



Natural Language Processing

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Part 1: Probability and Language

Probability and Language

Quick guide to probability theory

Entropy and Information Theory

Probability and Language

Assign a probability to an input sequence

Given a URL: choosespain.com. What is this website about?

Input	Scoring function
choose spain	-8.35
chooses pain	-9.88
⋮	⋮

The Goal

Find a good **scoring function** for input sequences.

Scoring Hypotheses in Speech Recognition

From acoustic signal to candidate transcriptions

Hypothesis	Score
the station signs are in deep in english	-14732
the stations signs are in deep in english	-14735
the station signs are in deep into english	-14739
the station 's signs are in deep in english	-14740
the station signs are in deep in the english	-14741
the station signs are indeed in english	-14757
the station 's signs are indeed in english	-14760
the station signs are indians in english	-14790
the station signs are indian in english	-14799
the stations signs are indians in english	-14807
the stations signs are indians and english	-14815

Scoring Hypotheses in Machine Translation

From source language to target language candidates

Hypothesis	Score
we must also discuss a vision .	-29.63
we must also discuss on a vision .	-31.58
it is also discuss a vision .	-31.96
we must discuss on greater vision .	-36.09
⋮	⋮

Scoring Hypotheses in Decryption

Character substitutions on ciphertext to plaintext candidates

Hypothesis	Score
Heopaj, zk ukq swjp pk gjks w oaynap?	-93
Urbcnw, mx hxd fjwc cx twxf j bnanc?	-92
Wtdepy, oz jzf hlye ez vyzh l dpncpe?	-91
Mjtufo, ep zpv xbou up lopx b tfdsfu?	-89
Nkuvgp, fq aqw ycpv vq mpqy c ugetgv?	-87
Gdnozi, yj tjp rvio oj fijr v nzxmzo?	-86
Czjkve, uf pfl nrek kf befn r jvtivk?	-85
Yvfgra, qb lbh jnag gb xabj n frperg?	-84
Zwghsb, rc mci kobh hc ybck o gsqfsh?	-83
Byijud, te oek mqdj je adem q iushuj?	-77
Jgqrcl, bm wms uylr rm ilmu y qcapcr?	-76
Listen, do you want to know a secret?	-25

The Goal

- ▶ Write down a **model** over sequences of words or letters.
- ▶ **Learn** the parameters of the model from data.
- ▶ Use the model to **predict** the probability of new sequences.

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Part 2: Quick guide to probability theory

Probability and Language

Quick guide to probability theory

Entropy and Information Theory

Probability: The Basics

- ▶ Sample space
- ▶ Event space
- ▶ Random variable

Probability distributions

- ▶ $P(X)$: probability of random variable X having a certain value.
 - ▶ $P(X = \text{killer}) = 1.05\text{e-}05$
 - ▶ $P(X = \text{app}) = 1.19\text{e-}05$

Joint probability

- ▶ $P(X,Y)$: probability that X and Y each have a certain value.
 - ▶ Let Y stand for choice of a word
 - ▶ Let X stand for the choice of a word that occurs before Y
 - ▶ $P(X = \text{killer}, Y = \text{app}) = 1.24\text{e-}10$

Joint Probability: $P(X=\text{value AND } Y=\text{value})$

- ▶ Since $X=\text{value AND } Y=\text{value}$, the order does not matter
- ▶ $P(X = \text{killer}, Y = \text{app}) \Leftrightarrow P(Y = \text{app}, X = \text{killer})$
- ▶ In both cases it is $P(X,Y) = P(Y,X) = P(\text{'killer app'})$
- ▶ In NLP, we often use numerical indices to express this:
 $P(W_{i-1} = \text{killer}, W_i = \text{app})$

Joint probability

Joint probability table

W_{i-1}	$W_i = \text{app}$	$P(W_{i-1}, W_i)$
$\langle S \rangle$	app	1.16e-19
an	app	1.76e-08
killer	app	1.24e-10
the	app	2.68e-07
this	app	3.74e-08
your	app	2.39e-08

There will be a similar table for each choice of W_i .

Get $P(W_i)$ from $P(W_{i-1}, W_i)$

$$P(W_i = \text{app}) = \sum_x P(W_{i-1} = x, W_i = \text{app}) = 1.19e - 05$$

Conditional probability

- ▶ $P(W_i \mid W_{i-1})$: probability that W_i has a certain value after fixing value of W_{i-1} .
- ▶ $P(W_i = \text{app} \mid W_{i-1} = \text{killer})$
- ▶ $P(W_i = \text{app} \mid W_{i-1} = \text{the})$

Conditional probability from Joint probability

$$P(W_i \mid W_{i-1}) = \frac{P(W_{i-1}, W_i)}{P(W_{i-1})}$$

- ▶ $P(\text{killer}) = 1.05\text{e-}5$
- ▶ $P(\text{killer}, \text{app}) = 1.24\text{e-}10$
- ▶ $P(\text{app} \mid \text{killer}) = 1.18\text{e-}5$

