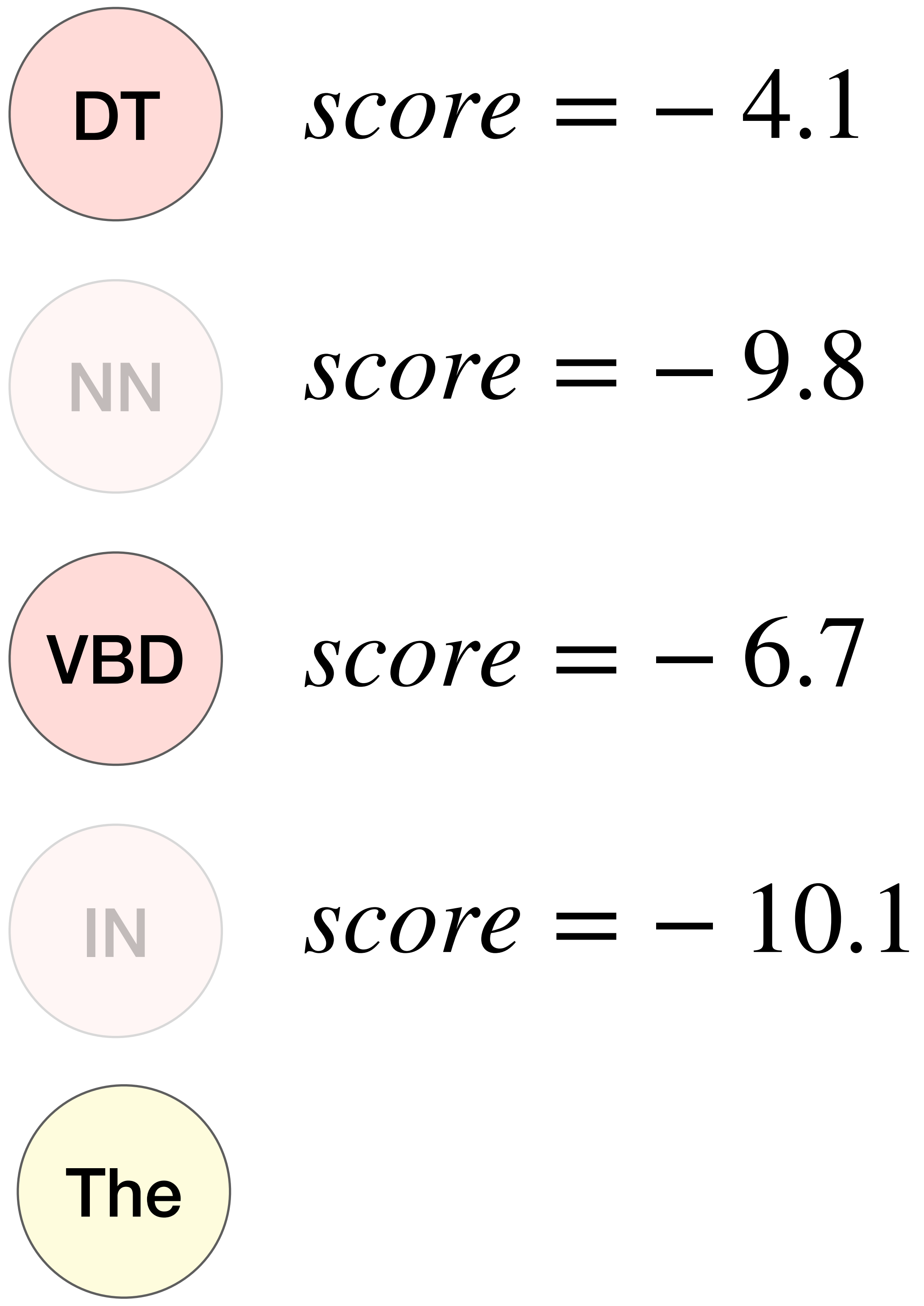
Beam Search

- If K (number of states) is too large, Viterbi is too expensive!
- Keep a fixed number of hypotheses at each point
  - Beam width, β

