
CMPT 825: Natural Language Processing

# Word Embeddings - Word2Vec

Fall 2020
2020-09-30

Adapted from slides from Dan Jurafsky, Chris Manning, Danqi Chen and Karthik Narasimhan

# Announcements

- Homework 1 due today
  - Both parts are due
  - Programming component has 2 grace days, but something must be turned in by tonight
- Single person groups - highly encouraged to team up with each other

- Video Lectures
  - Summary of logistic regression (optional)
  - Word vectors (required) - covers PPMI
  - Word vectors TF-IDF (required, not yet posted) - covers TF-IDF
  - Word vectors Summary (optional, not yet posted)
    - Using SVD to get dense word vectors, and connections to word2vec

- TA video summarizing key points about word vectors

# Representing words by their context

**Distributional hypothesis**: words that occur in similar contexts tend to have similar meanings

J.R.Firth 1957

- "You shall know a word by the company it keeps"
- One of the most successful ideas of modern statistical NLP!

*…government debt problems turning into* ***banking*** *crises as happened in 2009…*

*…saying that Europe needs unified* ***banking*** *regulation to replace the hodgepodge…*

*…India has just given its* ***banking*** *system a shot in the arm…*

These context words will represent ***banking***.

# Word Vectors

- One-hot vectors

  hotel = [0 0 0 0 0 0 0 0 0 0 0 **1** 0 0 0 0]

  motel = [0 0 0 **1** 0 0 0 0 0 0 0 0 0 0 0 0]

- Represent words by their context

word-word (term-context) co-occurrence matrix

context

(other words in the span around the target word)

term →

| | aardvark | computer | data | pinch | result | sugar | … |
|---|---|---|---|---|---|---|---|
| apricot | 0 | 0 | 0 | 1 | 0 | 1 | |
| pineapple | 0 | 0 | 0 | 1 | 0 | 1 | |
| digital | 0 | 2 | 1 | 0 | 1 | 0 | |
| information | 0 | 1 | 6 | 0 | 4 | 0 | |

$|V| \times |V|$ matrix

sugar, a sliced lemon, a tablespoonful of **apricot** jam, a pinch each of,
their enjoyment. Cautiously she sampled her first **pineapple** and another fruit whose taste she likened
well suited to programming on the digital **computer**. In finding the optimal R-stage policy from
for the purpose of gathering data and **information** necessary for the study authorized in the

# Sparse vs dense vectors

Vectors we get from word-word (term-context) co-occurrence matrix are

- **long** (length |V|= 20,000 to 50,000)
- **sparse** (most elements are zero)

True for both one-hot, tf-idf and PPMI vectors

Alternative: we want to represent words as

- **short** (50-300 dimensional)
- **dense** (real-valued) vectors

- The focus of this lecture
- The basis of all the modern NLP systems

# Dense vectors

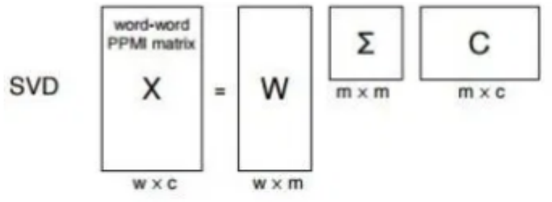
$$\text{employees} = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 10.109 \\ -0.542 \\ 0.349 \\ 0.271 \\ 0.487 \end{pmatrix}$$

**short + dense**

# Why dense vectors?

- Short vectors are easier to use as features in ML systems
- Dense vectors may generalize better than storing explicit counts
- They do better at capturing synonymy
  - $w_1$ co-occurs with “car”, $w_2$ co-occurs with “automobile”

- Different methods for getting dense vectors:
  - Singular value decomposition (SVD)
  - word2vec and friends: “learn” the vectors!



# Word2vec and friends

# Download pertained word embeddings

Google

**Word2vec** (Mikolov et al.)
https://code.google.com/archive/p/word2vec/

facebook

**Fasttext** http://www.fasttext.cc/

**Glove** (Pennington, Socher, Manning)
http://nlp.stanford.edu/projects/glove/

# Word2Vec

- Popular embedding method
- Very fast to train
- Idea: predict rather than count

(Mikolov et al, 2013): Distributed Representations of Words and Phrases and their Compositionality

