



Natural Language Processing

Anoop Sarkar

anoopsarkar.github.io/nlp-class

Simon Fraser University

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Part 1: Word Vectors

One-hot vectors

Singular Value Decomposition

Word2Vec

GloVe

Evaluation of Word Vectors

One-hot vectors

- ▶ Let $|V|$ be the size of the vocabulary
- ▶ Assign each word to a unique index from $1 \dots |V|$
- ▶ e.g. *aarvark* is 1, *a* is 2, etc.
- ▶ Represent each word as as a $\mathbb{R}^{|V| \times 1}$
- ▶ The vector has one at index i and all other values are 0

One-hot vectors

Figure from [1]

$$w^{aardvark} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, w^a = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, w^{at} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \dots w^{zebra} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

One-hot vectors

- ▶ Problems with similarity over one-hot vectors
- ▶ Consider similarity between words as dot product between their word vectors:

$$w_{\text{cat}} \cdot w_{\text{dog}} = w_{\text{joker}} \cdot w_{\text{dog}} = 0$$

- ▶ Idea: reduce the size of the large sparse one-hot vector
- ▶ Embed large sparse vector into a dense subspace.

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Window based co-occurrence matrix

- ▶ Assume a window around each word (window size 2, 5, ...)
- ▶ Collect co-occurrence counts for each pair of words in the vocabulary.
- ▶ Create a matrix X where each element $X_{i,j} = c(w_i, w_j)$
- ▶ $c(w_i, w_j)$ is the number of times we observe word w_i and w_j together
- ▶ X is going to be very sparse (lots of zeroes)

Window based co-occurrence matrix

DocID:		Title
doc0		Human machine interface for Lab ABC computer applications
doc1		A survey of user opinion of computer system response time
doc2		The EPS user interface management system
doc3		System and human system engineering testing of EPS
doc4		Relation of user-perceived response time to error measurement
doc5		The generation of random, binary, unordered trees
doc6		The intersection graph of paths in trees
doc7		Graph minors IV: Widths of trees and well-quasi-ordering
doc8		Graph minors: A survey

Window based co-occurrence matrix

	and	minors	generation	testing	engineering	computer	relation	human	measurement
and	0	1	0	1	1	0	0	1	0
minors	1	0	0	0	0	0	0	0	0
generation	0	0	0	0	0	0	0	0	0
testing	1	0	0	0	1	0	0	1	0
engineering	1	0	0	1	0	0	0	1	0
computer	0	0	0	0	0	0	0	1	0
relation	0	0	0	0	0	0	0	0	1
human	1	0	0	1	1	1	0	0	0
measurement	0	0	0	0	0	0	1	0	0
unordered	0	0	1	0	0	0	0	0	0

Singular Value Decomposition

- ▶ Collect $X = |V| \times |V|$ word co-occurrence matrix.
- ▶ Apply SVD on X to get $X = USV^T$

Transpose

Transpose of V is V^T which switches the row and column of V

- ▶ Select first k columns of U to get k -dimensional vectors
- ▶ The matrix S is a diagonal matrix with entries

$$\sigma_1, \dots, \sigma_i, \dots, \sigma_{|V|}$$

Variance

The amount of variance captured by the first k dimensions is given by

$$\frac{\sum_{i=1}^k \sigma_i}{\sum_{i=1}^{|V|} \sigma_i}$$

Dimensionality reduction with SVD

Figure from [1]

Applying SVD to X :

$$\begin{matrix} & |V| \\ & X \\ |V| \end{matrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{matrix} & |V| \\ & u_1 & u_2 & \dots \\ |V| \end{matrix} \begin{bmatrix} | \\ | \\ \end{bmatrix} \begin{matrix} & |V| \\ \begin{bmatrix} \sigma_1 & 0 & \dots \\ 0 & \sigma_2 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} & \\ |V| \end{matrix} \begin{matrix} & |V| \\ \begin{bmatrix} - & v_1 & - \\ - & v_2 & - \\ & \vdots & \end{bmatrix} & \\ |V| \end{matrix}$$

