SFU NatLangLab

Natural Language Processing

Angel Xuan Chang
angelxuanchang.github.io/nlp-class
adapted from lecture slides from
Anoop Sarkar, Danqi Chen and Karthik Narasimhan

Simon Fraser University

Naïve Bayes and Logistic Regression

Naïve Bayes

- Generative Model

  $$\hat{c} = \mathrm{argmax}_{\mathrm{c}}\, P(c)(d|c)$$

- Features assumed to be independent

Logistic Regression

- Discriminative Model

  $$\hat{c} = \mathrm{argmax}_{\mathrm{c}}\, P(c|d)$$

- Features don’t have to be independent

Logistic Regression Summary

- Input features: $f(x) \rightarrow [f_1, f_2, \dots, f_m]$
- Output: estimate $P(y = c|x)$ for each class c

  Need to model $P(y = c|x)$ with a family of functions

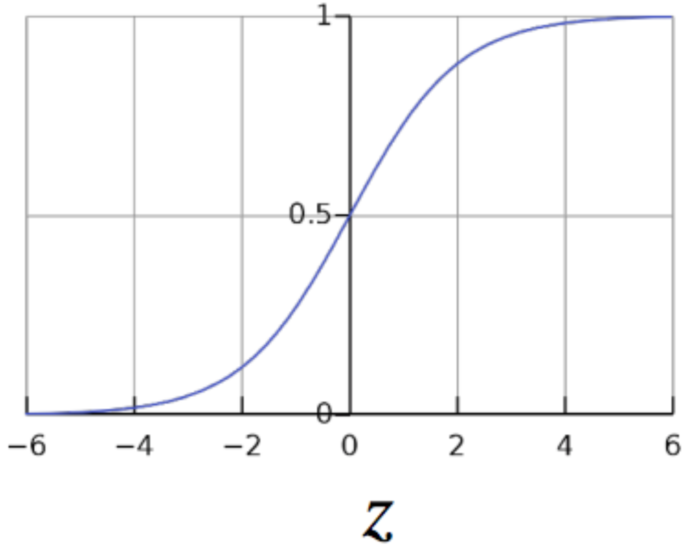
- Train phase: Learn parameters of model to minimize loss function
  - Need Loss function and Optimization algorithm
- Test phase: Apply parameters to predict class given a new input

Binary Logistic Regression

- Input features: $f(x) \rightarrow [f_1, f_2, \dots, f_m]$
- Output: $P(y = 1|x)$ and $P(y = 0|x)$
- Classification function: $\sigma(z) = \frac{1}{1+e^{-z}}$

  $z = \mathbf{v} \cdot \mathbf{f}(\mathbf{x})$

Sigmoid



bias term

Example

Features: $[1, \text{count}(\text{“}amazing\text{”}), \text{count}(\text{“}horrible), \dots]$

Weights: [-1.0, 0.8, -0.4, …]

Learning the weights

- Goal: predict label $\hat{y}$ as close as possible to actual label y
- Distance metric/Loss function: $L(\hat{y}, y)$
- Maximum likelihood estimate:

  Choose parameters so that $\log \mathrm{P}(\mathrm{y}|\mathrm{x})$ is maximized over the training dataset

$$\text{Maximize } \log \prod_{i=1}^{n} P\left(y^{(i)} \middle| x^{(i)}\right)$$

where $(x^{(i)}, y^{(i)})$ are paired documents and labels

Binary Cross Entropy Loss

- Let $\hat{y} = \sigma(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}))$
- Classifier probability: $P(y|x) = \hat{y}^{y}(1-\hat{y})^{1-y}$

$$y = 1: P(y|x) = \hat{y} \qquad\qquad y = 0: P(y|x) = 1 - \hat{y}$$

- Log probability: $\log P(y|x) = y \log \hat{y} + (1-y)\log(1-\hat{y})$

Binary Cross Entropy Loss

- Let $\hat{y} = \sigma(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}))$
- Classifier probability: $P(y|x) = \hat{y}^y(1-\hat{y})^{1-y}$
- Log probability: $\log P(y|x) = y \log\hat{y} + (1-y)\log(1-\hat{y})$
- Loss:

$$L(\hat{y}, y) = -\log \prod_{i=1}^{n} P\left(y^{(i)} \middle| x^{(i)}\right) = -\sum_{i=1}^{n} \log P\left(y^{(i)} \middle| x^{(i)}\right)$$

$$= -\sum_{i=1}^{n} [y^{(i)}\log\hat{y}^{(i)} + \left(1-y^{(i)}\right)\log\left(1-\hat{y}^{(i)}\right)]$$

