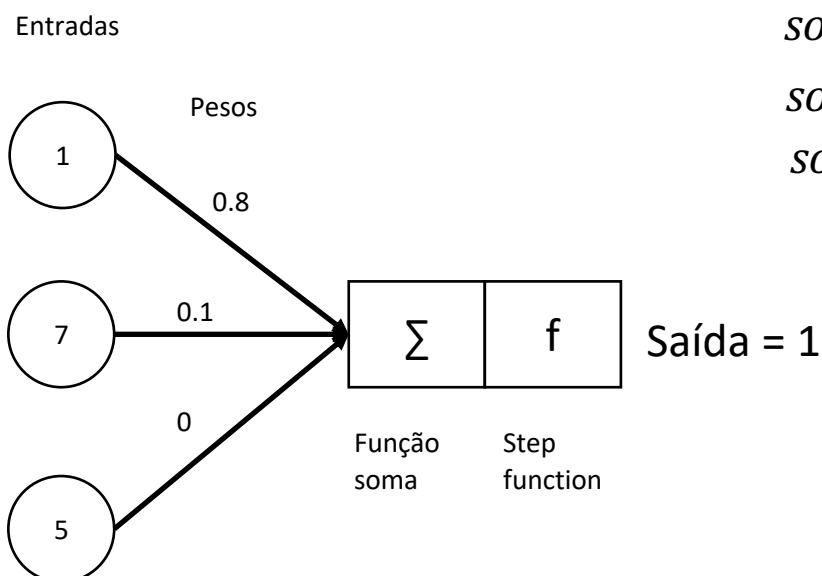


Perceptron de uma camada

Jones Granatyr



Neurônio artificial



$$soma = \sum_{i=1}^n xi * wi$$

$$soma = (1 * 0.8) + (7 * 0.1) + (5 * 0)$$

$$soma = 0.8 + 0.7 + 0$$

$$soma = 1.5$$

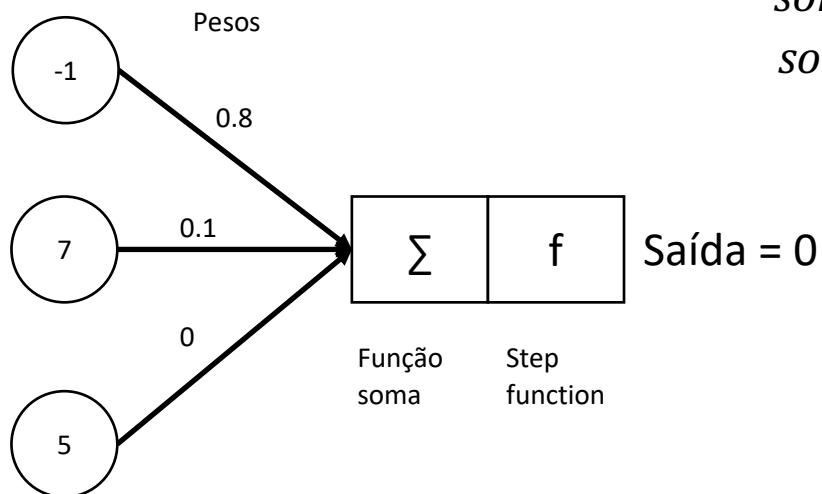
Step function (função Degrau)

Maior do que zero = 1
Caso contrário = 0

Neurônio artificial

$$soma = \sum_{i=1}^n xi * wi$$

Entradas



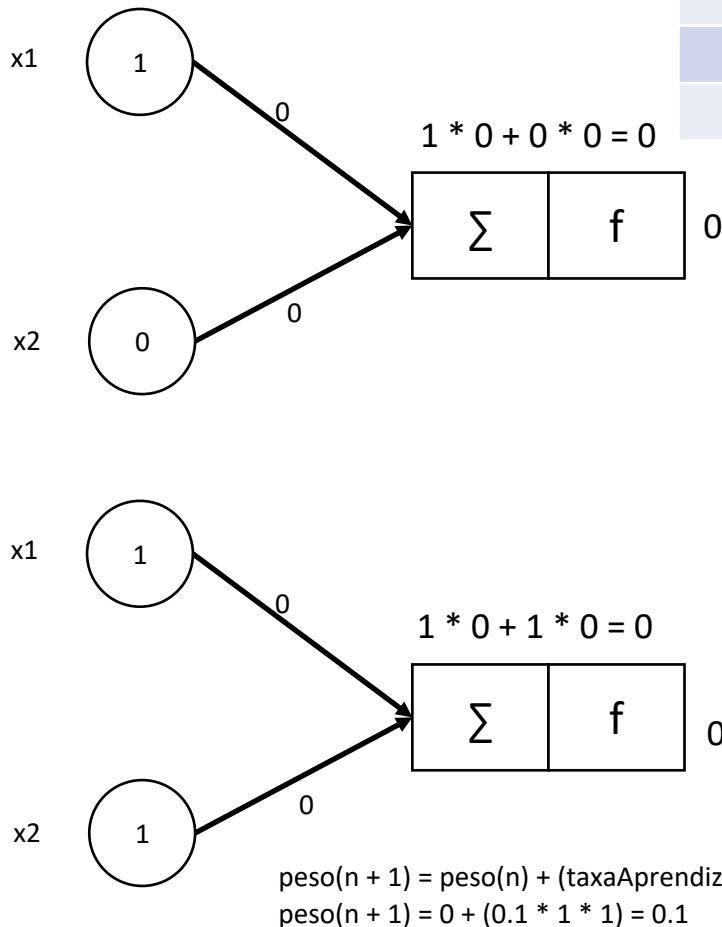
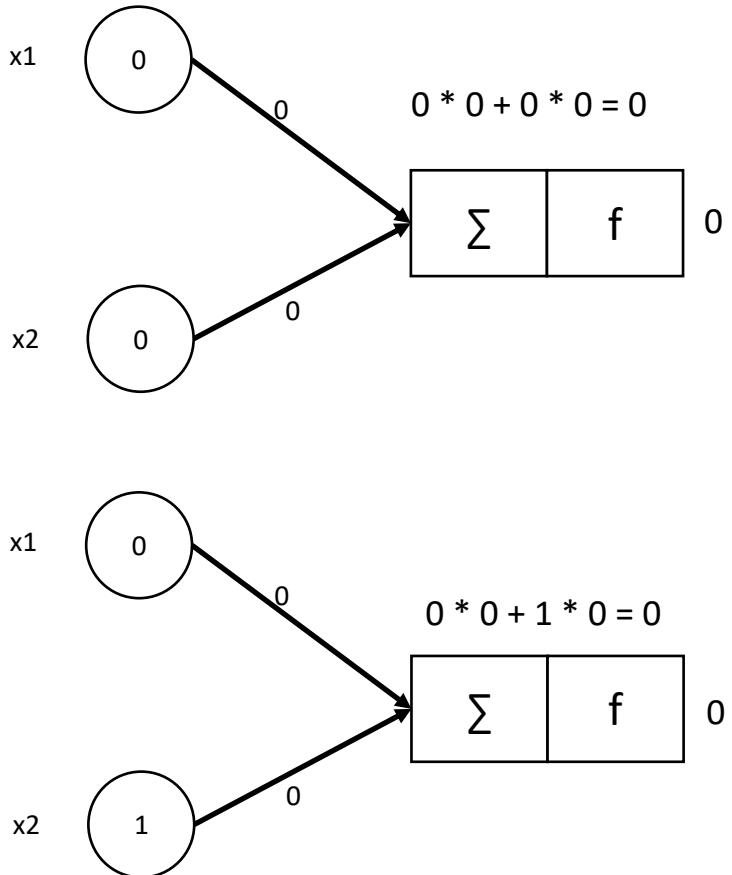
$$soma = (-1 * 0.8) + (7 * 0.1) + (5 * 0)$$

$$soma = -0.8 + 0.7 + 0$$

$$soma = -0.1$$

Operador E

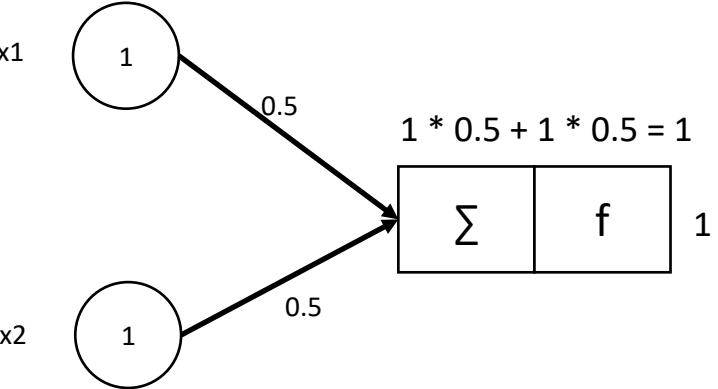
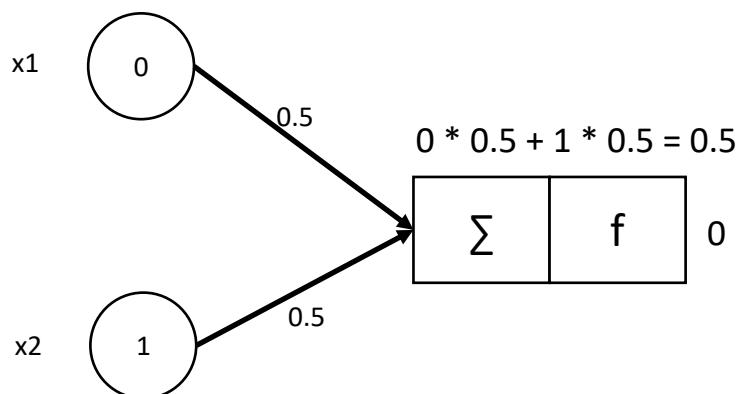
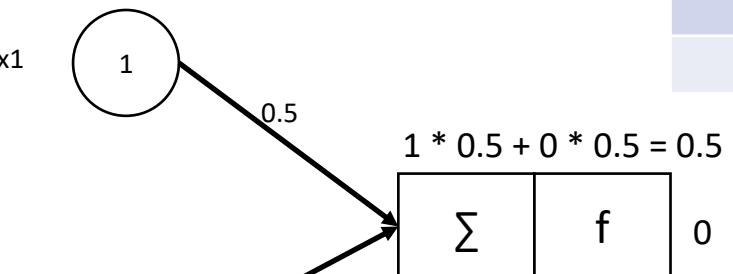
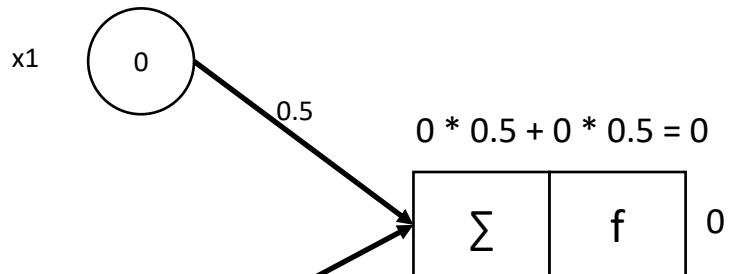
x1	x2	Classe
0	0	0
0	1	0
1	0	0
1	1	1



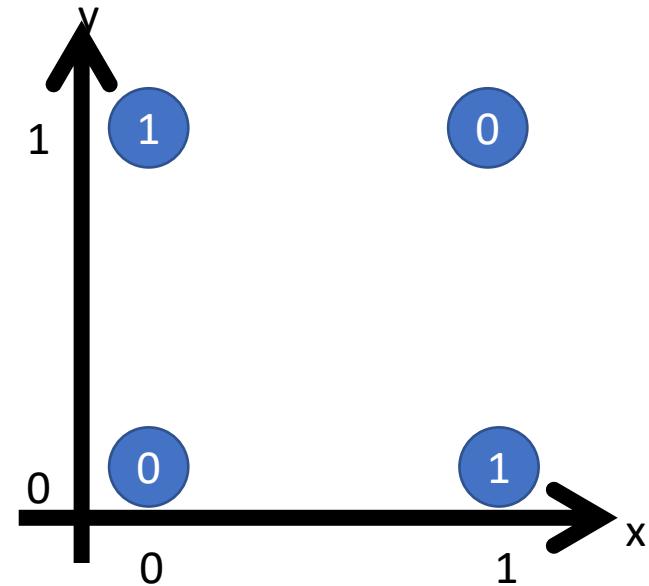
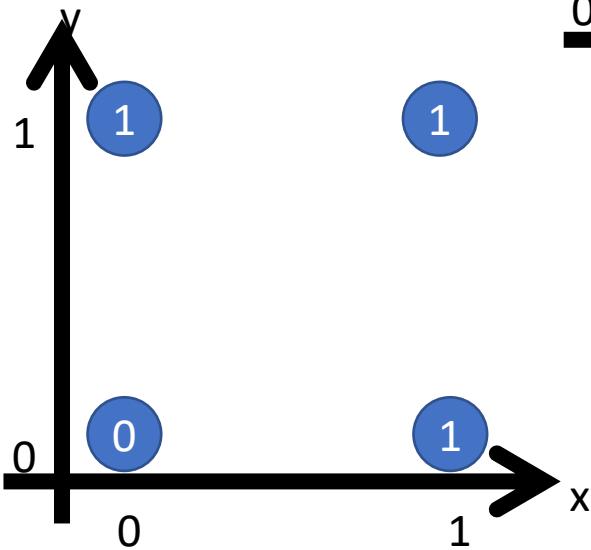
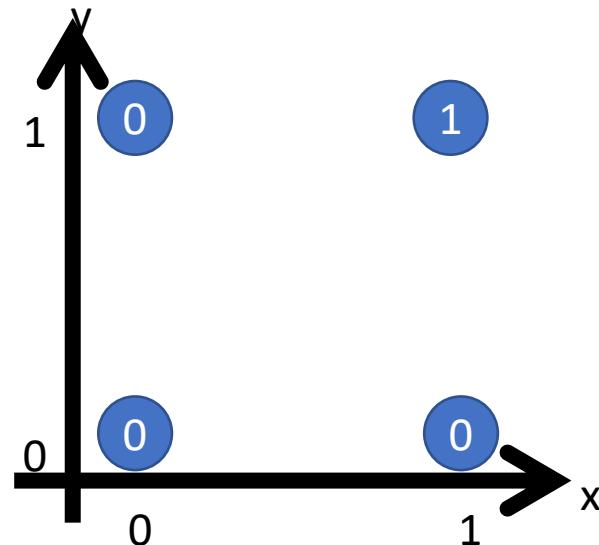
x1	x2	Classe
0	0	0
0	1	0
1	0	0
1	1	1

$$\begin{aligned}
 \text{peso}(n+1) &= \text{peso}(n) + (\text{taxaAprendizagem} * \text{entrada} * \text{erro}) \\
 \text{peso}(n+1) &= 0 + (0.1 * 1 * 1) = 0.1
 \end{aligned}$$

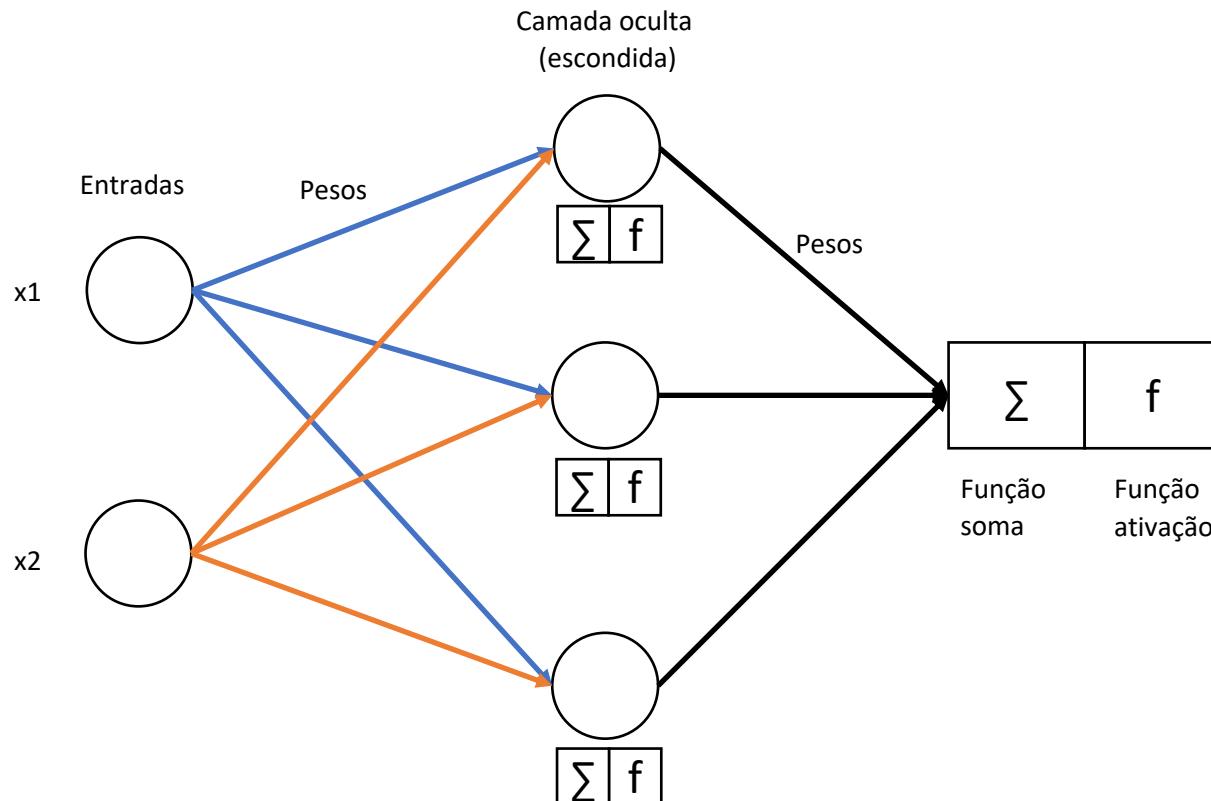
x1	x2	Classe
0	0	0
0	1	0
1	0	0
1	1	1



$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 1 - 1 &= 0 \end{aligned}$$