Dimensionality reduction with SVD

Figure from [1]

Reducing dimensionality by selecting first k singular vectors:

$$\begin{matrix} & |V| \\ & \hat{X} \\ |V| \left[\begin{array}{c} \end{array} \right] & = & \begin{matrix} & k \\ & u_1 & u_2 & \dots \\ | & | & \\ | & | & \end{matrix} \left[\begin{array}{c} \end{array} \right] & \begin{matrix} k \\ \sigma_1 & 0 & \dots \\ 0 & \sigma_2 & \dots \\ \vdots & \vdots & \ddots \end{matrix} \left[\begin{array}{c} \end{array} \right] & \begin{matrix} & |V| \\ & - & v_1 & - \\ & - & v_2 & - \\ & & \vdots & \end{matrix} \left[\begin{array}{c} \end{array} \right] \end{matrix}$$

Why SVD is not the ideal solution

- ▶ Computational complexity is high $\mathcal{O}(|V|^3)$
- ▶ Cannot be trained as part of a larger model.
- ▶ It is not a component that can be part of a larger neural network
- ▶ Cannot be trained discriminatively for a particular task

One-hot vectors

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Evaluation of Word Vectors

Word2Vec

- ▶ Word2Vec is a family of model + learning algorithm
- ▶ The goal is to learn dense word vectors

Continuous bag of words

- ▶ Takes the average of the context; predicts the target word
- ▶ Trained with gradient descent on cross entropy loss for word prediction

Skip-gram

- ▶ Considers each context word independently and constructs (target-word, context-word) pairs
- ▶ Trained using negative sampling and loss on predicting good vs. bad pairs

Word2Vec: Continuous Bag of Words

CBOW

the general _____ the troops

Predicting a center word from the surrounding words
(also window-based)

For each word we want to learn two vectors:

- ▶ $v_i \in \mathbb{R}^k$ (input vector) when the word w_i is in the context
- ▶ $u_i \in \mathbb{R}^k$ (output vector) when the word u_i is in the center

Word2Vec: Continuous Bag of Words

Algorithm

the general _____ the troops

v_{the} v_{general}

v_{the} v_{troops}

- ▶ Average the context vectors:

$$\hat{v} = \frac{v_{\text{the}} + v_{\text{general}} + v_{\text{the}} + v_{\text{troops}}}{4}$$

- ▶ For each word $i \in V$ we have a word vector $u_i \in \mathbb{R}^k$
- ▶ Compute the dot product $z_i = u_i \cdot \hat{v}$
- ▶ Convert $z_i \in \mathbb{R}$ into a probability:

$$\hat{y}_i = \frac{\exp(z_i)}{\sum_{k=1}^{|V|} \exp(z_k)}$$

- ▶ If the correct center word is w_i then the max should be \hat{y}_i .

Word2Vec: Continuous Bag of Words

the general _____ the troops

v_{the} v_{general}

v_{the} v_{troops}

- ▶ Average the context vectors to get \hat{v}
- ▶ Let matrix $U = [u_1, \dots, u_{|V|}] \in \mathbb{R}^{|V| \times k}$ with word vectors $u_i \in \mathbb{R}^k$
- ▶ Compute the matrix product $z = U \cdot \hat{v}$ where $z = [z_1, \dots, z_{|V|}] \in \mathbb{R}^{|V|}$ and each $z_i \in \mathbb{R}$
- ▶ Compute vector $\hat{y} \in \mathbb{R}^{|V|}$. Each element $\hat{y}_i = \frac{\exp(z_i)}{\sum_{k=1}^{|V|} \exp(z_k)}$
- ▶ We write this as $\hat{y} = \text{softmax}(z)$
- ▶ If the correct center word is w_i then the ideal output y is a one-hot vector with index i as 1 and all other elements are 0.

