

$$a_1 = 1$$

$$Q_2 = 0$$

$$a_2 = 0$$
$$a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + n \cdot 2^n$$

$$X^3 = 6X^2 - 12X + 8$$
$$X^3 - 6X^2 + 12X - 8 = 0$$

$$\begin{array}{c|ccc} & 1 & -6 & 12 & -8 \\ \hline 2 & 1 & -4 & 4 & 0 \\ 2 & 1 & -2 & 0 & \\ & 1 & 0 & & \end{array}$$

$$\left. \begin{array}{l} x_1 = 2 \\ x_2 = 2 \\ x_3 = 2 \end{array} \right\}$$

$$\begin{array}{r} 2880 \\ \hline 1440 \end{array}$$

$n=2$ Com multiplicidade 3.

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$$\begin{aligned} -3p_2 + 3p_1 - 3p_0 + 96p_2 - 48p_1 + 24p_0 - 243p_2 + 81p_1 - 27p_0 &= 0 \\ -150p_2 + 36p_1 - 6p_0 = 0 \quad \frac{-150}{60} \times \frac{108}{24} - 6p_0 = 0 \Rightarrow -6p_0 = -2 \quad \frac{8}{-24} \\ p_0 &= \frac{1}{3} \end{aligned}$$

Solução geral.

$$a_n = \alpha_1 \cdot 2^n + \alpha_2 n 2^n + \alpha_3 n^2 2^n$$

$$p_n = (\alpha_1 + \alpha_2 n + \alpha_3 n^2) 2^n$$

Procurando uma solução particular
tem uma raiz com apenas multiplicidade
de 3.

de 3

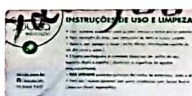
Vamos supor que $(a_n) = (p_2 n^2 + p_1 n + p_0) \cdot 2 \cdot n^3$

$a_{n-1} = (p_2 n^5 + p_1 n^4 + p_0 n^3) \cdot 2$

$a_{n-1} = (p_2 (n-1)^5 + p_1 (n-1)^4 + p_0 (n-1)^3) \cdot 2^{n-1}$

$a_{n-2} = (p_2 (n-2)^5 + p_1 (n-2)^4 + p_0 (n-2)^3) \cdot 2^{n-2}$

$a_{n-3} = (p_2 (n-3)^5 + p_1 (n-3)^4 + p_0 (n-3)^3) \cdot 2^{n-3}$



$$6a_{n-1} = 2^n \left[3(p_2(n^5 - 5n^4 + 10n^3 - 10n^2 + 5n - 1) + p_1(n^4 - 4n^3 + 6n^2 - 4n + 1) + p_0(n^3 - 3n^2 + 3n - 1)) \right]$$

$$- 12a_{n-2} = 2^n \left[3(p_2(n^5 - 10n^4 + 40n^3 - 80n^2 + 80n - 32) + p_1(n^4 - 8n^3 + 24n^2 - 32n + 16) + p_0(n^3 - 6n^2 + 12n - 8)) \right]$$

$$8a_{n-3} = 2^n \left[p_2(n^5 - 15n^4 + 80n^3 - 270n^2 + 405n - 243) + p_1(n^4 - 12n^3 + 54n^2 - 108n + 81) + p_0(n^3 - 9n^2 + 27n - 27) \right]$$

Daí a relação $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + n^2 \cdot 2^n$ se torna.

$$(p_2 n^5 + p_1 n^4 + p_0 n^3) \cdot 2^n = 2^n \left(3p_2(n^5 - 5n^4 + 10n^3 - 10n^2 + 5n - 1) + 3p_1(n^4 - 4n^3 + 6n^2 - 4n + 1) + 3p_0(n^3 - 3n^2 + 3n - 1) \right)$$

$$- 2^n \left(3p_2(n^5 - 10n^4 + 40n^3 - 80n^2 + 80n - 32) + 3p_1(n^4 - 8n^3 + 24n^2 - 32n + 16) + 3p_0(n^3 - 6n^2 + 12n - 8) \right)$$

$$+ 2^n \left(p_2(n^5 - 15n^4 + 80n^3 - 270n^2 + 405n - 243) + p_1(n^4 - 12n^3 + 54n^2 - 108n + 81) + p_0(n^3 - 9n^2 + 27n - 27) \right)$$

