Natural Language Processing

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Part 1: Feedforward neural networks

Log-linear models versus Neural networks

Feedforward neural networks

Stochastic Gradient Descent

Motivating example: XOR

Log linear model

- Let there be m features, $f_k(\mathbf{x}, y)$ for $k = 1, \ldots, m$
- Define a parameter vector $\mathbf{v} \in \mathbb{R}^m$
- A log-linear model for classification into labels $y \in \mathcal{Y}$:

$$\Pr(y \mid \mathbf{x}; \mathbf{v}) = \frac{exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y)\right))}{\sum_{y' \in \mathcal{Y}} exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y')\right))}$$

Advantages

The feature representation $\mathbf{f}(\mathbf{x}, y)$ can represent any aspect of the input that is useful for classification.

Disadvantages

The feature representation $\mathbf{f}(\mathbf{x}, y)$ has to be designed by hand which is time-consuming and error-prone.

Neural Networks

Advantages

- Neural networks replace hand-engineered features with **representation learning**
- Empirical results across many different domains show that learned representations give significant improvements in accuracy
- Neural networks allow end to end training for complex NLP tasks and do not have the limitations of multiple chained pipeline models

Disadvantages

For many tasks linear models are much faster to train compared to neural network models

Alternative Form of Log linear model

Log-linear model:

$$\Pr(y \mid \mathbf{x}; \mathbf{v}) = \frac{exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y)\right))}{\sum_{y' \in \mathcal{Y}} exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y')\right))}$$

Alternative form using functions:

$$\Pr(y \mid x; v) = \frac{exp\left(v(y) \cdot f(x) + \gamma_y\right)}{\sum_{y' \in \mathcal{Y}} exp\left(v(y') \cdot f(x) + \gamma_{y'}\right)}$$

- Feature vector $f(x)$ maps input x to $\mathbb{R}^d$
- Parameters $v(y) \in \mathbb{R}^d$ and $\gamma_y \in \mathbb{R}$ for each $y \in \mathcal{Y}$
- We use v to refer to the parameter vectors and bias values:

$$v = \{(v(y), \gamma_y) : y \in \mathcal{Y}\}$$

Representation Learning: Feedforward Neural Network

Replace hand-engineered features f with learned features ϕ:

$$\Pr(y \mid x; \theta, v) = \frac{exp\left(v(y) \cdot \phi(x;\theta) + \gamma_y\right)}{\sum_{y' \in \mathcal{Y}} exp\left(v(y') \cdot \phi(x;\theta) + \gamma_{y'}\right)}$$

- Replace $f(x)$ with $\phi(x;\theta) \in \mathbb{R}^d$ where θ are new parameters
- Parameters θ are learned from training data
- Using θ the model ϕ maps input x to $\mathbb{R}^d$: a learned representation of x
- x is assumed to be already represented as a vector of size d
- We will use feedforward neural networks to define $\phi(x;\theta)$
- $\phi(x;\theta)$ will be a **non-linear** mapping to $\mathbb{R}^d$ while f is a **linear** model

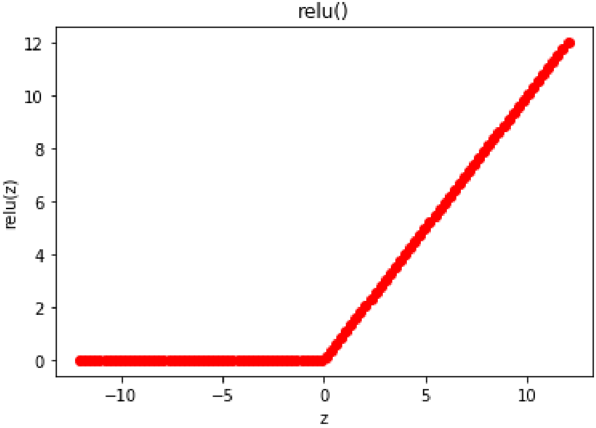
A Single Neuron aka Perceptron

A single neuron maps input $x \in \mathbb{R}^d$ to output h:

$$h = g(w \cdot x + b)$$

- Weight vector $w \in \mathbb{R}^d$, a bias $b \in \mathbb{R}$ are the parameters of the model learned from training data
- Transfer function $g : \mathbb{R} \to \mathbb{R}$
- It is important that g is a **non-linear** transfer function
- Linear $g(z) = \alpha \cdot z + \beta$ for constants α, β (linear perceptron)