# Word2Vec

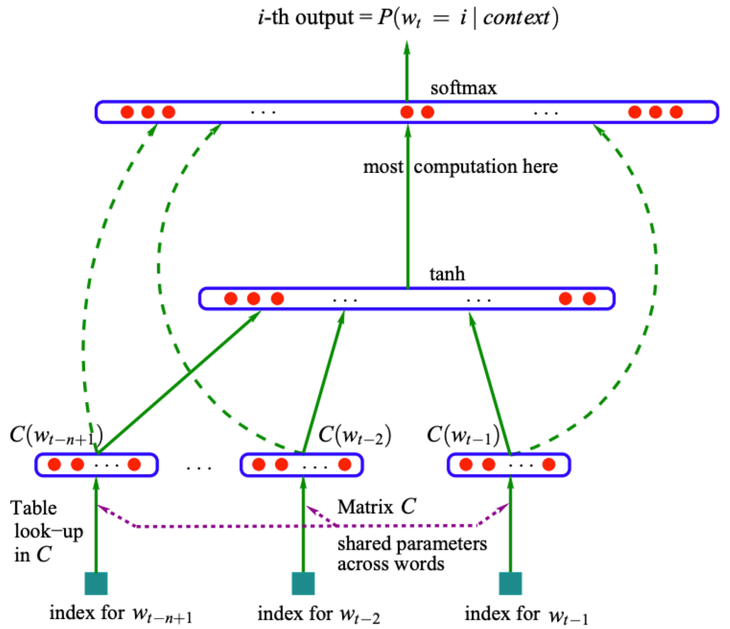
- Instead of counting how often each word *w* occurs near “apricot”
- Train a classifier on a binary prediction task:
  - Is *w* likely to show up near “apricot”?

- We don’t actually care about this task
  - But we’ll take the learned classifier weights as the word embeddings

# Word2Vec

Insight: use running text as implicitly supervised training data!

- A word *s* near apricot
  - Act as gold "correct answer" to the question "Is word w likely to show up near apricot?"
- No need for hand-labeled supervision
- The idea comes from neural language modelii
  - Bengio et al (2003)
  - Collobert et al (2011)




(Bengio et al, 2003): A Neural Probabilistic Language Model

# Word2vec

- Input: a large text corpora, $V$, $d$
  - $V$: a pre-defined vocabulary
  - $d$: dimension of word vectors (e.g. 300)
  - Text corpora:
    - Wikipedia + Gigaword 5: 6B
    - Twitter: 27B
    - Common Crawl: 840B
- Output: $f : V \to \mathbb{R}^d$

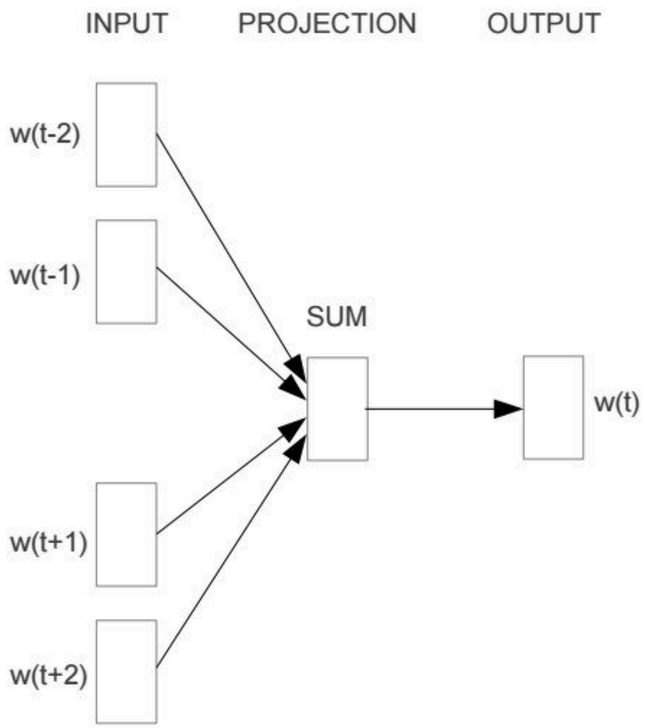
$$v_{\text{cat}} = \begin{pmatrix} -0.224 \\ 0.130 \\ -0.290 \\ 0.276 \end{pmatrix} \quad v_{\text{dog}} = \begin{pmatrix} -0.124 \\ 0.430 \\ -0.200 \\ 0.329 \end{pmatrix}$$

$$v_{\text{the}} = \begin{pmatrix} 0.234 \\ 0.266 \\ 0.239 \\ -0.199 \end{pmatrix} \quad v_{\text{language}} = \begin{pmatrix} 0.290 \\ -0.441 \\ 0.762 \\ 0.982 \end{pmatrix}$$

Learn *f* by training classifiers to predict words and take learned weights as word vectors.