$$+ 2^n \cdot n^2$$

$$p_2 n^5 + p_1 n^4 + p_0 n^3 = n^5 (3p_2 - 3p_2 + p_2) + n^4 (-15p_2 + 3p_1 + 30p_2 - 3p_1 - 15p_2 + p_1) + n^3 (30p_2 - 12p_1 + 3p_0$$

$$- 120p_2 + 24p_1 - 3p_0 + 90p_2 - 12p_1 + p_0) + n^2 (-30p_2 + 18p_1 - 9p_0 + 240p_2 - 72p_1$$

$$+ 18p_0 - 270p_2 + 54p_1 - 9p_0) + n (15p_2 - 12p_1 + 9p_0 - 240p_2 + 96p_1 - 36p_0 + 405p_2$$

$$- 108p_1 + 27p_0) + (-3p_2 + 3p_1 - 3p_0 + 96p_2 - 48p_1 + 24p_0 - 243p_2 + 81p_1 - 27p_0) + (n^2)$$

$p_2 n^5 = n^5 (p_2)$ não ajuda

$$- 60p_2 + 1 = 0 \Rightarrow 60p_2 = 1 \Rightarrow p_2 = \frac{1}{60}$$

$$15p_2 - 12p_1 + 9p_0 - 240p_2 + 96p_1 - 36p_0 + 405p_2 - 108p_1 + 27p_0 = 0$$

$$180p_2 - 24p_1 = 0 \Rightarrow 24p_1 = \frac{180}{60} \Rightarrow p_1 = \frac{3}{24}$$

$$a_n = (\alpha_1 + \alpha_2 n + \alpha_3 n^2) 2^n + \left(\frac{1}{60} n^2 + \frac{3}{24} n + \frac{1}{3} \right) 2^n \cdot n^3$$

é a fórmula fechada

b) Encontramos o décimo termo dessa sequência.

$$a_n = (\alpha_1 + \alpha_2 n + \alpha_3 n^2) 2^n$$

$$a_0 = (\alpha_1 + \alpha_2 \cdot 0 + \alpha_3 \cdot 0^2) 2^0$$

$$\boxed{a_0 = \alpha_1 = 10}$$

$$a_n = \alpha_1 2^n + \alpha_2 n 2^n + \alpha_3 n^2 2^n$$

$$1 = 20 + 2\alpha_2 + 2\alpha_3$$

$$2\alpha_2 + 2\alpha_3 = -19$$

$$a_2 = \alpha_1 2^2 + \alpha_2 n 2^n + \alpha_3 n^2 2^n$$

$$0 = 4\alpha_1 + 8\alpha_2 + 16\alpha_3$$

$$8\alpha_2 + 16\alpha_3 = -40$$

$$\begin{array}{rcl} 2\alpha_2 + 2\alpha_3 & = & -19 \quad | \cdot 8 \\ 8\alpha_2 + 16\alpha_3 & = & -40 \quad | \cdot 2 \end{array}$$

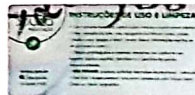
$$\begin{array}{rcl} 16\alpha_2 + 16\alpha_3 & = & -152 \\ -16\alpha_2 - 32\alpha_3 & = & -80 \end{array}$$

$$\begin{array}{rcl} -16\alpha_3 & = & -72 \\ \alpha_3 & = & \frac{72}{16} = \frac{9}{2} \end{array}$$

$$2\alpha_2 + 2 \times \frac{9}{2} = -19$$

$$2\alpha_2 = -28$$

$$\boxed{\alpha_2 = -14}$$



$$a_n = \left(10 - 14n + \frac{9}{2}n^2\right)2n^2 + \left(\frac{1}{60}n^2 + \frac{3}{24}n + \frac{1}{3}\right)2^n n^3$$

$$a_{10} = \left(10 - 14 \times 10 + \frac{9}{2} \times 10^2\right)2 \times 10^2 + \left(\frac{1}{60} \times 10^2 + \frac{3}{24} \times 10 + \frac{1}{3}\right)2^{10} \times 10^3$$

$$a_{10} = 3\,392\,000$$

