



# Natural Language Processing

Anoop Sarkar

[anoopsarkar.github.io/nlp-class](https://anoopsarkar.github.io/nlp-class)

Simon Fraser University

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Part 1: Feedforward neural networks

## Log-linear models versus Neural networks

Feedforward neural networks

Stochastic Gradient Descent

Motivating example: XOR

## Log linear model

- ▶ Let there be  $m$  features,  $f_k(\mathbf{x}, y)$  for  $k = 1, \dots, m$
- ▶ Define a parameter vector  $\mathbf{v} \in \mathbb{R}^m$
- ▶ A log-linear model for classification into labels  $y \in \mathcal{Y}$ :

$$\Pr(y \mid \mathbf{x}; \mathbf{v}) = \frac{\exp(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y))}{\sum_{y' \in \mathcal{Y}} \exp(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y'))}$$

### Advantages

The feature representation  $\mathbf{f}(\mathbf{x}, y)$  can represent any aspect of the input that is useful for classification.

### Disadvantages

The feature representation  $\mathbf{f}(\mathbf{x}, y)$  has to be designed by hand which is time-consuming and error-prone.

# Neural Networks

## Advantages

- ▶ Neural networks replace hand-engineered features with **representation learning**
- ▶ Empirical results across many different domains show that learned representations give significant improvements in accuracy
- ▶ Neural networks allow end to end training for complex NLP tasks and do not have the limitations of multiple chained pipeline models

## Disadvantages

For many tasks linear models are much faster to train compared to neural network models

# Alternative Form of Log linear model

Log-linear model:

$$\Pr(y \mid \mathbf{x}; \mathbf{v}) = \frac{\exp(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y))}{\sum_{y' \in \mathcal{Y}} \exp(\mathbf{v} \cdot \mathbf{f}(\mathbf{x}, y'))}$$

Alternative form using functions:

$$\Pr(y \mid x; v) = \frac{\exp(v(y) \cdot f(x) + \gamma_y)}{\sum_{y' \in \mathcal{Y}} \exp(v(y') \cdot f(x) + \gamma_{y'})}$$

- ▶ Feature vector  $f(x)$  maps input  $x$  to  $\mathbb{R}^d$
- ▶ Parameters  $v(y) \in \mathbb{R}^d$  and  $\gamma_y \in \mathbb{R}$  for each  $y \in \mathcal{Y}$
- ▶ We assume  $v(y) \cdot f(x)$  is a dot product. Using matrix multiplication it would be  $v(y) \cdot f(x)^T$
- ▶ Let  $v = \{(v(y), \gamma_y) : y \in \mathcal{Y}\}$

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# Representation Learning: Feedforward Neural Network

Replace hand-engineered features  $f$  with learned features  $\phi$ :

$$\Pr(y \mid x; \theta, v) = \frac{\exp(v(y) \cdot \phi(x; \theta) + \gamma_y)}{\sum_{y' \in \mathcal{Y}} \exp(v(y') \cdot \phi(x; \theta) + \gamma_{y'})}$$

- ▶ Replace  $f(x)$  with  $\phi(x; \theta) \in \mathbb{R}^d$  where  $\theta$  are new parameters
- ▶ Parameters  $\theta$  are learned from training data
- ▶ Using  $\theta$  the model  $\phi$  maps input  $x$  to  $\mathbb{R}^d$ : a learned representation of  $x$
- ▶  $x$  is assumed to be already represented as a vector of size  $d$
- ▶ We will use feedforward neural networks to define  $\phi(x; \theta)$
- ▶  $\phi(x; \theta)$  will be a **non-linear** mapping to  $\mathbb{R}^d$  while  $f$  is a **linear** model



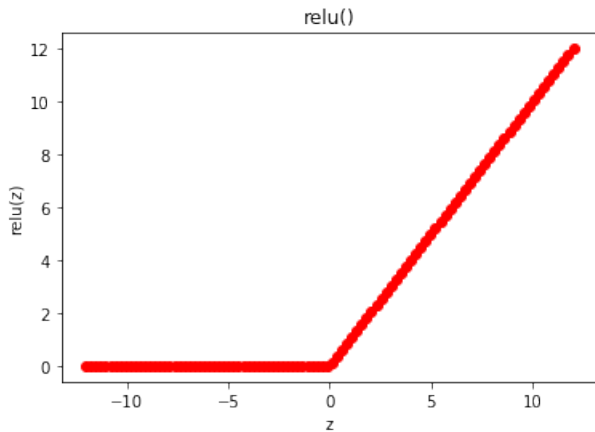
# A Single Neuron aka Perceptron

A single neuron maps input  $x \in \mathbb{R}^d$  to output  $h$ :