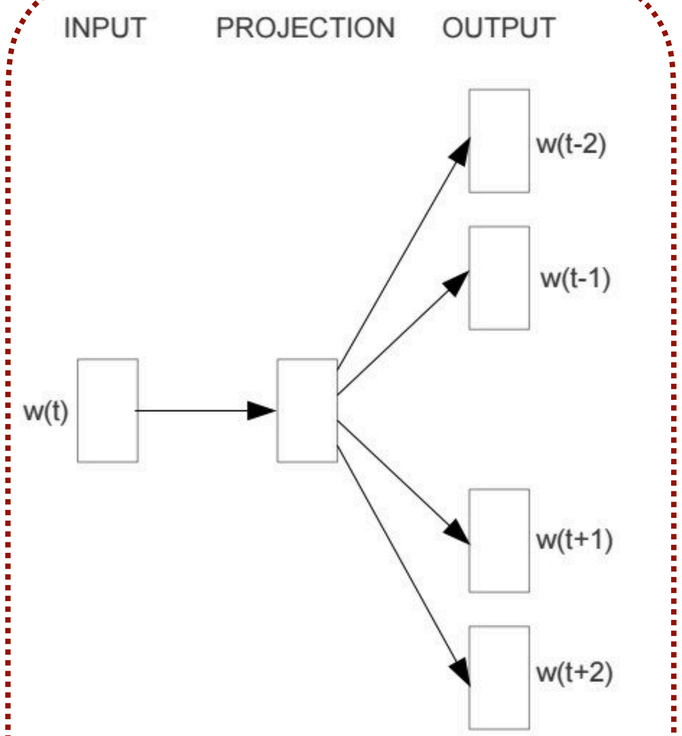
# Word2vec

Predict center word from context words




Continuous Bag of Words (CBOW)

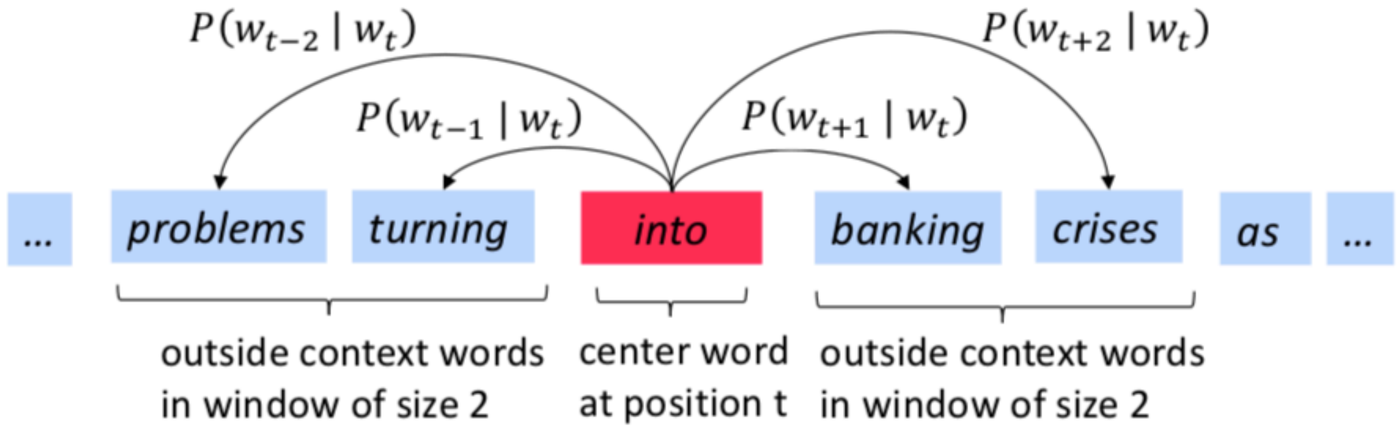
Predict context words from center word




Skip-grams

# Skip-gram

- The idea: we want to use words to **predict** their context words
- Context: a fixed window of size 2



# Skip-gram: Basic Setup

Let's represent words as vectors of some length (say 300), randomly initialized.

So we start with 300 * V random parameters

Over the entire training set, we'd like to adjust those word vectors such that we

- Predict the probability distribution for how likely a word is to be a context word

$$P(c \mid t)$$

# Skip-gram

## Training data

Training sentence:

... lemon, a tablespoon of **apricot** jam a pinch ...

c1 c2 t c3 c4

Training data: input/output pairs centering on apricot

assume a +/- 2 word window

## Goal

Given a tuple $(t, c)$ = target, context

(apricot, jam)

(apricot, aardvark)

Return probability that $c$ is a real context word:

$$P(c \mid t)$$

# Skip-gram: objective function

- For each position $t = 1,2,\ldots T$, predict context words within context size $m$, given center word $w_j$:

all the parameters to be optimized

$$\mathcal{L}(\theta) = \prod_{t=1}^{T} \prod_{-m \leq j \leq m, j \neq 0} P(w_{t+j} \mid w_t; \theta)$$

