# Natural Language Processing

Anoop Sarkar
anoopsarkar.github.io/nlp-class

Simon Fraser University

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Part 1: Feedforward neural networks

Log-linear models versus Neural networks

Feedforward neural networks

Stochastic Gradient Descent

Motivating example: XOR

# Log linear model

- Let there be $m$ features, $f_k(\mathbf{x}, y)$ for $k = 1, \ldots, m$
- Define a parameter vector $\mathbf{v} \in \mathbb{R}^m$
- A log-linear model for classification into labels $y \in \mathcal{Y}$:

$$\Pr(y \mid \mathbf{x}; \mathbf{v}) = \frac{exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y)\right))}{\sum_{y' \in \mathcal{Y}} exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y')\right))}$$

### Advantages

The feature representation $\mathbf{f}(\mathbf{x}, y)$ can represent any aspect of the input that is useful for classification.

### Disadvantages

The feature representation $\mathbf{f}(\mathbf{x}, y)$ has to be designed by hand which is time-consuming and error-prone.

# Neural Networks

## Advantages

- Neural networks replace hand-engineered features with **representation learning**
- Empirical results across many different domains show that learned representations give significant improvements in accuracy
- Neural networks allow end to end training for complex NLP tasks and do not have the limitations of multiple chained pipeline models

## Disadvantages

For many tasks linear models are much faster to train compared to neural network models

# Alternative Form of Log linear model

## Log-linear model:

$$\Pr(y \mid \mathbf{x}; \mathbf{v}) = \frac{exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y)\right))}{\sum_{y' \in \mathcal{Y}} exp\left(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y')\right))}$$

## Alternative form using functions:

$$\Pr(y \mid x; v) = \frac{exp\left(v(y) \cdot f(x) + \gamma_y\right)}{\sum_{y' \in \mathcal{Y}} exp\left(v(y') \cdot f(x) + \gamma_{y'}\right))}$$

- Feature vector $f(x)$ maps input $x$ to $\mathbb{R}^d$
- Parameters $v(y) \in \mathbb{R}^d$ and $\gamma_y \in \mathbb{R}$ for each $y \in \mathcal{Y}$
- We use $v$ to refer to the parameter vectors and bias values:

$$v = \{(v(y), \gamma_y) : y \in \mathcal{Y}\}$$

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# Representation Learning: Feedforward Neural Network

Replace hand-engineered features $f$ with learned features $\phi$:

$$\Pr(y \mid x; \theta, v) = \frac{exp\left(v(y) \cdot \phi(x;\theta) + \gamma_y\right)}{\sum_{y' \in \mathcal{Y}} exp\left(v(y') \cdot \phi(x;\theta) + \gamma_{y'}\right)}$$

- Replace $f(x)$ with $\phi(x;\theta) \in \mathbb{R}^d$ where $\theta$ are new parameters
- Parameters $\theta$ are learned from training data
- Using $\theta$ the model $\phi$ maps input $x$ to $\mathbb{R}^d$: a learned representation of $x$
- $x$ is assumed to be already represented as a vector of size $d$
- We will use feedforward neural networks to define $\phi(x;\theta)$
- $\phi(x;\theta)$ will be a **non-linear** mapping to $\mathbb{R}^d$ while $f$ is a **linear** model

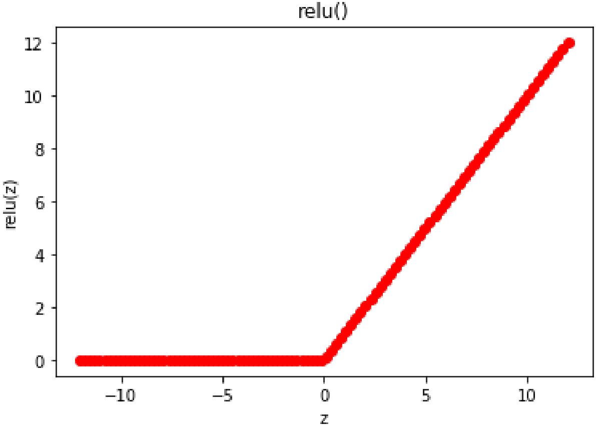
# A Single Neuron aka Perceptron

A single neuron maps input $x \in \mathbb{R}^d$ to output $h$:

$$h = g(w \cdot x + b)$$

- Weight vector $w \in \mathbb{R}^d$, a bias $b \in \mathbb{R}$ are the parameters of the model learned from training data
- Transfer function $g : \mathbb{R} \to \mathbb{R}$
- It is important that $g$ is a **non-linear** transfer function
- Linear $g(z) = \alpha \cdot z + \beta$ for constants $\alpha, \beta$ (linear perceptron)

