Mais um exemplo: Encontrar o 10.º termo de

$$a_n = \underbrace{7a_{n-1} - 15a_{n-2} + 9a_{n-3}} + (-1)^n \quad \begin{cases} a_0 = 1 \\ a_1 = -1 \end{cases} \quad a_2 = 1$$

1.º) Encontrar a solução geral da parte homogênea

2.º) Encontrar uma solução particular da relação. $(a_n)^p = C \cdot (-1)^n$

3.º) Encontrar os parâmetros da sequência.

4.º) Calcular o termo desejado.

Discreta, aula 18 parte 2 20/11

1.º) Encontrar a solução geral de

$$a_n = 7a_{n-1} - 15a_{n-2} + 9a_{n-3} \cancel{\alpha}$$

Polinômio característico: $r^3 = 7r^2 - 15r + 9$

Polinômio característico: $r^3 = 7r^2 - 15r + 9$

$\pm 1, \pm 3, \pm 9$

Raízes: $r^3 - 7r^2 + 15r - 9 = 0$

$r = 1 \Rightarrow 1^3 - 7 \cdot 1^2 + 15 - 9 = 1 - 7 + 15 - 9 = 16 - 16 = 0$

1	1	-7	15	-9
	1	-6	9	0

$r^3 - 7r^2 + 15r - 9 = (r-1) \cdot \underbrace{(r^2 - 6r + 9)}_{} = 0$

$r^2 - 6r + 9 = 0 \Leftrightarrow r = \dfrac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$

$r = \dfrac{6 \pm 0}{2}$ $\left\langle \begin{array}{l} r = 3 \checkmark \\ r = 3 \checkmark \end{array} \right.$

Raízes:

$r = 1$ com multiplicidade 1

$r = 3$ com multiplicidade 2 ←

Solução geral da parte homogênea:

$$(a_n)^h = \alpha \cdot 1^n + (\beta + \gamma \cdot n) \cdot (3)^n = \boxed{\alpha + (\beta + \gamma n) \cdot 3^n = (a_n)^h}$$

2.) Solução particular de $a_n = 7a_{n-1} - 15a_{n-2} + 9a_{n-3} + (-1)^n$

$$(a_n)^p = C \cdot (-1)^n$$

$$a_n = C \cdot (-1)^n$$

$$a_{n-1} = C \cdot (-1)^{n-1} = C \cdot (-1)^n \cdot (-1)^{-1} = C \cdot (-1)^n \cdot \frac{1}{(-1)^1} = -C \cdot (-1)^n$$

$$a_{n-2} = C \cdot (-1)^{n-2} = C \cdot (-1)^n \cdot (-1)^{-2} = C \cdot (-1)^n \cdot \frac{1}{(-1)^2} = C \cdot (-1)^n$$

$$a_{n-3} = C \cdot (-1)^{n-3} = C \cdot (-1)^n \cdot (-1)^{-3} = C \cdot (-1)^n \cdot \frac{1}{(-1)^3} = -C \cdot (-1)^n$$

$$a_n = 7 \cdot a_{n-1} - 15 \cdot a_{n-2} + 9 \cdot a_{n-3} + (-1)^n$$

$$C \cdot \cancel{(-1)^n} = 7 \cdot (-C \cdot \cancel{(-1)^n}) - 15 \cdot C \cdot \cancel{(-1)^n} + 9 \cdot (-C \cdot \cancel{(-1)^n}) + \cancel{(-1)^n} \cdot 1$$

$$C = \underbrace{-7C - 15C - 9C}_{} + 1 \Rightarrow C + 31C = 1$$

$$32C = 1$$

$$\boxed{C = 1/32}$$

A solução particular é $(a_n)^p = \frac{1}{32}(-1)^n$.

E a solução geral é $(a_n) = (a_n)^h + (a_n)^p$

$$a_n = \alpha + (\beta + \gamma n) \cdot 3^n + \frac{1}{32} \cdot (-1)^n$$

3: Encontrando α, β e γ: Lembre: $a_0 = 1$, $a_1 = -1$, $a_2 = 1$

$$a_0 = \alpha + (\beta + \cancel{\gamma \cdot 0}) \cdot 3^0 + \frac{1}{32} \cdot (-1)^0 = \boxed{\alpha + \beta + \frac{1}{32} = 1}$$

$$\alpha + \beta = 1 - \frac{1}{32} = \frac{31}{32}$$

$$a_1 = \alpha + (\beta + \gamma \cdot 1) \cdot 3^1 + \frac{1}{32} \cdot (-1)^1 = \boxed{\alpha + 3\beta + 3\gamma - \frac{1}{32} = -1}$$

$$\alpha + 3\beta + 3\gamma = -\frac{31}{32}$$

$$a_2 = \alpha + (\beta + \gamma \cdot 2) \cdot (3)^2 + \frac{(-1)^2}{32} = \alpha + 9\beta + 18\gamma + \frac{1}{32} = 1$$

$$\alpha + 9\beta + 18\gamma = 1 - \frac{1}{32} = \frac{32}{32} - \frac{1}{32} = \boxed{\frac{31}{32} = A}$$

$$\begin{cases} \boxed{\alpha + 9\beta + 18\gamma = \frac{31}{32}} = A \\ \boxed{\alpha + 3\beta + 3\gamma = \frac{-31}{32}} = -A \\ \boxed{\alpha + \beta + 0\gamma = \frac{31}{32} = A} \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 9 & 18 & A \\ 1 & 3 & 3 & -A \\ 1 & 1 & 0 & A \end{array}\right]$$