- The objective function $J(\theta)$ is the (average) negative log likelihood:

$$J(\theta) = -\frac{1}{T} \log \mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{-m \leq j \leq m, j \neq 0} \log P(w_{t+j} \mid w_t; \theta)$$

# How to define $P(w_{t+j} \mid w_t; \theta)$?

- We have two sets of vectors for each word in the vocabulary

$$\mathbf{u}_i \in \mathbb{R}^d \text{ : embedding for target word } i$$

$$\mathbf{v}_{i'} \in \mathbb{R}^d \text{ : embedding for context word } i'$$

- Use inner product $\mathbf{u}_i \cdot \mathbf{v}_{i'}$ to measure how likely word $i$ appears with context word $i'$, the larger the better

"softmax" we learned last time!

$$P(w_{t+j} \mid w_t) = \frac{\exp(\mathbf{u}_{w_t} \cdot \mathbf{v}_{w_{t+j}})}{\sum_{k \in V} \exp(\mathbf{u}_{w_t} \cdot \mathbf{v}_k)}$$

Normalized over entire vocabulary

$\theta = \{\{\mathbf{u}_k\}, \{\mathbf{v}_k\}\}$ are all the parameters in this model!

Q: Why two sets of vectors?

# How to train the model

Calculating all the gradients together!

$$\theta = \{\{\mathbf{u}_k\}, \{\mathbf{v}_k\}\}$$

$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\leq j\leq m, j\neq 0} \log P(w_{t+j} \mid w_t; \theta) \qquad \nabla_\theta J(\theta) = ?$$

Q: How many parameters are in total?

We can apply stochastic gradient descent (SGD)!

$$\theta^{(t+1)} = \theta^{(t)} - \eta \nabla_\theta J(\theta)$$

Let's walk through the math..

# Warm-up

$$f(x) = \exp(x) \qquad \frac{df}{dx} = \exp(x)$$

$$f(x) = \log(x) \qquad \frac{df}{dx} = \frac{1}{x}$$

chain rule:

$$f(x) = f_1(f_2(x)) \qquad \frac{df}{dx} = \frac{df_1(z)}{dz}\frac{df_2(x)}{dx} \qquad z = f_2(x)$$

$$f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{a} \qquad \frac{\partial f}{\partial \mathbf{x}} = \mathbf{a}$$

$$\frac{\partial f}{\partial \mathbf{x}} = [\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}]$$

# Computing the gradients

Consider one pair of target/context words (t, c):

$$y = -\log\left(\frac{\exp(\mathbf{u}_t \cdot \mathbf{v}_c)}{\sum_{k\in V}\exp(\mathbf{u}_t \cdot \mathbf{v}_k)}\right)$$

$$\begin{aligned}
\frac{\partial y}{\partial \mathbf{u}_t} &= \frac{\partial\left(-\mathbf{u}_t \cdot \mathbf{v}_c + \log\left(\sum_{k\in V}\exp\left(\mathbf{u}_t \cdot \mathbf{v}_k\right)\right)\right)}{\partial \mathbf{u}_t} \\
&= -\mathbf{v}_c + \frac{\sum_{k\in V}\frac{\partial \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}{\partial \mathbf{u}_t}}{\sum_{k\in V}\exp\left(\mathbf{u}_t \cdot \mathbf{v}_k\right)} \\
&= -\mathbf{v}_c + \frac{\sum_{k\in V}\exp(\mathbf{u}_t \cdot \mathbf{v}_k)\mathbf{v}_k}{\sum_{k\in V}\exp\left(\mathbf{u}_t \cdot \mathbf{v}_k\right)} \\
&= -\mathbf{v}_c + \sum_{k\in V} P(k \mid t)\mathbf{v}_k
\end{aligned}$$

$$\frac{\partial y}{\partial \mathbf{v}_k} = -1(k = c)\mathbf{u}_t + P(k \mid t)\mathbf{u}_t$$

Make sure you know how to do this!

# Putting it together

- Input: text corpus, context size $m$, embedding size $d$, $V$
- Initialize $\mathbf{u}_i$, $\mathbf{v}_i$ randomly
- Walk through the training corpus and collect training data (t, c):
  - Update $\mathbf{u}_t \leftarrow \mathbf{u}_t - \eta \frac{\partial y}{\partial \mathbf{u_t}}$
  - Update $\mathbf{v}_k \leftarrow \mathbf{v}_k - \eta \frac{\partial y}{\partial \mathbf{v_k}}, \forall k \in V$

Any issues?