Cross-entropy between the true distribution $P(y|x)$ and predicted distribution $P(\hat{y}|x)$

Binary Cross Entropy Loss

- Cross Entropy Loss:

$$\mathrm{L}_{\mathrm{CE}} = -\sum_{i=1}^{n} \log\left[y^{(i)} \log \hat{y}^{(i)} + \left(1 - y^{(i)}\right) \log\left(1 - \hat{y}^{(i)}\right)\right]$$

- Ranges from 0 (perfect predictions) to $+\infty$
- Lower loss = better classifier

Multinomial Logistic Regression

- Input features: $f(x) \rightarrow [f_1, f_2, \dots, f_m]$
- Output: $P(y = c|x)$ for each class c
- Classification function Softmax

$$\frac{\exp(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y))}{\underbrace{\sum_{y'} \exp(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y'))}_{\text{Normalization}}}$$

Features are a function of both input x and output class c

Multinomial Logistic Regression

Var	Definition	Wt
$f_1(0,x)$	$\begin{cases} 1 & \text{if “!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	-4.5
$f_1(+,x)$	$\begin{cases} 1 & \text{if “!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	2.6
$f_1(-,x)$	$\begin{cases} 1 & \text{if “!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1.3

Multinomial Logistic Regression

- Generalize binary loss to multinomial CE loss

$$\mathrm{L}_{\mathrm{CE}}(\hat{y}, \mathrm{y}) = -\sum_{c=1}^{k} 1\{\mathrm{y} = \mathrm{c}\} \log P(y = c|x)$$

$$= \sum_{c=1}^{k} 1\{\mathrm{y} = \mathrm{c}\} \frac{\exp(\mathbf{v_c} \cdot \mathbf{f}(\mathbf{x}, c))}{\sum_{y'=1}^{k} \exp(\mathbf{v_{y'}} \cdot \mathbf{f}(\mathbf{x}, y'))}$$

Multinomial Logistic Regression

- Generalize binary loss to multinomial CE loss

$$\mathrm{L_{CE}}(\hat{y}, \mathrm{y}) = -\sum_{c=1}^{k} 1\{\mathrm{y} = \mathrm{c}\} \log P(y = c|x)$$

$$= \sum_{c=1}^{k} 1\{\mathrm{y} = \mathrm{c}\} \frac{\exp(\mathbf{v_c} \cdot \mathbf{f}(\mathbf{x}, c))}{\sum_{y'=1}^{k} \exp(\mathbf{v_{y'}} \cdot \mathbf{f}(\mathbf{x}, y'))}$$

