Natural Language Processing

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Part 1: Word Vectors

One-hot vectors

Singular Value Decomposition

Word2Vec

GloVe

One-hot vectors

- Let $|V|$ be the size of the vocabulary
- Assign each word to a unique index from $1 \ldots |V|$
- e.g. *aarvark* is 1, *a* is 2, etc.
- Represent each word as as a $\mathbb{R}^{|V| \times 1}$
- The vector has one at index i and all other values are 0

One-hot vectors

Figure from [1]

$$w^{aardvark} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, w^{a} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, w^{at} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \cdots \; w^{zebra} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

One-hot vectors

- Problems with similarity over one-hot vectors
- Consider similarity between words as dot product between their word vectors:

$$w_{\mathrm{cat}} \cdot w_{\mathrm{dog}} = w_{\mathrm{joker}} \cdot w_{\mathrm{dog}} = 0$$

- Idea: reduce the size of the large sparse one-hot vector
- Embed large sparse vector into a dense subspace.

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Window based co-occurrence matrix

- Assume a window around each word (window size 2, 5, ...)
- Collect co-occurrence counts for each pair of words in the vocabulary.
- Create a matrix X where each element $X_{i,j} = c(w_i, w_j)$
- $c(w_i, w_j)$ is the number of times we observe word w_i and w_j together
- X is going to be very sparse (lots of zeroes)

Window based co-occurrence matrix

DocID:	Title
doc0	Human machine interface for Lab ABC computer applications
doc1	A survey of user opinion of computer system response time
doc2	The EPS user interface management system
doc3	System and human system engineering testing of EPS
doc4	Relation of user-perceived response time to error measurement
doc5	The generation of random, binary, unordered trees
doc6	The intersection graph of paths in trees
doc7	Graph minors IV: Widths of trees and well-quasi-ordering
doc8	Graph minors: A survey

Window based co-occurrence matrix

	and	minors	generation	testing	engineering	computer	relation	human	measurement
and	0	1	0	1	1	0	0	1	0
minors	1	0	0	0	0	0	0	0	0
generation	0	0	0	0	0	0	0	0	0
testing	1	0	0	0	1	0	0	1	0
engineering	1	0	0	1	0	0	0	1	0
computer	0	0	0	0	0	0	0	1	0
relation	0	0	0	0	0	0	0	0	1
human	1	0	0	1	1	1	0	0	0
measurement	0	0	0	0	0	0	1	0	0
unordered	0	0	1	0	0	0	0	0	0

Singular Value Decomposition

- Collect $X = |V| \times |V|$ word co-occurrence matrix.
- Apply SVD on X to get $X = USV^T$

Transpose

Transpose of V is V^T which switches the row and column of V

- Select first k columns of U to get k-dimensional vectors
- The matrix S is a diagonal matrix with entries $\sigma_1, \ldots, \sigma_i, \ldots, \sigma_{|V|}$

Variance

The amount of variance captured by the first k dimensions is given by

$$\frac{\sum_{i=1}^{k} \sigma_i}{\sum_{i=1}^{|V|} \sigma_i}$$

Dimensionality reduction with SVD

Figure from [1]

Applying SVD to X:

$$
\begin{array}{cc} & |V| \\ |V| & \begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix} \end{array}
= \begin{array}{cc} & |V| \\ |V| & \begin{bmatrix} | & | & \\ u_1 & u_2 & \cdots \\ | & | & \end{bmatrix} \end{array}
\begin{array}{cc} & |V| \\ |V| & \begin{bmatrix} \sigma_1 & 0 & \cdots \\ 0 & \sigma_2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \end{array}
\begin{array}{cc} & |V| \\ |V| & \begin{bmatrix} - & v_1 & - \\ - & v_2 & - \\ & \vdots & \end{bmatrix} \end{array}
$$

Dimensionality reduction with SVD

Figure from [1]

Reducing dimensionality by selecting first k singular vectors:

$$\begin{array}{cc} & |V| \\ |V| & \begin{bmatrix} & & \\ & \hat{X} & \\ & & \end{bmatrix}\end{array} = \begin{array}{cc} & k \\ |V| & \begin{bmatrix} | & | & \\ u_1 & u_2 & \cdots \\ | & | & \end{bmatrix}\end{array} \begin{array}{cc} & k \\ k & \begin{bmatrix} \sigma_1 & 0 & \cdots \\ 0 & \sigma_2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}\end{array} \begin{array}{cc} & |V| \\ k & \begin{bmatrix} - & v_1 & - \\ - & v_2 & - \\ & \vdots & \end{bmatrix}\end{array}$$