Beam Search

- Keep a fixed number of hypotheses at each point

$\beta = 2$



$score = -4.1$

$score = -9.8$

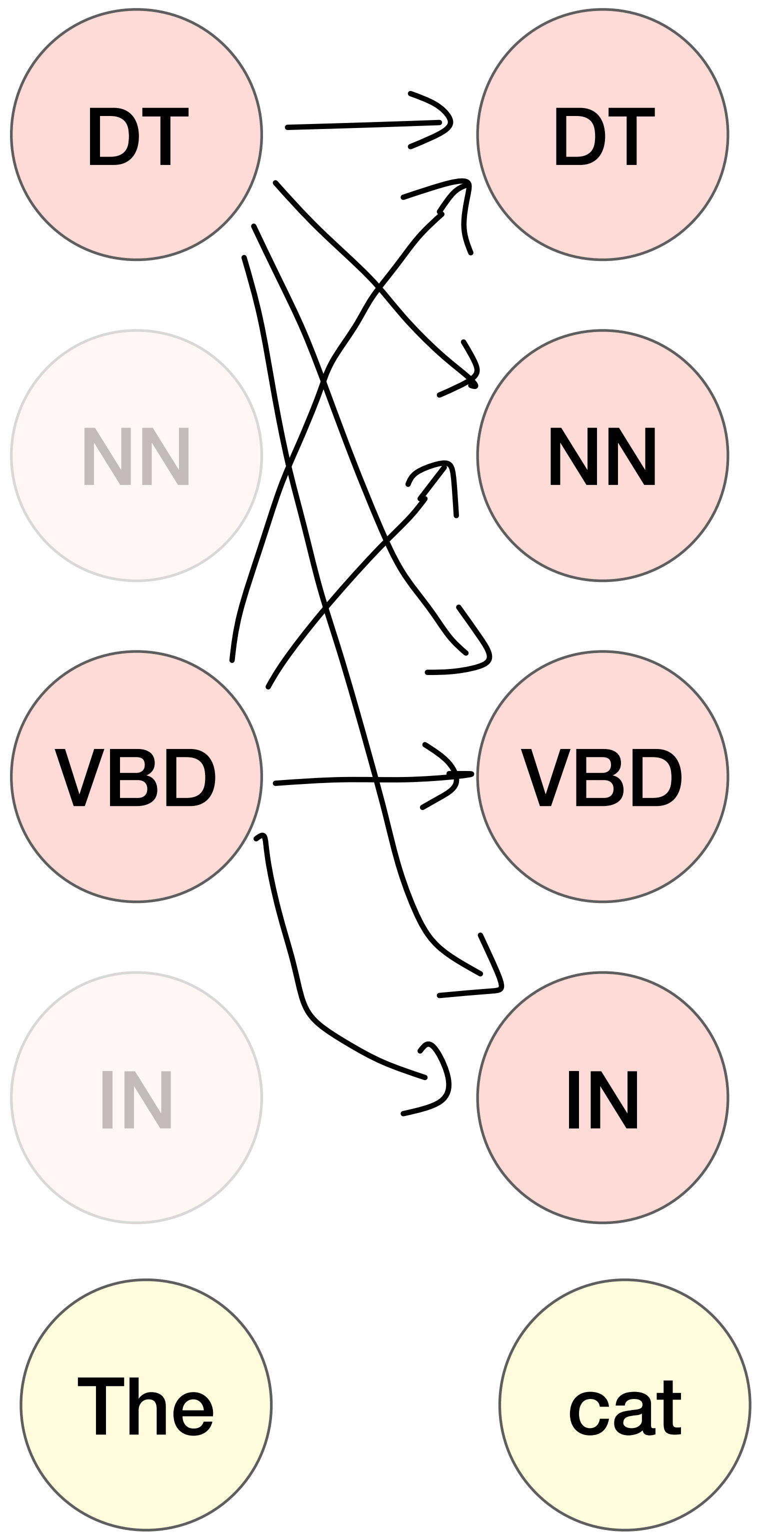
$score = -6.7$

$score = -10.1$

Beam Search

- Keep a fixed number of hypotheses at each point

$\beta = 2$



$score = -16.5$

$score = -6.5$

$score = -13.0$

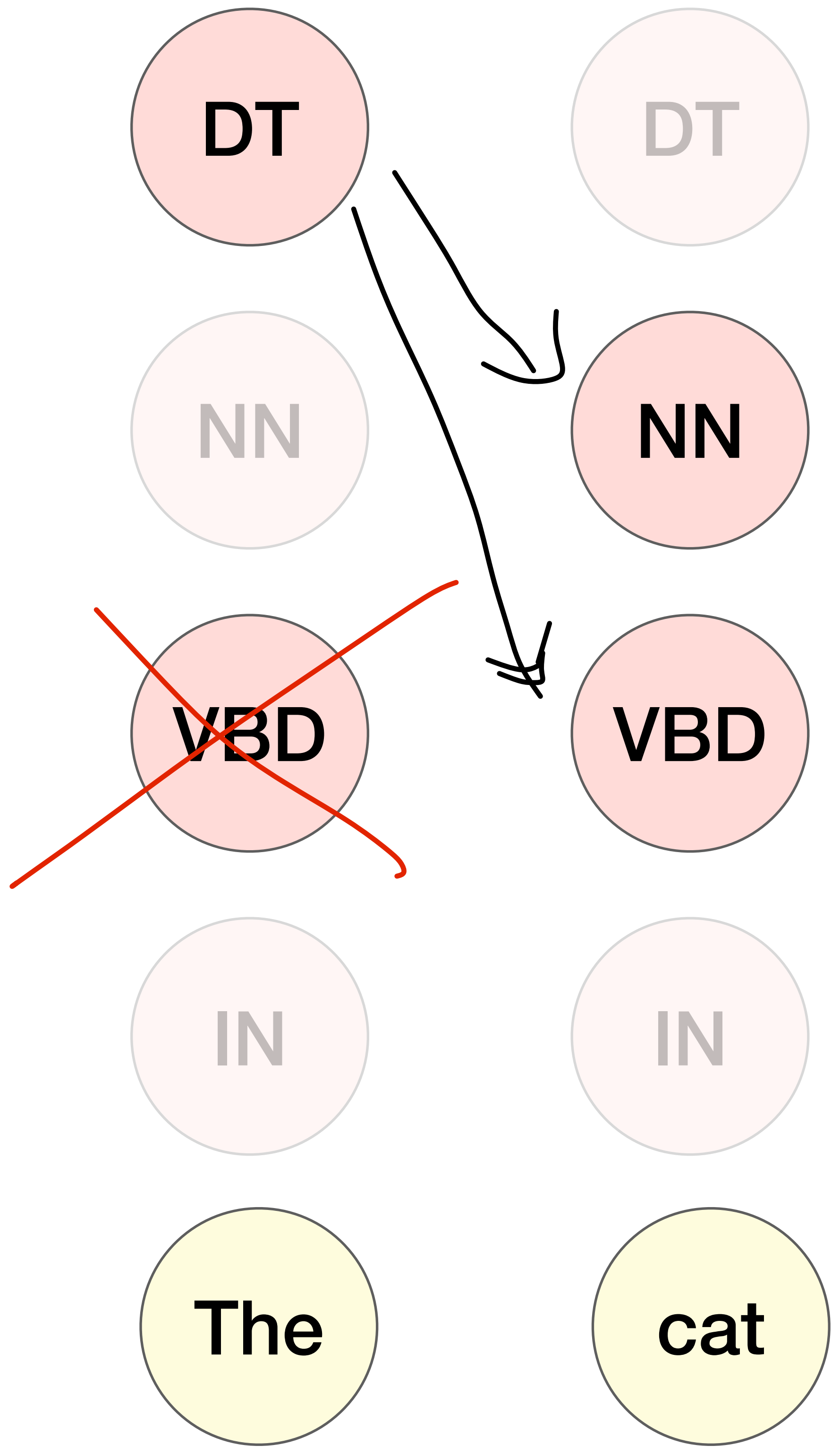
$score = -22.1$

Step 1: Expand all partial sequences in current beam

Beam Search

- Keep a fixed number of hypotheses at each point

$\beta = 2$



$score = -16.5$

$score = -6.5$

$score = -13.0$

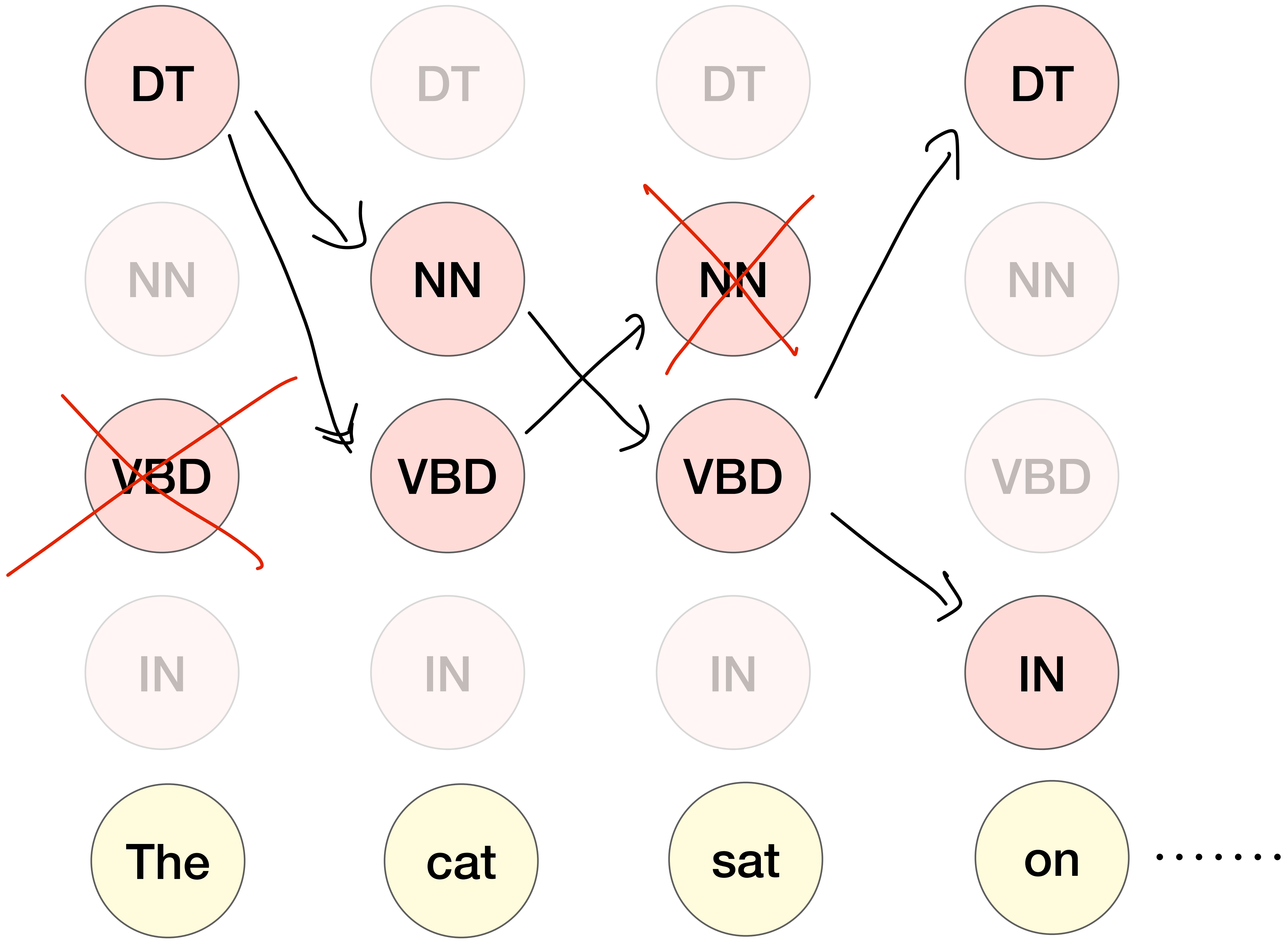
$score = -22.1$

Step 2: Prune set back to top β sequences

Beam Search

- Keep a fixed number of hypotheses at each point

$\beta = 2$



Pick $\max_k M[n, k]$ from within beam and backtrack

Beam Search

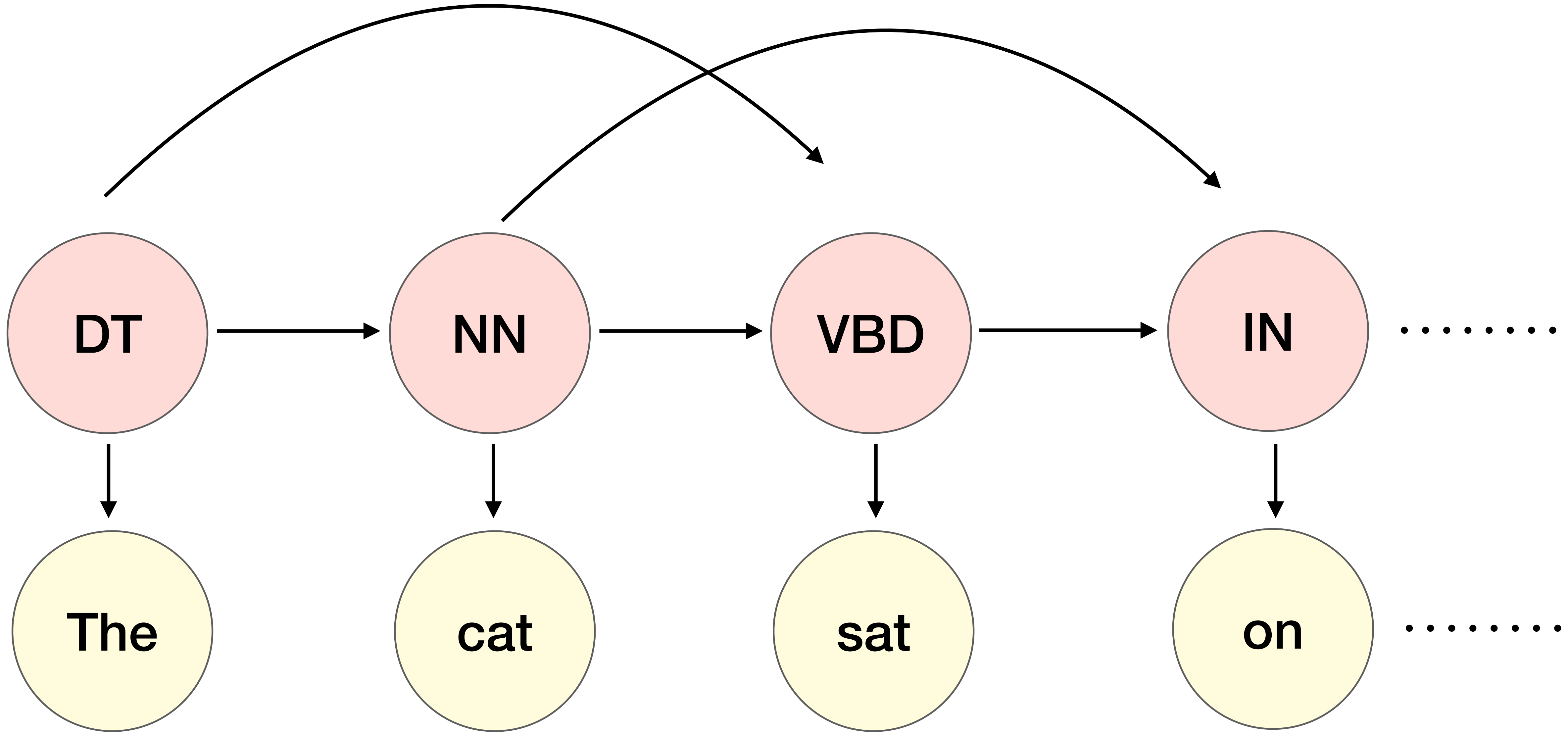
- If K (number of states) is too large, Viterbi is too expensive!