# The ReLU Transfer Function $[0, z]$



# The ReLU Transfer Function

## Rectified Linear Unit (ReLU):

$$g(z) = \{z \text{ if } z \geq 0 \text{ or } 0 \text{ if } z < 0\}$$

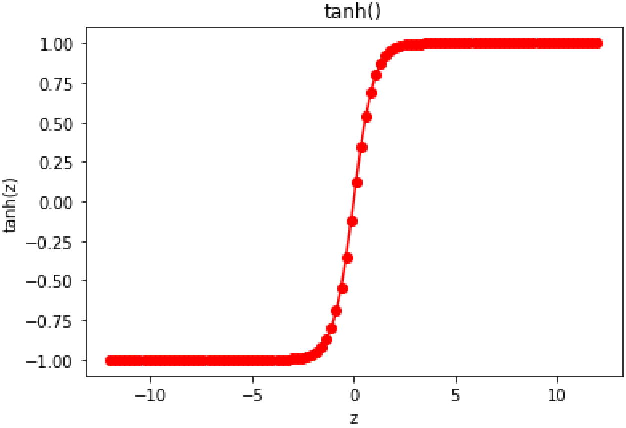
or equivalently $g(z) = \max\{0, z\}$

## Derivative of ReLU:

$$\frac{dg(z)}{dz} = \{1 \text{ if } z > 0 \text{ or } 0 \text{ if } z < 0\}$$

non-differentiable or undefined if $z = 0$
(in practice: choose a value for $z = 0$)

# The tanh Transfer Function $[-1, 1]$



# The tanh Transfer Function

tanh transfer function:

$$g(z) = \frac{e^{2z} - 1}{e^{2z} + 1}$$

Derivative of tanh:

$$\frac{dg(z)}{dz} = 1 - g(z)^2$$

# Derivatives w.r.t. parameters

## Derivatives w.r.t. $w$:

Given

$$h = g(w \cdot x + b)$$

derivatives w.r.t. $w_1, \ldots, w_j, \ldots w_d$:

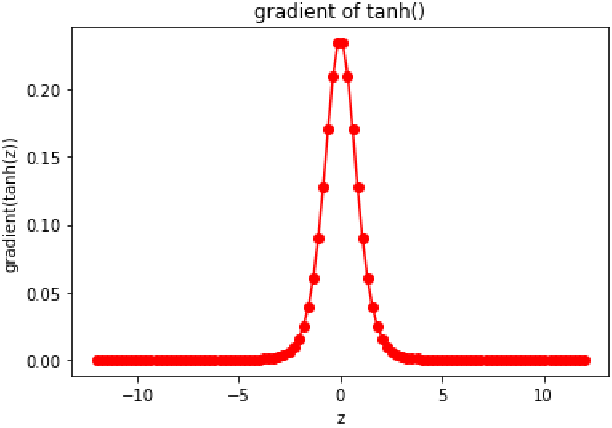
$$\frac{dh}{dw_j}$$

## Derivatives w.r.t. $b$:

derivatives w.r.t. $b$:

$$\frac{dh}{db}$$

# tanh Gradient



# Chain Rule of Differentiation

Introduce an intermediate variable $z \in \mathbb{R}$

$$z = w \cdot x + b$$

$$h = g(z)$$

Then by the chain rule to differentiate w.r.t. $w$:

$$\frac{dh}{dw_j} = \frac{dh}{dz}\frac{dz}{dw_j} = \frac{dg(z)}{dz} \times x_j$$

And similarly for $b$:

$$\frac{dh}{db} = \frac{dh}{dz}\frac{dz}{db} = \frac{dg(z)}{dz} \times 1$$

# Single Layer Feedforward model

A single layer feedforward model consists of:

- An integer $d$ specifying the input dimension. Each input to the network is $x \in \mathbb{R}^d$
- An integer $m$ specifying the number of hidden units
- A parameter matrix $W \in \mathbb{R}^{m \times d}$. The vector $W_k \in \mathbb{R}^d$ for $1 \leq k \leq m$ is the $k$th row of $W$
- A vector $b \in \mathbb{R}^d$ of bias parameters
- A transfer function $g : \mathbb{R} \to \mathbb{R}$
  $g(z) = \mathrm{ReLU}(z)$ or $g(z) = \tanh(z)$

# Single Layer Feedforward model (continued)

For $k = 1, \ldots, m$:

- The input to the $k$th neuron is: $z_k = W_k \cdot x + b_k$
- The output from the $k$th neuron is: $h_k = g(z_k)$
- Define vector $\phi(x; \theta) \in \mathbb{R}^m$ as: $\phi(x; \theta) = h_k$
- $\theta = (W, b)$ where $W \in \mathbb{R}^{m \times d}$ and $b \in \mathbb{R}^d$
- Size of $\theta$ is $m \times (d + 1)$ parameters

Some intuition

The neural network employs $m$ hidden units, each with their own parameters $W_k$ and $b_k$, and these neurons are used to construct a *hidden* representation $h \in \mathbb{R}^m$

# Matrix Form

We can replace the operation:

$$z_k = W_k \cdot x + b \text{ for } k = 1, \ldots, m$$

with

$$z = Wx + b$$

where the dimensions are as follows (vector of size $m$ equals a matrix of size $m \times 1$):

$$\underbrace{z}_{m\times 1} = \underbrace{\underbrace{W}_{m\times d}\underbrace{x}_{d\times 1}}_{m\times 1} + \underbrace{b}_{m\times 1}$$

# Single Layer Feedforward model (matrix form)

A single layer feedforward model consists of:

- An integer $d$ specifying the input dimension. Each input to the network is $x \in \mathbb{R}^d$
- An integer $m$ specifying the number of hidden units
- A parameter matrix $W \in \mathbb{R}^{m \times d}$
- A vector $b \in \mathbb{R}^d$ of bias parameters
- A transfer function $g : \mathbb{R}^m \to \mathbb{R}^m$
  $g(z) = [\ldots, \mathrm{ReLU}(z_i), \ldots]$ or $g(z) = [\ldots, \tanh(z_i), \ldots]$ for $i = 1, \ldots, m$

# Single Layer Feedforward model (matrix form, continued)

Define $\phi$ in matrix form:

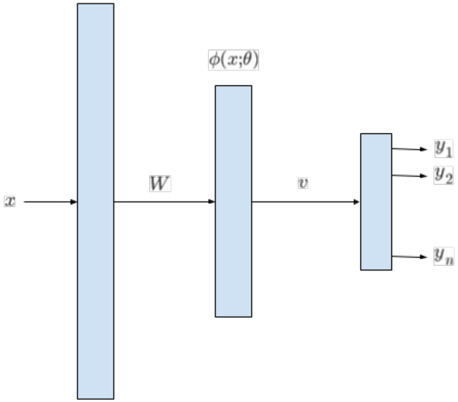
- Vector of inputs to the hidden layer $z \in \mathbb{R}^m$: $z = Wx + b$
- Vector of outputs from hidden layer $h \in \mathbb{R}^m$: $h = g(z)$
- Define $\phi(x; \theta) = h$ where $\theta = (W, b)$

$$\phi(x; \theta) = g(Wx + b)$$

Putting it all together:

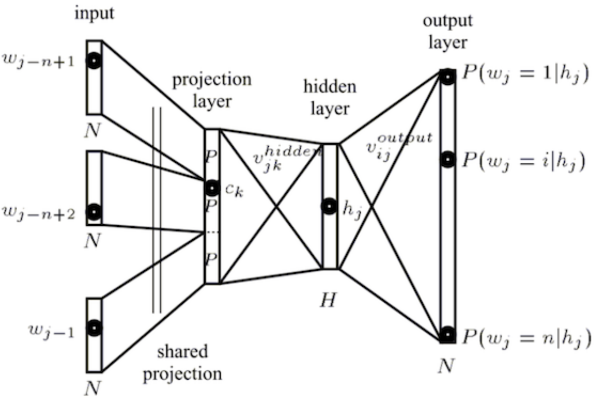
$$\Pr(y \mid x; \theta, v) = \frac{exp\left(v(y) \cdot \phi(x; \theta) + \gamma_y\right)}{\sum_{y' \in \mathcal{Y}} exp\left(v(y') \cdot \phi(x; \theta) + \gamma_{y'}\right)}$$

# Feedforward neural network



# n-gram Feedforward neural network

(Bengio and Schwenk 2013)



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# Simple stochastic gradient descent

Inputs:

- Training examples $(x^i, y^i)$ for $i = 1, \ldots, n$
- A feedforward representation $\phi(x; \theta)$
- Integer $T$ specifying the number of updates
- A sequence of learning rates: $\eta^1, \ldots, \eta^T$ where $\eta^t > 0$

Initialization:
Set $v = (v(y), \gamma_y)$ for all $y$, and $\theta$ to random values

