Context-free Grammars: In-class Exercise

(1) Consider the CFG G with S' as the start symbol:

$$\begin{array}{rcl} S' & \to & S \mid \epsilon \\ S & \to & T \mid (\; N \; , \; C \;) \\ C & \to & C \; , \; S \mid S \\ T & \to & a \mid b \mid c \\ N & \to & x \mid y \mid z \end{array}$$

a. List the set of terminal symbols and the set of non-terminal symbols in G.

b. For each of the following strings, write down `true` if the string is in the language $L(G)$ generated by G, `false` otherwise.

1. `y`
2. `c`
3. `(x)`
4. `(x,y)`
5. `(z,a,b,a,b,c)`
6. `(x,a,(y,b),c)`
7. `(x,(y,a),(z,b))`
8. `(x,(x,(x,(x,a))`

(2) Consider the family of CFGs G_k with S as the start symbol and k is some arbitrary non-zero positive integer such that $G_1, G_2, G_3, \ldots$ are individual CFGs with the rules:

$$\begin{array}{rcl} S & \to & A \; B \\ B & \to & C \; A \; A \\ C & \to & c \\ A & \to & a_i \quad \text{defines } i \text{ rules, where } i \in [1, k] \end{array}$$

For example, in G_3 the rules with A as left-hand side are: $A \to a_1 \mid a_2 \mid a_3$ with three terminal symbols.

a. Provide the number of terminal symbols in a grammar G_k.

b. If the string $a_4 c a_3 a_2$ is accepted by grammar G_3 then provide a derivation for it.

c. If the string $a_4 c a_3 a_2$ is accepted by grammar G_4 then provide a derivation for it.

d. Provide the total number of strings that can be generated for a grammar G_k.

(3) One of the rules in the CFG below is redundant: any sentence that can be generated using this rule can already be generated by a combination of other rules. Write down the redundant rule.

S	→	NP VP	IV	→	runs	N	→	John
NP	→	N	IV	→	sits	N	→	he
NP	→	D N	TV	→	chases	N	→	Mary
VP	→	VP PP	TV	→	eats	N	→	dog
VP	→	VP CONJ VP	TV	→	catches	N	→	tree
VP	→	IV	TV	→	tells	N	→	squirrel
VP	→	IV PP	TV	→	sees	D	→	the
VP	→	TV NP	CONJ	→	and			
VP	→	TV C S	C	→	that			
NP	→	NP CONJ NP	P	→	in			
PP	→	P	P	→	away			
PP	→	P NP						