$$h = g(w \cdot x + b)$$

- ▶ Weight vector  $w \in \mathbb{R}^d$ , a bias  $b \in \mathbb{R}$  are the parameters of the model learned from training data
- ▶ Transfer function  $g : \mathbb{R} \rightarrow \mathbb{R}$
- ▶ It is important that  $g$  is a **non-linear** transfer function
- ▶ Linear  $g(z) = \alpha \cdot z + \beta$  for constants  $\alpha, \beta$  (linear perceptron)

# The ReLU Transfer Function $[0, z]$



# The ReLU Transfer Function

Rectified Linear Unit (ReLU):

$$g(z) = \{z \text{ if } z \geq 0 \text{ or } 0 \text{ if } z < 0\}$$

or equivalently  $g(z) = \max\{0, z\}$

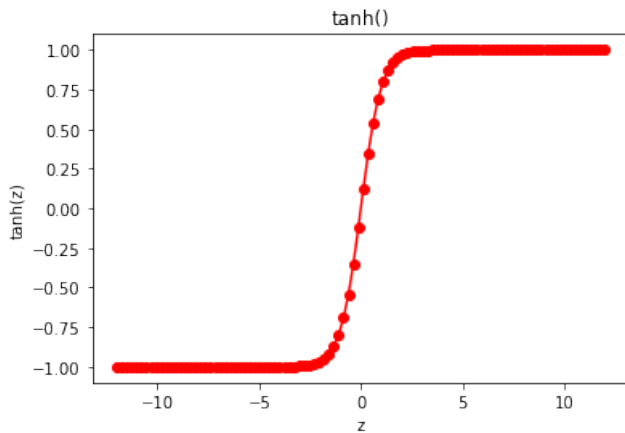
Derivative of ReLU:

$$\frac{dg(z)}{dz} = \{1 \text{ if } z > 0 \text{ or } 0 \text{ if } z < 0\}$$

non-differentiable or undefined if  $z = 0$

(in practice: choose a value for  $z = 0$ )

# The tanh Transfer Function $[-1, 1]$



# The tanh Transfer Function

tanh transfer function:

$$g(z) = \frac{e^{2z} - 1}{e^{2z} + 1}$$

Derivative of tanh:

$$\frac{dg(z)}{dz} = 1 - g(z)^2$$

# Derivatives w.r.t. parameters

## Derivatives w.r.t. $w$ :

Given

$$h = g(w \cdot x + b)$$

derivatives w.r.t.  $w_1, \dots, w_j, \dots, w_d$ :

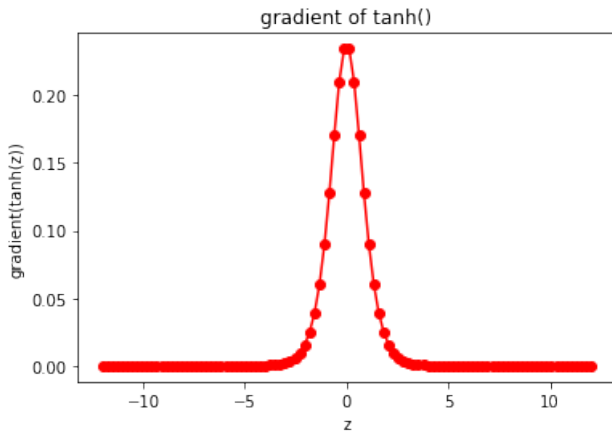
$$\frac{dh}{dw_j}$$

## Derivatives w.r.t. $b$ :

derivatives w.r.t.  $b$ :