- Keep a fixed number of hypotheses at each point
  - Beam width, β
- Trade-off computation for (some) accuracy

Time complexity?

$$\beta \times K \times T$$

Beyond bigrams

- Real-world HMM taggers have more relaxed assumptions
- Trigram HMM: $P(s_{t+1} \mid s_1, s_2, \ldots, s_t) \approx P(s_{t+1} \mid s_{t-1}, s_t)$



Pros?

Cons?

Other sequence tagging problems

BIO encoding

- Named Entity Recognition

B-PER	I-PER	O	O	O	O	B-ORG	I-ORG
Michael	Jordan	is	a	professor	at	UC	Berkeley.

- Shallow Phrase Chunking

B-NP	I-NP	B-VP	B-PP	B-NP	I-NP	B-PP	B-NP	I-NP
The	cat	sat	on	the	mat	under	the	sun

HMMs for language modeling

- Language modeling: estimate probability of sentence

P(the, cat, sat, on, the, mat) = ??

- Need to sum over the probabilities of the possible states

$$P(O) = P(o_1, o_2, \ldots, o_n) = \sum_{s_1, \ldots, s_n} \prod_{t=1}^{n} P(o_t|s_t) P(s_t|s_{t-1})$$

- Use Viterbi-like algorithm, but take sum instead of max!