The ReLU Transfer Function $[0, z]$



The ReLU Transfer Function

Rectified Linear Unit (ReLU):

$$g(z) = \{z \text{ if } z \geq 0 \text{ or } 0 \text{ if } z < 0\}$$

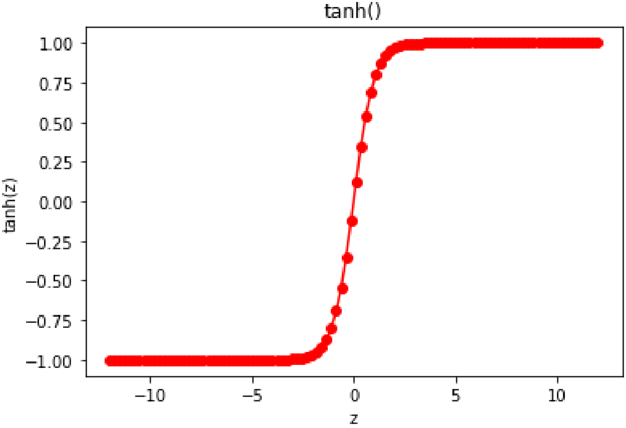
or equivalently $g(z) = \max\{0, z\}$

Derivative of ReLU:

$$\frac{dg(z)}{dz} = \{1 \text{ if } z > 0 \text{ or } 0 \text{ if } z < 0\}$$

non-differentiable or undefined if $z = 0$
(in practice: choose a value for $z = 0$)

The tanh Transfer Function $[-1, 1]$



The tanh Transfer Function

tanh transfer function:

$$g(z) = \frac{e^{2z} - 1}{e^{2z} + 1}$$

Derivative of tanh:

$$\frac{dg(z)}{dz} = 1 - g(z)^2$$

Derivatives w.r.t. parameters

Derivatives w.r.t. w:

Given

$$h = g(w \cdot x + b)$$

derivatives w.r.t. $w_1, \ldots, w_j, \ldots w_d$:

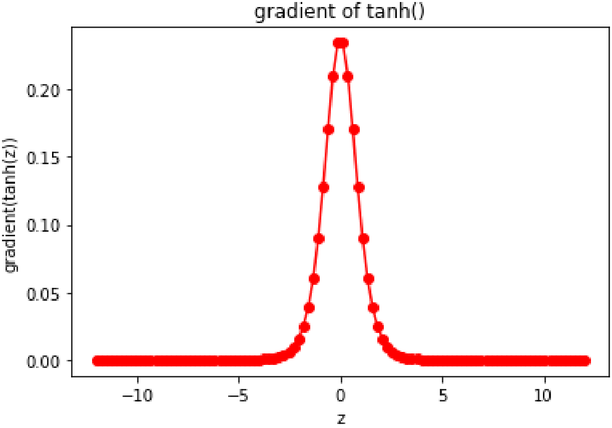
$$\frac{dh}{dw_j}$$

Derivatives w.r.t. b:

derivatives w.r.t. b:

$$\frac{dh}{db}$$

tanh Gradient



Chain Rule of Differentiation

Introduce an intermediate variable $z \in \mathbb{R}$

$$z = w \cdot x + b$$

$$h = g(z)$$

Then by the chain rule to differentiate w.r.t. w:

$$\frac{dh}{dw_j} = \frac{dh}{dz}\frac{dz}{dw_j} = \frac{dg(z)}{dz} \times x_j$$

And similarly for b:

$$\frac{dh}{db} = \frac{dh}{dz}\frac{dz}{db} = \frac{dg(z)}{dz} \times 1$$

Single Layer Feedforward model

A single layer feedforward model consists of:

- An integer d specifying the input dimension. Each input to the network is $x \in \mathbb{R}^d$
- An integer m specifying the number of hidden units
- A parameter matrix $W \in \mathbb{R}^{m \times d}$. The vector $W_k \in \mathbb{R}^d$ for $1 \leq k \leq m$ is the kth row of W
- A vector $b \in \mathbb{R}^d$ of bias parameters
- A transfer function $g : \mathbb{R} \to \mathbb{R}$
  $g(z) = \mathrm{ReLU}(z)$ or $g(z) = \tanh(z)$