# Gradient descent

## Algorithm:

- For $t = 1, \ldots, T$
  - Select an integer $i$ uniformly at random from $\{1, \ldots, n\}$
  - Define $L(\theta, v) = -\log P(y_i \mid x_i; \theta, v)$
  - For each parameter $\theta_j$ and $v_k(y)$ and $\gamma_y$ (for each label $y$):

$$\begin{aligned} \theta_j &= \theta_j - \eta^t \times \frac{dL(\theta, v)}{d\theta_j} \\ v_k(y) &= v_k(y) - \eta^t \times \frac{dL(\theta, v)}{dv_k(y)} \\ \gamma(y) &= \gamma(y) - \eta^t \times \frac{dL(\theta, v)}{d\gamma(y)} \end{aligned}$$

- **Output**: parameters $\theta$, $v = (v(y), \gamma_y)$ for all $y$

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# Motivating example: the XOR problem

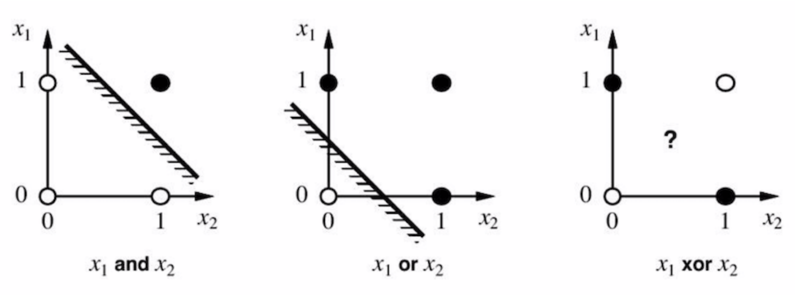
From *Deep Learning* by Goodfellow, Bengio, Courville

We will assume a training set where each label is in the set $\mathcal{Y} = \{-1, +1\}$

There are four training examples:

$$\begin{aligned} x^1 &= [0,0], y^1 = -1 \\ x^2 &= [0,1], y^2 = +1 \\ x^3 &= [1,0], y^3 = +1 \\ x^4 &= [1,1], y^4 = -1 \end{aligned}$$

# Motivating example: the XOR problem



# Motivating example: the XOR problem

## Theorem

For examples $(x^i, y^i)$ for $i = 1, \ldots, 4$ as defined previously for the feedforward neural network:

$$\Pr(y \mid x; W, b, v) = \frac{exp\,(v(y) \cdot g(Wx + b) + \gamma_y)}{\sum_{y' \in \mathcal{Y}} exp\,(v(y') \cdot g(Wx + b) + \gamma_{y'}))}$$

where $x \in \mathbb{R}^2$ ($d = 2$) and let $m = 2$ so $W \in \mathbb{R}^{2\times 2}$ and $b \in \mathbb{R}^2$ and $g$ is a ReLU transfer function.

Then there are parameter settings $v(-1)$, $v(+1)$, $\gamma_{-1}$, $\gamma_{+1}$, $W, b$ such that

$$p(y^i \mid x^i; v) > 0.5 \text{ for } i = 1, \ldots, 4$$

# Motivating example: the XOR problem

### Proof Sketch

Define $W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ Then for each input $x$ calculate values of $z = Wx + b$ and $h = g(z)$:

$$\begin{aligned} x = [0, 0] \quad &\Rightarrow \quad z = [0, -1] \Rightarrow h = [0, 0] \\ x = [1, 0] \quad &\Rightarrow \quad z = [1, 0] \Rightarrow h = [1, 0] \\ x = [0, 1] \quad &\Rightarrow \quad z = [1, 0] \Rightarrow h = [1, 0] \\ x = [1, 1] \quad &\Rightarrow \quad z = [2, 1] \Rightarrow h = [2, 1] \end{aligned}$$

# Motivating example: the XOR problem

## Proof Sketch (continued)

$$\begin{aligned} p(+1 \mid x; v) &= \frac{exp(v(+1) \cdot h + \gamma_{+1})}{exp(v(+1) \cdot h + \gamma_{+1}) + exp(v(-1) \cdot h + \gamma_{-1})} \\ &= \frac{1}{1 + exp(-(u \cdot h + \gamma))} \end{aligned}$$

To satisfy $P(y^i \mid x^i; v) > 0.5$ for $i = 1, \ldots, 4$ we have to find parameters $u = v(+1) - v(-1)$ and $\gamma = \gamma_{+1} - \gamma_{-1}$ such that:

$$\begin{aligned} u \cdot [0,0] + \gamma &< 0 \\ u \cdot [1,0] + \gamma &> 0 \\ u \cdot [1,0] + \gamma &> 0 \\ u \cdot [2,1] + \gamma &< 0 \end{aligned}$$

$u = [1, -2]$ and $\gamma = -0.5$ satisfies these constraints.

## Acknowledgements

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