$L_1^* = L_1$

$L_2^* = \left(-\frac{1}{1} \cdot L_1 + L_2\right) \cdot (-1/3)$

$L_3^* = \left(-\frac{1}{1} \cdot L_1 + L_3\right)(-1)$

$$\left[\begin{array}{ccc|c} 1 & 9 & 18 & A \\ 0 & 2 & 5 & \frac{2A}{3} \\ 0 & 8 & 18 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 9 & 18 & A \\ 0 & 2 & 5 & \frac{2A}{3} \\ 0 & 8 & 18 & 0 \end{array}\right]$$

$L_1^* = -\frac{9}{2} \cdot L_2 + L_1$

$L_3^* = -\frac{8}{2} \cdot L_2 + L_3$

$\boxed{L_3 - 4L_2}$

$-\frac{9}{2} \cdot 0 + 1 = 1$

$-\frac{9}{2} \cdot 2 + 9 = 0$

$-\frac{9}{2} \cdot 5 + 18 = \frac{-45+36}{2} = \frac{-9}{2}$

$$\left[\begin{array}{cccc} 1 & 0 & -\frac{9}{2} & -2A \\ 0 & 2 & 5 & \frac{2A}{3} \\ 0 & 0 & -2 & -\frac{8A}{3} \end{array}\right]$$

$-\frac{\cancel{9}^{3}}{\cancel{2}} \cdot \frac{\cancel{2}A}{\cancel{3}} + A = -3A + A = -2A$

$-4 \cdot 0 + 0 = 0$

$8 - 4 \cdot 2 = 0$

$18 - 4 \cdot 5 = -2$

$0 - 4 \cdot \frac{2A}{3} = -\frac{8}{3}A$

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$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{9}{2} & -2A \\ 0 & 2 & 5 & \frac{2A}{3} \\ 0 & 0 & -2 & -\frac{8A}{3} \end{array}\right]$$

$$L_1^* = -\frac{-\frac{9}{2}}{\frac{-2}{1}} \cdot L_3 + L_1 = -\frac{9}{4} \cdot L_3 + L_1 = \boxed{L_1 - \frac{9}{4} \cdot L_3}$$

$$L_2^* = -\frac{5}{-2} \cdot L_3 + L_2 = \boxed{L_2 + \frac{5}{2} \cdot L_3} \qquad \boxed{L_3^* = -\frac{1}{2} \cdot L_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4A \\ 0 & 2 & 0 & -6A \\ 0 & 0 & 1 & \frac{4A}{3} \end{array}\right]$$

$$1 - \frac{9}{4} \cdot 0 = 1$$

$$\begin{array}{r} 72 \\ -24 \\ \hline 48 \end{array}$$

$$0 - \frac{9}{4} \cdot 0 = 0$$

$$-\frac{9}{2} - \frac{9}{\cancel{4}} \cdot (-\cancel{2}) = \frac{9}{2} + \frac{9}{2} = 0$$

$$\frac{2A}{3} + \frac{5}{2} \cdot \left(\frac{-8A}{3}\right) = \frac{4A - 40A}{6} = \frac{-36A}{6} = -6A$$

$$-2A - \frac{9}{4} \cdot \left(-\frac{8}{3} A\right) = \frac{-24A + 72A}{12} = \frac{48}{12} \cdot A = 4A$$

$$
\left[\begin{array}{ccc|c}
1 & 0 & 0 & 4A \\
0 & 2 & 0 & -6A \\
0 & 0 & 1 & \frac{4A}{3}
\end{array}\right]
$$

$\alpha \quad \beta \quad \gamma$

$1\alpha + 0\beta + 0\gamma = 4A$ $\quad \boxed{\alpha = 4A}$

$2\beta = -6A \Rightarrow \boxed{\beta = -3A}$

$\boxed{1\gamma = \frac{4A}{3}}$

Lembre: $A = \frac{31}{32} \Rightarrow \alpha = 4 \cdot \frac{31}{32} = \boxed{\frac{31}{8} = \alpha}$

$\beta = -3A = \frac{-3 \cdot 31}{32} = \boxed{\frac{-93}{32} = \beta}$

$\gamma = \frac{\cancel{4}}{3} \cdot \frac{31}{\cancel{32}} = \boxed{\frac{31}{24} = \gamma}$

Lembre que

$$a_n = \alpha + (\beta + \gamma n) \cdot 3^n + \frac{1}{32} \cdot (-1)^n$$

e $\alpha = \frac{31}{8}$ e $\gamma = \frac{31}{24}$

$\beta = -\frac{93}{32}$

Então

$$a_n = \frac{31}{8} + \left(-\frac{93}{32} + \frac{31}{24} n\right) \cdot 3^n + \frac{(-1)^n}{32}$$

24, 32	2
12, 16	2
6, 8	2
3, 4	2
3, 2	2
3, 1	3
1, 1	

$32 \cdot 3$

96

$$a_n = \frac{124 + (-1)^n}{32} + 3^n \cdot \left(\frac{31}{24} n - \frac{93}{32}\right)$$

4?) $a_9 = 171\,615$ //

Pode acontecer que em

$$a_n = \underbrace{c_1 a_{n-1} + \dots + c_k a_{n-k}}_{} + s^n \cdot t$$

que s seja raiz do polinômio característico!

mult. - 1

$$\alpha \cdot r^n + \beta \cdot n \cdot r^n + \gamma \cdot n^2 \cdot r^n + C \cdot r^n \cdot n^m$$

Exemplo: $a_n = \underbrace{7a_{n-1} - 15a_{n-2} + 9a_{n-3}} + 3^n$

$$(a_n)^p = C \cdot 3^n \qquad (a_n)^h = \alpha(1)^n + (\beta + \gamma n) \cdot 3^n$$

Encontrado $C = C_0$ | $a_n = \alpha + \underbrace{(\beta + \gamma n)} \cdot 3^n + C_0 \cdot 3^n \cdot n^2$