# Skip-gram with negative sampling (SGNS)

Problem: every time you get one pair of ($t$, $c$), you need to update $\mathbf{v}_k$ with all the words in the vocabulary! It is very computationally expensive.

$$\frac{\partial y}{\partial \mathbf{u}_t} = -\mathbf{v}_c + \sum_{k \in V} P(k \mid t)\mathbf{v}_k$$
$$\frac{\partial y}{\partial \mathbf{v}_k} = -1(k = c)\mathbf{u}_t + P(k \mid t)\mathbf{u}_t$$

**Negative sampling**: instead of considering all the words in V, let's randomly sample K (5-20) negative examples.

softmax: $$y = -\log\left(\frac{\exp(\mathbf{u}_t \cdot \mathbf{v}_c)}{\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}\right)$$

Q: Why is it faster?

NS: $$y = -\log(\sigma(\mathbf{u}_t \cdot \mathbf{v}_c)) - \sum_{i=1}^{K} \mathbb{E}_{j \sim P(w)} \log(\sigma(-\mathbf{u}_t \cdot \mathbf{v}_j))$$

# Skip-gram with negative sampling (SGNS)

$$y = -\log(\sigma(\mathbf{u}_t \cdot \mathbf{v}_c)) - \sum_{i=1}^{K} \mathbb{E}_{j \sim P(w)} \log(\sigma(-\mathbf{u}_t \cdot \mathbf{v}_j))$$

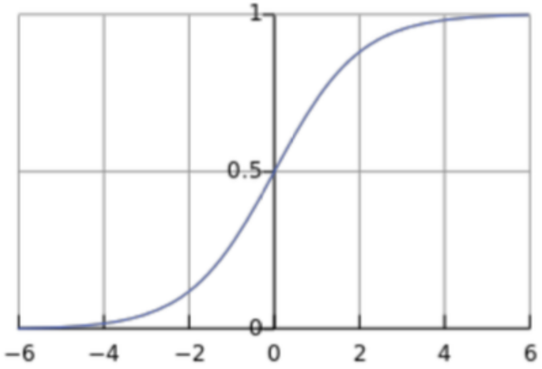
**positive examples +**

| t | c |
|---|---|
| apricot | tablespoon |
| apricot | of |
| apricot | jam |
| apricot | a |

**negative examples -**

| t | c | t | c |
|---|---|---|---|
| apricot | aardvark | apricot | seven |
| apricot | my | apricot | forever |
| apricot | where | apricot | dear |
| apricot | coaxial | apricot | if |

$$\sigma(x) = \frac{1}{1 + \exp(-x)}$$




Same as training a **logistic regression** for binary classification!

$$P(D = 1 \mid t, c) = \sigma(\mathbf{u}_t \cdot \mathbf{v}_c)$$

# Skip-gram with negative sampling (SGNS)

1. Treat the target word and a neighboring context word as positive examples.
2. Randomly sample other words in the lexicon to get negative samples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the weights as the embeddings

# Skip-gram with negative sampling

## Training data

Training sentence:

... lemon, a tablespoon of **apricot** jam a pinch ...

c1 c2 t c3 c4

Training data: input/output pairs centering on apricot

assume a +/- 2 word window

## Goal

Given a tuple $(t, c)$ = target, context

(apricot, jam)

(apricot, aardvark)

Return probability that $c$ is a real context word:

$$P(+|t,c)$$
$$P(-|t,c) = 1 - P(+|t,c)$$

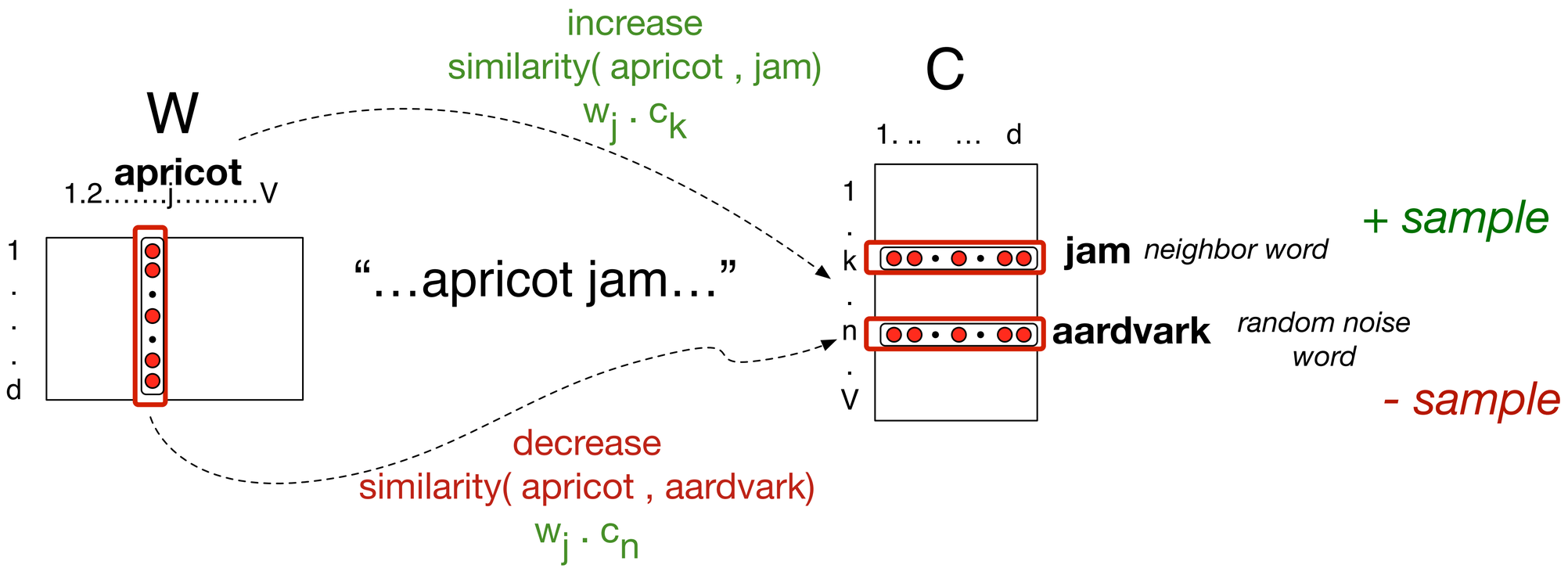
# SGNS: Basic Setup

Let's represent words as vectors of some length (say 300), randomly initialized.

So we start with 300 * V random parameters

Over the entire training set, we'd like to adjust those word vectors such that we

- Maximize the similarity of the target word, context word pairs (t,c) drawn from the positive data
- Minimize the similarity of the (t,c) pairs drawn from the negative data.


increase
similarity( apricot , jam)
$w_j \cdot c_k$
W
apricot
1.2.....j.....V
1
.
.
.
d
"…apricot jam…"
C
1. ..
...
d
1
.
k
.
n
.
V
jam
neighbor word
aardvark
random noise word
+ sample
- sample
decrease
similarity( apricot , aardvark)
$w_j \cdot c_n$

# SGNS: Learning the classifier

Iterative process.

We'll start with 0 or random weights

Then adjust the word weights to

- make the positive pairs more likely
- and the negative pairs less likely

over the entire training set

# SGNS Training

Training sentence:
... lemon, a tablespoon of **apricot** jam a pinch ...
c1 c2 t c3 c4

**positive examples +**

| t | c |
|---|---|
| apricot | tablespoon |
| apricot | of |
| apricot | preserves |
| apricot | or |

- For each positive example, we'll create $K$ negative examples.
- Using noise words
- Any random word that isn't $t$

# Choosing noise words

Could pick $w$ according to their unigram frequency $P(w)$

More common to chose them according to $P_\alpha(w)$

$$P_\alpha(w) = \frac{count(w)^\alpha}{\sum_w count(w)^\alpha}$$

$\alpha$= ¾ works well because it gives rare noise words slightly higher probability

To show this, imagine two events $p(a) = 0.99$ and $p(b) = 0.01$:

$$P_\alpha(a) = \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97$$

$$P_\alpha(b) = \frac{.01^{.75}}{.99^{.75} + .01^{.75}} = .03$$

# SGNS: objective function

We want to maximize…

$$\sum_{(t,c)\in+} logP(+|t,c) + \sum_{(t,c)\in-} logP(-|t,c)$$

Maximize the + label for the pairs from the positive training data,

and the – label for the pairs sample from the negative data.

# Focusing on one target word t:

$$
\begin{aligned}
L(\theta) &= \log P(+|t,c) + \sum_{i=1} \log P(-|t,n_i) \\
&= \log \sigma(c \cdot t) + \sum_{i=1}^{k} \log \sigma(-n_i \cdot t) \\
&= \log \frac{1}{1+e^{-c \cdot t}} + \sum_{i=1}^{k} \log \frac{1}{1+e^{n_i \cdot t}}
\end{aligned}
$$

# Skip-gram with negative sampling (SGNS)

$$y = -\log(\sigma(\mathbf{u}_t \cdot \mathbf{v}_c)) - \sum_{i=1}^{K} \mathbb{E}_{j \sim P(w)} \log(\sigma(-\mathbf{u}_t \cdot \mathbf{v}_j))$$

**positive examples +**

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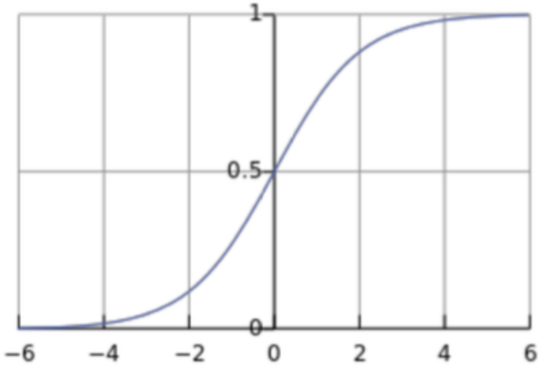
$$\sigma(x) = \frac{1}{1 + \exp(-x)}$$

