Universitas Mulawarman
Fakultas Keguruan dan Ilmu Pendidikan
Pendidikan Matematika

Modul Mata Kuliah Aljabar

Semester 1

Anggara Duta Medika

Seri Pertama 2020

Modul Mata Kuliah Aljabar

Edisi Pertama

Anggara Duta Medika
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Modul Mata Kuliah Aljabar Edisi Pertama
Anggara Duta Medika

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Kata Pengantar

Puji syukur kehadirat Tuhan Yang Maha Esa, karena atas rahmat dan hidayah-Nya penulis dapat menulis dan menyelesaikan modul ini dengan senang hati. Beragam halangan dan rintangan telah dilalui dalam proses pembuatan modul pembelajaran ini, dengan harapan mahasiswa program studi Pendidikan Matematika, terutama mahasiswa baru dapat terbantu dalam memahami mata kuliah Aljabar ini.

Di Prodi Pendidikan Matematika ini, terutama untuk mata kuliah Aljabar, referensi yang dapat digunakan untuk mahasiswa masih kurang. Mahasiswa sebenarnya dapat mencari di internet mengenai materi aljabar. Tetapi yang penulis lihat di internet, informasi yang diberikan ada yang kurang relevan, bahkan penulisannya pun tidak diformat dengan baik sehingga hal ini juga akan membuat minat mahasiswa turun untuk mencari materi Aljabar. Oleh karena itu, penulis membuat modul ini agar dapat memudahkan mahasiswa untuk mengulas kembali materi-materi yang telah dipelajari sebelumnya. Selain itu, mahasiswa juga dapat menggunakan modul ini sebagai persiapan kuis, ujian tengah semester, maupun ujian akhir semester.

Buku ini dibuat dengan menggunakan LaTeX untuk memudahkan dalam penulisan formula matematika. Penggunaan LaTeX juga sangat simpel untuk pembuatan modul seperti ini, karena *template* yang sangat bervariasi di internet. Pembuatan modul ini juga menjadi sarana bagi penulis untuk dapat berlatih dalam menggunakan LaTeX lebih lanjut. Oleh karena itu, apabila terdapat kesalahan dalam buku ini, penulis akan sangat senang mendengar kritik dan saran dari pembaca. Kritik dan saran dari pembaca itulah yang membuat buku ini akan menjadi lebih baik lagi kedepannya.

Penjelasan dalam buku ini dibuat sedetail mungkin sehingga mungkin bukan juga sebagai modul, tetapi juga dapat digunakan sebagai buku penuntun pembelajaran. Penulisan yang detail juga mungkin akan membuat pembaca kesulitan dalam memahami tulisan dalam buku ini. Untuk menghadapi hal ini, penulis juga membuat penjelasan yang detail itu dalam bentuk kalimat yang sederhana dan mudah dicerna. Oleh karena itu, jika ada suatu istilah yang kurang familiar, penulis juga terkadang membuat catatan kaki untuk memperjelas makna dari istilah tersebut.

Selain penjelasan yang mendetail, dalam modul ini juga terdapat soal-soal latihan yang dapat dikerjakan oleh pembaca agar dapat lebih memahami materi yang diajarkan. Soal-soal latihan yang terdapat pada buku ini dibagi menjadi dua klasifikasi, yaitu soal-soal rutin dan soal-soal tidak rutin (soal tantangan). Tujuannya adalah, agar pembaca selain lebih mengetahui konsep-konsep aljabar, pembaca juga dapat mengasah kemampuannya agar lebih mahir dalam bermatematika. Selain itu, dengan mengerjakan soal-soal yang beragam, pembaca juga dapat mempersiapkan diri untuk menghadapi ilmu matematika yang lebih mendalam.

Anggara Duta Medika

Ucapan Terima Kasih

Terima kasih kepada teman-teman satu program studi yang membantu mengoreksi segala kesalahan yang terdapat pada buku ini demi terciptanya modul pembelajaran aljabar yang sempurna.

Selain itu, terima kasih juga kepada para dosen yang membantu untuk meningkatkan kualitas modul pembelajaran ini serta saran-saran membangunnya yang menjadi sumber inspirasi dalam penulisan modul ini menjadi lebih baik.