Why SVD is not the ideal solution

- Computational complexity is high $\mathcal{O}(|V|^3)$
- Cannot be trained as part of a larger model.
- It is not a component that can be part of a larger neural network
- Cannot be trained discriminatively for a particular task

One-hot vectors

Singular Value Decomposition

Word2Vec

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Word2Vec

- Word2Vec is a family of model + learning algorithm
- The goal is to learn dense word vectors

Continuous bag of words

- Takes the average of the context; predicts the target word
- Trained with gradient descent on cross entropy loss for word prediction

Skip-gram

- Considers each context word independently and constructs (target-word, context-word) pairs
- Trained using negative sampling and loss on predicting good vs. bad pairs

Word2Vec: Continuous Bag of Words

CBOW

the general __________ the troops

Predicting a center word from the surrounding words
(also window-based)

For each word we want to learn two vectors:

- $v_i \in \mathbb{R}^k$ (input vector) when the word w_i is in the context
- $u_i \in \mathbb{R}^k$ (output vector) when the word u_i is in the center

Word2Vec: Continuous Bag of Words

Algorithm

the general __________ the troops

$v_{\text{the}}\, v_{\text{general}}$ $v_{\text{the}}\, v_{\text{troops}}$

- Average the context vectors:
$$\hat{v} = \frac{v_{\text{the}} + v_{\text{general}} + v_{\text{the}} + v_{\text{troops}}}{4}$$
- For each word $i \in V$ we have a word vector $u_i \in \mathbb{R}^k$
- Compute the dot product $z_i = u_i \cdot \hat{v}$
- Convert $z_i \in \mathbb{R}$ into a probability:
$$\hat{y}_i = \frac{exp(z_i)}{\sum_{k=1}^{|V|} exp(z_k)}$$
- If the correct center word is w_i then the max should be $\hat{y}_i$.

Word2Vec: Continuous Bag of Words

the general ___________ the troops

$v_{\text{the}}\, v_{\text{general}}$ $v_{\text{the}}\, v_{\text{troops}}$

- Average the context vectors to get $\hat{v}$
- Let matrix $U = [u_1, \ldots, u_{|V|}] \in \mathbb{R}^{|V| \times k}$ with word vectors $u_i \in \mathbb{R}^k$
- Compute the matrix product $z = U \cdot \hat{v}$ where $z = [z_1, \ldots, z_{|V|}] \in \mathbb{R}^{|V|}$ and each $z_i \in \mathbb{R}$
- Compute vector $\hat{y} \in \mathbb{R}^{|V|}$. Each element $\hat{y}_i = \frac{exp(z_i)}{\sum_{k=1}^{|V|} exp(z_k)}$
- We write this as $\hat{y} = \mathrm{softmax}(z)$
- If the correct center word is w_i then the ideal output y is a one-hot vector with index i as 1 and all other elements are 0.

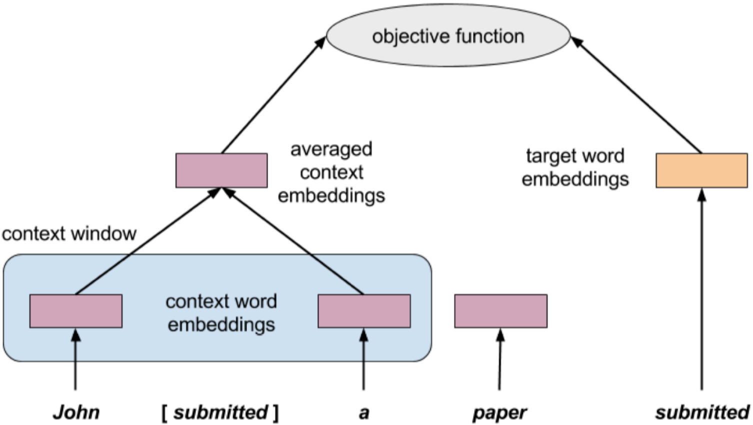
Word2Vec: Continuous Bag of Words

Learning

- Goal: learn k-dimensional word vectors u_i, v_i for each $i = 1, \ldots |V|$
- For each training example the correct center word w_j is represented as a one-hot vector y where $y_j = 1$.
- $\hat{y} = \mathrm{softmax}(U \cdot \hat{v})$ where $\hat{v}$ is the average of the context words
- Loss function is the cross entropy:
$$H(\hat{y}, y) = -\log(\hat{y}_j) \text{ for } j \text{ where } y_j = 1$$
- If c is the index of the correct word, consider case where prediction $\hat{y}_c = 0.99$ then the loss or penalty is low $H(\hat{y}, y) = -1 \cdot \log(0.99) = 0.01$
- If the prediction was bad $\hat{y}_c = 0.01$ then the loss is high $H(\hat{y}, y) = -1 \cdot \log(0.01) = 4.6$