Maximum Entropy Markov Models

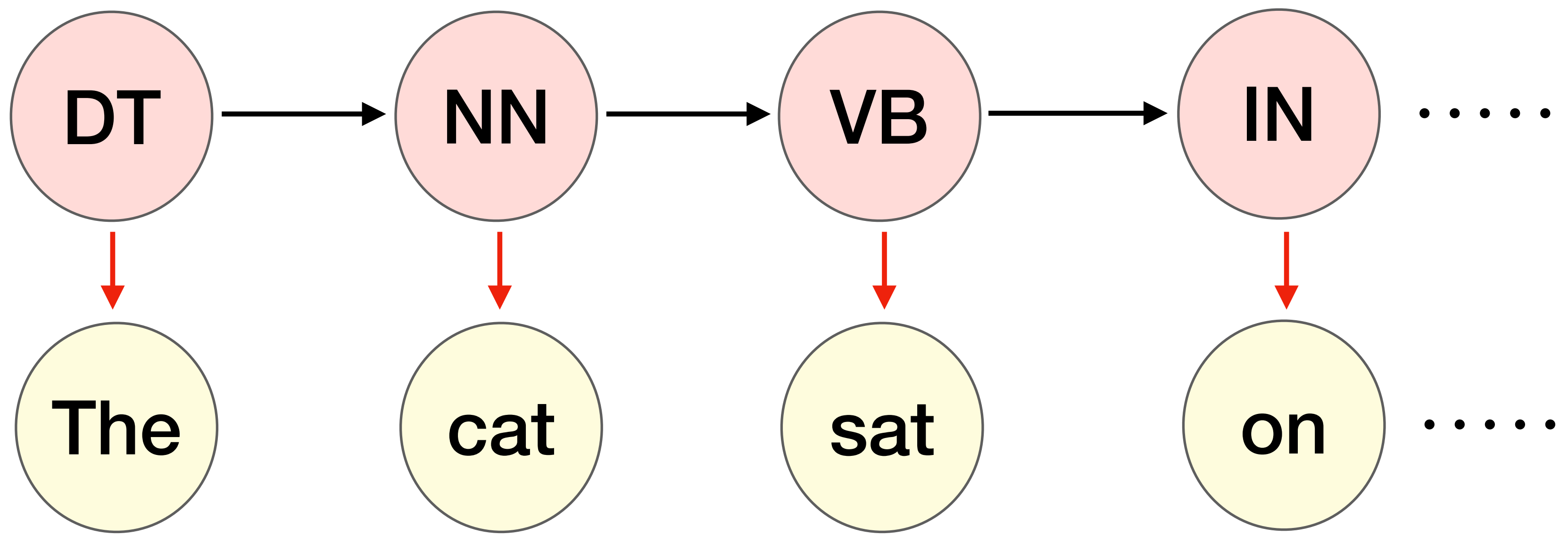
(extra content - not covered)

Generative vs Discriminative

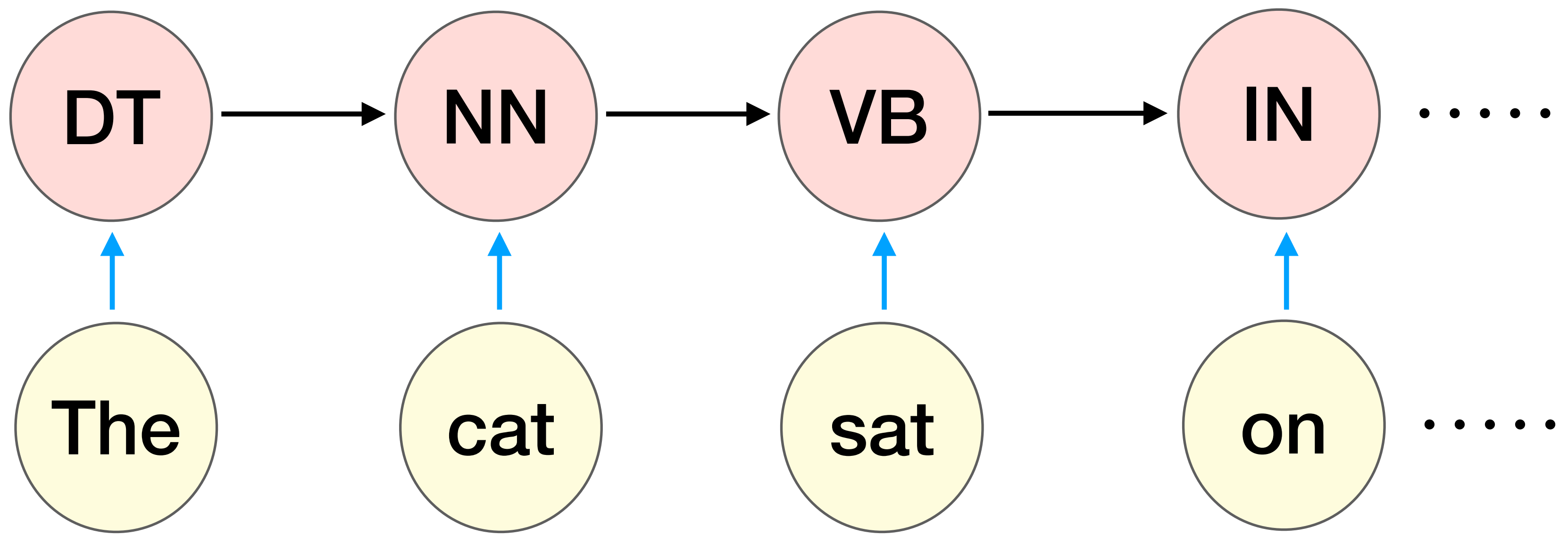
- HMM is a *generative* model
- Can we model $P(s_1, \ldots, s_n \mid o_1, \ldots, o_n)$ directly?

Generative	Discriminative
Naive Bayes: $P(c)P(d \mid c)$	Logistic Regression: $P(c \mid d)$
HMM: $P(s_1, \ldots, s_n)P(o_1, \ldots, o_n \mid s_1, \ldots, s_n)$	MEMM: $P(s_1, \ldots, s_n \mid o_1, \ldots, o_n)$

MEMM



HMM



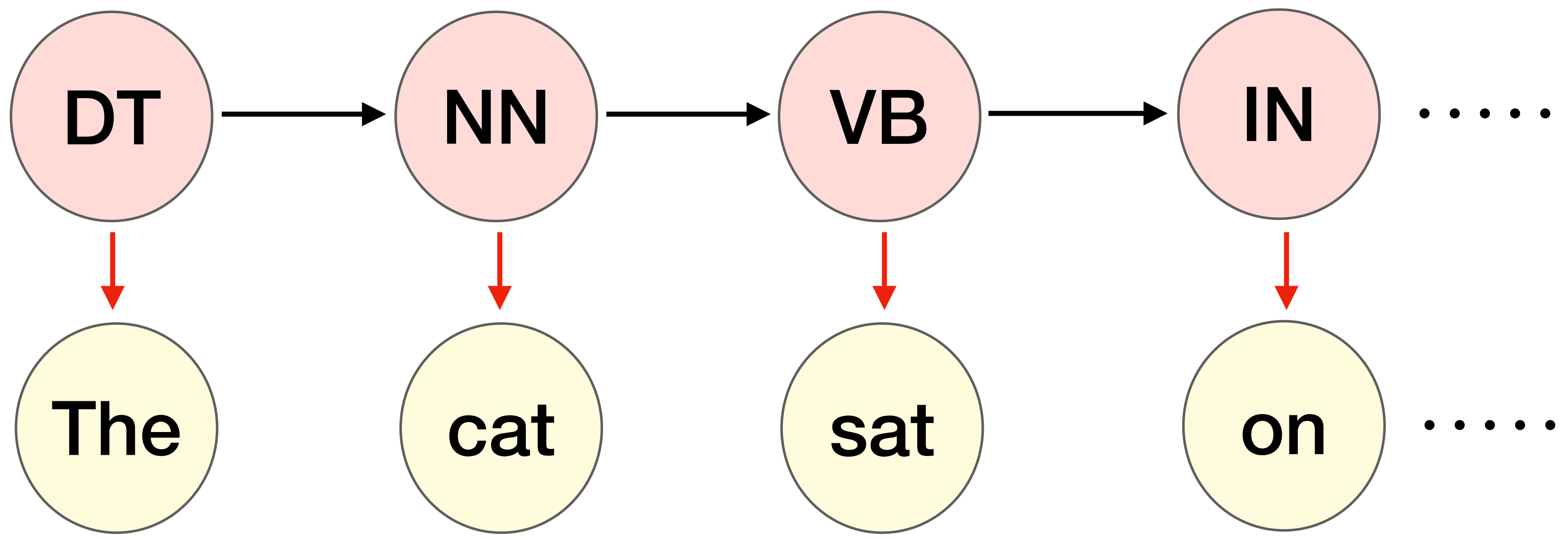
MEMM

- Compute the posterior directly:

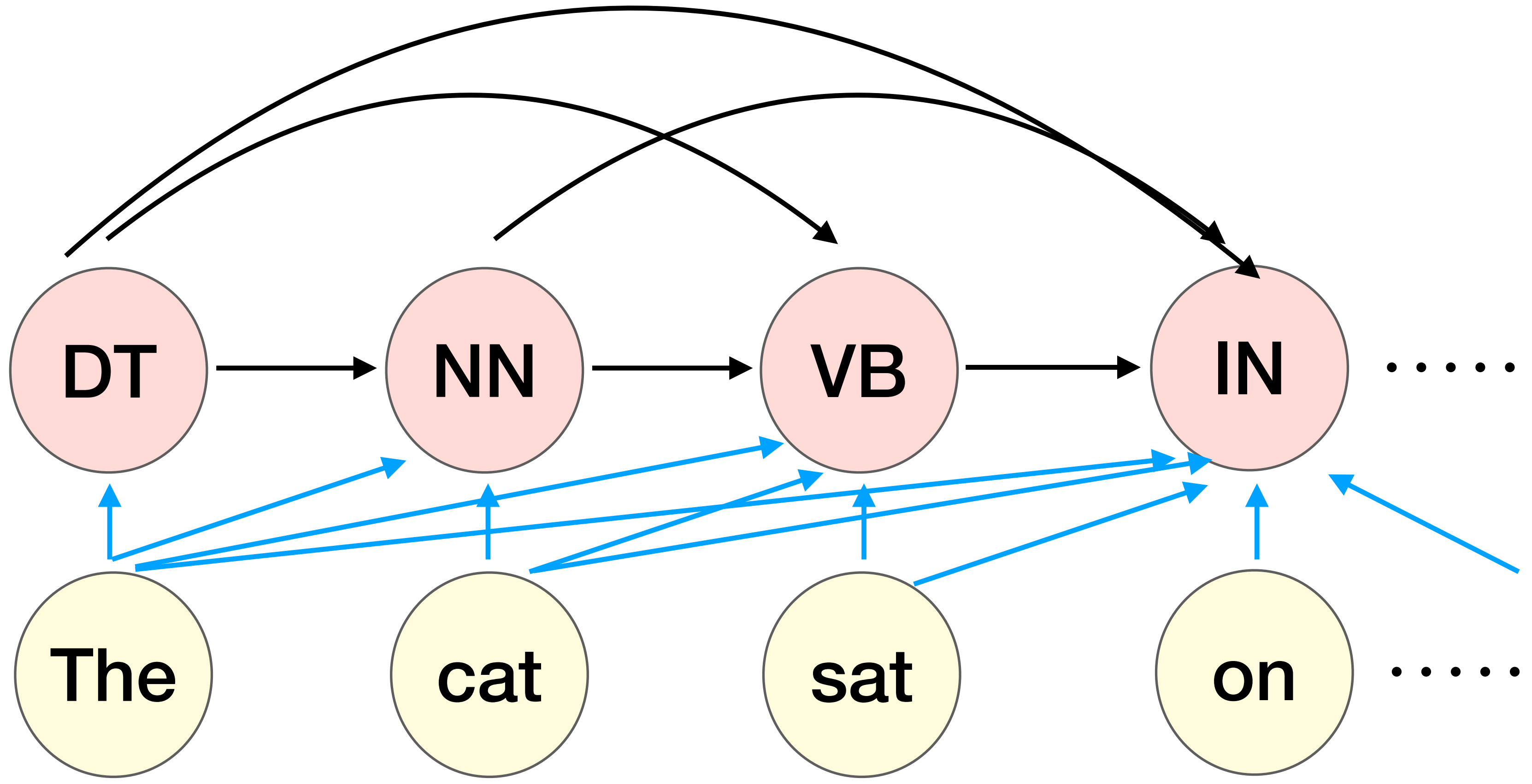
$$\hat{S} = \arg\max_S P(S\,|\,O) = \arg\max_S \prod_i P(s_i\,|\,o_i, s_{i-1})$$

- Use features: $P(s_i\,|\,o_i, s_{i-1}) \propto \exp(\underset{\text{weights}}{w} \cdot \overset{\text{Features}}{f}(s_i, o_i, s_{i-1}))$

MEMM



HMM



MEMM

- In general, we can use all observations and all previous states:

$$\hat{S} = \arg\max_{S} P(S \mid O) = \arg\max_{S} \prod_{i} P(s_i \mid o_n, o_{i-1}, \ldots, o_1, s_{i-1}, \ldots, s_1)$$

$$P(s_i \mid s_{i-1}, \ldots, s_1, O) \propto \exp(w \cdot f(s_i, s_{i-1}, \ldots, s_1, O)$$

Features in an MEMM

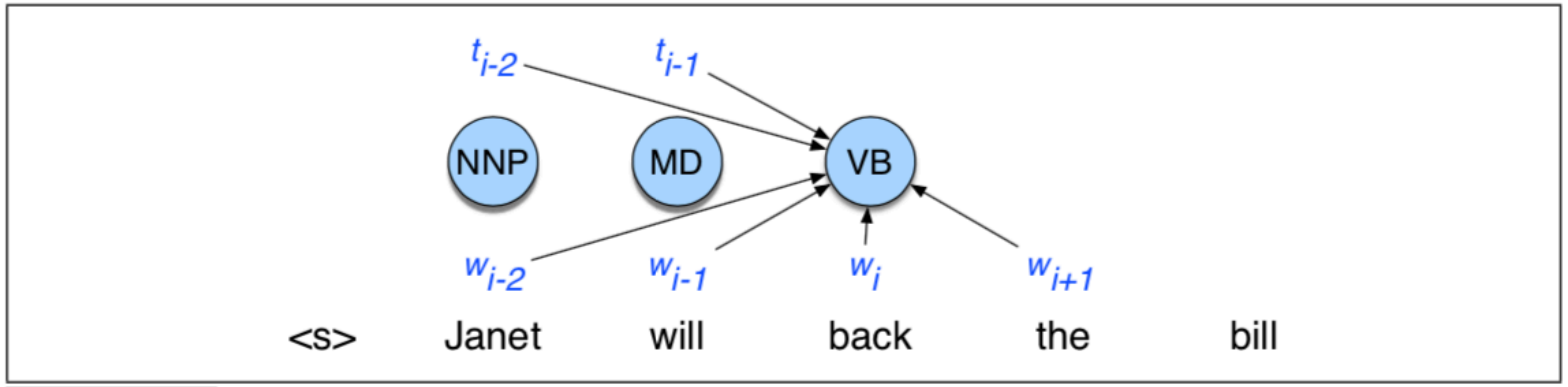


Figure 8.13 An MEMM for part-of-speech tagging showing the ability to condition on more features.