Word2Vec: Continuous Bag of Words

Learning

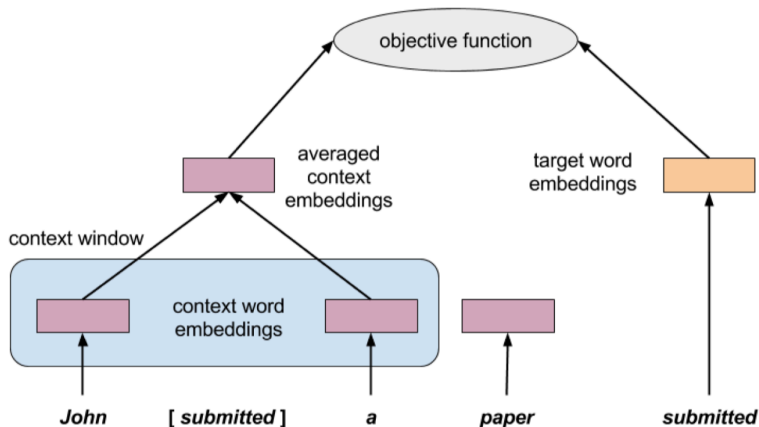
- ▶ Goal: learn k -dimensional word vectors u_i, v_i for each $i = 1, \dots, |V|$
- ▶ For each training example the correct center word w_j is represented as a one-hot vector y where $y_j = 1$.
- ▶ $\hat{y} = \text{softmax}(U \cdot \hat{v})$ where \hat{v} is the average of the context words
- ▶ Loss function is the cross entropy:

$$H(\hat{y}, y) = -\log(\hat{y}_j) \text{ for } j \text{ where } y_j = 1$$

- ▶ If c is the index of the correct word, consider case where prediction $\hat{y}_c = 0.99$ then the loss or penalty is low
 $H(\hat{y}, y) = -1 \cdot \log(0.99) = 0.01$
- ▶ If the prediction was bad $\hat{y}_c = 0.01$ then the loss is high
 $H(\hat{y}, y) = -1 \cdot \log(0.01) = 4.6$

CBOW Loss Function

Figure from [2]



Gradient descent

Objective function

$$\begin{aligned} &\text{Minimize } J \\ &= -\log P(u_c \mid \hat{v}) \\ &= -u_c \cdot \hat{v} + \log \sum_{j=1}^{|V|} \exp(u_j \cdot \hat{v}) \end{aligned}$$

Gradient descent

- ▶ Initialize $u^{(0)}$ and $v^{(0)}$
- ▶ $J(u, v) = -u_c \cdot \hat{v} + \log \sum_{j=1}^{|V|} \exp(u_j \cdot \hat{v})$
- ▶ $t \leftarrow 0$
- ▶ Iterate to minimize loss $H(\hat{y}, y)$ on each training example:
 - ▶ Pick a training example at random
 - ▶ Calculate:

$$\begin{aligned}\hat{y} &= \text{softmax}(U \cdot \hat{v}) \\ \Delta_u &= \left. \frac{dJ(u, v)}{du} \right|_{u, v=u^{(t)}, v^{(t)}} \\ \Delta_v &= \left. \frac{dJ(u, v)}{dv} \right|_{u, v=u^{(t)}, v^{(t)}}\end{aligned}$$

- ▶ Using a learning rate γ find new parameter values:

$$\begin{aligned}\mathbf{u}^{(t+1)} &\leftarrow \mathbf{u}^{(t)} - \gamma \Delta_u \\ \mathbf{v}^{(t+1)} &\leftarrow \mathbf{v}^{(t)} - \gamma \Delta_v\end{aligned}$$

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Co-occurrence matrix

Let X denote the word-word co-occurrence matrix.

X_{ij} is number of times word j occurs in the context of word i .