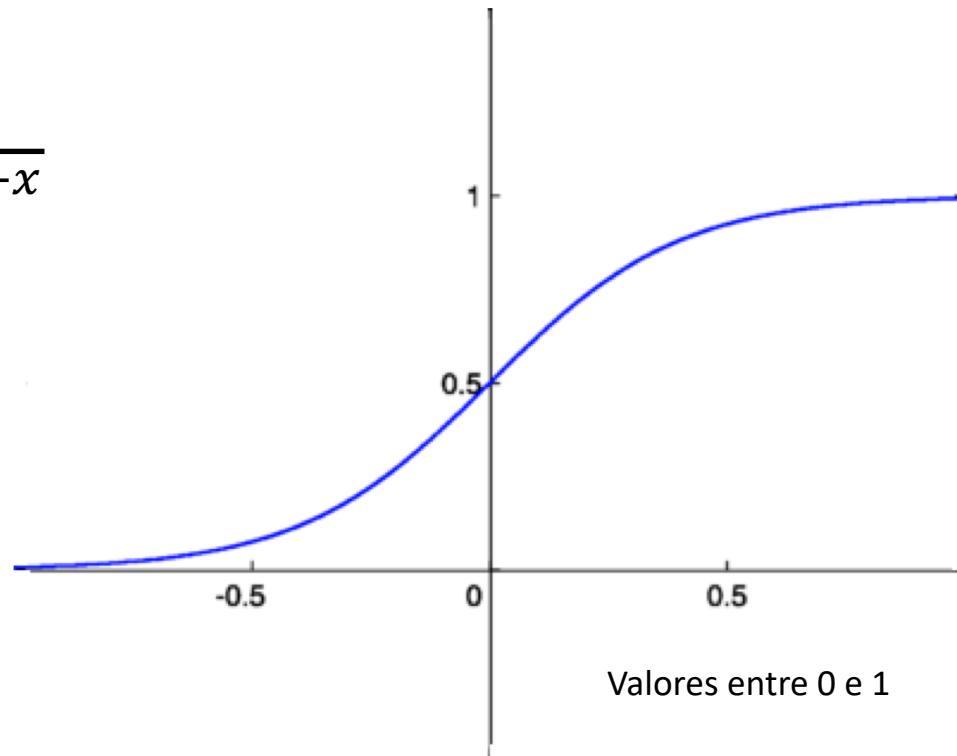


Redes multicamada (multilayer perceptron)



Sigmoid (função sigmoide)

$$y = \frac{1}{1 + e^{-x}}$$



Valores entre 0 e 1

Se X for alto o valor será aproximadamente 1

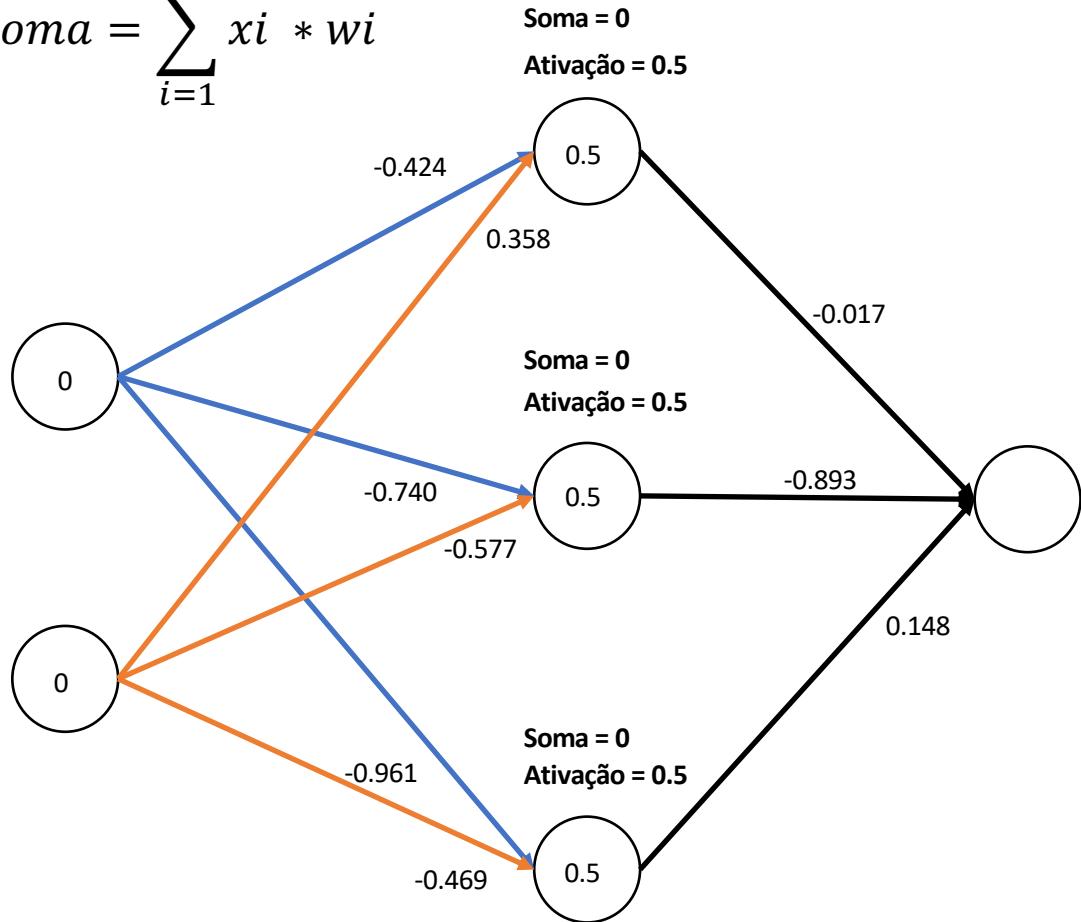
Se X for pequeno o valor será aproximadamente 0
Não retorna valores negativos

Operador XOR

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

Cálculos camada entrada
para camada oculta

$$soma = \sum_{i=1}^n xi * wi$$



$$y = \frac{1}{1 + e^{-x}}$$

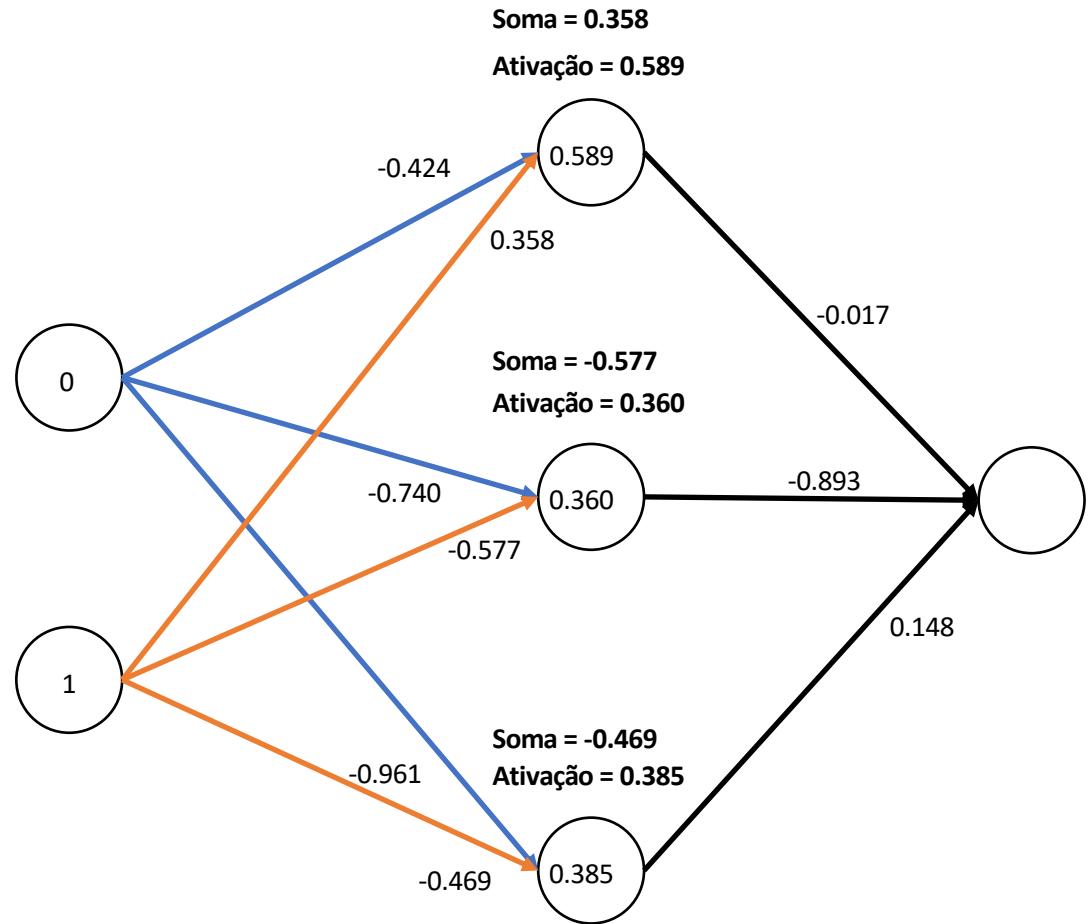
$$0 * (-0.424) + 0 * 0.358 = 0$$

$$0 * (-0.740) + 0 * (-0.577) = 0$$

$$0 * (-0.961) + 0 * (-0.469) = 0$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

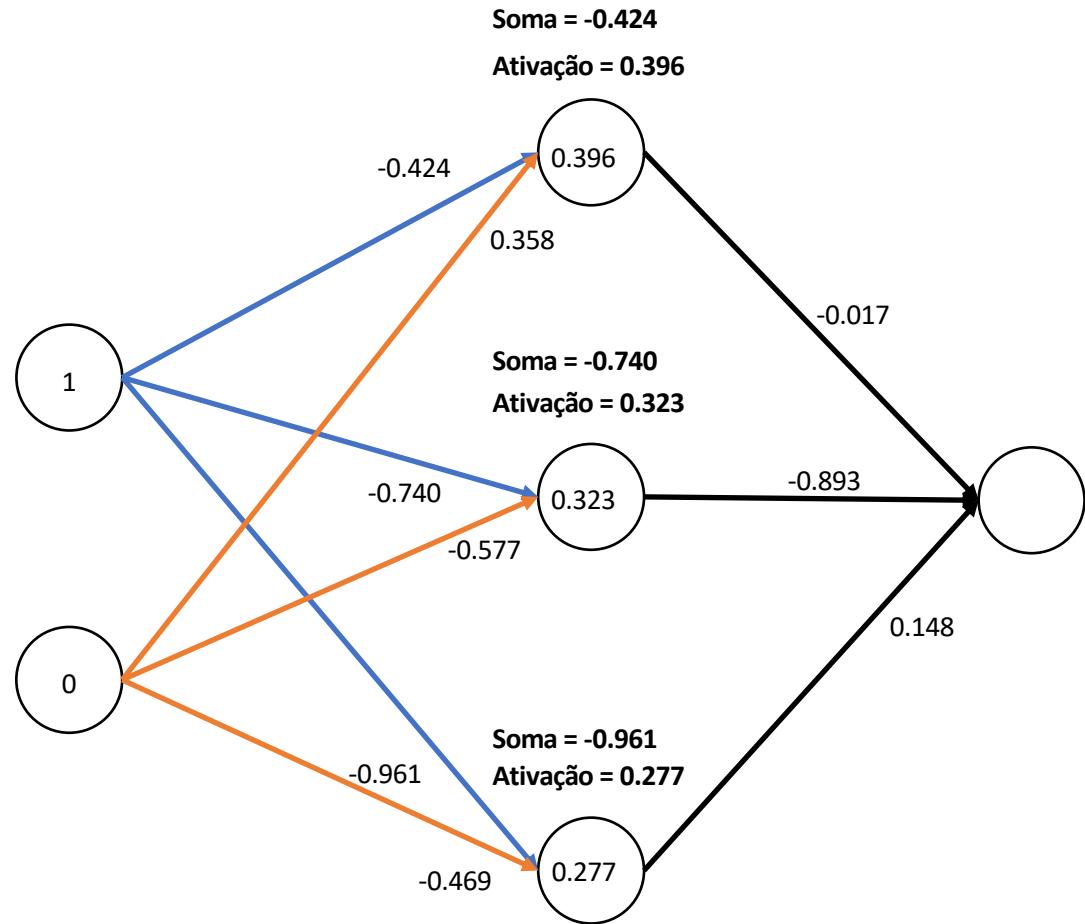


$$0 * (-0.424) + 1 * 0.358 = 0.358$$

$$0 * (-0.740) + 1 * (-0.577) = -0.577$$

$$0 * (-0.961) + 1 * (-0.469) = -0.469$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

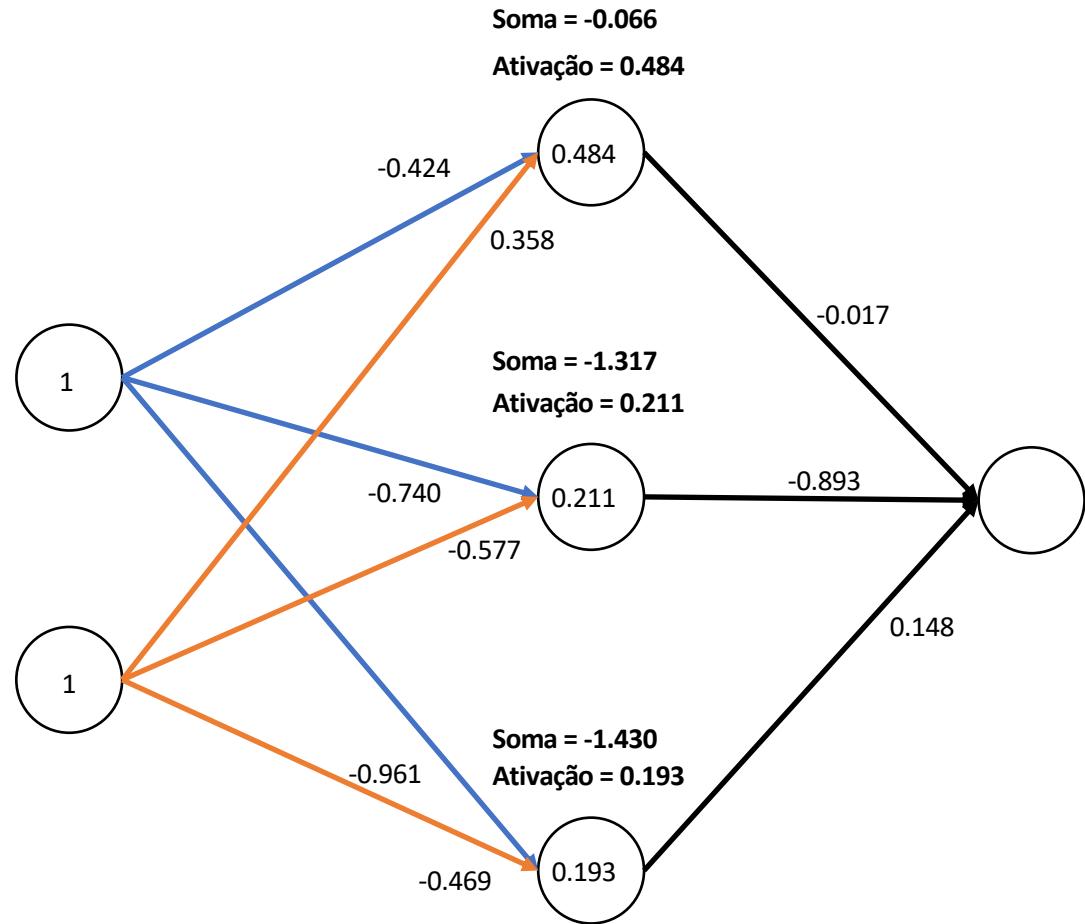


$$1 * (-0.424) + 0 * 0.358 = -0.424$$

$$1 * (-0.740) + 0 * (-0.577) = -0.740$$

$$1 * (-0.961) + 0 * (-0.469) = -0.961$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0