$$\begin{aligned} \langle t_i, w_{i-2}\rangle, \langle t_i, w_{i-1}\rangle, \langle t_i, w_i\rangle, \langle t_i, w_{i+1}\rangle, \langle t_i, w_{i+2}\rangle \\ \langle t_i, t_{i-1}\rangle, \langle t_i, t_{i-2}, t_{i-1}\rangle, \\ \langle t_i, t_{i-1}, w_i\rangle, \langle t_i, w_{i-1}, w_i\rangle \langle t_i, w_i, w_{i+1}\rangle, \end{aligned}$$

Feature templates

t_i = VB and w_{i-2} = Janet
t_i = VB and w_{i-1} = will
t_i = VB and w_i = back
t_i = VB and w_{i+1} = the
t_i = VB and w_{i+2} = bill
t_i = VB and t_{i-1} = MD
t_i = VB and t_{i-1} = MD and t_{i-2} = NNP
t_i = VB and w_i = back and w_{i+1} = the

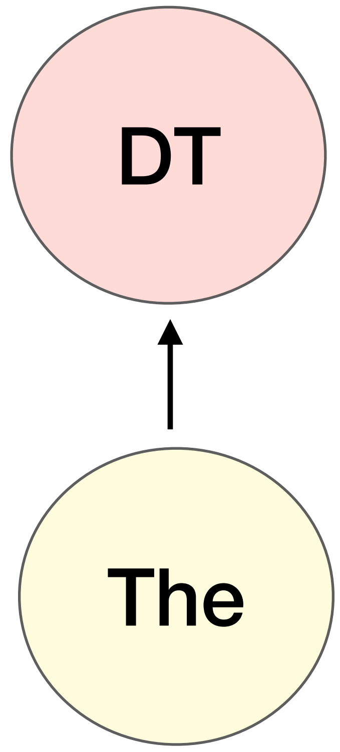
Features

MEMMs: Decoding

$$\hat{S} = \arg\max_S P(S \mid O) = \arg\max_S \Pi_i P(s_i \mid o_i, s_{i-1})$$

(assume features only on previous time step and current obs)

- Greedy decoding:



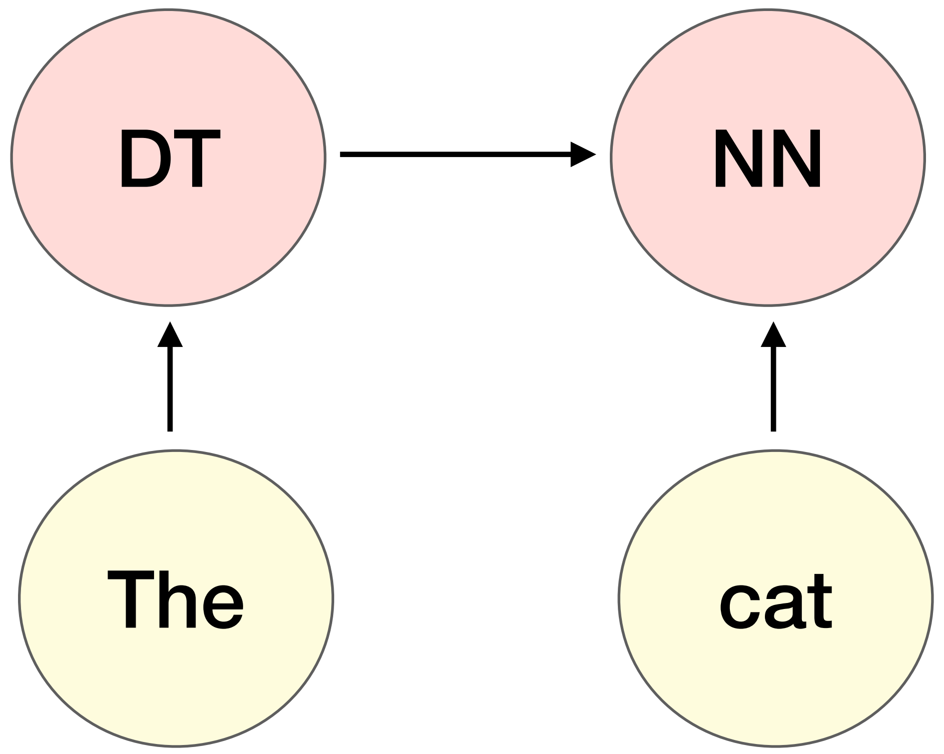
$$\hat{s}_1 = \arg\max_s P(s \mid \text{The})$$

$$= DT$$

MEMMs: Decoding

$$\hat{S} = \arg\max_{S} P(S \,|\, O) = \arg\max_{S} \Pi_i P(s_i \,|\, o_i, s_{i-1})$$

- Greedy decoding:



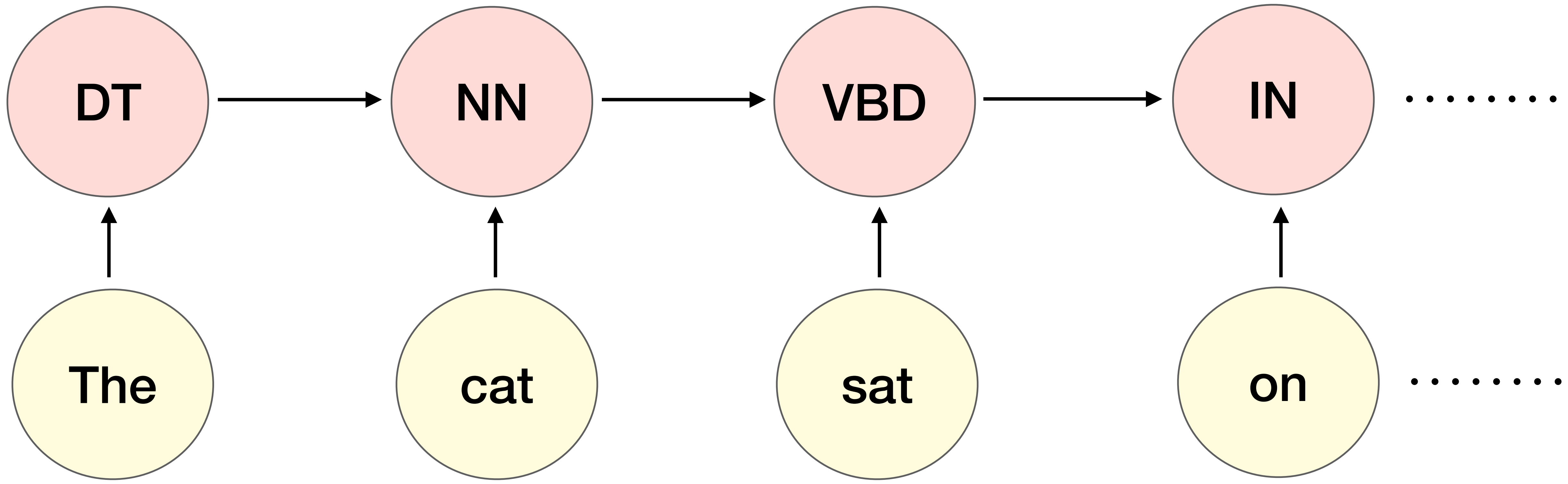
$$\hat{s}_2 = \underset{s}{\mathrm{argmax}}\; P(s \,|\, \mathrm{cat}, \mathrm{DT})$$

$$= \mathrm{NN}$$

MEMMs: Decoding

$$\hat{S} = \arg\max_S P(S \mid O) = \arg\max_S \Pi_i P(s_i \mid o_i, s_{i-1})$$

- Greedy decoding:



$$\forall t, \quad \hat{s}_{t+1} = \arg\max_s P(s \mid o_{t+1}, \hat{s}_t)$$

MEMMs: Decoding

$$\hat{S} = \arg\max_{S} P(S \,|\, O) = \arg\max_{S} \Pi_i P(s_i \,|\, o_i, s_{i-1})$$

- Greedy decoding

- Viterbi decoding:

$$M[i,j] = \max_{k} M[i-1,k]\ P(s_j \,|\, o_i, s_k) \quad 1 \leq k \leq K \quad 1 \leq i \leq n$$

DP Lattice

states

timesteps

MEMM: Learning

- Gradient descent: similar to logistic regression!