Basic Terms

- ▶ $P(e)$ – *a priori* probability or just *prior*
- ▶ $P(f \mid e)$ – *conditional* probability. The chance of f given e
- ▶ $P(e, f)$ – *joint* probability. The chance of e and f both happening.
- ▶ If e and f are *independent* then we can write
$$P(e, f) = P(e) \times P(f)$$
- ▶ If e and f are not *independent* then we can write
$$P(e, f) = P(e) \times P(f \mid e)$$
$$P(e, f) = P(f) \times ?$$

Basic Terms

- Addition of integers:

$$\sum_{i=1}^n i = 1 + 2 + 3 + \dots + n$$

- Product of integers:

$$\prod_{i=1}^n i = 1 \times 2 \times 3 \times \dots \times n$$

- Factoring:

$$\sum_{i=1}^n i \times k = k + 2k + 3k + \dots + nk = k \sum_{i=1}^n i$$

- Product with constant:

$$\prod_{i=1}^n i \times k = 1k \times 2k \dots \times nk = k^n \times \prod_{i=1}^n i$$

Probability: Axioms

- ▶ P measures total probability of a set of events
- ▶ $P(\emptyset) = 0$
- ▶ $P(\text{all events}) = 1$
- ▶ $P(X) \leq P(Y)$ for any $X \subseteq Y$
- ▶ $P(X) + P(Y) = P(X \cup Y)$ provided that $X \cap Y = \emptyset$

Probability Axioms

- ▶ All events sum to 1:

$$\sum_e P(e) = 1$$

- ▶ Marginal probability $P(f)$:

$$P(f) = \sum_e P(e, f)$$

- ▶ Conditional probability:

$$\sum_e P(e \mid f) = \sum_e \frac{P(e, f)}{P(f)} = \frac{1}{P(f)} \sum_e P(e, f) = 1$$

- ▶ Computing $P(f)$ from axioms:

$$P(f) = \sum_e P(e) \times P(f \mid e)$$

Probability: The Chain Rule

- ▶ $P(a, b, c, d \mid e)$
- ▶ We can simplify this using the Chain Rule:
- ▶ $P(a, b, c, d \mid e) =$
 $P(d \mid e) \cdot P(c \mid d, e) \cdot P(b \mid c, d, e) \cdot P(a \mid b, c, d, e)$
- ▶ To see why this is possible, recall that $P(X \mid Y) = \frac{p(X, Y)}{p(Y)}$
 - ▶ $\frac{p(a, b, c, d, e)}{p(e)} = \frac{p(d, e)}{p(e)} \cdot \frac{p(c, d, e)}{p(d, e)} \cdot \frac{p(b, c, d, e)}{p(c, d, e)} \cdot \frac{p(a, b, c, d, e)}{p(b, c, d, e)}$
- ▶ We can approximate the probability by removing some random variables from the context. For example, we can keep at most two variables to get:

$$P(a, b, c, d \mid e) \approx P(d \mid e) \cdot P(c \mid d, e) \cdot P(b \mid c, e) \cdot P(a \mid b, e)$$

Probability: The Chain Rule

► $P(e_1, e_2, \dots, e_n) = P(e_1) \times P(e_2 \mid e_1) \times P(e_3 \mid e_1, e_2) \dots$

$$P(e_1, e_2, \dots, e_n) = \prod_{i=1}^n P(e_i \mid e_{i-1}, e_{i-2}, \dots, e_1)$$

► In NLP, we call:

- $P(e_i)$: unigram probability
- $P(e_i \mid e_{i-1})$: bigram probability
- $P(e_i \mid e_{i-1}, e_{i-2})$: trigram probability
- $P(e_i \mid e_{i-1}, e_{i-2}, \dots, e_{i-(n-1)})$: n-gram probability

Probability: Random Variables and Events

- ▶ What is y in $P(y)$?
- ▶ Shorthand for value assigned to a random variable Y , e.g.
 $Y = y$
- ▶ y is an element of some implicit **event space**: \mathcal{E}

Probability: Random Variables and Events

- ▶ The *marginal probability* $P(y)$ can be computed from $P(x, y)$ as follows:

$$P(y) = \sum_{x \in \mathcal{E}} P(x, y)$$

- ▶ Finding the value that maximizes the probability value:

$$\hat{x} = \arg \max_{x \in \mathcal{E}} P(x)$$

Log Probability Arithmetic

- ▶ Practical problem with tiny $P(e)$ numbers: underflow
- ▶ One solution is to use log probabilities:

$$\begin{aligned}\log(P(e)) &= \log(p_1 \times p_2 \times \dots \times p_n) \\ &= \log(p_1) + \log(p_2) + \dots + \log(p_n)\end{aligned}$$

- ▶ Note that:

$$x = \exp(\log(x))$$

- ▶ Also more efficient: addition instead of multiplication

Log Probability Arithmetic

p	$\log(p)$
0.0	$-\infty$
0.1	-3.32
0.2	-2.32
0.3	-1.74
0.4	-1.32
0.5	-1.00
0.6	-0.74
0.7	-0.51
0.8	-0.32
0.9	-0.15
1.0	0.00

Log Probability Arithmetic

- ▶ So: $(0.5 \times 0.5 \times \dots 0.5) = (0.5)^n$ might get too small but $(-1 - 1 - 1 - 1) = -n$ is manageable
- ▶ Another useful fact when writing code (\log_2 is *log to the base 2*):

$$\log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)}$$

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Part 3: Entropy and Information Theory

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Quick guide to probability theory

Entropy and Information Theory

Information Theory

- ▶ Information theory is the use of probability theory to quantify and measure “information”.
- ▶ Consider the task of efficiently sending a message. Sender Alice wants to send several messages to Receiver Bob. Alice wants to do this as efficiently as possible.
- ▶ Let's say that Alice is sending a message where the entire message is just one character a , e.g. $aaaa\dots$. In this case we can save space by simply sending the length of the message and the single character.