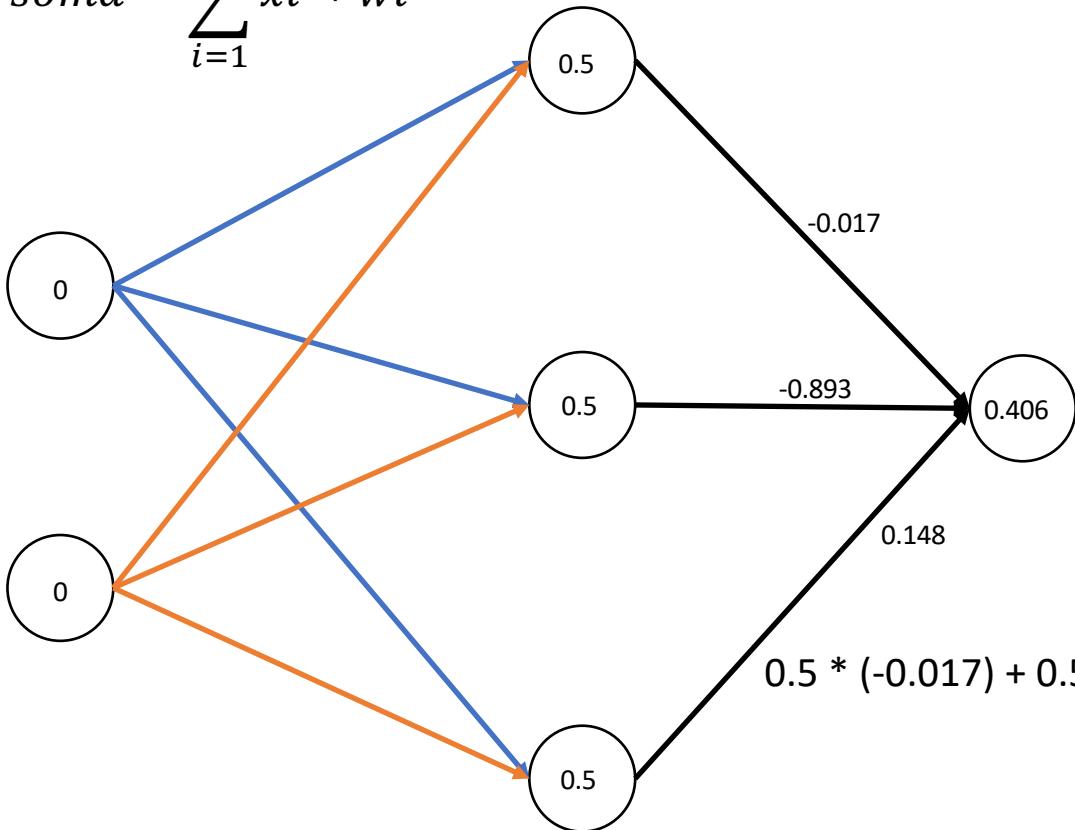
$$1 * (-0.424) + 1 * 0.358 = -0.066$$

$$1 * (-0.740) + 1 * (-0.577) = -1.317$$

$$1 * (-0.961) + 1 * (-0.469) = -1.430$$

Cálculos camada oculta para
camada de saída

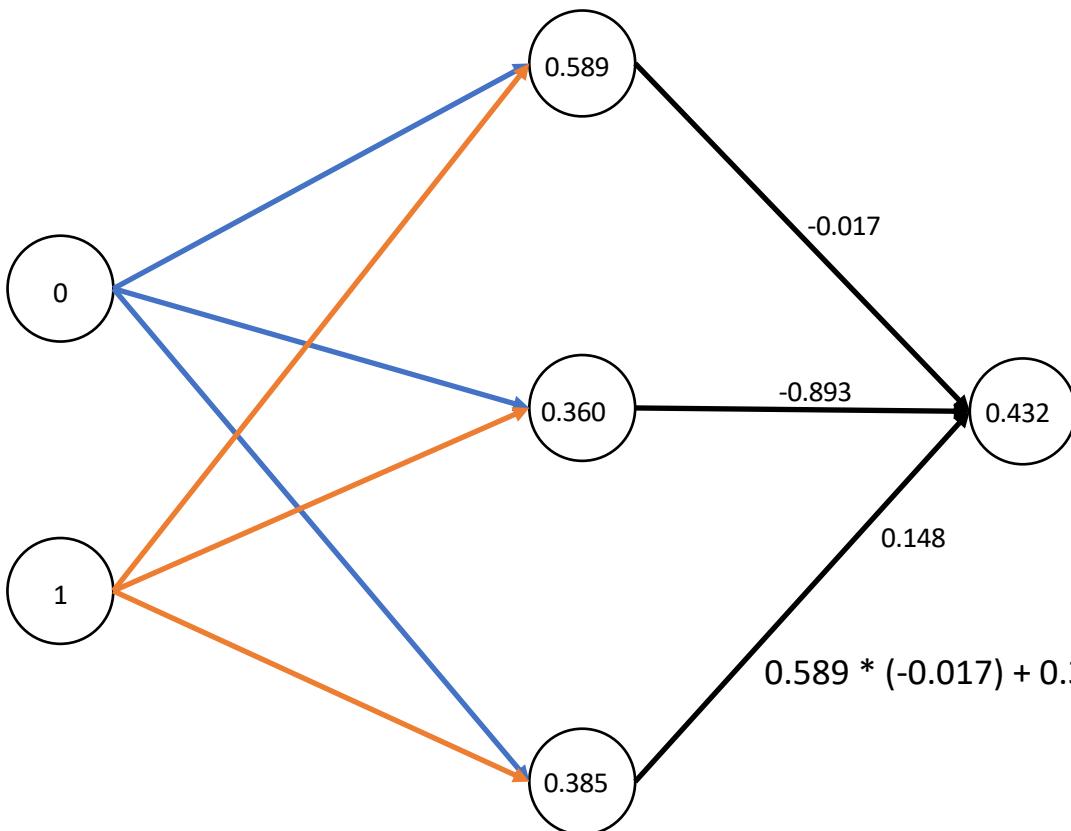
$$soma = \sum_{i=1}^n xi * wi$$



$$y = \frac{1}{1 + e^{-x}}$$

Soma = -0.381
Ativação = 0.406

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

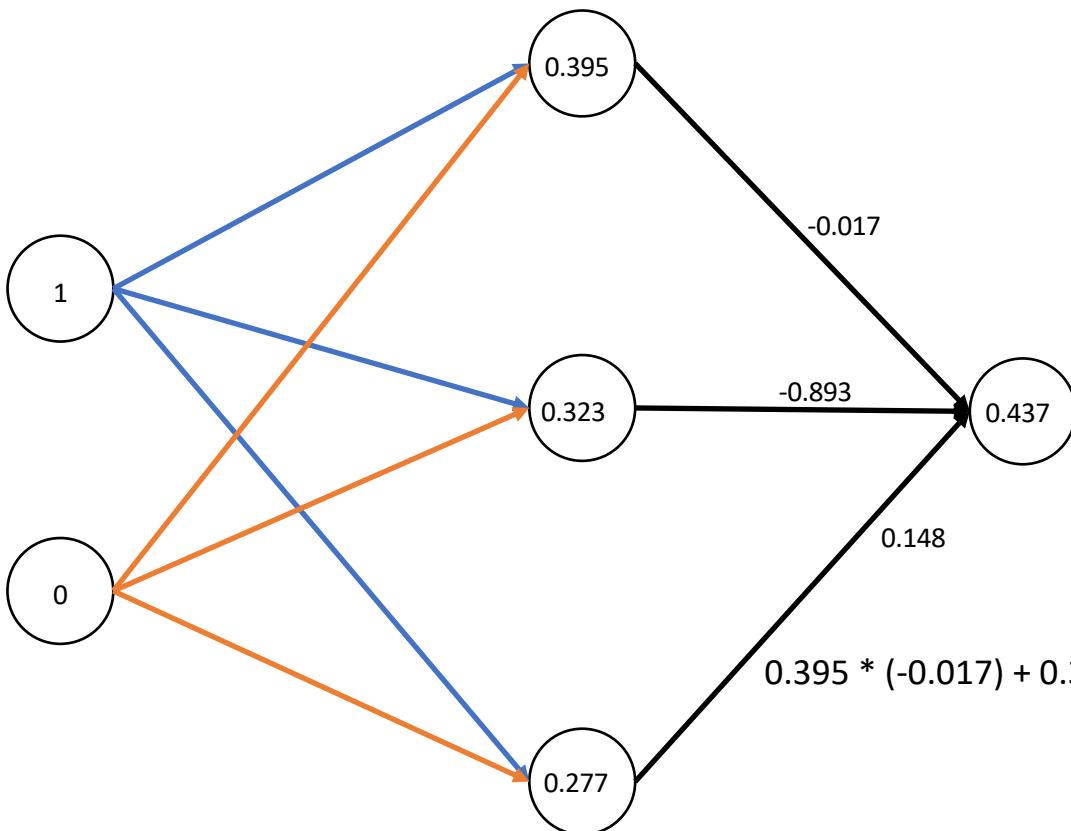


$$\text{Soma} = -0.274$$

$$\text{Ativação} = 0.432$$

$$0.589 * (-0.017) + 0.360 * (-0.893) + 0.385 * 0.148 = -0.274$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0



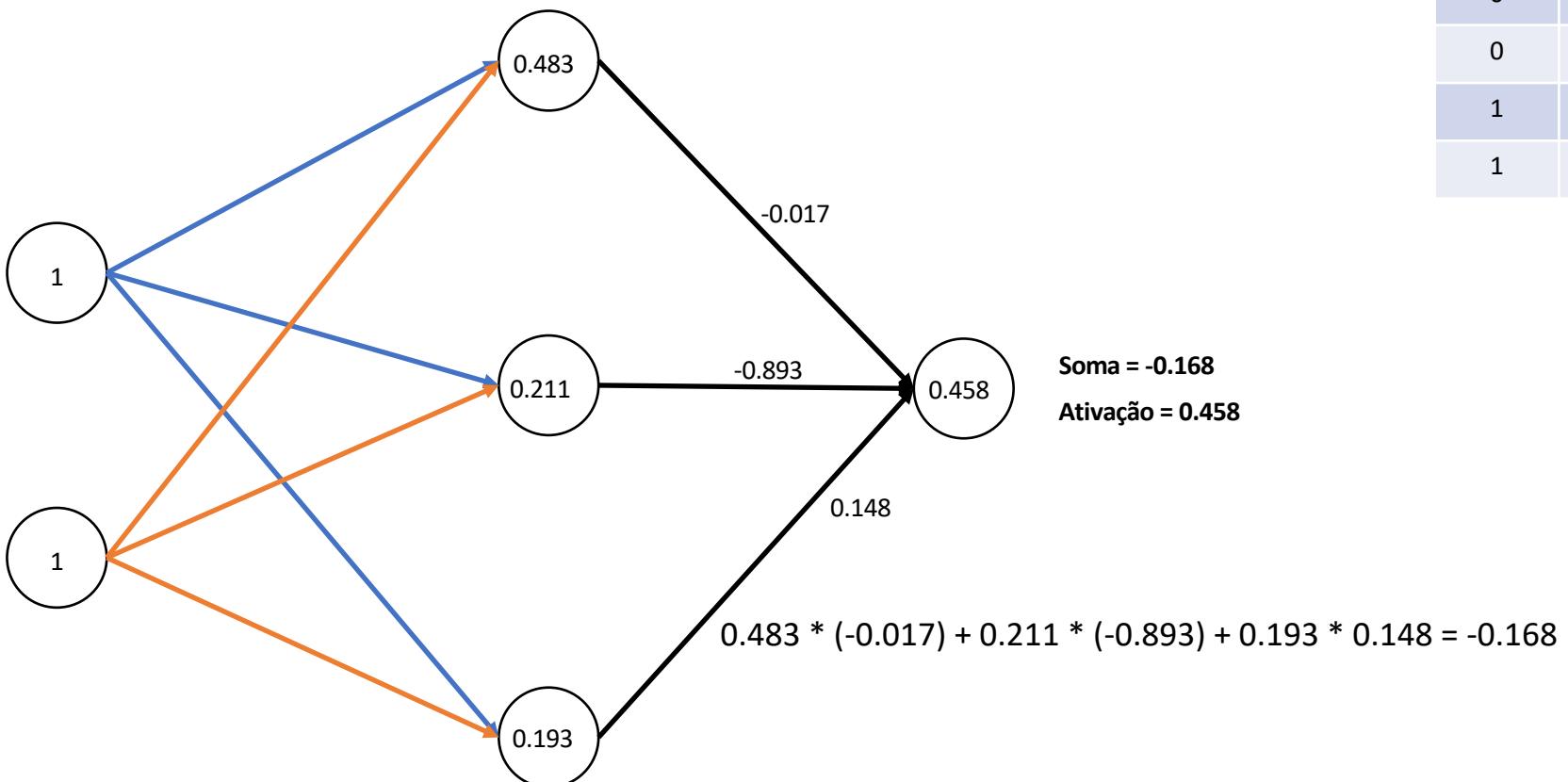
$$\text{Soma} = -0.254$$

$$\text{Ativação} = 0.437$$

$$0.395 * (-0.017) + 0.323 * (-0.893) + 0.277 * 0.148 = -0.254$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0



Erro

- Algoritmo mais simples
 - erro = respostaCorreta – respostaCalculada

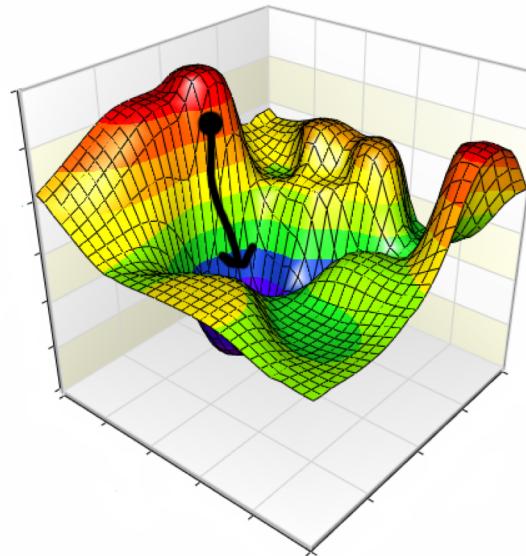
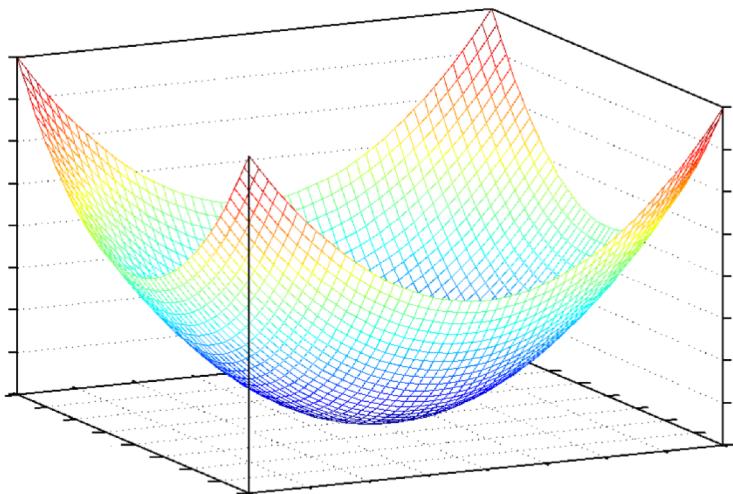
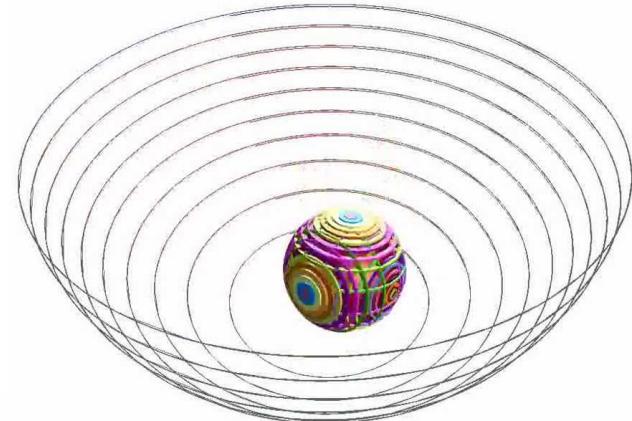
x1	x2	Classe	Calculado	Erro
0	0	0	0.406	-0.406
0	1	1	0.432	0.568
1	0	1	0.437	0.563
1	1	0	0.458	-0.458

Média absoluta = 0.49

Gradiente

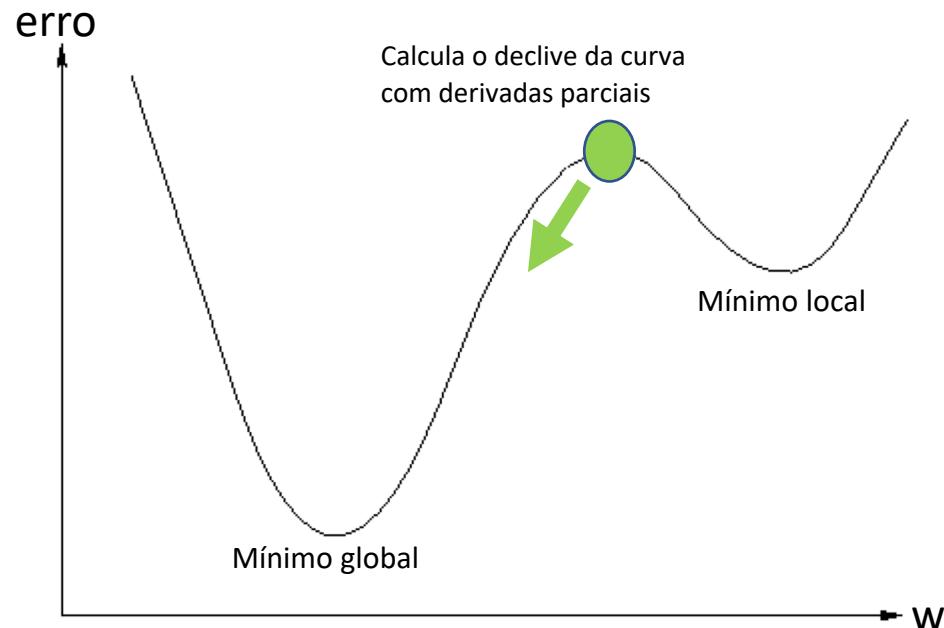
$$\min C(w_1, w_2 \dots w_n)$$

Calcular a derivada parcial para mover para a direção do gradiente



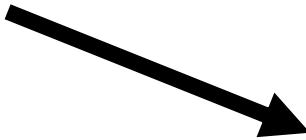
Gradiente

- Encontrar a combinação de pesos que o erro é o menor possível
- Gradiente é calculado para saber quanto ajustar os pesos