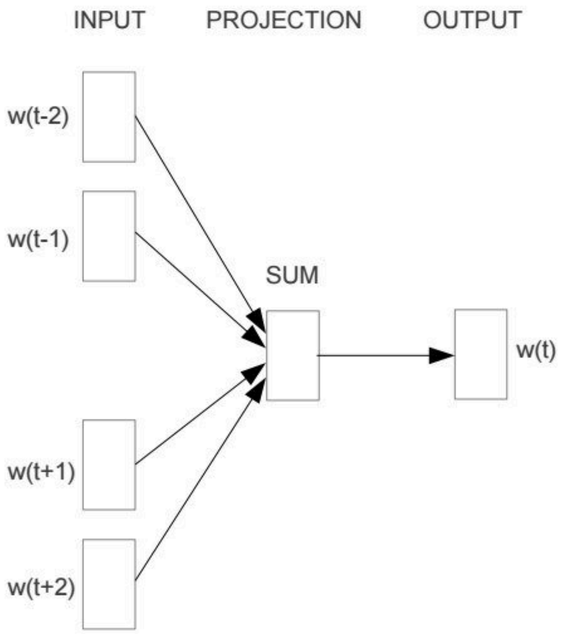



Same as training a **logistic regression** for binary classification!

$$P(D = 1 \mid t, c) = \sigma(\mathbf{u}_t \cdot \mathbf{v}_c)$$

(see also http://jalammar.github.io/illustrated-word2vec/)

# Continuous Bag of Words (CBOW)




$$L(\theta) = \prod_{t=1}^{T} P\left(w_t \mid \{w_{t+j}\}, -m \leq j \leq m, j \neq 0\right)$$

$$\bar{\mathbf{v}}_t = \frac{1}{2m} \sum_{-m \leq j \leq m, j \neq 0} \mathbf{v}_{t+j}$$

$$P(w_t \mid \{w_{t+j}\}) = \frac{\exp(\mathbf{u}_{w_t} \cdot \bar{\mathbf{v}}_t)}{\sum_{k \in V} \exp(\mathbf{u}_k \cdot \bar{\mathbf{v}}_t)}$$

# GloVe: **Glo**bal **Ve**ctors

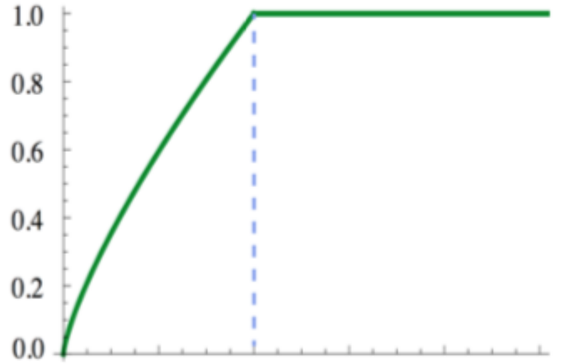
- Let's take the global co-occurrence statistics: $X_{i,j}$

$$J = \sum_{i,j=1}^{V} f\left(X_{ij}\right)\left(w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij}\right)^2$$

- Training faster

- Scalable to very large corpora

$f \sim$




(Pennington et al, 2014): GloVe: Global Vectors for Word Representation

# GloVe: **Glo**bal **Ve**ctors

Nearest words to frog:

1. frogs
2. toad
3. litoria
4. leptodactylidae
5. rana
6. lizard
7. eleutherodactylus

litoria


leptodactylidae

rana

eleutherodactylus

(Pennington et al, 2014): GloVe: Global Vectors for Word Representation

# FastText: Sub-Word Embeddings

- Similar as Skip-gram, but break words into n-grams with n = 3 to 6

where:

3-grams: <wh, whe, her, ere, re>

4-grams: <whe, wher, here, ere>

5-grams: <wher, where, here>

6-grams: <where, where>

- Replace $\mathbf{u}_i \cdot \mathbf{v}_j$ by $\sum_{g \in n\text{-grams}(w_i)} \mathbf{u}_g \cdot \mathbf{v}_j$

- More to come! Contextualized word embeddings

(Bojanowski et al, 2017): Enriching Word Vectors with Subword Information

# Trained word embeddings available

- word2vec: https://code.google.com/archive/p/word2vec/
- GloVe: https://nlp.stanford.edu/projects/glove/
- FastText: https://fasttext.cc/