Single Layer Feedforward model (continued)

For $k = 1, \ldots, m$:

- The input to the kth neuron is: $z_k = W_k \cdot x + b_k$
- The output from the kth neuron is: $h_k = g(z_k)$
- Define vector $\phi(x; \theta) \in \mathbb{R}^m$ as: $\phi(x; \theta) = h_k$
- $\theta = (W, b)$ where $W \in \mathbb{R}^{m \times d}$ and $b \in \mathbb{R}^d$
- Size of θ is $m \times (d + 1)$ parameters

Some intuition

The neural network employs m hidden units, each with their own parameters W_k and b_k, and these neurons are used to construct a *hidden* representation $h \in \mathbb{R}^m$

Matrix Form

We can replace the operation:

$$z_k = W_k \cdot x + b \text{ for } k = 1, \ldots, m$$

with

$$z = Wx + b$$

where the dimensions are as follows (vector of size m equals a matrix of size $m \times 1$):

$$\underbrace{z}_{m\times 1} = \underbrace{\underbrace{W}_{m\times d}\underbrace{x}_{d\times 1}}_{m\times 1} + \underbrace{b}_{m\times 1}$$

Single Layer Feedforward model (matrix form)

A single layer feedforward model consists of:

- An integer d specifying the input dimension. Each input to the network is $x \in \mathbb{R}^d$
- An integer m specifying the number of hidden units
- A parameter matrix $W \in \mathbb{R}^{m \times d}$
- A vector $b \in \mathbb{R}^d$ of bias parameters
- A transfer function $g : \mathbb{R}^m \to \mathbb{R}^m$
  $g(z) = [\ldots, \mathrm{ReLU}(z_i), \ldots]$ or $g(z) = [\ldots, \tanh(z_i), \ldots]$ for $i = 1, \ldots, m$

Single Layer Feedforward model (matrix form, continued)

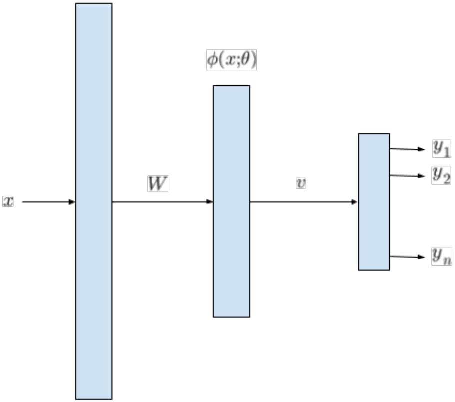
Define ϕ in matrix form:

- Vector of inputs to the hidden layer $z \in \mathbb{R}^m$: $z = Wx + b$
- Vector of outputs from hidden layer $h \in \mathbb{R}^m$: $h = g(z)$
- Define $\phi(x;\theta) = h$ where $\theta = (W, b)$
$$\phi(x;\theta) = g(Wx + b)$$

Putting it all together:

$$\Pr(y \mid x; \theta, v) = \frac{exp\left(v(y) \cdot \phi(x;\theta) + \gamma_y\right)}{\sum_{y' \in \mathcal{Y}} exp\left(v(y') \cdot \phi(x;\theta) + \gamma_{y'}\right)}$$

Feedforward neural network



Log-linear models versus Neural networks

Feedforward neural networks

Stochastic Gradient Descent

Motivating example: XOR

Simple stochastic gradient descent

Inputs:

- Training examples (x^i, y^i) for $i = 1, \ldots, n$
- A feedforward representation $\phi(x; \theta)$
- Integer T specifying the number of updates
- A sequence of learning rates: $\eta^1, \ldots, \eta^T$ where $\eta^t > 0$

Initialization:
Set $v = (v(y), \gamma_y)$ for all y, and θ to random values

Gradient descent

Algorithm:

- For $t = 1, \ldots, T$
  - Select an integer i uniformly at random from $\{1, \ldots, n\}$
  - Define $L(\theta, v) = -\log P(y_i \mid x_i; \theta, v)$
  - For each parameter θ_j and $v_k(y)$ and γ_y (for each label y):

$$
\begin{aligned}
\theta_j &= \theta_j - \eta^t \times \frac{dL(\theta, v)}{d\theta_j} \\
v_k(y) &= v_k(y) - \eta^t \times \frac{dL(\theta, v)}{dv_k(y)} \\
\gamma(y) &= \gamma(y) - \eta^t \times \frac{dL(\theta, v)}{d\gamma(y)}
\end{aligned}
$$