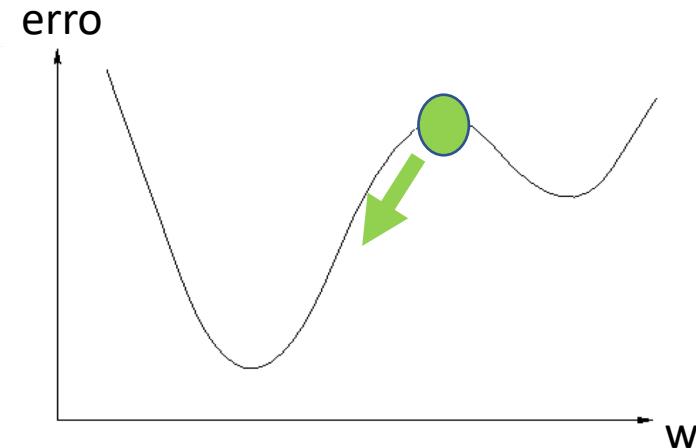
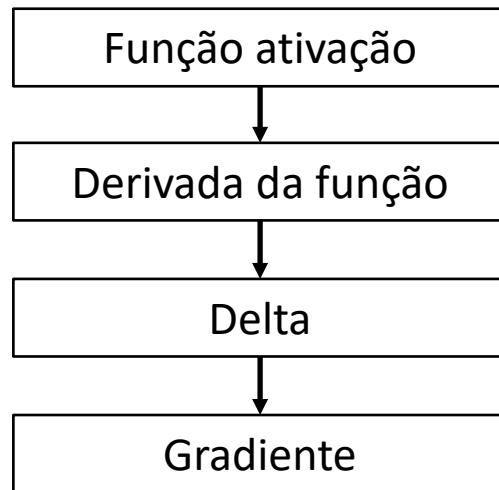
Gradiente (derivada)

$$y = \frac{1}{1 + e^{-x}}$$



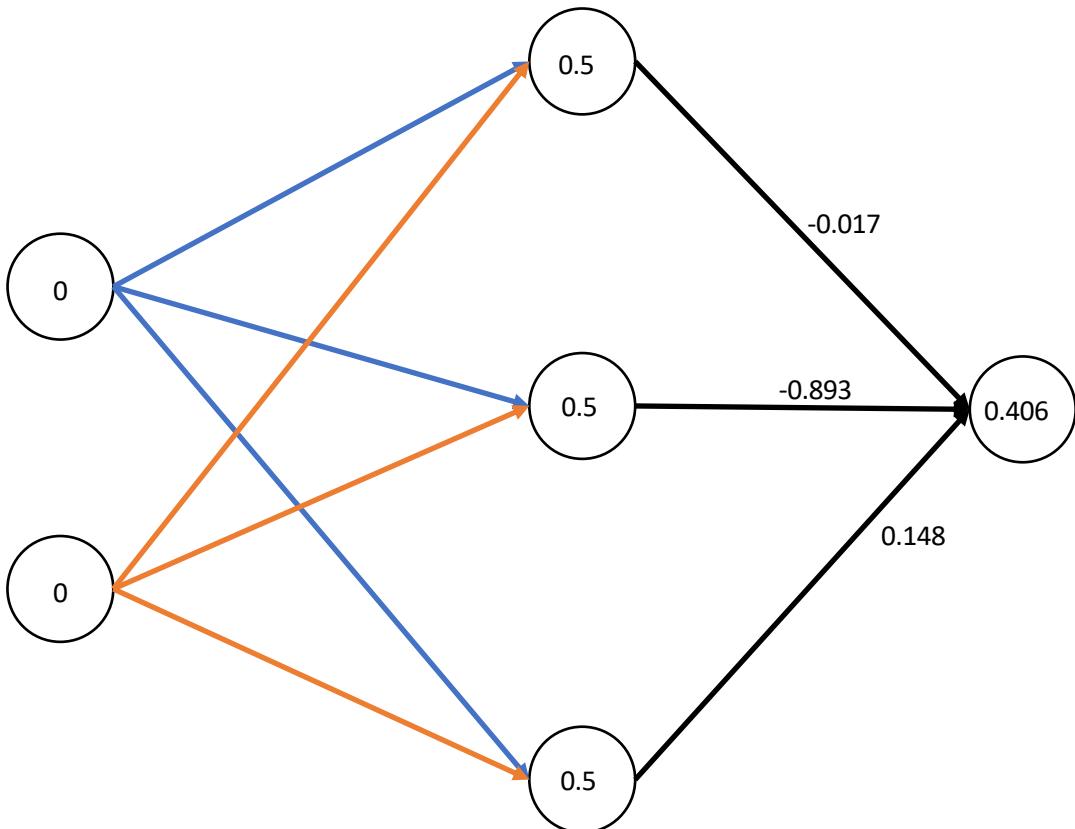
$$d = y * (1 - y)$$

Cálculo do parâmetro Delta



Delta camada saída

$$\Delta_{Saida} = Erro * DerivadaSigmoid$$



Soma = -0.381

Ativação = 0.406

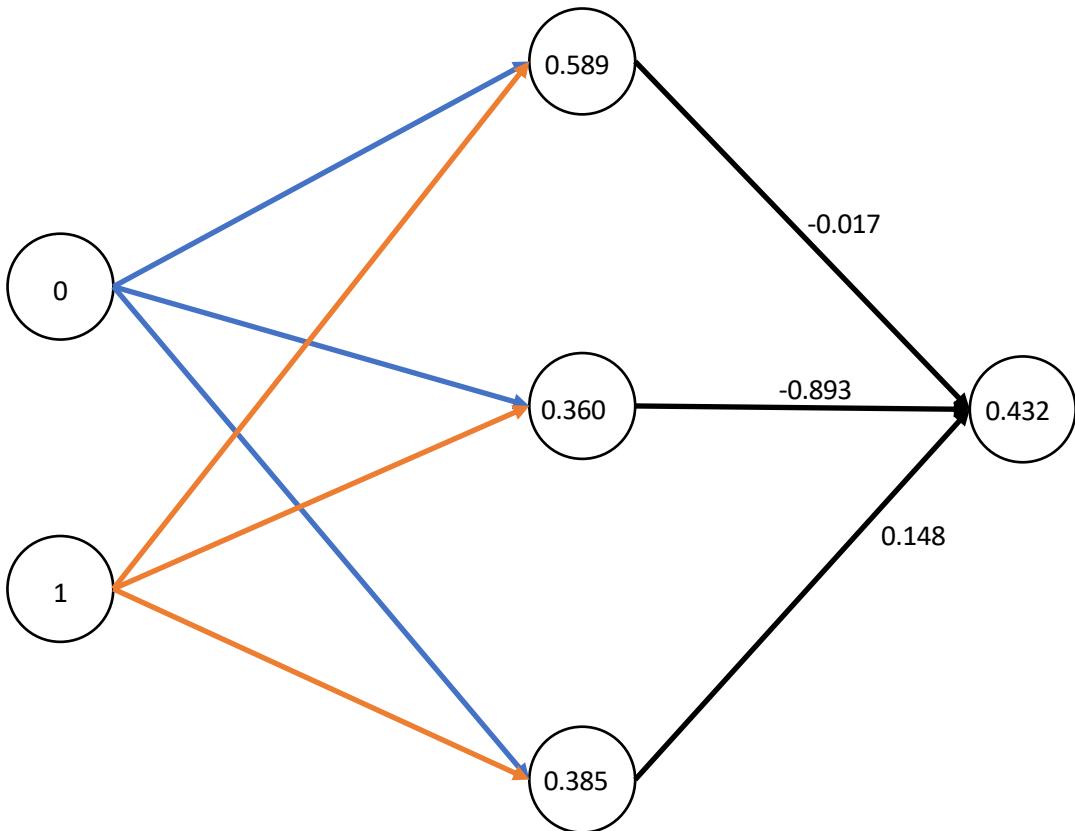
Erro = 0 - 0.406 = -0.406

Derivada ativação (sigmoide) = 0.241

$\Delta_{Saida} = -0.406 * 0.241 = -0.098$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

$$\Delta_{Saida} = Erro * DerivadaSigmoid$$



Soma = -0.274

Ativação = 0.432

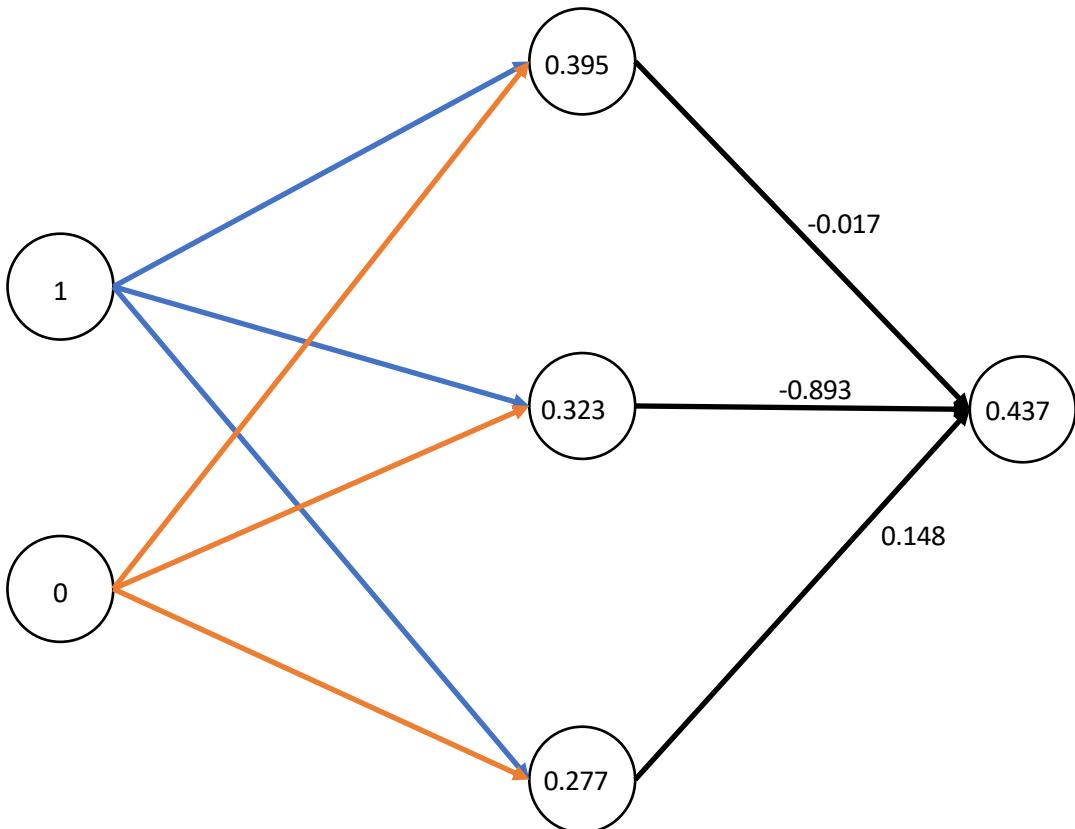
Erro = 1 - 0.432 = 0.568

Derivada ativação (sigmoide) = 0.245

$\Delta_{Saida} = 0.568 * 0.245 = 0.139$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

$$\Delta_{Saida} = Erro * DerivadaSigmoid$$



Soma = -0.254

Ativação = 0.437

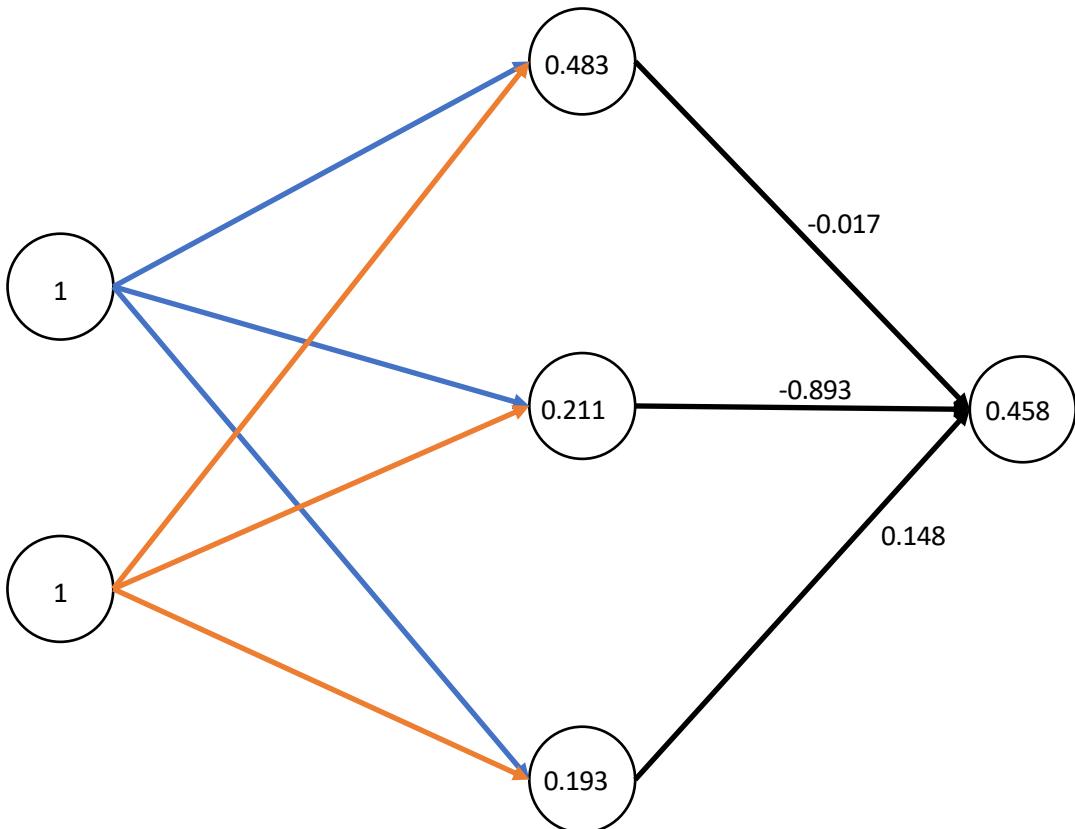
Erro = 1 - 0.437 = 0.563

Derivada ativação (sigmoide) = 0.246

$\Delta_{Saida} = 0.563 * 0.246 = 0.139$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

$$\Delta_{\text{Saída}} = \text{Erro} * \text{DerivadaSigmoid}$$



Soma = -0.168

Ativação = 0.458

Erro = 0 - 0.458 = -0.458

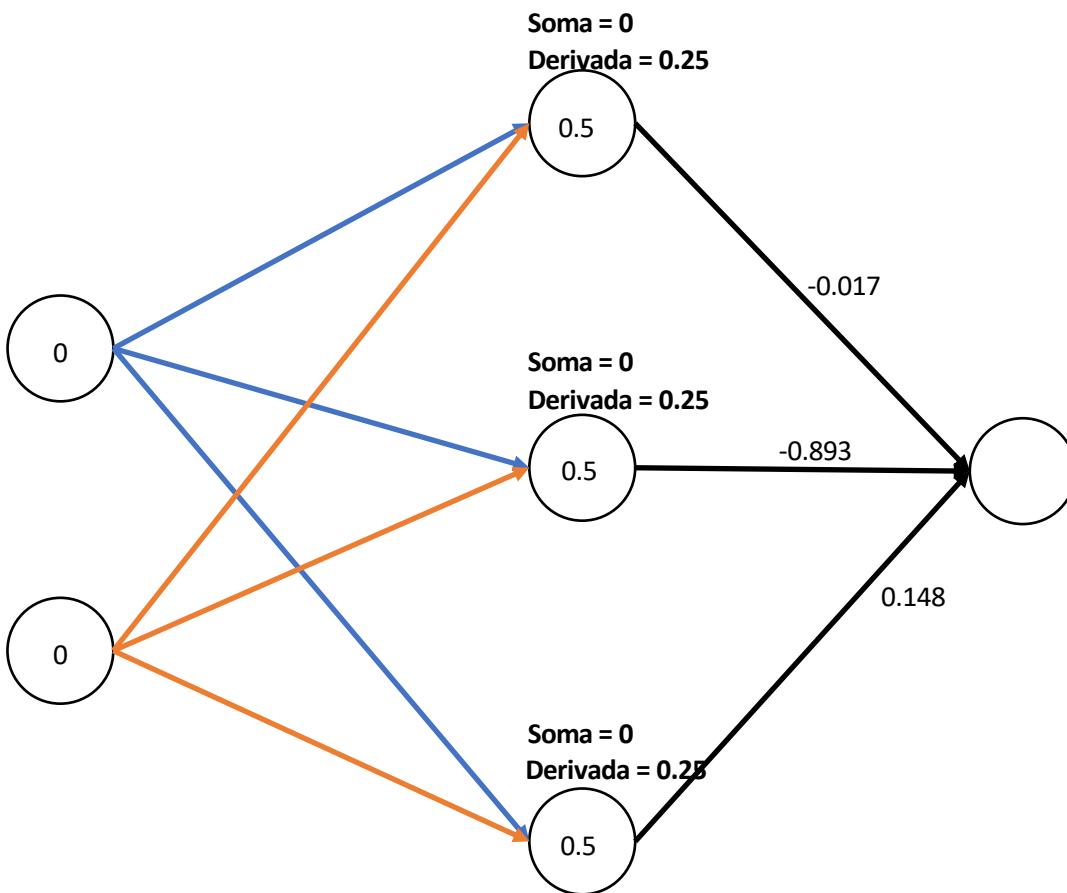
Derivada ativação (sigmoide) = 0.248

$\Delta_{\text{Saída}} = -0.458 * 0.248 = -0.114$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

