$a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + m4^n$

$a_0 = -2$, $a_1 = 0$ $a_2 = 5$

$x^3 - 7x^2 + 16x - 12 = 0$

$x_1 = 2$ $x_2 = 3$ $x_2 = 3$

$(x-2)(x-2)(x-3)$

Solução geral.

$a_n = \alpha_1 2^n + \alpha_2 n 2^n + \varphi 3^n$

$a_n = (\alpha_1 + \alpha_2 n) 2^n + \varphi 3^n$

$a_0 = (\alpha_1 + \alpha_2 0) 2^0 + \varphi 3^0$

$a_0 = \alpha_1 + \varphi$

$a_1 = 2\alpha_1 + 2\alpha_2 + 3\varphi = 0$

$a_2 = 4\alpha_1 + 8\alpha_2 + 9 \cdot \varphi$

$\alpha_1 + \varphi = -2$

$2\alpha_1 + 2\alpha_2 + 3\varphi = 0$

$4\alpha_1 + 8\alpha_2 + 9 \cdot \varphi = 5$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 2 & 2 & 3 & 0 \\ 4 & 8 & 9 & 5 \end{array}\right]$$

$L_1^* = L_1$

$L_2^* = -2L_1 + L_2$

$L_3^* = -4L_1 + L_3$

$L_3^* = -4L_2 + L_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 2 & 1 & 4 \\ 0 & 8 & 5 & 13 \end{array}\right]$$

$+\frac{16}{3}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -2 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$\alpha_1 + \varphi = -2$

$2\alpha_2 + \varphi = -4$

$\varphi = -3$

$\alpha_1 = 1$

$2\alpha_2 = 4 + 3$

$\alpha_2 = x/2$

$\alpha_1 = 1$

$\alpha_2 = 7/2$

$\varphi = -3$

$$\boxed{a_n = \left(1 + \frac{7}{2}n\right)2^n - 3 \cdot 3^n}$$

Em busca de uma solução particular.

$$a_n^p = n^2(p_1 n + p_0)4^n$$

$$a_{n-1} = (n-1)^2\left(p_1(n-1) + p_0\right)4^{(n-1)}$$

$$a_{n-1} = \left[p_1(n-1)^3 + p_0(n-1)^2\right] \times \frac{4^n}{4}$$

$$(4+3)a_{n-1} = \left[p_1(n^3 - 3n^2 + 3n - 1) + p_0(n^2 - 2n + 1)\right](4^n + 3)$$

$$a_{n-2} = \left[p_1(n-2)^3 + p_0(n-2)^2\right] \times 4^{n-2}$$

$$-16a_{n-2} = \left[p_1(n^3 - 6n^2 + 12n - 8) + p_0(n^2 - 4n + 4)\right] \times -4^n$$

$$a_{n-3} = \left[p_1(n-3)^3 + p_0(n-3)^2\right] + \frac{4^n}{4^3}$$

$$(3\times4)a_{n-3} = 3\left[p_1(n^3 - 9n^2 + 27n - 27) + p_0(n^2 - 6n + 9)\right] \times \frac{4^n}{16}$$

$a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n)$

$\cancel{4^n}(p_1n^3+p_0n^2) = (7p_1(n^3-3n^2+3n-1)+7p_0(n^2-2n+1))(\frac{4^n}{4}) - p_1(n^3-6n^2+12n-8)$

$p_0(n^2-4n+4))\cancel{4^n} + (3p_1(n^3-9n^2+27n-27)+3p_0(n^2-6n+9))\frac{4^n}{16}$

$+\cancel{4^n}n$

$p_1n^3+p_0n^2 = \frac{7}{4}p_1(n^3-3n^2+3n-1)+\frac{7}{4}p_0(n^2-2n+1) - p_1(n^3-6n^2+12n-8)$

$-p_0(n^2-4n+4)+\frac{3}{16}p_1(n^3-9n^2+27n-27)+\frac{3}{16}p_0(n^2-6n+9)$

$= n^3\left(\frac{7}{4}p_1 - p_1 + \frac{3}{16}p_1\right) + n^2\left(-\frac{21}{4}p_1+\frac{7}{4}p_0+6p_1-p_0-\frac{27}{16}p_1+\frac{3}{16}p_0\right)$

$+n\left(\frac{21}{4}p_1 - \frac{14}{4}p_0 - 12p_1 + 4p_0 + \frac{81}{16}p_1 - \frac{18}{16}p_0 + 1\right)$

$\left(-\frac{7}{4}p_1 + \frac{7}{4}p_0 + 8p_1 - 4p_0 - \frac{81}{16}p_1 + \frac{27}{16}p_0\right)$

$\cancel{35}$

81

$p_1\cancel{n^3} = \cancel{n^3}\left(\frac{7}{4}p_1 - p_1 + \frac{3}{16}p_1\right)$

$\cancel{p_1} = \frac{15}{16}\cancel{p_1}$ não ajuda

$p_0\cancel{n^2} = \cancel{n^2}\left(-\frac{21}{4}p_1+\frac{7}{4}p_0+6p_1-p_0-\frac{27}{16}p_1+\frac{3}{16}p_0\right)$

$p_0 - \frac{15}{16}p_0 = \frac{15}{16}p_1$ $\quad \frac{19}{16}p_1 - \frac{9}{16}p_0 = 0$

$p_0 = 15p_1$

$$\frac{21}{4}p_1 - \frac{14}{7}p_0 - 12p_1 + 4p_0 + \frac{81}{16}p_1 - \frac{18}{16}p_0 + 1 = 0$$

$$p_0 = -15p_1$$

$$p_1 = \frac{14}{27}p_0 + \frac{16}{27}$$

$$-\frac{80}{79} = p_0 \quad \text{e} \quad p_1 = \frac{16}{237}$$

$$a_n = \left(1 + \frac{7}{2}n\right)^n - 3.3^n + n^2\left(p_1 n + p_0\right)4^n$$

$$a_n = \left(1 + \frac{7}{2}n\right)^n - 3.3^n + n^2\left(\frac{16}{237}n - \frac{80}{79}\right)4^n$$