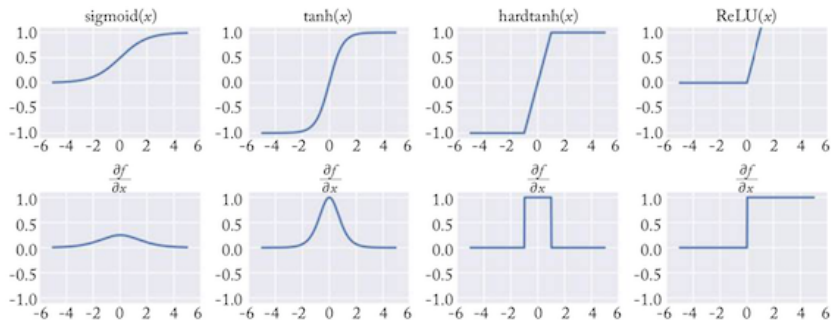
$$\frac{dh}{db}$$

## tanh Gradient



# Activation Functions and their Gradients

from Goldberg 2017, Fig. 4.3





# Chain Rule of Differentiation

Introduce an intermediate variable  $z \in \mathbb{R}$

$$z = w \cdot x + b$$

$$h = g(z)$$

Then by the chain rule to differentiate w.r.t.  $w$ :

$$\frac{dh}{dw_j} = \frac{dh}{dz} \frac{dz}{dw_j} = \frac{dg(z)}{dz} \times x_j$$

And similarly for  $b$ :

$$\frac{dh}{db} = \frac{dh}{dz} \frac{dz}{db} = \frac{dg(z)}{dz} \times 1$$

# Single Layer Feedforward model

A single layer feedforward model consists of:

- ▶ An integer  $d$  specifying the input dimension. Each input to the network is  $x \in \mathbb{R}^d$
- ▶ An integer  $m$  specifying the number of hidden units
- ▶ A parameter matrix  $W \in \mathbb{R}^{m \times d}$ . The vector  $W_k \in \mathbb{R}^d$  for  $1 \leq k \leq m$  is the  $k$ th row of  $W$
- ▶ A vector  $b \in \mathbb{R}^d$  of bias parameters
- ▶ A transfer function  $g : \mathbb{R} \rightarrow \mathbb{R}$   
 $g(z) = \text{ReLU}(z)$  or  $g(z) = \tanh(z)$

# Single Layer Feedforward model (continued)

For  $k = 1, \dots, m$ :

- ▶ The input to the  $k$ th neuron is:  $z_k = W_k \cdot x + b_k$
- ▶ The output from the  $k$ th neuron is:  $h_k = g(z_k)$
- ▶ Define vector  $\phi(x; \theta) \in \mathbb{R}^m$  as:  $\phi(x; \theta) = h_k$
- ▶  $\theta = (W, b)$  where  $W \in \mathbb{R}^{m \times d}$  and  $b \in \mathbb{R}^d$
- ▶ Size of  $\theta$  is  $m \times (d + 1)$  parameters

## Some intuition

The neural network employs  $m$  hidden units, each with their own parameters  $W_k$  and  $b_k$ , and these neurons are used to construct a *hidden* representation  $h \in \mathbb{R}^m$

# Matrix Form

We can replace the operation:

$$z_k = W_k \cdot x + b \text{ for } k = 1, \dots, m$$

with

$$z = Wx + b$$

where the dimensions are as follows (vector of size  $m$  equals a matrix of size  $m \times 1$ ):

$$\underbrace{z}_{m \times 1} = \underbrace{W}_{m \times d} \underbrace{x}_{d \times 1} + \underbrace{b}_{m \times 1}$$

$\underbrace{\hspace{10em}}_{m \times 1}$

# Single Layer Feedforward model (matrix form)

A single layer feedforward model consists of:

- ▶ An integer  $d$  specifying the input dimension. Each input to the network is  $x \in \mathbb{R}^d$
- ▶ An integer  $m$  specifying the number of hidden units
- ▶ A parameter matrix  $W \in \mathbb{R}^{m \times d}$
- ▶ A vector  $b \in \mathbb{R}^d$  of bias parameters
- ▶ A transfer function  $g : \mathbb{R}^m \rightarrow \mathbb{R}^m$   
 $g(z) = [\dots, \text{ReLU}(z_i), \dots]$  or  $g(z) = [\dots, \tanh(z_i), \dots]$  for  $i = 1, \dots, m$

# Single Layer Feedforward model (matrix form, continued)

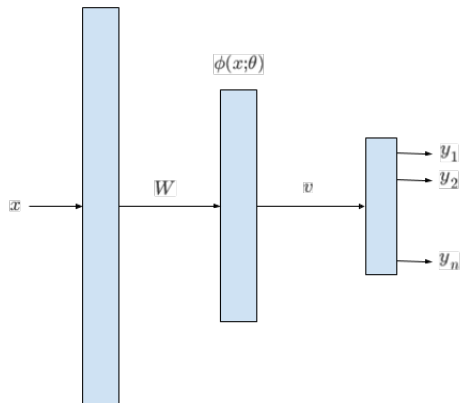
Define  $\phi$  in matrix form:

- ▶ Vector of inputs to the hidden layer  $z \in \mathbb{R}^m$ :  $z = Wx + b$
- ▶ Vector of outputs from hidden layer  $h \in \mathbb{R}^m$ :  $h = g(z)$
- ▶ Define  $\phi(x; \theta) = h$  where  $\theta = (W, b)$   
$$\phi(x; \theta) = g(Wx + b)$$
- ▶ Define  $\text{softmax}_y(r) = \frac{\exp(r_y)}{\sum_{y'} \exp(r_{y'})}$  for  $r \in \mathbb{R}^m$

Putting it all together:

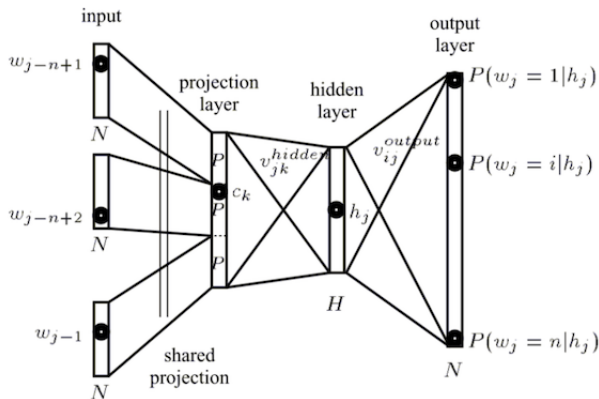
$$\begin{aligned}\Pr(y \mid x; \theta, v) &= \frac{\exp(v(y) \cdot \phi(x; \theta) + \gamma_y)}{\sum_{y' \in \mathcal{Y}} \exp(v(y') \cdot \phi(x; \theta) + \gamma_{y'})} \\ &= \underbrace{\text{softmax}\left(\underbrace{v(y) \cdot \phi(x; \theta) + \gamma_y}_{\text{for each } y \in \mathcal{Y} \text{ an } \mathbb{R} \text{ value}}\right)}_{\text{A vector of size } \mathbb{R}^{\mathcal{Y}} \text{ that sums to 1}}\end{aligned}$$

# Feedforward neural network



# n-gram Feedforward neural network

(Bengio and Schwenk 2013)





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Motivating example: XOR

# Simple stochastic gradient descent

## Inputs:

- ▶ Training examples  $(x^i, y^i)$  for  $i = 1, \dots, n$
- ▶ A feedforward representation  $\phi(x; \theta)$
- ▶ Integer  $T$  specifying the number of updates
- ▶ A sequence of learning rates:  $\eta^1, \dots, \eta^T$  where  $\eta^t \in [0, 1]$ 
  - ▶ One should experiment with learning rates: 0.001, 0.01, 0.1, 1
  - ▶ Bottou (2012) suggests a learning rate  $\eta^t = \frac{\eta^1}{1 + \eta^1 \times \lambda \times t}$  where  $\lambda$  is a hyperparameter that can be tuned experimentally

## Initialization:

Set  $v = (v(y), \gamma_y)$  for all  $y$ , and  $\theta$  to random values

# Gradient descent

## Algorithm:

- ▶ For  $t = 1, \dots, T$ 
  - ▶ Select an integer  $i$  uniformly at random from  $\{1, \dots, n\}$
  - ▶ Define  $L(\theta, \nu) = -\log P(y_i \mid x_i; \theta, \nu)$
  - ▶ For each parameter  $\theta_j$  and  $\nu_k(y)$  and  $\gamma_y$  (for each label  $y$ ):

$$\theta_j = \theta_j - \eta^t \times \frac{dL(\theta, \nu)}{d\theta_j}$$

$$\nu_k(y) = \nu_k(y) - \eta^t \times \frac{dL(\theta, \nu)}{d\nu_k(y)}$$

$$\gamma(y) = \gamma(y) - \eta^t \times \frac{dL(\theta, \nu)}{d\gamma(y)}$$

- ▶ **Output:** parameters  $\theta, \nu = (\nu(y), \gamma_y)$  for all  $y$

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# Motivating example: the XOR problem