Delta camada escondida

$$\Delta_{\text{Escondida}} = \text{DerivadaSigmoid} * \text{peso} * \Delta_{\text{Saída}}$$



$$\Delta_{\text{Saída}} = -0.098$$

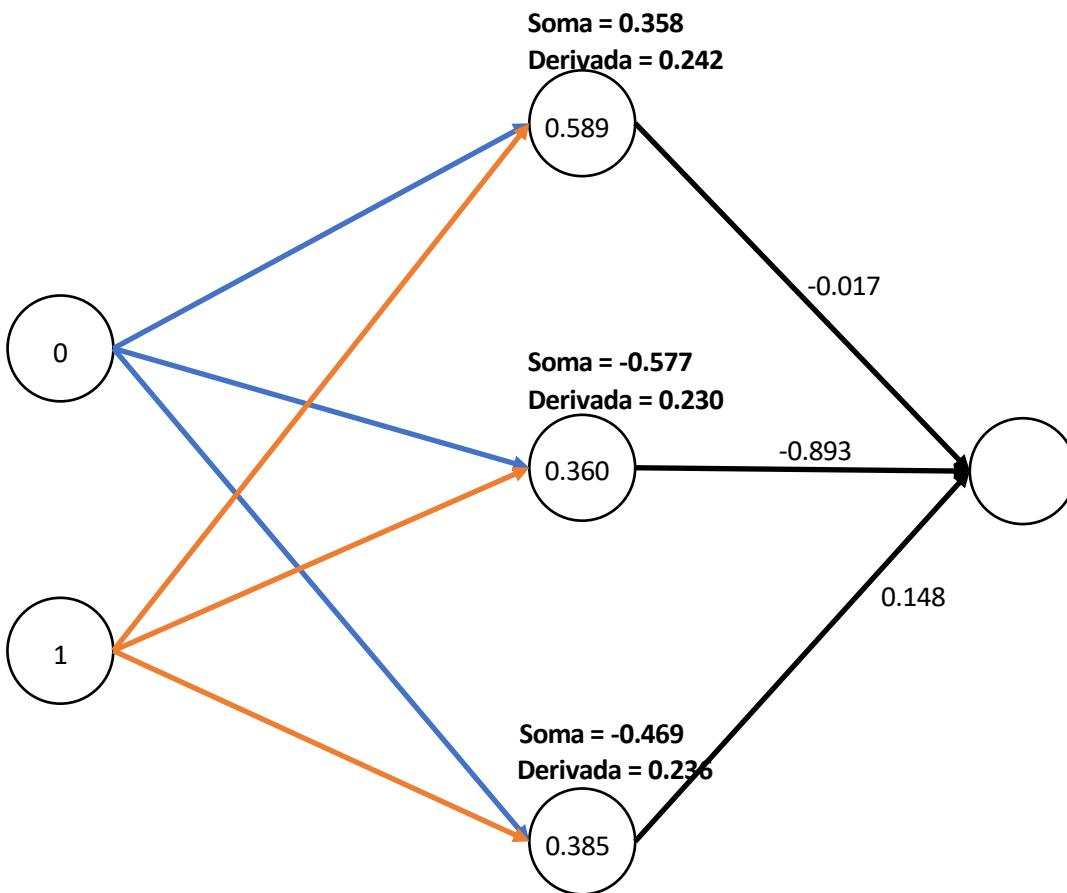
$$0.25 * (-0.017) * (-0.098) = 0.000$$

$$0.25 * (-0.893) * (-0.098) = 0.022$$

$$0.25 * 0.148 * (-0.098) = -0.004$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

$$\Delta_{\text{Escondida}} = \text{DerivadaSigmoid} * \text{peso} * \Delta_{\text{Saída}}$$



$$\Delta_{\text{Saída}} = 0.139$$

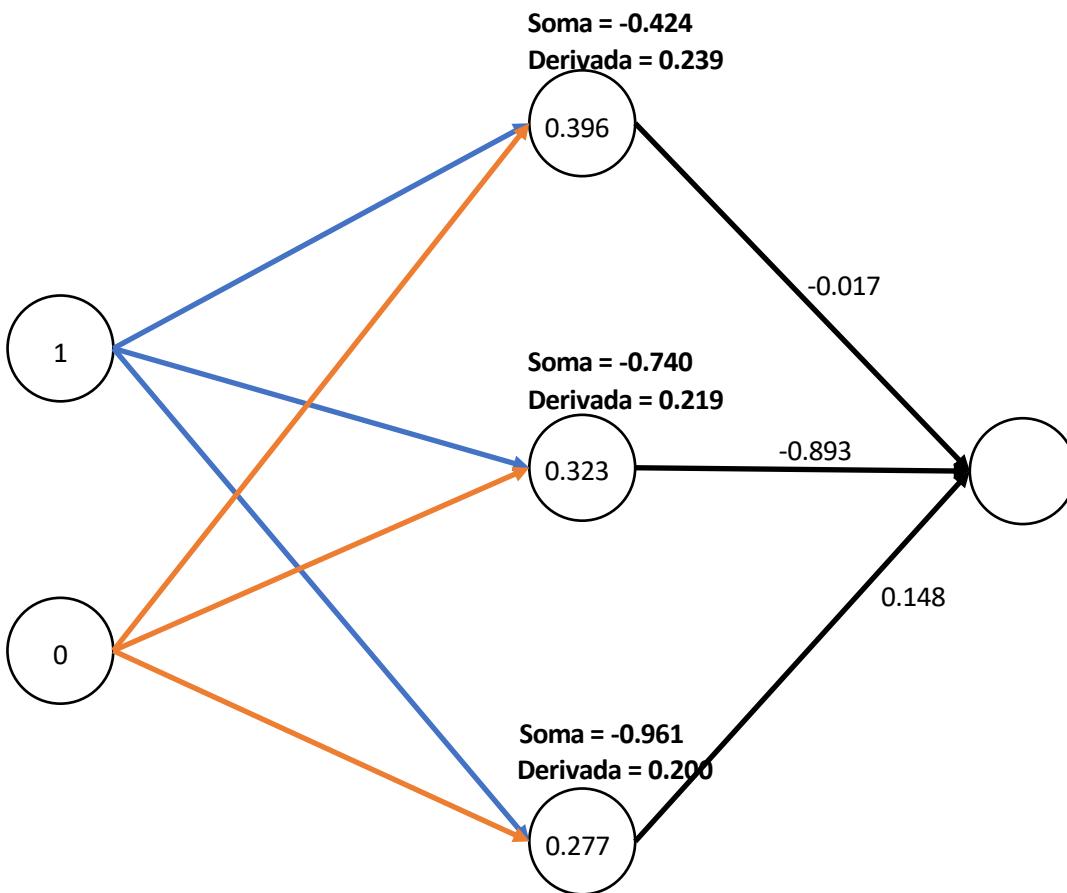
$$0.242 * (-0.017) * 0.139 = -0.001$$

$$0.230 * (-0.893) * 0.139 = -0.029$$

$$0.236 * 0.148 * 0.139 = 0.005$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

$$\Delta_{\text{Escondida}} = \text{DerivadaSísmoide} * \text{peso} * \Delta_{\text{Saída}}$$



$$\Delta_{\text{Saída}} = 0.139$$

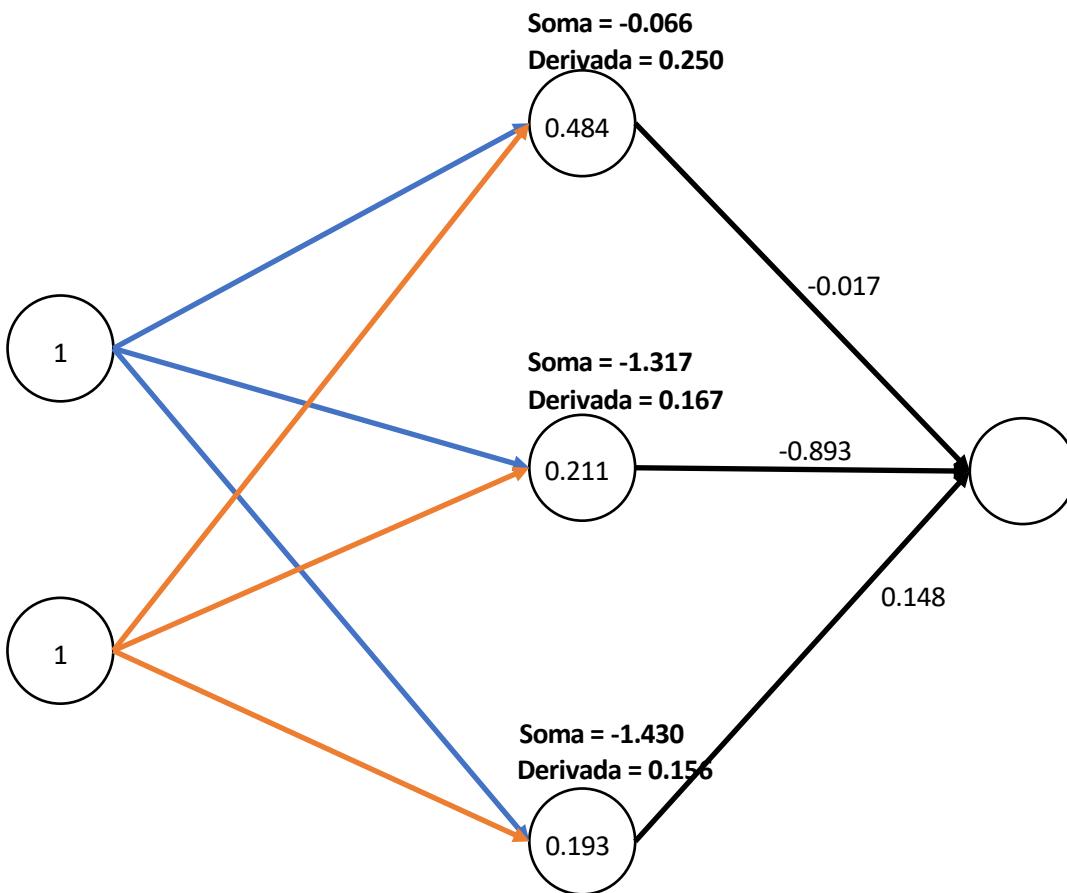
$$0.239 * (-0.017) * 0.139 = -0.001$$

$$0.219 * (-0.893) * 0.139 = -0.027$$

$$0.200 * 0.148 * 0.139 = 0.004$$

x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

$$\Delta_{\text{Escondida}} = \text{DerivadaSigmoid} * \text{peso} * \Delta_{\text{Saída}}$$