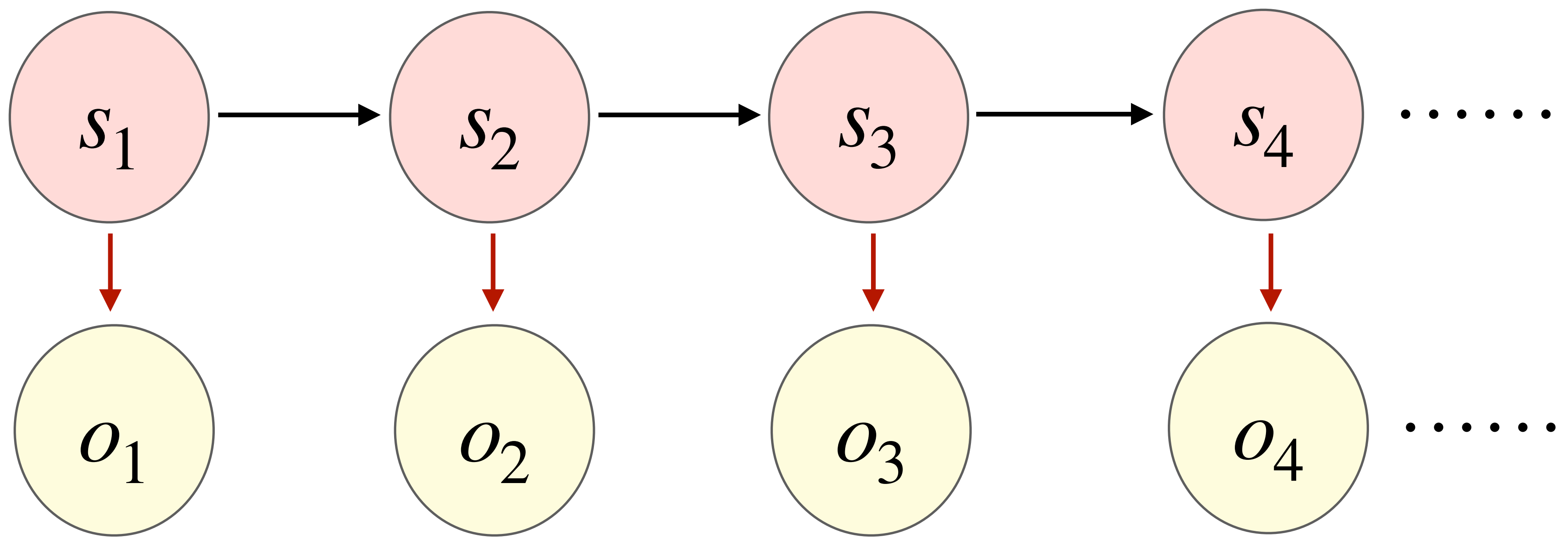
$$P(s_i \,|\, s_1, \ldots, s_{i-1}, O) \propto \exp(w \cdot f(s_1, \ldots, s_i, O))$$

- Given: pairs of (S, O) where each $S = \langle s_1, s_2, \ldots, s_n \rangle$

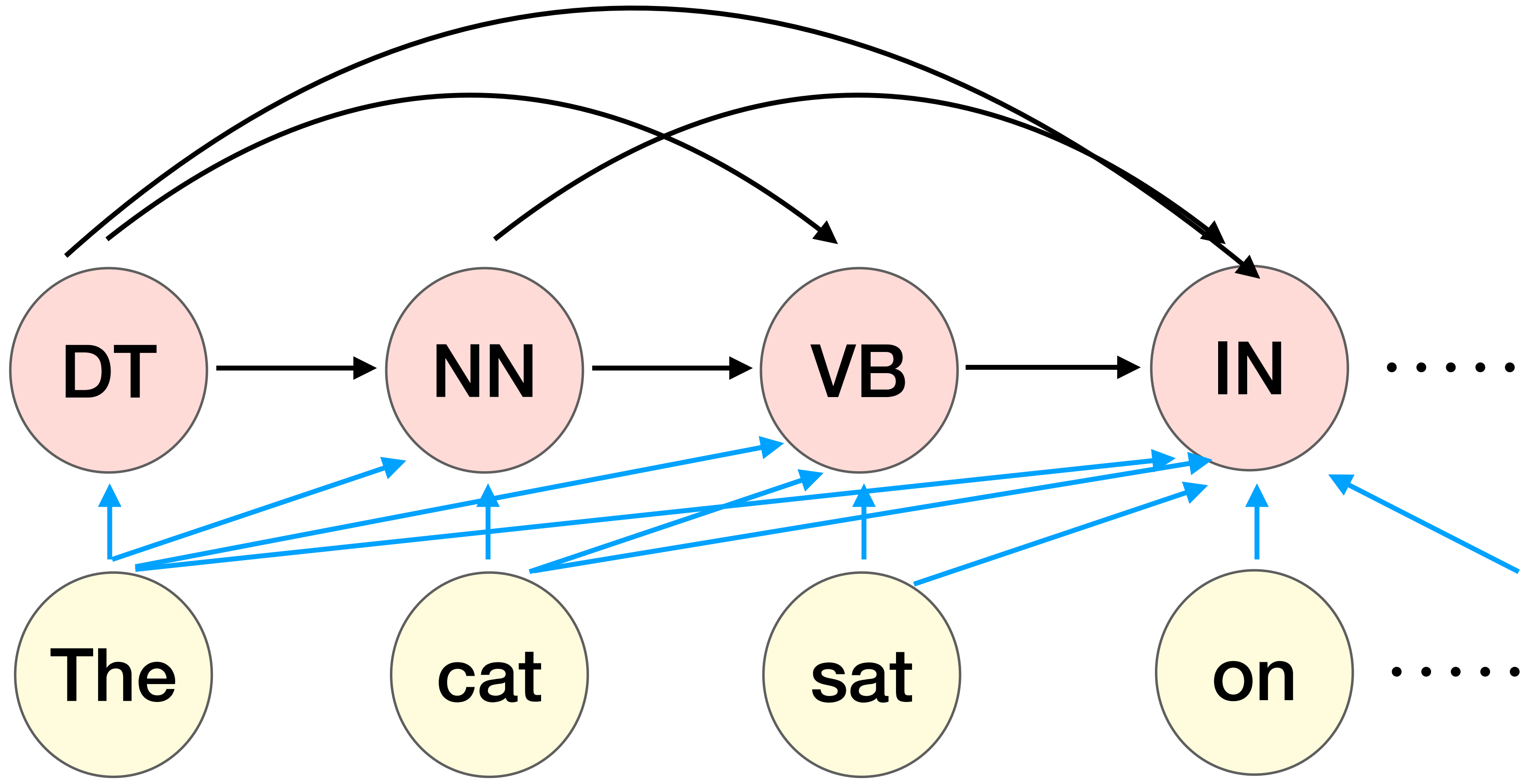
Loss for one sequence, $L = -\sum_i \log P(s_i \,|\, s_1, \ldots, s_{i-1}, O)$