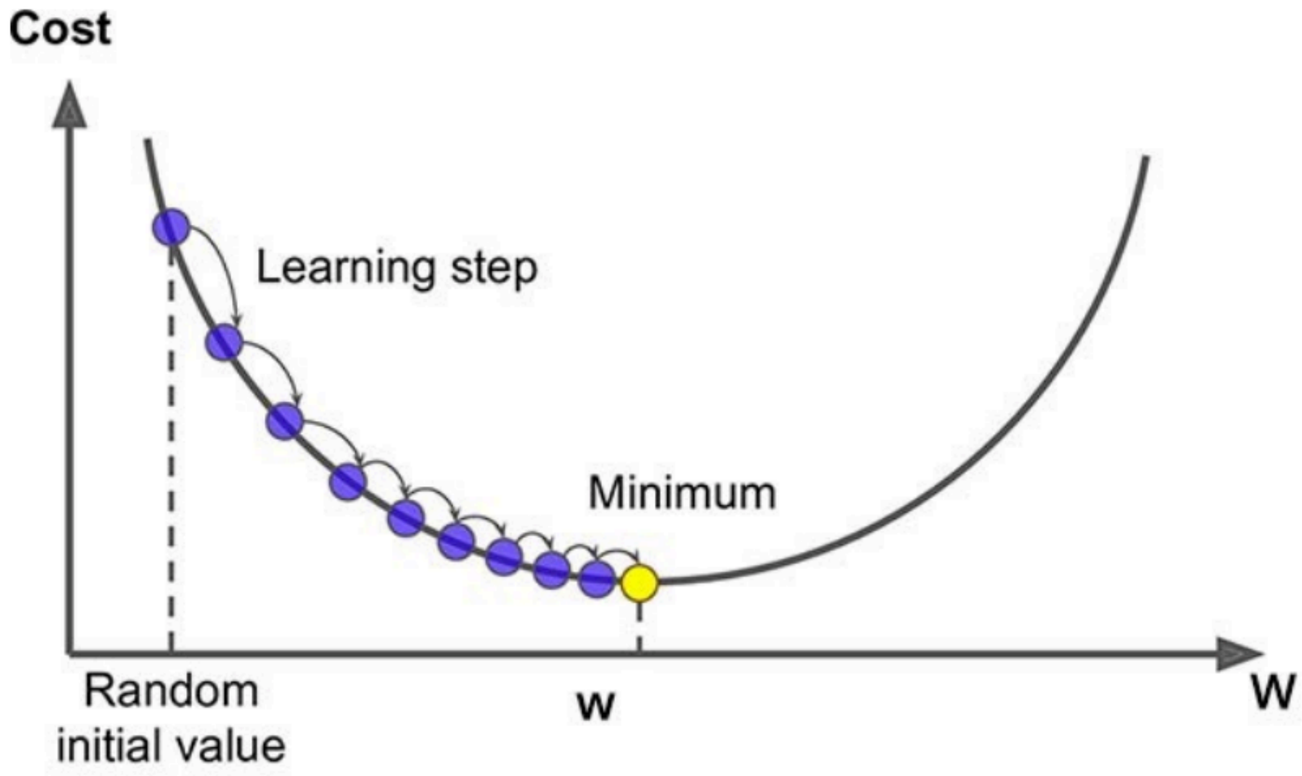
Optimization

- We have our loss function and our estimator $\hat{y}$ = $\sigma(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}))$
- How do we find the best set of parameters/weights: $\mathbf{v}$

$$\hat{\mathbf{v}} = \hat{\theta} = \arg\min \frac{1}{n} \sum_{i=1}^{n} L_{CE}(y^{(i)}, x^{(i)}; \theta)$$

- Use gradient descent!
  - Find direction of steepest slope
  - Move in opposite direction

Gradient descent (1-D)

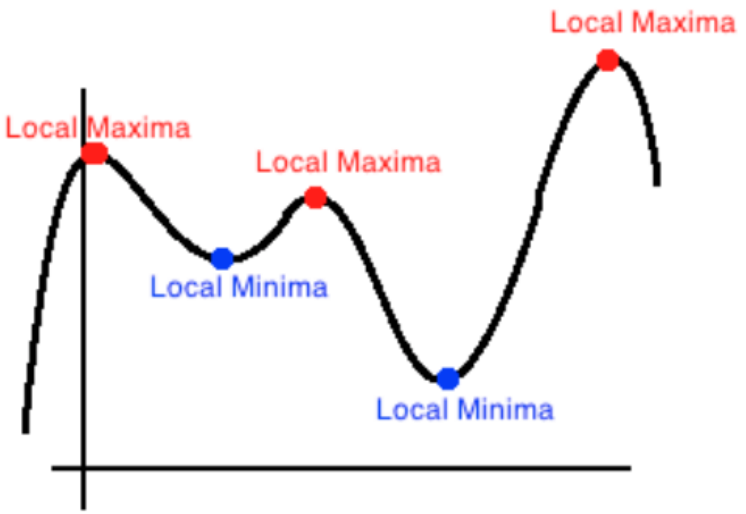


$$\theta^{t+1} = \theta^{t} - \eta \frac{d}{d\theta} f(x;\theta)$$

Gradient descent for LR

- Cross entropy loss for logistic regression is convex (i.e. has only one global minimum)
  - No local minima to get stuck in