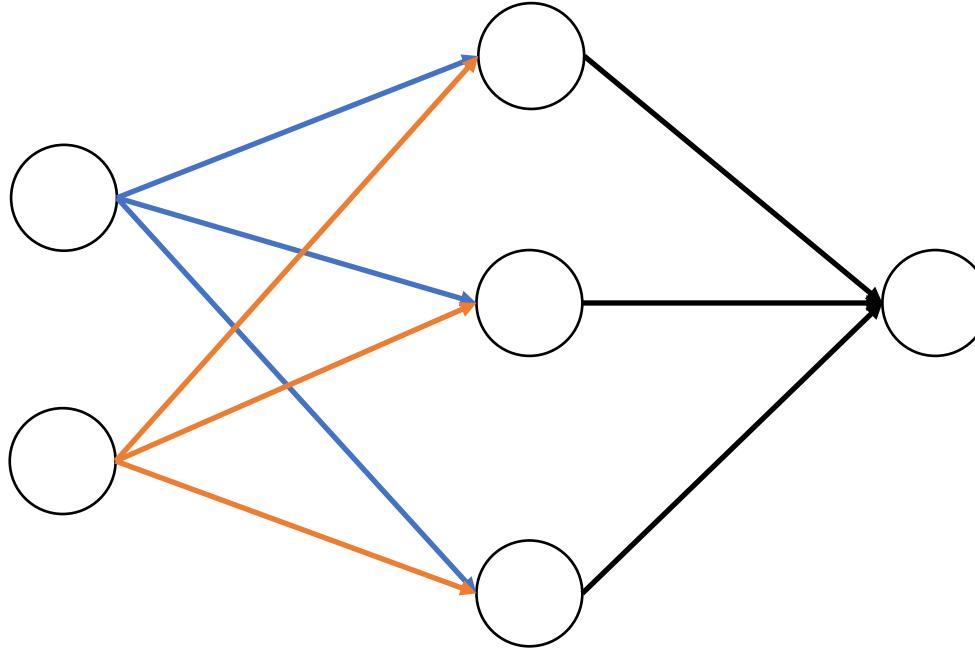
x1	x2	Classe
0	0	0
0	1	1
1	0	1
1	1	0

$$\Delta_{\text{Saída}} = -0.114$$

$$0.250 * (-0.017) * (-0.114) = 0.000$$

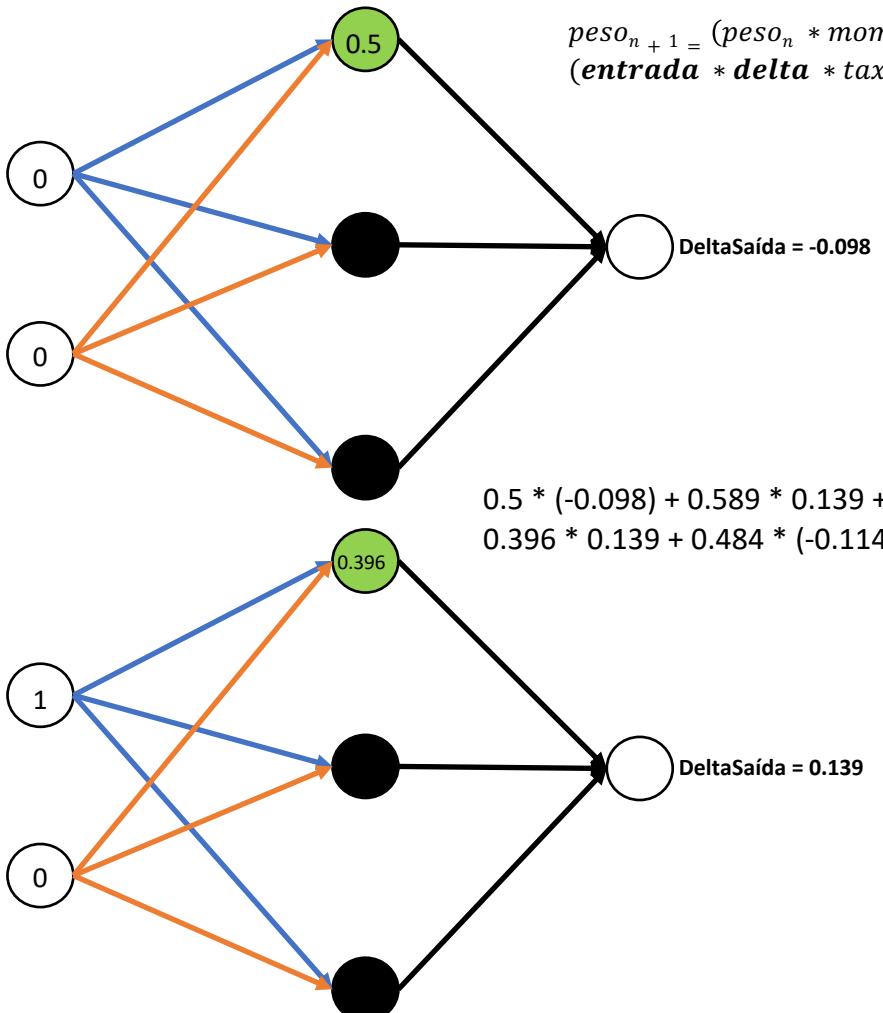
$$0.167 * (-0.893) * (-0.114) = 0.017$$

$$0.156 * 0.148 * (-0.114) = -0.003$$

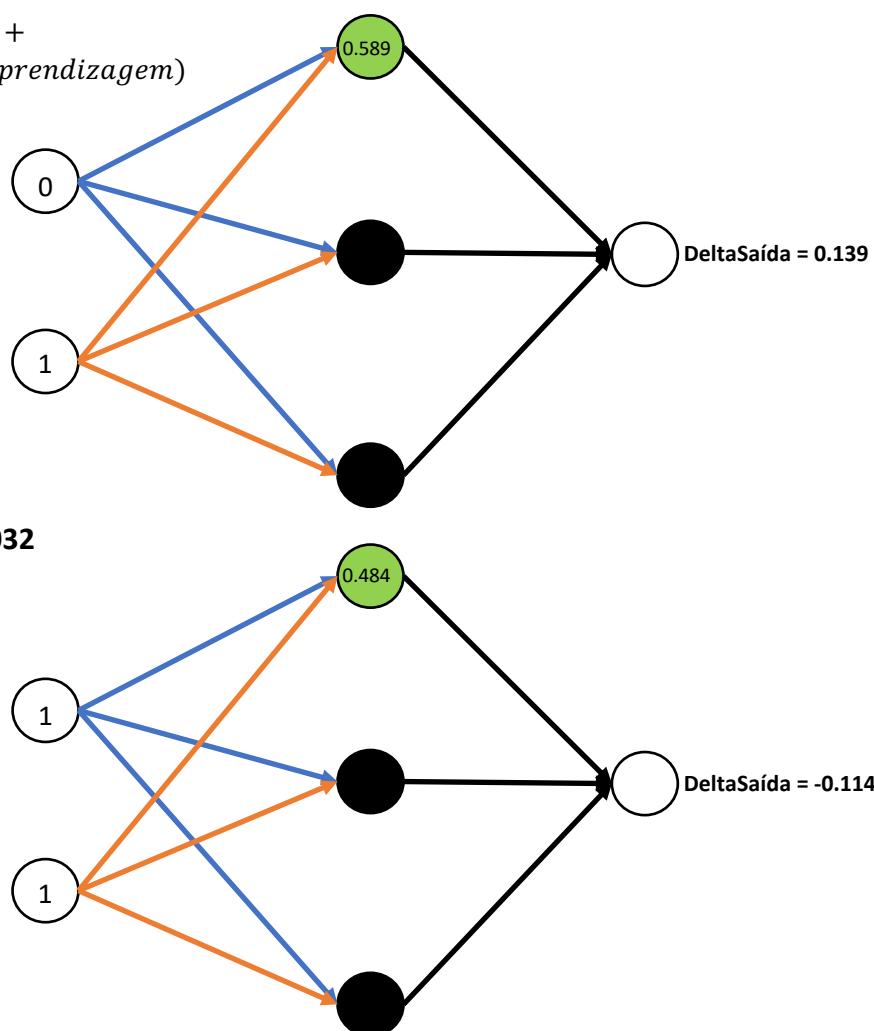


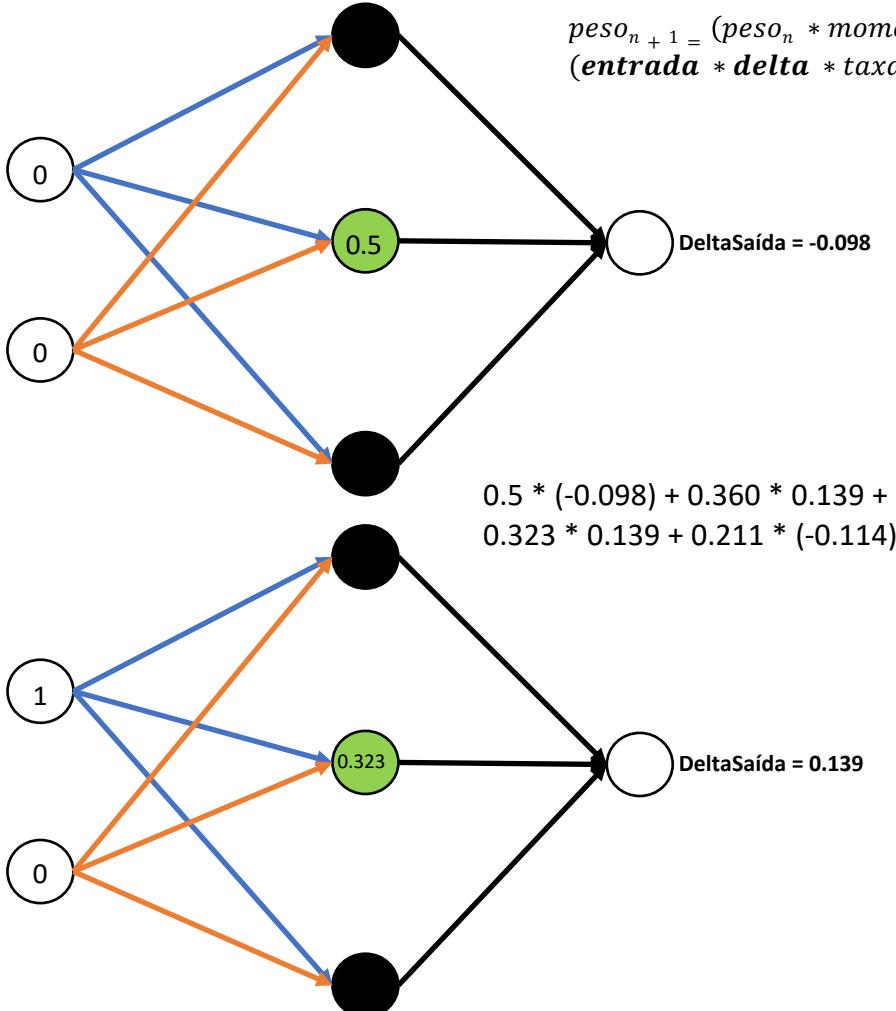
$$peso_{n+1} = (peso_n * momento) + (entrada * delta * taxa\ de\ aprendizagem)$$

Ajuste dos pesos da camada
oculta para a camada de saída



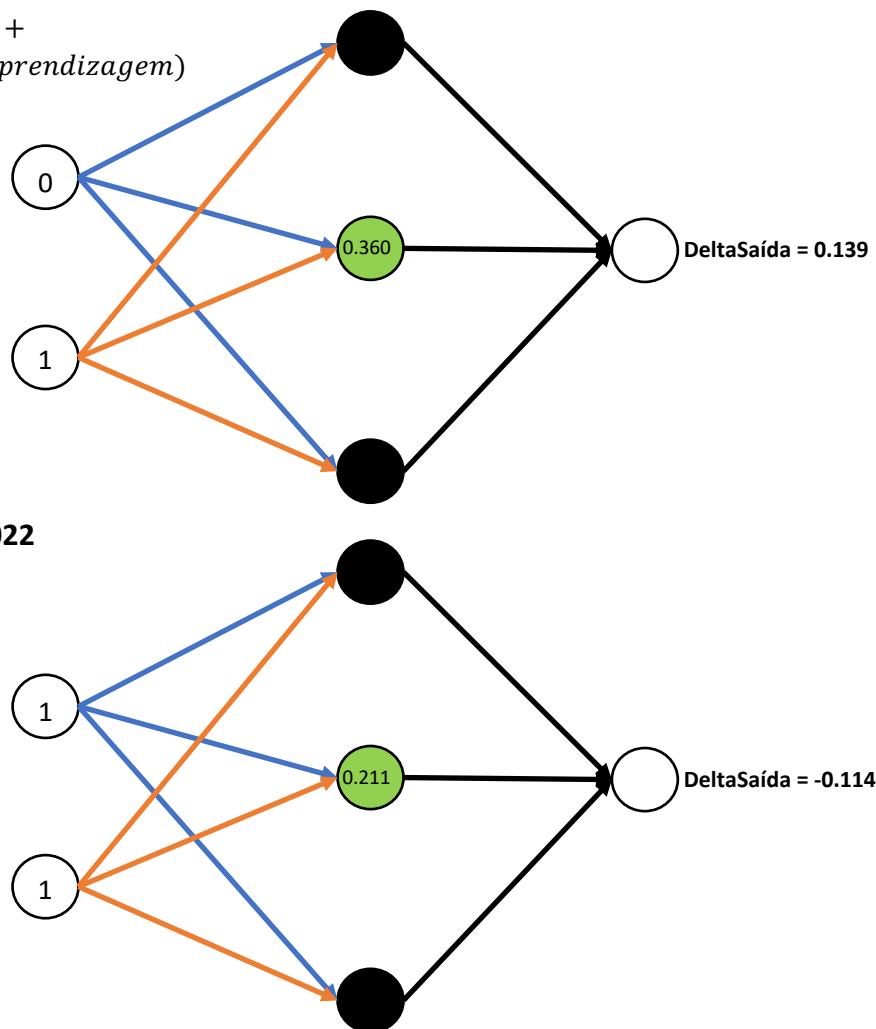
$$\text{peso}_{n+1} = (\text{peso}_n * \text{momento}) + (\text{entrada} * \text{delta} * \text{taxa de aprendizagem})$$

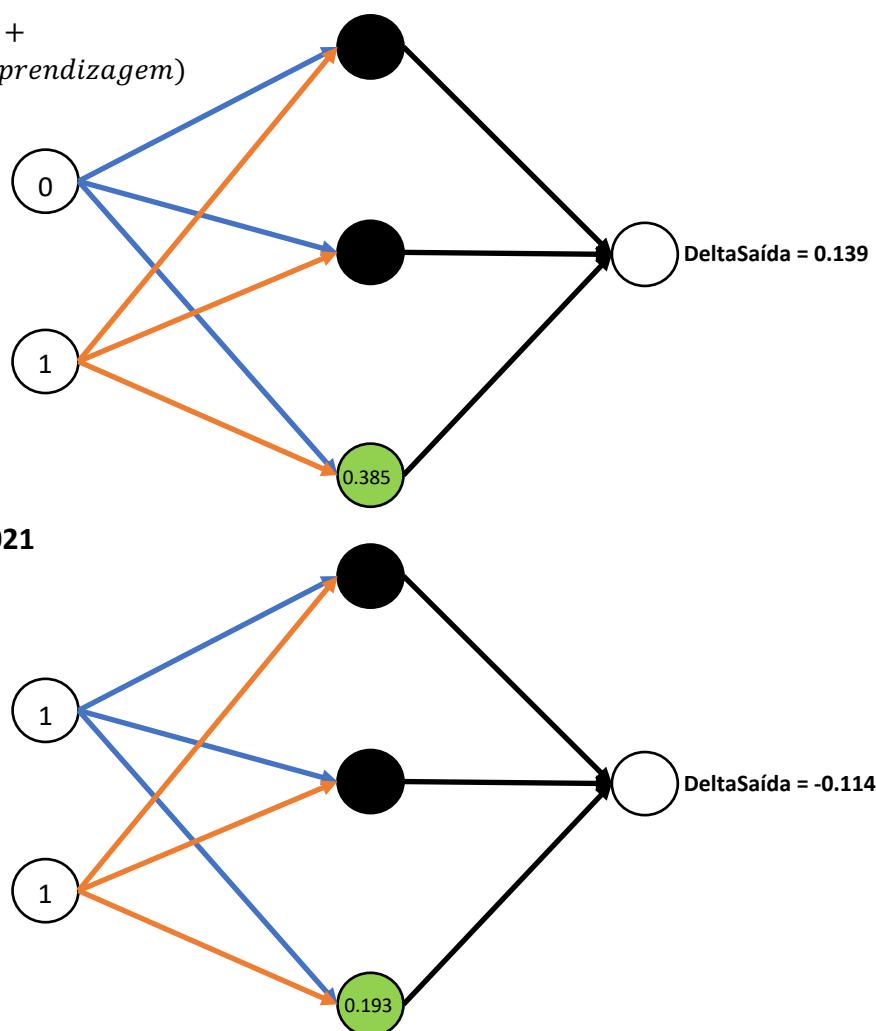
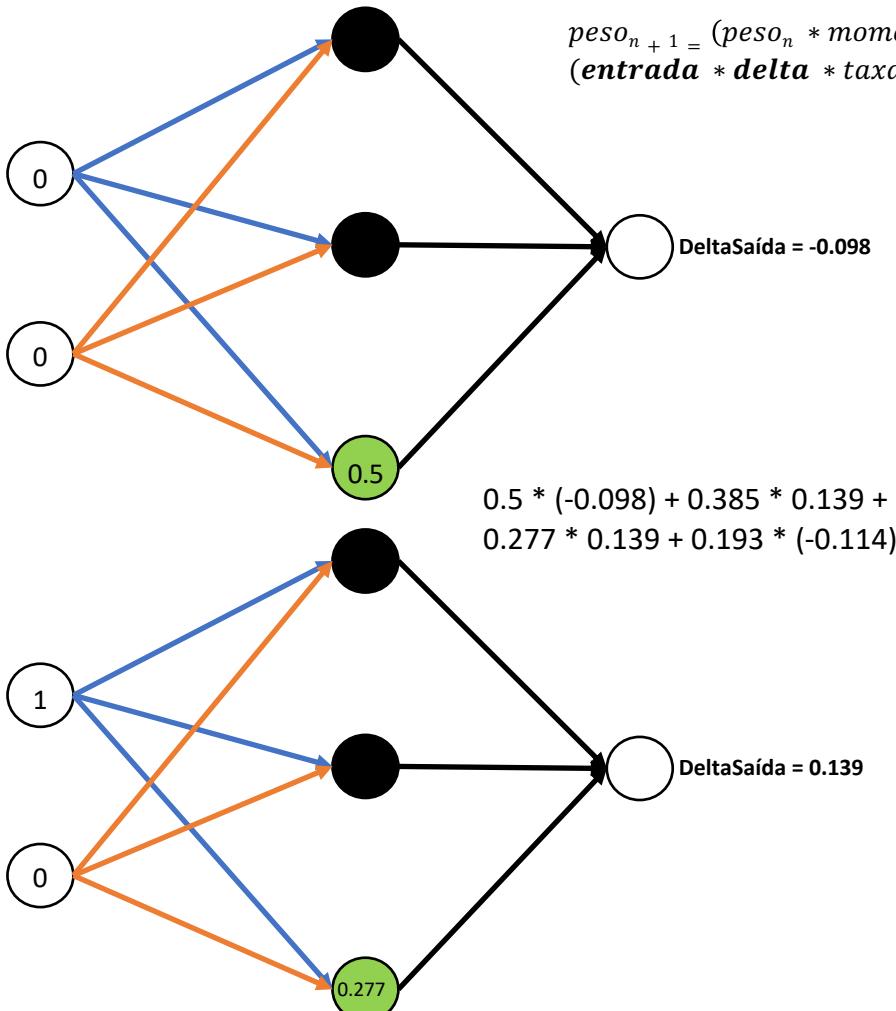




$$peso_{n+1} = (peso_n * momento) + (entrada * delta * taxa\ de\ aprendizagem)$$

$$0.5 * (-0.098) + 0.360 * 0.139 + 0.323 * 0.139 + 0.211 * (-0.114) = \mathbf{0.022}$$





Taxa de aprendizagem = 0.3

Momento = 1

Entrada x delta

0.032

0.022

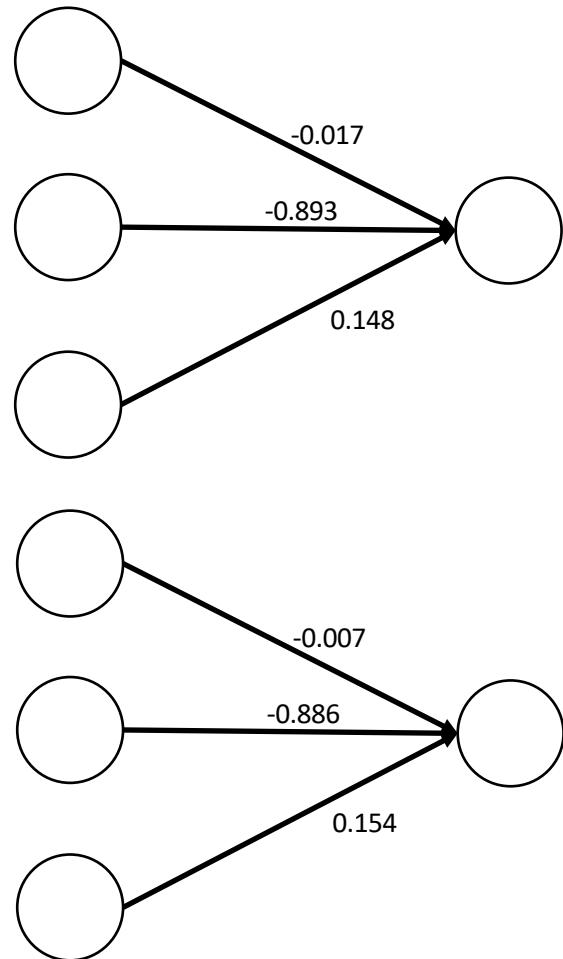
0.021

$$Peso_{n+1} = (peso_n * momento) + (entrada * delta * taxa\ de\ aprendizagem)$$

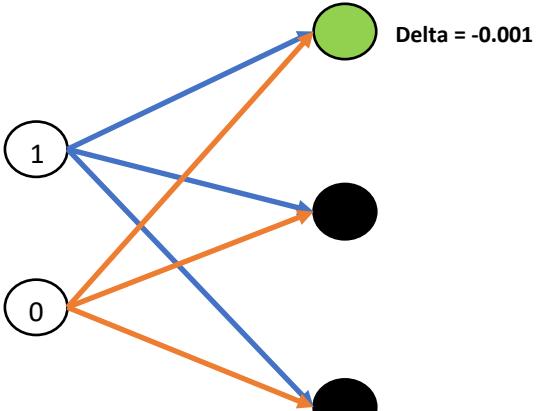
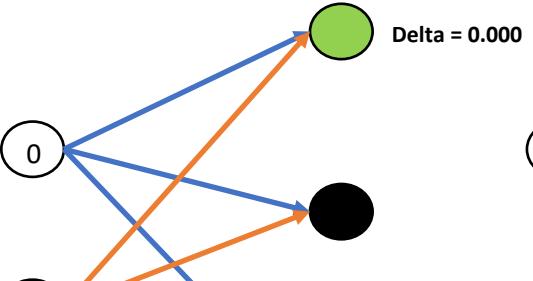
$$(-0.017 * 1) + 0.032 * 0.3 = \mathbf{-0.007}$$

$$(-0.893 * 1) + 0.022 * 0.3 = \mathbf{-0.886}$$

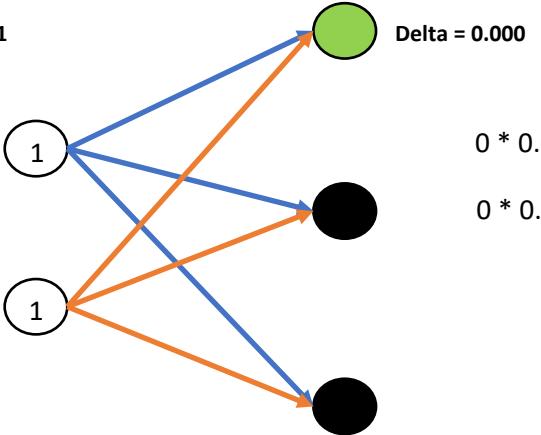
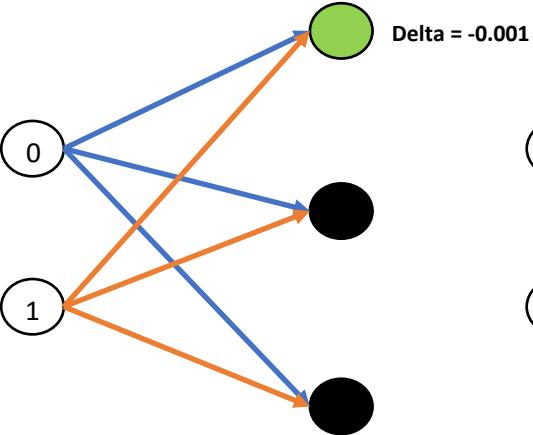
$$(0.148 * 1) + 0.021 * 0.3 = \mathbf{0.154}$$