- Compute gradients with respect to weights w and update

Bidirectionality



HMM

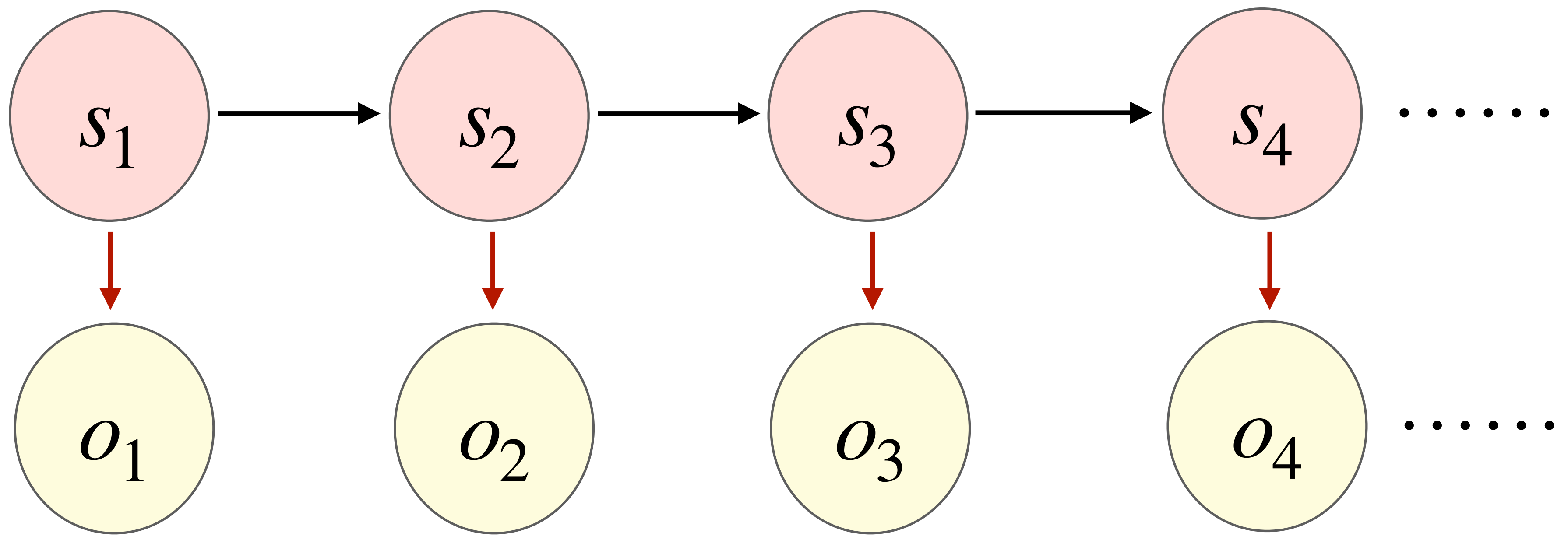


MEMM

Both HMM and MEMM assume left-to-right processing

Why can this be undesirable?

Bidirectionality



HMM

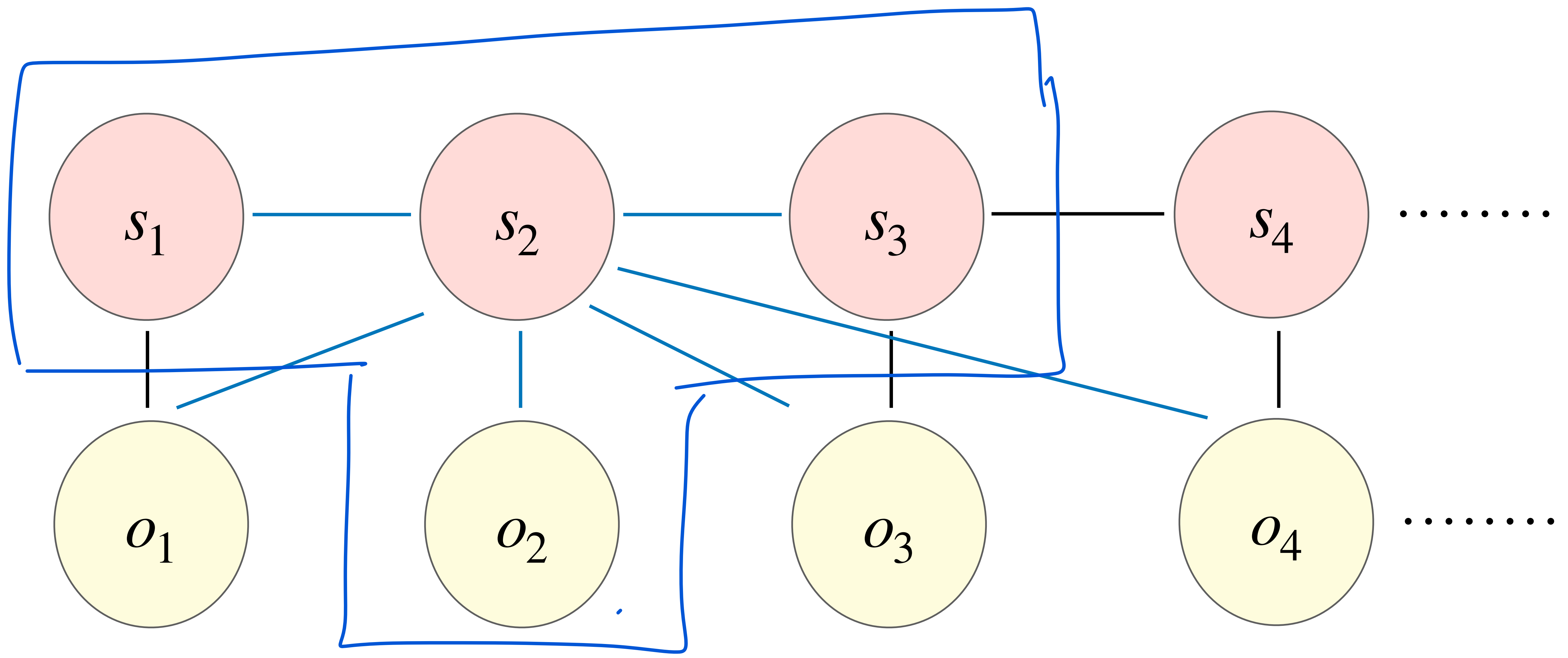
The/? old/? man/? the/? boat/?

$$P(JJ\,|\,DT)\boxed{P(\text{old}\,|\,JJ)}P(NN\,|\,JJ)\boxed{P(\text{man}\,|\,NN)}P(DT\,|\,NN)$$

$$P(NN\,|\,DT)\boxed{P(\text{old}\,|\,NN)}P(VB\,|\,NN)\boxed{P(\text{man}\,|\,VB)}P(DT\,|\,VB)$$

Observation bias

Conditional Random Field (advanced)



- Compute log-linear functions over cliques
- Lesser independence assumptions
- Ex: $P(s_t \,|\, \text{everything else}) \propto \exp(w \cdot f(s_{t-1}, s_t, s_{t+1}, O))$