- **Output**: parameters θ, $v = (v(y), \gamma_y)$ for all y

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Motivating example: XOR

Motivating example: the XOR problem

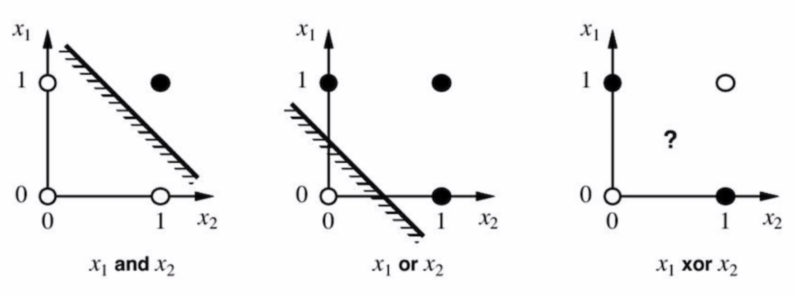
From *Deep Learning* by Goodfellow, Bengio, Courville

We will assume a training set where each label is in the set $\mathcal{Y} = \{-1, +1\}$

There are four training examples:

$$\begin{aligned} x^1 &= [0,0], y^1 = -1 \\ x^2 &= [0,1], y^2 = +1 \\ x^3 &= [1,0], y^3 = +1 \\ x^4 &= [1,1], y^4 = -1 \end{aligned}$$

Motivating example: the XOR problem



Motivating example: the XOR problem

Theorem

For examples (x^i, y^i) for $i = 1, \ldots, 4$ as defined previously for the feedforward neural network:

$$\Pr(y \mid x; W, b, v) = \frac{exp\left(v(y) \cdot g(Wx + b) + \gamma_y\right)}{\sum_{y' \in \mathcal{Y}} exp\left(v(y') \cdot g(Wx + b) + \gamma_{y'}\right)}$$

where $x \in \mathbb{R}^2$ $(d = 2)$ and let $m = 2$ so $W \in \mathbb{R}^{2 \times 2}$ and $b \in \mathbb{R}^2$ and g is a ReLU transfer function.
Then there are parameter settings $v(-1)$, $v(+1)$, γ_{-1}, γ_{+1}, W, b such that

$$p(y^i \mid x^i; v) > 0.5 \text{ for } i = 1, \ldots, 4$$

Motivating example: the XOR problem

Proof Sketch

Define $W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ Then for each input x calculate values of $z = Wx + b$ and $h = g(z)$:

$$\begin{aligned}
x = [0,0] \quad &\Rightarrow \quad z = [0,-1] \Rightarrow h = [0,0] \\
x = [1,0] \quad &\Rightarrow \quad z = [1,0] \Rightarrow h = [1,0] \\
x = [0,1] \quad &\Rightarrow \quad z = [1,0] \Rightarrow h = [1,0] \\
x = [1,1] \quad &\Rightarrow \quad z = [2,1] \Rightarrow h = [2,1]
\end{aligned}$$

Motivating example: the XOR problem

Proof Sketch (continued)

$$
\begin{aligned}
p(+1 \mid x; v) &= \frac{exp(v(+1) \cdot h + \gamma_{+1})}{exp(v(+1) \cdot h + \gamma_{+1}) + exp(v(-1) \cdot h + \gamma_{-1})} \\
&= \frac{1}{1 + exp(-(u \cdot h + \gamma))}
\end{aligned}
$$

To satisfy $P(y^i \mid x^i; v) > 0.5$ for $i = 1, \ldots, 4$ we have to find parameters $u = v(+1) - v(-1)$ and $\gamma = \gamma_{+1} - \gamma_{-1}$ such that:

$$
\begin{aligned}
u \cdot [0, 0] + \gamma &< 0 \\
u \cdot [1, 0] + \gamma &> 0 \\
u \cdot [1, 0] + \gamma &> 0 \\
u \cdot [2, 1] + \gamma &< 0
\end{aligned}
$$

$u = [1, -2]$ and $\gamma = -0.5$ satisfies these constraints.

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