**Download pre-trained word vectors**

- Pre-trained word vectors. This data is made available under the Public Domain Dedication and License v1.0 whose full text can be found at: http://www.opendatacommons.org/licenses/pddl/1.0/.
  - Wikipedia 2014 + Gigaword 5 (6B tokens, 400K vocab, uncased, 50d, 100d, 200d, & 300d vectors, 822 MB download): glove.6B.zip
  - Common Crawl (42B tokens, 1.9M vocab, uncased, 300d vectors, 1.75 GB download): glove.42B.300d.zip
  - Common Crawl (840B tokens, 2.2M vocab, cased, 300d vectors, 2.03 GB download): glove.840B.300d.zip
  - Twitter (2B tweets, 27B tokens, 1.2M vocab, uncased, 25d, 50d, 100d, & 200d vectors, 1.42 GB download): glove.twitter.27B.zip
- Ruby script for preprocessing Twitter data

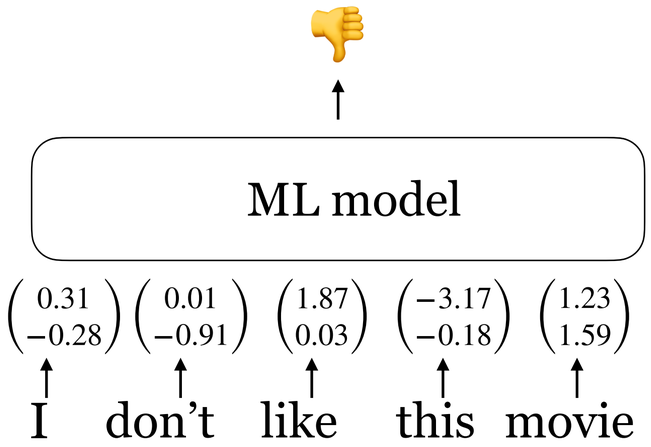
Differ in algorithms, text corpora, dimensions, cased/uncased…

# Evaluating Word Embeddings

# Extrinsic vs intrinsic evaluation

Extrinsic evaluation

- Let's plug these word embeddings into a real NLP system and see whether this improves performance
- Could take a long time but still the most important evaluation metric




Intrinsic evaluation

- Evaluate on a specific/intermediate subtask
- Fast to compute
- Not clear if it really helps the downstream task

# Intrinsic evaluation

## Word similarity

Example dataset: wordsim-353
353 pairs of words with human judgement
http://www.cs.technion.ac.il/~gabr/resources/data/wordsim353/

| Word 1 | Word 2 | Human (mean) |
|---|---|---|
| tiger | cat | 7.35 |
| tiger | tiger | 10 |
| book | paper | 7.46 |
| computer | internet | 7.58 |
| plane | car | 5.77 |
| professor | doctor | 6.62 |
| stock | phone | 1.62 |
| stock | CD | 1.31 |
| stock | jaguar | 0.92 |

Cosine similarity:

$$\cos(\boldsymbol{u}_i, \boldsymbol{u}_j) = \frac{\boldsymbol{u}_i \cdot \boldsymbol{u}_j}{||\boldsymbol{u}_i||_2 \times ||\boldsymbol{u}_j||_2}.$$

Metric: Spearman rank correlation

# Intrinsic evaluation

**Word Similarity**

| Model | Size | WS353 | MC | RG | SCWS | RW |
|---|---|---|---|---|---|---|
| SVD | 6B | 35.3 | 35.1 | 42.5 | 38.3 | 25.6 |
| SVD-S | 6B | 56.5 | 71.5 | 71.0 | 53.6 | 34.7 |
| SVD-L | 6B | 65.7 | 72.7 | 75.1 | 56.5 | 37.0 |
| CBOW† | 6B | 57.2 | 65.6 | 68.2 | 57.0 | 32.5 |
| SG† | 6B | 62.8 | 65.2 | 69.7 | 58.1 | 37.2 |
| GloVe | 6B | 65.8 | 72.7 | 77.8 | 53.9 | 38.1 |
| SVD-L | 42B | 74.0 | 76.4 | 74.1 | 58.3 | 39.9 |
| GloVe | 42B | **75.9** | **83.6** | **82.9** | **59.6** | **47.8** |
| CBOW* | 100B | 68.4 | 79.6 | 75.4 | 59.4 | 45.5 |

# Intrinsic evaluation

**Word analogy**

man: woman ≈ king: ?

$$\arg\max_i \left(\cos(\mathbf{u}_i, \mathbf{u}_b - \mathbf{u}_a + \mathbf{u}_c)\right)$$

| semantic | syntactic |
| --- | --- |
| Chicago:Illinois≈Philadelphia: ? | bad:worst ≈ cool: ? |

More examples at
http://download.tensorflow.org/data/questions-words.txt