From *Deep Learning* by Goodfellow, Bengio, Courville

We will assume a training set where each label is in the set  $\mathcal{Y} = \{-1, +1\}$

There are four training examples:

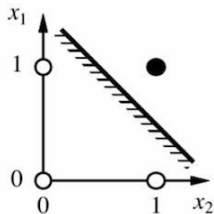
$$x^1 = [0, 0], y^1 = -1$$

$$x^2 = [0, 1], y^2 = +1$$

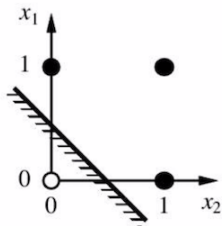
$$x^3 = [1, 0], y^3 = +1$$

$$x^4 = [1, 1], y^4 = -1$$

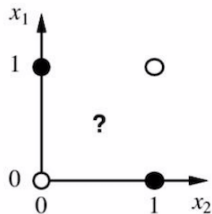
## Motivating example: the XOR problem



$x_1$  and  $x_2$



$x_1$  or  $x_2$



$x_1$  xor  $x_2$

## Motivating example: the XOR problem

### Theorem

For examples  $(x^i, y^i)$  for  $i = 1, \dots, 4$  as defined previously for the feedforward neural network:

$$\Pr(y \mid x; W, b, v) = \frac{\exp(v(y) \cdot g(Wx + b) + \gamma_y)}{\sum_{y' \in \mathcal{Y}} \exp(v(y') \cdot g(Wx + b) + \gamma_{y'})}$$

where  $x \in \mathbb{R}^2$  ( $d = 2$ ) and let  $m = 2$  so  $W \in \mathbb{R}^{2 \times 2}$  and  $b \in \mathbb{R}^2$  and  $g$  is a ReLU transfer function.

Then there are parameter settings  $v(-1)$ ,  $v(+1)$ ,  $\gamma_{-1}$ ,  $\gamma_{+1}$ ,  $W$ ,  $b$  such that

$$p(y^i \mid x^i; v) > 0.5 \text{ for } i = 1, \dots, 4$$

## Motivating example: the XOR problem

### Proof Sketch

Define  $W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$  and  $b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$  Then for each input  $x$  calculate values of  $z = Wx + b$  and  $h = g(z)$ :

$$x = [0, 0] \Rightarrow z = [0, -1] \Rightarrow h = [0, 0]$$

$$x = [1, 0] \Rightarrow z = [1, 0] \Rightarrow h = [1, 0]$$

$$x = [0, 1] \Rightarrow z = [1, 0] \Rightarrow h = [1, 0]$$

$$x = [1, 1] \Rightarrow z = [2, 1] \Rightarrow h = [2, 1]$$



# Motivating example: the XOR problem

## Proof Sketch (continued)

$$\begin{aligned} p(+1 \mid x; v) &= \frac{\exp(v(+1) \cdot h + \gamma_{+1})}{\exp(v(+1) \cdot h + \gamma_{+1}) + \exp(v(-1) \cdot h + \gamma_{-1})} \\ &= \frac{1}{1 + \exp(-(u \cdot h + \gamma))} \end{aligned}$$

To satisfy  $P(y^i \mid x^i; v) > 0.5$  for  $i = 1, \dots, 4$  we have to find parameters  $u = v(+1) - v(-1)$  and  $\gamma = \gamma_{+1} - \gamma_{-1}$  such that:

$$u \cdot [0, 0] + \gamma < 0$$

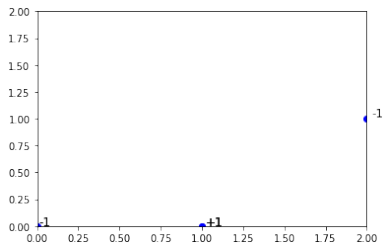
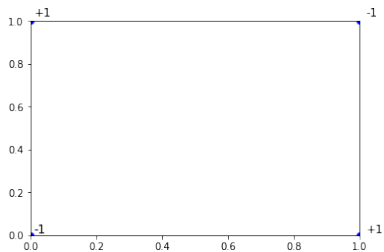
$$u \cdot [1, 0] + \gamma > 0$$

$$u \cdot [1, 0] + \gamma > 0$$

$$u \cdot [2, 1] + \gamma < 0$$

$u = [1, -2]$  and  $\gamma = -0.5$  satisfies these constraints.

# Solving the XOR problem



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