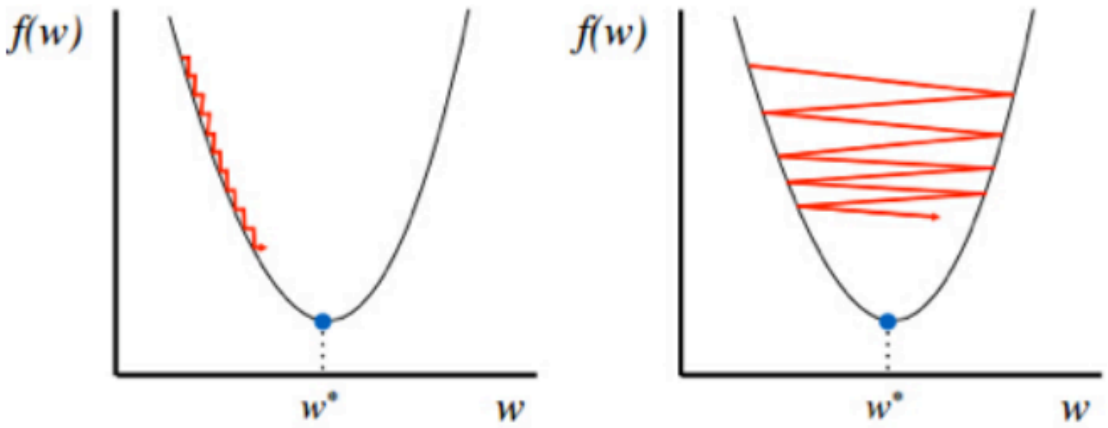
- Deep neural networks are not so easy
  - Non-convex



Learning Rate

- Updates: $\theta^{t+1} = \theta^t - \eta \frac{d}{d\theta} f(x;\theta)$ Magnitude of movement

- Higher/faster learning rate = larger update



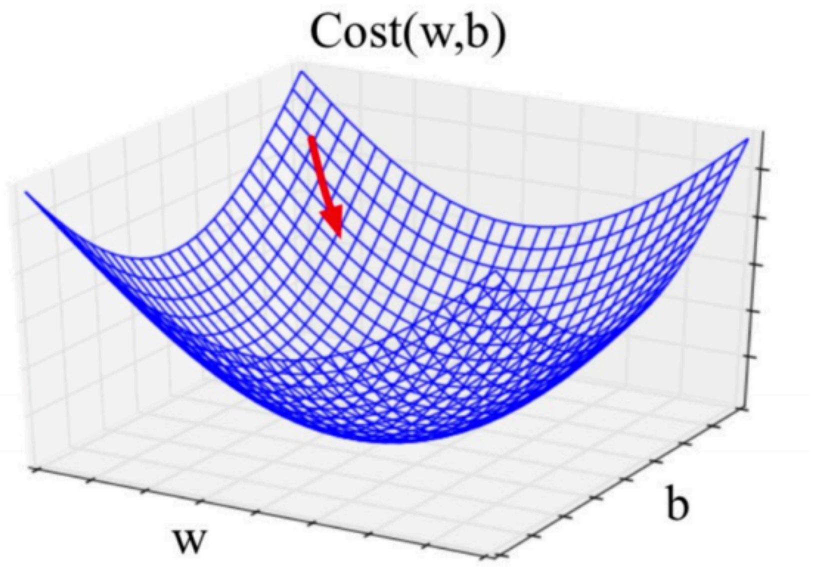
Too small: converge very slowly

Too big: overshoot and even diverge

Gradient descent with vector weights

Express slope as a partial derivative of loss w.r.t each weight:

$$\nabla_\theta L(f(x;\theta),y)) = \begin{bmatrix} \frac{\partial}{\partial w_1} L(f(x;\theta),y) \\ \frac{\partial}{\partial w_2} L(f(x;\theta),y) \\ \vdots \\ \frac{\partial}{\partial w_n} L(f(x;\theta),y) \end{bmatrix}$$