Ajuste dos pesos da camada de
entrada para a camada oculta



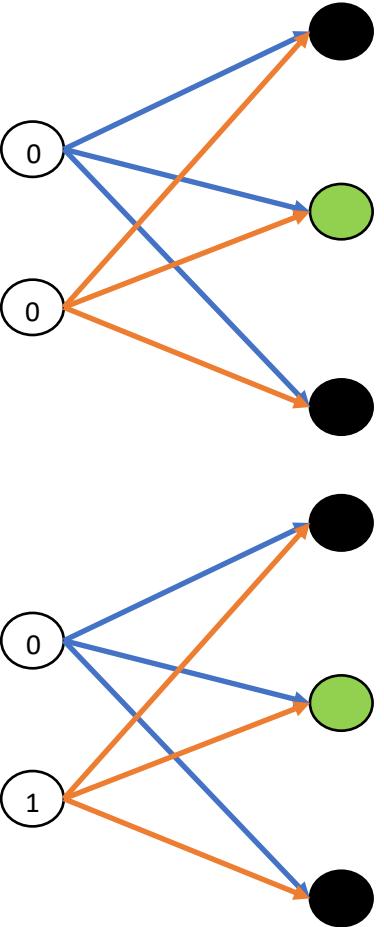
$peso_{n+1} = (peso_n * momento) + (entrada * delta * taxa\ de\ aprendizagem)$



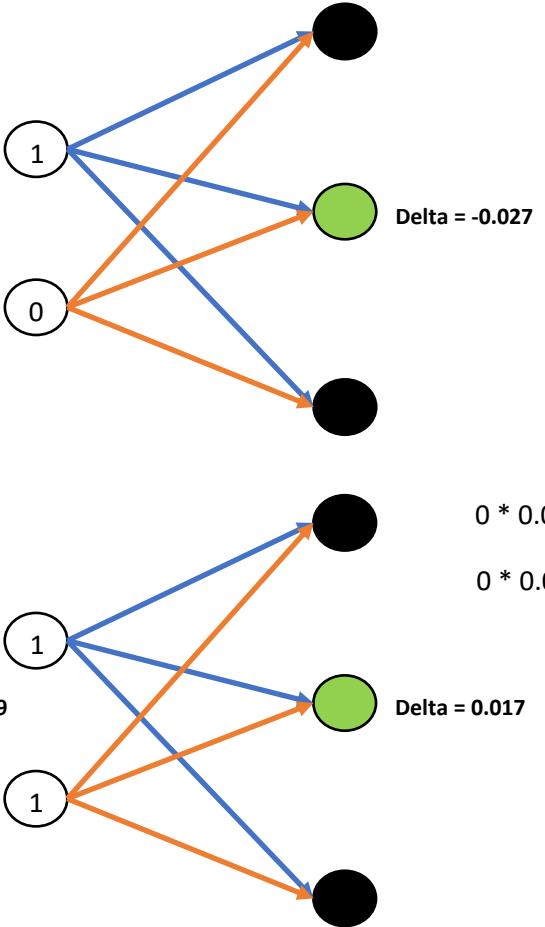
$$0 * 0.000 + 0 * (-0.001) + 1 * (-0.001) + 1 * 0.000 = -0.000$$

$$0 * 0.000 + 1 * (-0.001) + 0 * (-0.001) + 1 * 0.000 = -0.000$$

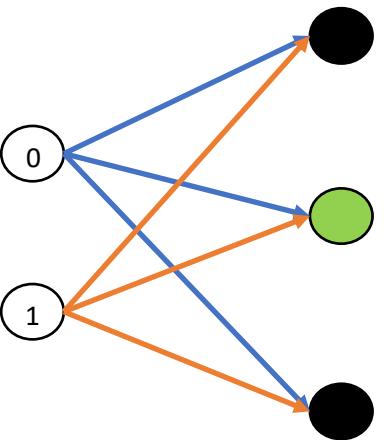
Arredondado!



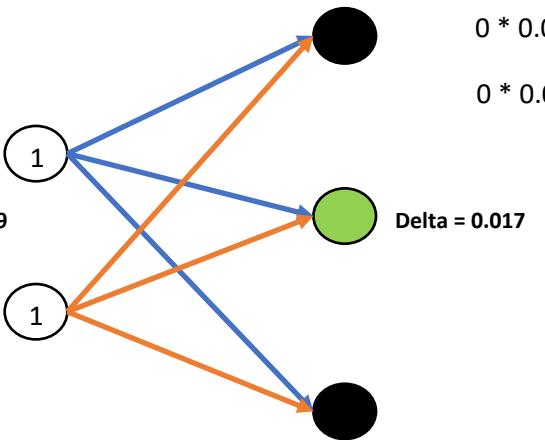
Delta = 0.022



Delta = -0.027



Delta = -0.029

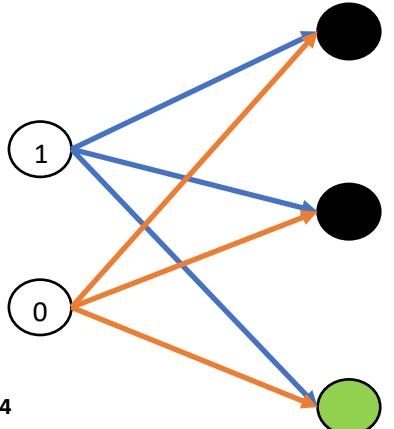
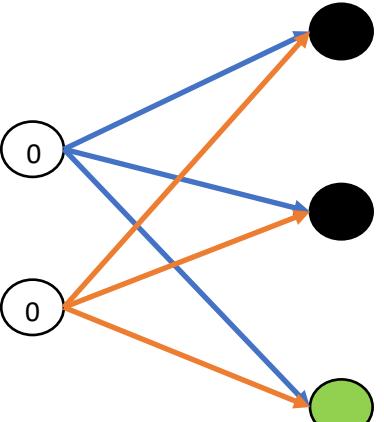


Delta = 0.017

$peso_{n+1} = (peso_n * momento) + (entrada * delta * taxa\ de\ aprendizagem)$

$$0 * 0.022 + 0 * (-0.029) + 1 * (-0.027) + 1 * 0.017 = -0.010$$

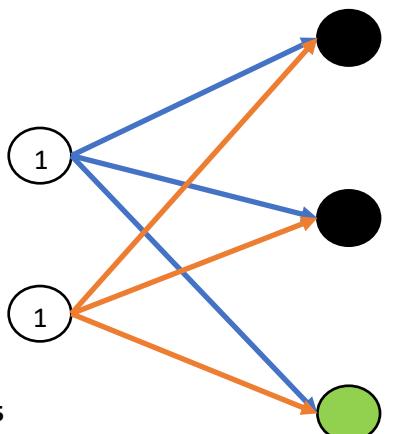
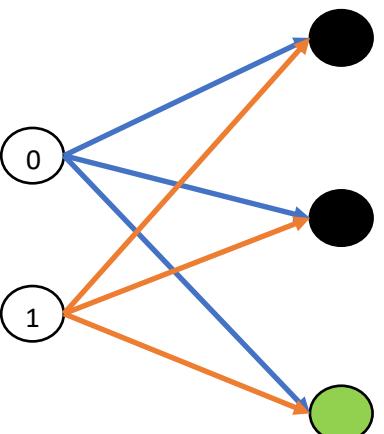
$$0 * 0.022 + 1 * (-0.029) + 0 * (-0.027) + 1 * 0.017 = -0.012$$



$$peso_{n+1} = (peso_n * momento) + (entrada * \delta * taxa\ de\ aprendizagem)$$

Delta = -0.004

Delta = 0.004



$$0 * (-0.004) + 0 * 0.005 + 1 * 0.004 + 1 * (-0.003) = 0.001$$

$$0 * (-0.004) + 1 * 0.005 + 0 * 0.004 + 1 * (-0.003) = 0.002$$

Delta = -0.003

Delta = 0.005

Taxa de aprendizagem = 0.3

Momento = 1

Entrada x delta

-0.000 -0.010 0.001

-0.000 -0.012 0.002

$$\text{Peso}_{n+1} = (\text{peso}_n * \text{momento}) + (\text{entrada} * \text{delta} * \text{taxa de aprendizagem})$$

$$(-0.424 * 1) + (-0.000) * 0.3 = \mathbf{-0.424}$$

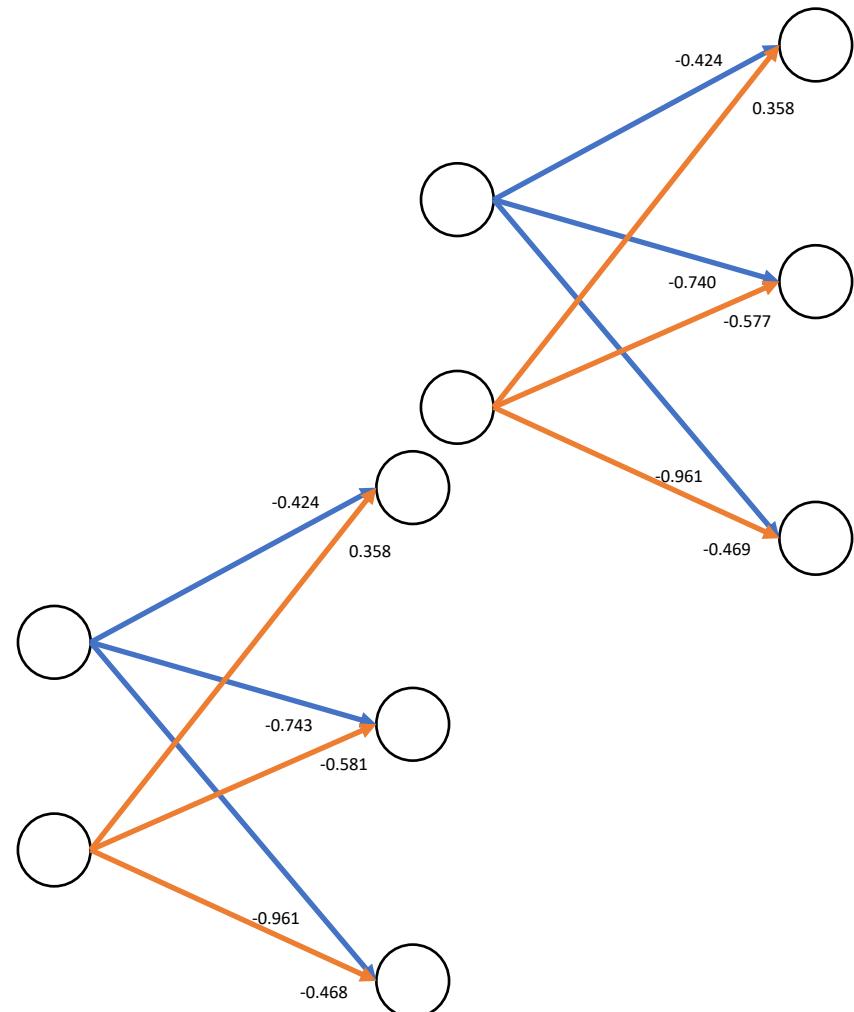
$$(0.358 * 1) + (-0.000) * 0.3 = \mathbf{0.358}$$

$$(-0.740 * 1) + (-0.010) * 0.3 = \mathbf{-0.743}$$

$$(-0.577 * 1) + (-0.012) * 0.3 = \mathbf{-0.581}$$

$$(-0.961 * 1) + 0.001 * 0.3 = \mathbf{-0.961}$$

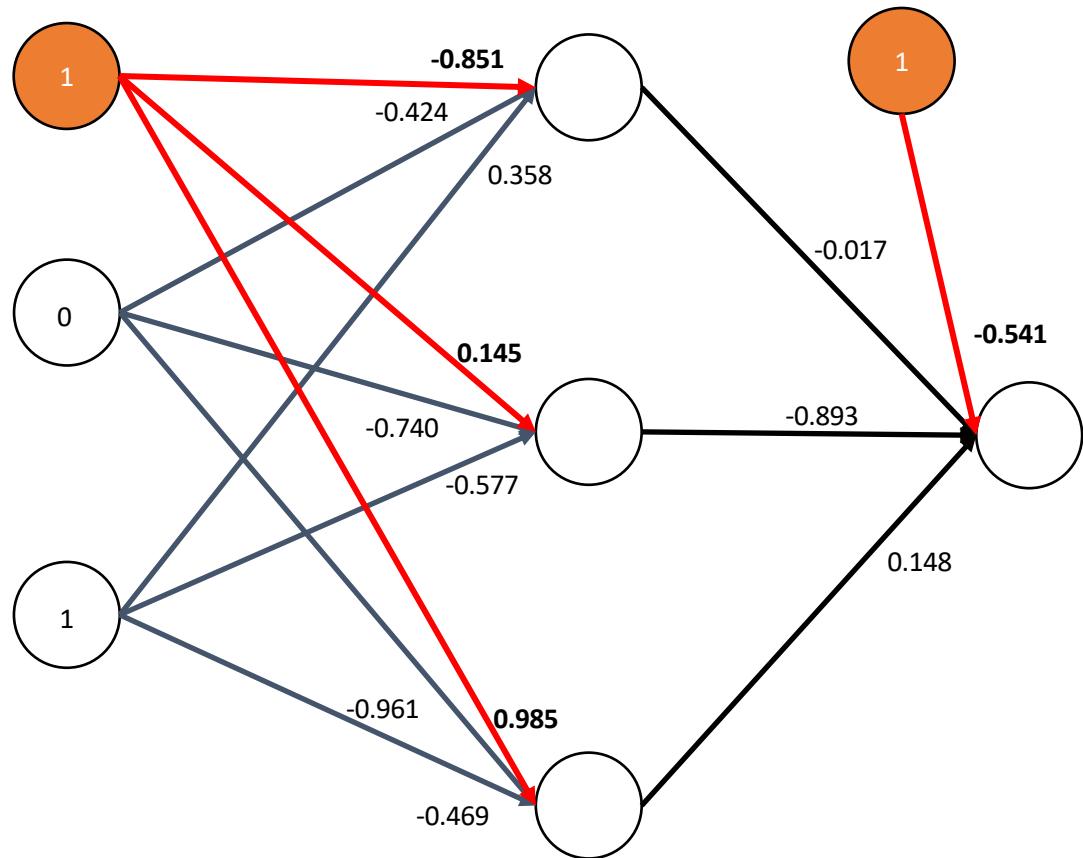
$$(-0.469 * 1) + 0.002 * 0.3 = \mathbf{-0.468}$$



Bias

Valores diferentes mesmo se todas as entradas forem zero

Muda a saída com a unidade de bias



Erro

- Algoritmo mais simples
 - $\text{erro} = \text{respostaCorreta} - \text{respostaCalculada}$

x1	x2	Classe	Calculado	Erro
0	0	0	0.406	-0.406
0	1	1	0.432	0.568
1	0	1	0.437	0.563
1	1	0	0.458	-0.458

Mean square error (MSE) e Root mean square error (RMSE)

$$MSE = \frac{1}{N} \sum_{i=1}^N (f_i - y_i)^2$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (f_i - o_i)^2}$$

x1	x2	Classe	Calculado	Erro
0	0	0	0.406	$(0 - 0.406)^2 = 0.164$
0	1	1	0.432	$(1 - 0.432)^2 = 0.322$
1	0	1	0.437	$(1 - 0.437)^2 = 0.316$
1	1	0	0.458	$(0 - 0.458)^2 = 0.209$

$$\text{Soma} = 1.011$$

$$MSE = 1.011 / 4 = 0.252$$

$$RMSE = 0.501$$

História do crédito	Dívida	Garantias	Renda anual	Risco
3	1	1	1	100
2	1	1	2	100
2	2	1	2	010
2	2	1	3	100
2	2	1	3	001
2	2	2	3	001
3	2	1	1	100
3	2	2	3	010
1	2	1	3	001
1	1	2	3	001
1	1	1	1	100
1	1	1	2	010
1	1	1	3	001
3	1	1	2	100

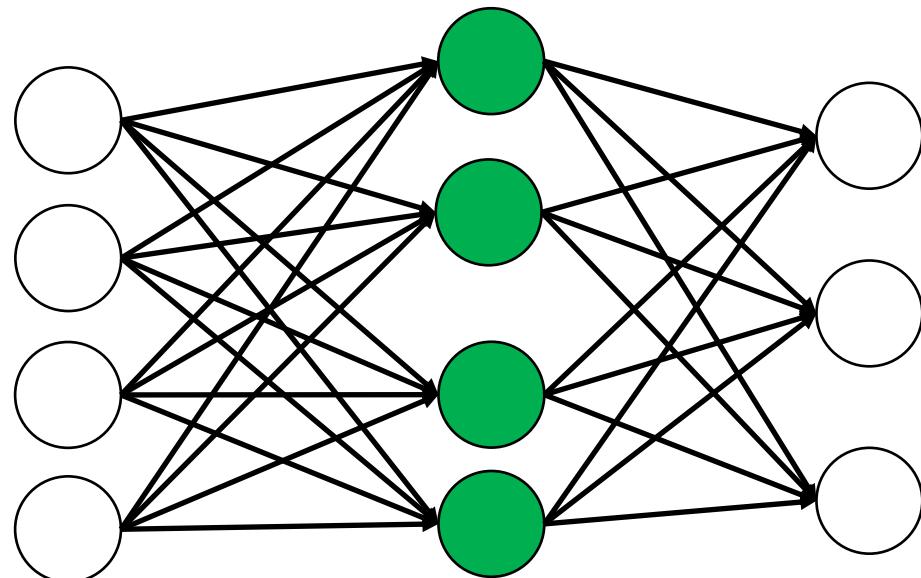
Calcula o erro para todos os registros e atualiza os pesos