CBOW Loss Function

Figure from [2]



Gradient descent

Objective function

$$
\begin{aligned}
&\text{Minimize } J \\
&\quad = \; -\log P(u_c \mid \hat{v}) \\
&\quad = \; -u_c \cdot \hat{v} + \log \sum_{j=1}^{|V|} exp(u_j \cdot \hat{v})
\end{aligned}
$$

Gradient descent

- Initialize $u^{(0)}$ and $v^{(0)}$
- $J(u, v) = -u_c \cdot \hat{v} + \log \sum_{j=1}^{|V|} exp(u_j \cdot \hat{v})$
- $t \leftarrow 0$
- Iterate to minimize loss $H(\hat{y}, y)$ on each training example:
  - Pick a training example at random
  - Calculate:

$$\begin{aligned} \hat{y} &= \mathrm{softmax}(U \cdot \hat{v}) \\ \Delta_u &= \left.\frac{dJ(u,v)}{du}\right|_{u,v=u^{(t)},v^{(t)}} \\ \Delta_v &= \left.\frac{dJ(u,v)}{dv}\right|_{u,v=u^{(t)},v^{(t)}} \end{aligned}$$

  - Using a learning rate γ find new parameter values:

$$\begin{aligned} \mathbf{u}^{(t+1)} &\leftarrow \mathbf{u}^{(t)} - \gamma\Delta_u \\ \mathbf{v}^{(t+1)} &\leftarrow \mathbf{v}^{(t)} - \gamma\Delta_v \end{aligned}$$

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Co-occurrence matrix

Let X denote the word-word co-occurrence matrix.
X_{ij} is number of times word j occurs in the context of word i.
Let $X_i = \sum_k X_{ik}$
And $P_{ij} = P(w_j \mid w_i) = \frac{X_{ij}}{X_i}$

GloVe objective

Probability that word j occurs in context of word i:

$$Q_{ij} = \frac{exp(u_i \cdot v_j)}{\sum_{w=1}^{|V|} exp(u_w \cdot v_i)}$$

Compute global cross-entropy loss:

$$J = -\sum_{i=1}^{|V|} \sum_{j=1}^{|V|} X_{ij} \log Q_{ij}$$

GloVe

Cross Entropy Loss

$$J = -\sum_{i=1}^{|V|} \sum_{j=1}^{|V|} \underbrace{X_{ij}}_{X_i P_{ij}} \log Q_{ij}$$

$$X_{i,j} = X_i P_{ij} \text{ because: } P_{ij} = \frac{X_{ij}}{\sum_k X_{ik}} = \frac{X_{ij}}{X_i}$$

$$J = -\sum_i X_i \underbrace{\sum_j P_{ij} \log Q_{ij}}_{H(P_i, Q_i)}$$

where H is the cross entropy of Q_{ij} which uses the parameters u, v wrt the observed frequencies P_{ij}.

GloVe

Simplify objective function

In the objective $-\sum_{ij} X_i \cdot P_{ij} \log Q_{ij}$ the distribution Q_{ij} requires an expensive normalization over the entire vocabulary.

Simplify J to $\hat{J}$ using the squared error of the logs of $\hat{P}$ and $\hat{Q}$ without normalization:

$$\hat{J} = -\sum_{i,j=1}^{|V|} \underbrace{X_i}_{\text{replace with function } f(X_{ij})} \left(\log \underbrace{\hat{Q}_{ij}}_{exp(u_i \cdot v_j)} - \log \underbrace{\hat{P}_{ij}}_{X_{ij}} \right)^2$$

$$\hat{J} = -\sum_{ij} f(X_{ij})(u_i \cdot v_j - \log X_{ij})^2$$

The GloVe model efficiently leverages global statistical information by training only on the nonzero elements in a word-word co-occurrence matrix.

[1] Christopher Manning, Richard Socher, Francois Chaubard, Michael Fang, Guillaume Genthial, Rohit Mundra.
Natural Language Processing with Deep Learning: Word Vectors I: Introduction, SVD and Word2Vec
Winter 2019.

[2] O. Melamud and J. Goldberger and I. Dagan
context2vec: Learning Generic Context Embedding with Bidirectional LSTM.
CoNLL 2016.

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