Updates: $\theta^{(t+1)} = \theta^t - \eta \nabla L(f(x;\theta), y)$

Computing the gradients

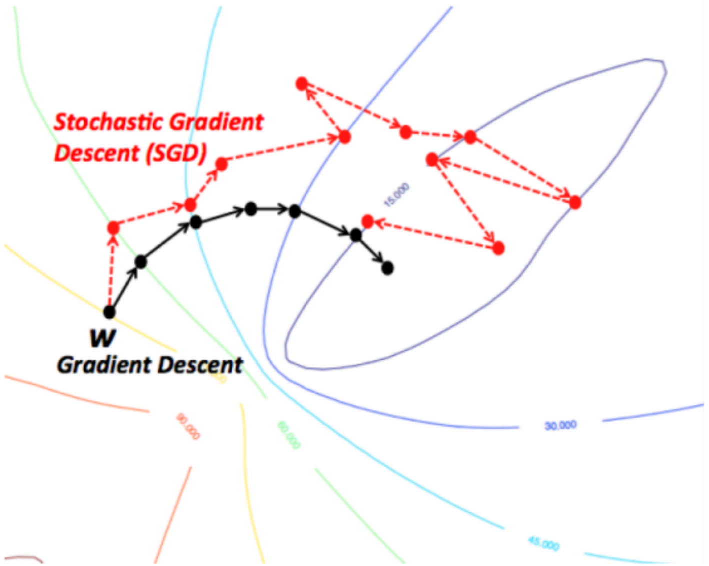
- From last lecture:

$$\arg\max \sum_{i=1}^{n} \log P\left(y^{(i)} | x^{(i)}; \theta\right)$$

$$\begin{aligned}
&\left.\frac{dL(\mathbf{v})}{d\mathbf{v}}\right|_{\mathbf{v}} \\
&= \sum_i \mathbf{f}(\mathbf{x}_i, y_i) - \sum_i \frac{1}{\sum_{y''} exp\left(\mathbf{v} \cdot \mathbf{f}(x_i, y'')\right)} \\
&\quad \sum_{y'} \mathbf{f}(\mathbf{x}_i, y') \cdot exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}_i, y')\right) \\
&= \sum_i \mathbf{f}(\mathbf{x}_i, y_i) - \sum_i \sum_{y'} \mathbf{f}(\mathbf{x}_i, y') \frac{exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}_i, y')\right)}{\sum_{y''} exp\left(\mathbf{v} \cdot \mathbf{f}(x_i, y'')\right)} \\
&= \underbrace{\sum_i \mathbf{f}(\mathbf{x}_i, y_i)}_{\text{Observed counts}} - \underbrace{\sum_i \sum_{y'} \mathbf{f}(\mathbf{x}_i, y') \Pr(y' \mid \mathbf{x}_i; \mathbf{v})}_{\text{Expected counts}}
\end{aligned}$$

Stochastic Gradient Descent

- Online optimization
- Compute loss and minimize after each training examples
  (or mini-batch)



Regularization

- May overfit on the training data!
- Use regularization to prevent overfitting!
- Objective function:

$$\hat{\theta} = \arg\max \sum_{i=1}^{n} \log P\left(y^{(i)} \middle| x^{(i)}\right) - \alpha\, R(\theta)$$

L2 Regularization

$$R(\theta) = ||\theta||^2 = \sum_{j=1}^{d} \theta_j^2$$

Euclidean distance of weight vector θ from origin

L2 regularized objective:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(y^{(i)} | x^{(i)}) - \alpha \sum_{j=1}^{d} \theta_j^2$$

L1 Regularization

$$R(\theta) = ||\theta||_1 = \sum_{j=1}^{d} |\theta_j|$$

Manhattan distance of weight vector θ from origin

L1 regularized objective:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(y^{(i)} | x^{(i)}) - \alpha \sum_{j=1}^{d} |\theta_j|$$

L2 vs L1 regularization

- L2 is easier to optimize
  - L1 is complex since the derivative of |θ| is not continuous at 0
- L2 leads to many small weights
  - L1 prefers sparse weight vectors with many weights set to 0