Batch gradient descent

História do crédito	Dívida	Garantias	Renda anual	Risco
3	1	1	1	100
2	1	1	2	100
2	2	1	2	010
2	2	1	3	100
2	2	2	3	001
3	2	1	1	100
3	2	2	3	010
1	2	1	3	001
1	1	2	3	001
1	1	1	1	100
1	1	1	2	010
1	1	1	3	001
3	1	1	2	100

Calcula o erro para cada registro e atualiza os pesos

Stochastic gradient descent



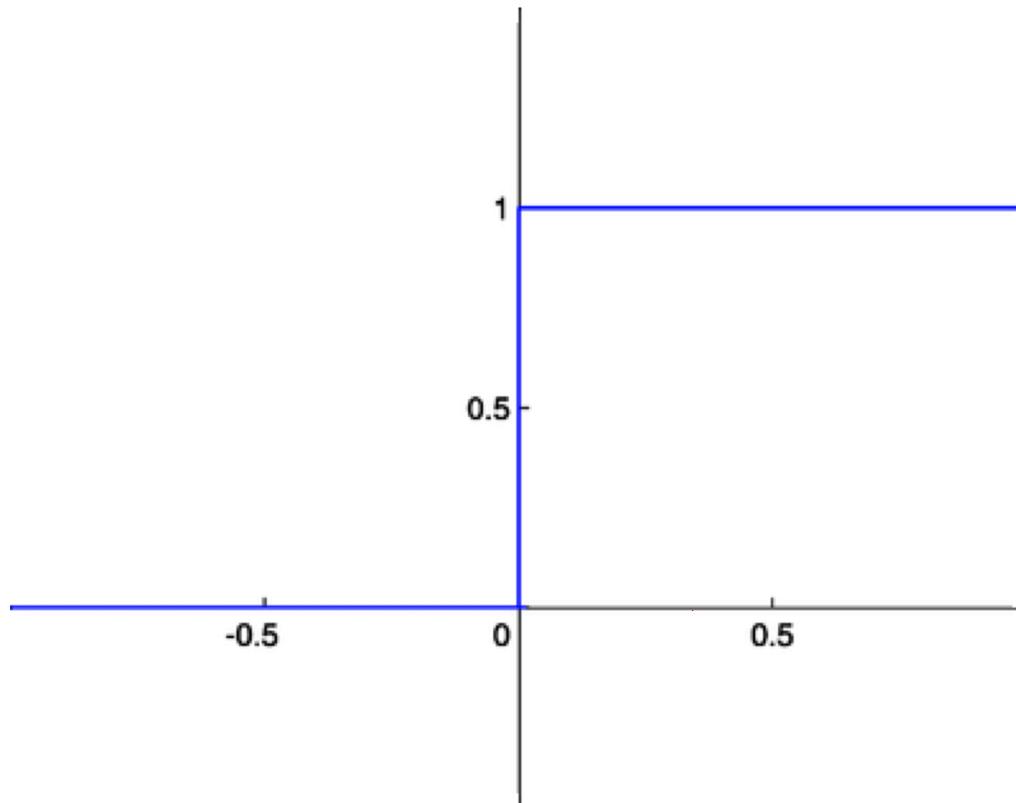
Gradient descent

- Stochastic
 - Ajuda a prevenir mínimos locais (superfícies não convexas)
 - Mais rápido (não precisa carregar todos os dados em memória)
- Mini batch gradient descent
 - Escolhe um número de registros para rodar e atualizar os pesos

Parâmetros

- Learning rate (taxa de aprendizagem)
- Batch size (tamanho do lote)
- Epochs (épocas)

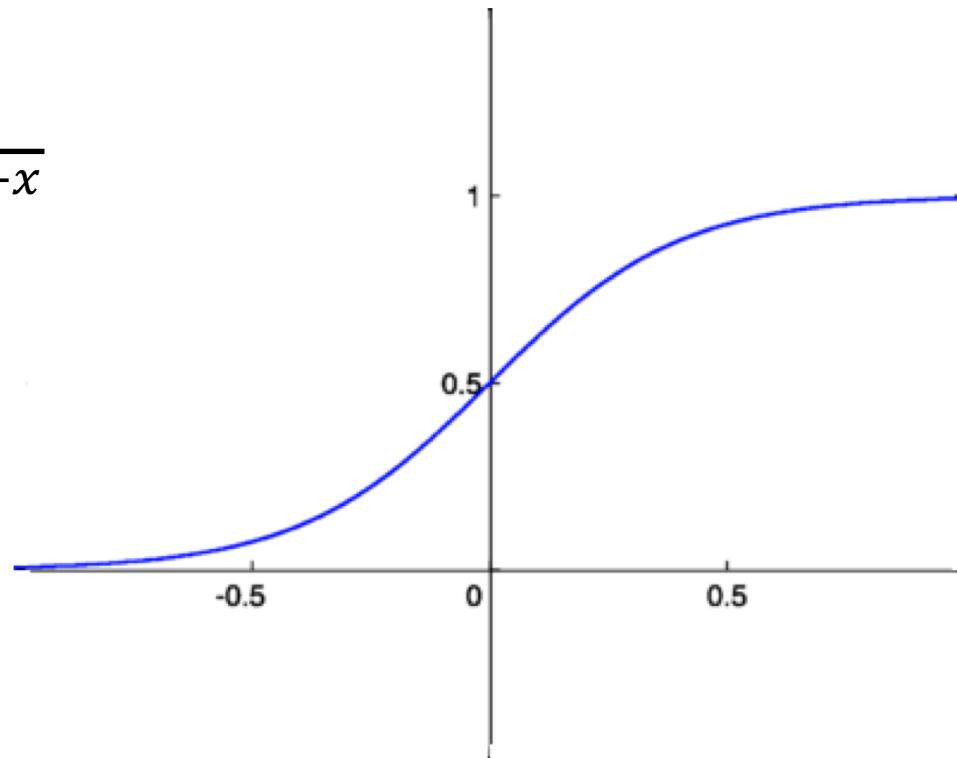
Step (função degrau)



Valor 0 ou 1

Sigmoid (função sigmoide)

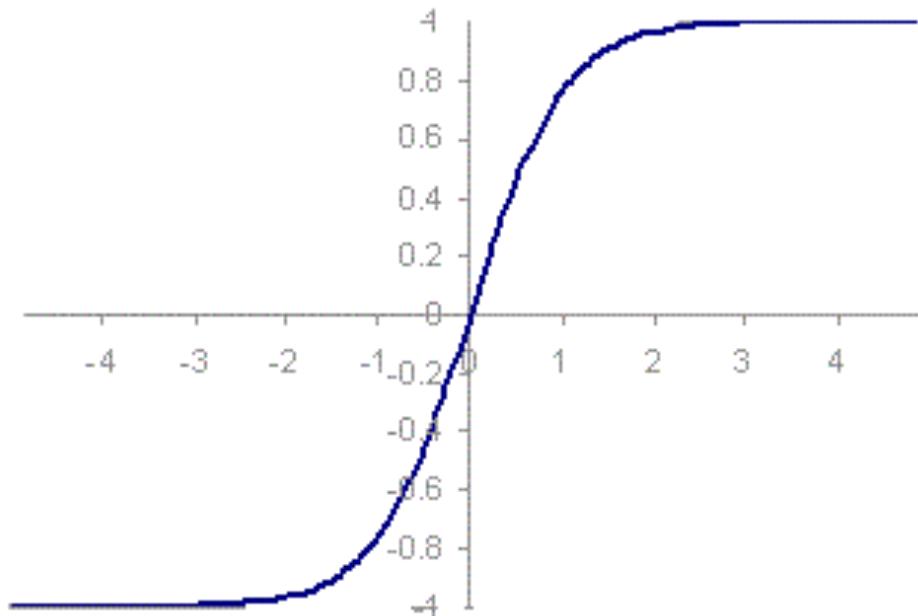
$$y = \frac{1}{1 + e^{-x}}$$



Valores entre 0 e 1

Hyperbolic tangent (função tangente hiperbólica)

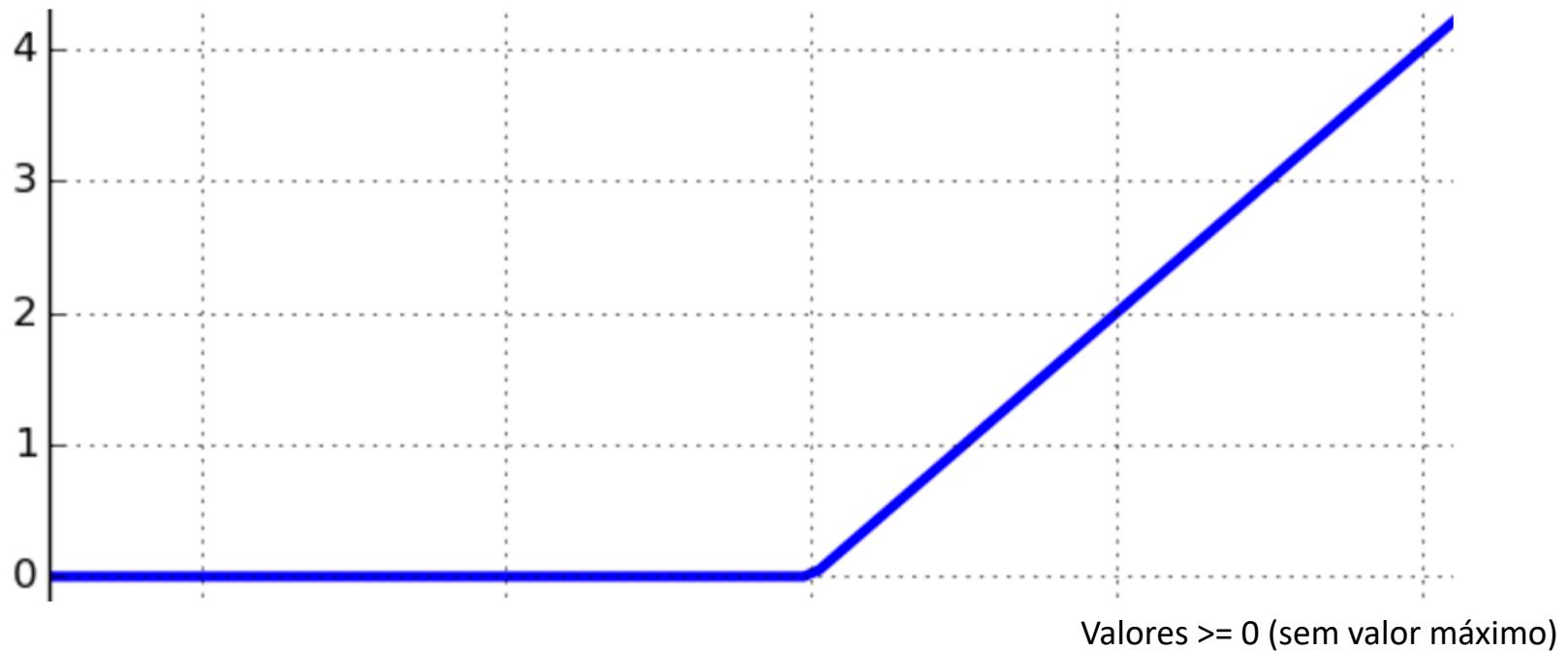
$$Y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



Valores entre -1 e 1

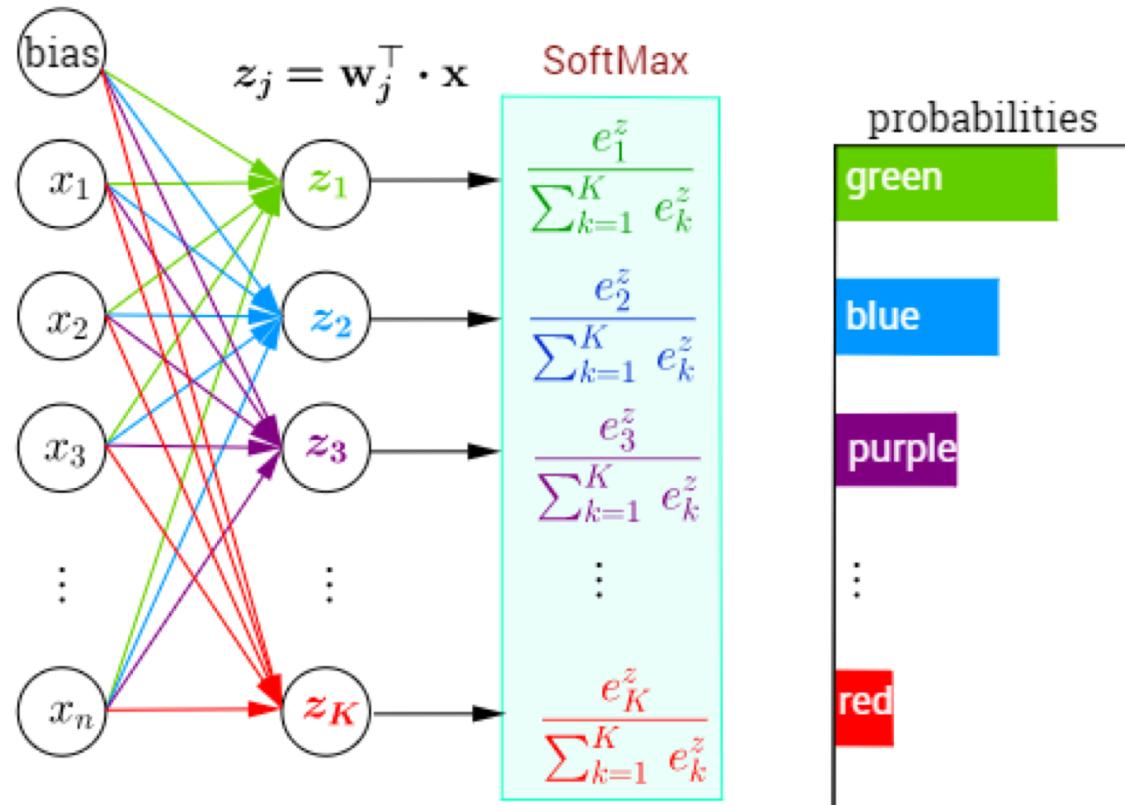
ReLU (rectified linear units)

$$Y = \max(0, x)$$



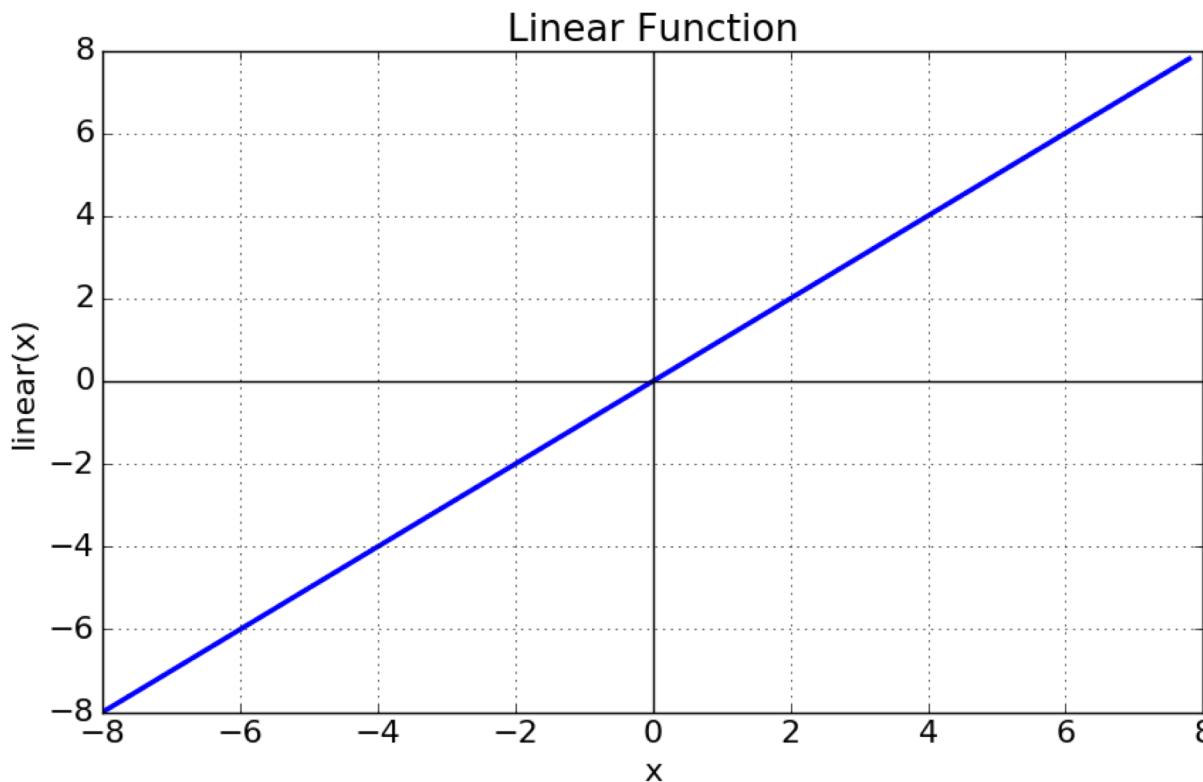
Softmax

$$Y = \frac{e(x)}{\sum e(x)}$$



Fonte: <https://deepnotes.io/category/cnn-series>

Linear



Conclusão