Let $X_i = \sum_k X_{ik}$

And $P_{ij} = P(w_j \mid w_i) = \frac{X_{ij}}{X_i}$

GloVe objective

Probability that word j occurs in context of word i :

$$Q_{ij} = \frac{\exp(u_j \cdot v_i)}{\sum_{w=1}^{|V|} \exp(u_w \cdot v_i)}$$

Compute global cross-entropy loss:

$$J = - \sum_{i=1}^{|V|} \sum_{j=1}^{|V|} X_{ij} \log Q_{ij}$$

Cross Entropy Loss

$$J = - \sum_{i=1}^{|V|} \sum_{j=1}^{|V|} \underbrace{X_{ij}}_{X_i P_{ij}} \log Q_{ij}$$

$$X_{i,j} = X_i P_{ij} \text{ because: } P_{ij} = \frac{X_{ij}}{\sum_k X_{ik}} = \frac{X_{ij}}{X_i}$$

$$J = - \sum_i X_i \underbrace{\sum_j P_{ij} \log Q_{ij}}_{H(P_i, Q_i)}$$

where H is the cross entropy of Q_{ij} which uses the parameters u, v wrt the observed frequencies P_{ij} .

Simplify objective function

In the objective $-\sum_{ij} X_{ij} \cdot P_{ij} \log Q_{ij}$ the distribution Q_{ij} requires an expensive normalization over the entire vocabulary.

Simplify J to \hat{J} using the squared error of the logs of \hat{P} and \hat{Q} without normalization:

$$\hat{J} = - \sum_{i,j=1}^{|V|} \underbrace{X_{ij}}_{\text{replace with function } f(X_{ij})} \left(\log \underbrace{\hat{Q}_{ij}}_{\exp(u_j \cdot v_i)} - \log \underbrace{\hat{P}_{ij}}_{X_{ij}} \right)^2$$

$$\hat{J} = - \sum_{ij} f(X_{ij}) (u_j \cdot v_i - \log X_{ij})^2$$

The GloVe model efficiently leverages global statistical information by training only on the nonzero elements in a word-word co-occurrence matrix.

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Evaluation of Word Vectors

Intrinsic Evaluation

- ▶ Evaluation on a specific intermediate task
- ▶ Fast to compute performance
- ▶ Helps us understand the model flaws and strengths
- ▶ However can fool us into thinking our model is good at extrinsic tasks
- ▶ *nokia* can be close to *samsung* but also to *finland* (Nokia is Finnish)

Intrinsic Evaluation

a : b :: c : ?

An intrinsic evaluation can be to identify the word vector which maximizes the cosine similarity for an analogy task:

$$d = \operatorname{argmax}_i \frac{(x_b - x_a + x_c) \cdot x_i}{\|x_b - x_a + x_c\|}$$

we identify the vector x_d which maximizes the normalized dot-product between the two word vectors (cosine similarity).

Intrinsic Evaluation

Obtain data from external source for validation e.g. geography data.

Input	Result Produced
Chicago : Illinois : : Houston	Texas
Chicago : Illinois : : Philadelphia	Pennsylvania
Chicago : Illinois : : Phoenix	Arizona
Chicago : Illinois : : Dallas	Texas
Chicago : Illinois : : Jacksonville	Florida
Chicago : Illinois : : Indianapolis	Indiana
Chicago : Illinois : : Austin	Texas
Chicago : Illinois : : Detroit	Michigan
Chicago : Illinois : : Memphis	Tennessee
Chicago : Illinois : : Boston	Massachusetts

Extrinsic Evaluation

- ▶ Evaluation on a “real” task
- ▶ Slow to compute performance
- ▶ If the word vectors fail on this task it is often unclear exactly why
- ▶ Can experiment with various training hyperparameters or model choices to improve task performance

Parameters

Some parameters we can consider tuning on intrinsic evaluation tasks:

- ▶ Dimension of word vectors
- ▶ Corpus size
- ▶ Corpus source / domain / type
- ▶ Context window size
- ▶ Context symmetry

Can you think of any other parameters to tune in a word vector model?

- [1] Christopher Manning, Richard Socher, Francois Chaubard, Michael Fang, Guillaume Genthial, Rohit Mundra.
Natural Language Processing with Deep Learning: Word Vectors I: Introduction, SVD and Word2Vec
Winter 2019.
- [2] O. Melamud and J. Goldberger and I. Dagan
context2vec: Learning Generic Context Embedding with Bidirectional LSTM.
CoNLL 2016.

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