Information Theory

- ▶ Now let's say that Alice is sending a completely random signal to Bob. If it is random then we cannot exploit anything in the message to compress it any further.
- ▶ The *expected* number of bits it takes to transmit some infinite set of messages is what is called entropy.
- ▶ This formulation of entropy by Claude Shannon was adapted from thermodynamics, converting information into a quantity that can be measured.
- ▶ Information theory is built around this notion of message compression as a way to evaluate the amount of information.

Expectation

- ▶ For a probability distribution p
- ▶ **Expectation** with respect to p is a weighted average:

$$\begin{aligned}E_p[x] &= \frac{x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + \dots + x_n p(x_n)}{p(x_1) + p(x_2) + \dots + p(x_n)} \\&= x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + \dots + x_n p(x_n) \\&= \sum_{x \in \mathcal{E}} x \cdot p(x)\end{aligned}$$

- ▶ Example: for a six-sided die the expectation is:

$$E_p[x] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots + 6 \cdot \frac{1}{6} = 3.5$$

Entropy

- ▶ For a probability distribution p
- ▶ **Entropy** of p is:

$$H(p) = - \sum_{x \in \mathcal{E}} p(x) \cdot \log_2 p(x)$$

- ▶ Any base can be used for the log, but base 2 means that entropy is measured in bits.
- ▶ What is the *expected* number of bits with respect to p :

$$-E_p[\log_2 p(x)] = H(p)$$

- ▶ Entropy answers the question: *What is the expected number of bits needed to transmit messages from event space \mathcal{E} , where $p(x)$ defines the probability of observing x ?*

Perplexity

- ▶ The value $2^{H(p)}$ is called the **perplexity** of a distribution p
- ▶ Perplexity is the weighted average number of choices a random variable has to make.
- ▶ Choosing between 8 equally likely options ($H=3$) is $2^3 = 8$.

Relative Entropy

- ▶ We often wish to determine the divergence of a distribution q from another distribution p
- ▶ Let's say q is the estimate and p is the true probability
- ▶ We define the *divergence* from q to p as the **relative entropy**: written as $D(p\|q)$

$$D(p\|q) = - \sum_{x \in \mathcal{E}} p(x) \log_2 \frac{q(x)}{p(x)}$$

- ▶ Note that

$$D(p\|q) = -E_{p(x)} \left[\log_2 \frac{q(x)}{p(x)} \right]$$

- ▶ The relative entropy is also called the *Kullback-Leibler divergence*.

Cross Entropy and Relative Entropy

- ▶ The **relative entropy** can be written as the sum of two terms:

$$\begin{aligned} D(p\|q) &= - \sum_{x \in \mathcal{E}} p(x) \log_2 \frac{q(x)}{p(x)} \\ &= - \sum_x p(x) \log_2 q(x) + \sum_x p(x) \log_2 p(x) \end{aligned}$$

- ▶ We know that $H(p) = - \sum_x p(x) \log_2 p(x)$
- ▶ Similarly define $H(p, q) = - \sum_x p(x) \log_2 q(x)$

$$\begin{aligned} D(p\|q) &= H(p, q) - H(p) \\ \text{relative entropy}(p, q) &= \text{cross entropy}(p, q) - \text{entropy}(p) \end{aligned}$$

- ▶ The term $H(p, q)$ is called the **cross entropy**.

Cross Entropy and Relative Entropy

- ▶ $H(p, q) \geq H(p)$ always.
- ▶ $D(p\|q) \geq 0$ always, and $D(p\|q) = 0$ iff $p = q$
- ▶ $D(p\|q)$ is not a true distance:
 - ▶ It is asymmetric: $D(p\|q) \neq D(q\|p)$,
 - ▶ It does not obey the triangle inequality:
 $D(p\|q) \not\leq D(p\|r) + D(r\|q)$

Conditional Entropy and Mutual Information

- ▶ *Entropy* of a random variable X :

$$H(X) = - \sum_{x \in \mathcal{E}} p(x) \log_2 p(x)$$

- ▶ *Conditional Entropy* between two random variables X and Y :

$$H(X | Y) = - \sum_{x,y \in \mathcal{E}} p(x,y) \log_2 p(x | y)$$

- ▶ *Mutual Information* between two random variables X and Y :

$$I(X; Y) = D(p(x,y) \| p(x)p(y)) = \sum_x \sum_y p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}$$

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