$a_0 = 10$

$a_1 = 1$

$a_2 = 0$

$a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + n^2 \cdot 2^n$

$x^3 = 6x^2 - 12x + 8$

$x^3 - 6x^2 + 12 - 8 = 0$

	1	-6	12	-8
2	1	-4	4	0
2	1	-2	0	
	1	0		

$0 = (x-2)(x-2)(x-2)$

$\alpha_1 = 2$

$\alpha_2 = 2$

$\alpha_3 = 2$

$r = 2$ Com multiplicidade 3.

$\frac{2880}{1440}$

solução geral.

$a_n = \alpha_1 \cdot 2^n + \alpha_2 n 2^n + \alpha_3 n^2 2^n$

$a_n = (\alpha_1 + \alpha_2 n + \alpha_3 n^2) 2^n$

Procurando uma solução particular

tem uma raiz com apenas multiplicidade de 3

Vamos supor que $(a_n)^p = (p_2 n^2 + p_1 n + p_0) \cdot 2^n \cdot n^3$

$a_{n-1} = (p_2 n^5 + p_1 n^4 + p_0 n^3) \cdot 2^n$

$a_{n-1} = (p_2(n-1)^5 + p_1(n-1)^4 + p_0(n-1)^3) \cdot 2^{n-1}$

$a_{n-2} = (p_2(n-2)^5 + p_1(n-2)^4 + p_0(n-2)^3) \cdot 2^{n-2}$

$a_{n-3} = (p_2(n-3)^5 + p_1(n-3)^4 + p_0(n-3)^3) \cdot 2^{n-3}$

$-3p_2 + 3p_1 - 3p_0 + 96p_2 - 48p_1 + 24p_0 - 243p_2 + 81p_1 - 27p_0 = 0$

$-150p_2 + 36p_1 - 6p_0 = 0$ $\quad \frac{-150}{60} \cancel{+} \frac{108}{24} - 6p_0 = 0 \Rightarrow -6p_0 = -2$ $\quad \boxed{p_0 = \frac{1}{3}}$ $\quad \frac{8}{-24}$

$$6a_{n-1} = 2^n\left[3\left(p_2(n^5-5n^4+10n^3-10n^2+5n-1)+p_1(n^4-4n^3+6n^2-4n+1)+p_0(n^3-3n^2+3n-1)\right)\right]$$

$$-12a_{n-2} = 2^n\left[3\left(p_2(n^5-10n^4+40n^3-80n^2+80n-32)+p_1(n^4-8n^3+24n^2-32n+16)+p_0(n^3-6n^2+12n-8)\right)\right]$$

$$8a_{n-3} = 2^n\left[p_2(n^5-15n^4+90n^3-270n^2+405n-243)+p_1(n^4-12n^3+54n^2-108n+81)+p_0(n^3-9n^2+27n-27)\right]$$

Daí a relação $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + n^2 \cdot 2^n$ se torna.

$$(p_2n^5+p_1n^4+p_0n^3)\cdot \cancel{2^n} = \cancel{2^n}\left(3p_2(n^5-5n^4+10n^3-10n^2+5n-1)+3p_1(n^4-4n^3+6n^2-4n+1)+3p_0(n^3-3n^2+3n-1)\right)$$
$$-\cancel{2^n}\left(3p_2(n^5-10n^4+40n^3-80n^2+80n-32)+3p_1(n^4-8n^3+24n^2-32n+16)+3p_0(n^3-6n^2+12n-8)\right)$$
$$+\cancel{2^n}\left(p_2(n^5-15n^4+90n^3-270n^2+405n-243)+p_1(n^4-12n^3+54n^2-108n+81)+p_0(n^3-9n^2+27n-27)\right)$$
$$+\cancel{2^n}\cdot n^2$$

$$p_2n^5+p_1n^4+p_0n^3 = n^5(3p_2-3p_2+p_2)+n^4(-15p_2+3p_1+30p_2-3p_1-15p_2+p_1)+n^3(30p_2-12p_1+3p_0$$
$$-120p_2+24p_1-3p_0+90p_2-12p_1+p_0)+n^2(-30p_2+18p_1-9p_0+240p_2-72p_1$$
$$+18p_0-270p_2+54p_1-9p_0)+n(15p_2-12p_1+9p_0-240p_2+96p_1-36p_0+405p_2$$
$$-108p_1+27p_0)+(-3p_2+3p_1-3p_0+96p_2-48p_1+24p_0-243p_2+81p_1-27p_0)+(n^2)$$

$p_2n^5 = n^5(p_2)$ não ajuda

$p_1\cdot n^4 = p_1$

$p_0n^3 = p_0$

$$-30p_2+18p_1-9p_0+240p_2-72p_1+18p_0-270p_2+54p_1-9p_0+1=0$$
$$-60p_2+1=0 \Rightarrow 60p_2=1 \Rightarrow p_2=\frac{1}{60}$$
$$15p_2-12p_1+9p_0-240p_2+96p_1-36p_0+405p_2-108p_1+27p_0=0$$
$$180p_2-24p_1=0 \Rightarrow 24p_1=\frac{180}{60} \Rightarrow p_1=\frac{3}{24}$$

$$a_n = \left(\alpha_1 + \alpha_2 n + \frac{\alpha_3}{3} n^2\right) 2^n + \left(\frac{1}{60} n^2 + \frac{3}{24} n + \frac{1}{3}\right) 2^n \cdot n^3$$

é a formula fechada

b) Encontramos o décimo termo dessa sequência.

$$a_n = (\alpha_1 + \alpha_2 n + \alpha_3 n^2) 2^n$$

$$a_0 = (\alpha_1 + \alpha_2 \times 0 + \alpha_3 \times 0^2) 2^0$$

$$\boxed{a_0 = \alpha_1 = 10}$$

$$a_1 = \alpha_1 2^n + \alpha_2 n 2^n + \alpha_3 n^2 2^n$$

$$1 = 20 + 2\alpha_2 + 2\alpha_3$$

$$2\alpha_2 + 2\alpha_3 = -19$$

$$a_2 = \alpha_1 2^n + \alpha_2 n 2^n + \alpha_3 n^2 2^n$$

$$0 = 4\alpha_1 + 8\alpha_2 + 16\alpha_3$$

$$8\alpha_2 + 16\alpha_3 = -40$$

$$\begin{cases} 2\alpha_2 + 2\alpha_3 = -19 & \cdot 8 \\ 8\alpha_2 + 16\alpha_3 = -40 & \cdot (-2) \end{cases}$$

$$\begin{cases} 16\alpha_2 + 16\alpha_3 = -152 \\ -16\alpha_2 - 32\alpha_3 = -80 \end{cases}$$

$$-16\alpha_3 = 72$$

$$\alpha_3 = \frac{72}{16} = \frac{9}{2}$$

$$2\alpha_2 + \cancel{2} \times \frac{9}{\cancel{2}} = -19$$

$$2\alpha_2 = -28$$

$$\boxed{\alpha_2 = -14}$$

$$a_n = \left(10 - 14n + \frac{9}{2}n^2\right)2n^2 + \left(\frac{1}{60}n^2 + \frac{3}{24}n + \frac{1}{3}\right)2^n n^3$$

$$a_{10} = \left(10 - 14\times 10 + \frac{9}{2}\times 10^2\right)2\times 10^2 + \left(\frac{1}{60}\times 10^2 + \frac{3}{24}\times 10 + \frac{1}{3}\right)2^{10}\times 10^3$$

$$\boxed{a_{10} = 3\,392\,000}$$

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