Banyak soal-soal latihan dalam modul ini yang terinspirasi ataupun diambil dari olimpiade-olimpiade matematika baik dari tingkat kota hingga tingkat internasional seperti Olimpiade (Kompetisi) Sains tingkat Kota, Olimpiade (Kompetisi) Sains tingkat Provinsi, Olimpiade (Kompetisi) Sains tingkat Nasional, International Mathematical Olympiad, dan masih banyak yang lainnya.

Penulis melakukan yang terbaik untuk mengutip semua sumber asli soal-soal tersebut. Penulis juga menyampaikan apresiasi mendalam kepada para pengusul soal yang asli.

Selain itu, penulis juga tidak akan dapat menyelesaikan buku ini tanpa buku-buku utama dan modul-modul perkuliahan yang ada. Oleh karena itu, penulis juga menyampaikan terima kasih kepada para pembuat buku dan modul yang digunakan dalam pembuatan buku ini.

Cara Menggunakan Buku

Dalam buku ini, terdapat beberapa istilah-istilah yang digunakan agar pembaca dapat lebih memahami isi buku ini lebih baik. Selain itu, hal ini juga dapat membuat pembaca memiliki fokus yang lebih terarah ketika mempelajari buku ini.

Eksplorasi Dalam kotak eksplorasi, pembaca diharapkan dapat mengeksplorasi mengenai permasalahan yang diberikan pada kotak tersebut. Jika pembaca tidak memahami mengenai cara menjawab permasalahan dalam kotak ini, pembaca dapat mencari referensi lain seperti buku matematika lainnya atau internet. Itulah tujuan dari kotak eksplorasi ini, agar pembaca dapat mencari referensi lain dan tidak terpatok pada buku ini saja.

> **Eksplorasi**
>
> Coba cari tahu mengenai Generasi Steiner. Apa hubungannya dengan proses pensketsaan parabola?

Peringatan Dalam kotak peringatan, pembaca dapat melihat suatu informasi penting yang kadang dilewatkan. Terkadang dalam matematika, terdapat beberapa miskonsepsi. Oleh karena itu, kotak peringatan ini bertujuan agar pembaca tidak terjerumus ke dalam miskonsepsi tersebut.

> **Peringatan**
>
> Perhatikan bahwa $\displaystyle\int f(x)\,g(x)\,\mathrm{d}x \neq \int f(x)\,\mathrm{d}x \cdot \int g(x)\,\mathrm{d}x.$

Informasi Dalam kotak informasi, pembaca dapat mengetahui hal-hal mengenai suatu materi. Biasanya berisi mengenai tips dan trik, cara cepat, atau asal-usul formula tertentu dalam materi tersebut.

Informasi

Penyebutan rumus kuadratik sebagai rumus abc di Indonesia terkadang dijadikan candaan bagi dosen/guru matematika.

> "Jika Anda mencari 'abc formula' di internet, tidak akan ketemu itu. Orang luar tidak ada yang menyebutnya sebagai abc formula, tetapi quadratic formula."
>
> - Prof. Hendra Gunawan, Dosen Matematika ITB

Tokoh Dalam kotak tokoh, pembaca dapat mengetahui tokoh-tokoh matematika yang berperan dalam pengembangan materi yang diajarkan dalam buku ini. Pembaca juga diharapkan dapat meneladani tokoh-tokoh tersebut dan bisa menjadikannya sumber inspirasi dalam karier matematika Anda.

Tokoh — Francois Viete

Viete adalah matematikawan Prancis yang mendalami aljabar. Viete dilahirkan di Fontenay-le-Comte, yang saat ini dikenal sebagai Vendee pada tahun 1540. Ia dikenal karena teorema Vieta yang dikembangkannya. Viete sendiri memiliki nama lain (nama latin) Fransiscus Vieta. Nama latin inilah yang merupakan dasar penamaan teorema Vieta tersebut.

Simbol Khusus Soal Latihan Dalam soal-soal latihan, terdapat beberapa simbol khusus pada beberapa soal. Tujuan dari simbol khusus ini adalah agar pembaca dapat mengetahui jenis soal dan teknik penyelesaiannya. Beberapa simbol khusus tersebut adalah sebagai berikut.

- $\boxed{\approx}$, yang berarti soal tersebut dapat diselesaikan dengan aproksimasi numerik.
- $\boxed{\text{C}}$, yang berarti penggunaan kalkulator (biasa) disarankan unuk menjawab soal tersebut.
- $\boxed{\text{CALC}}$, yang berarti soal tersebut kemungkinan besar harus diselesaikan dengan pengetahuan yang cukup mengenai kalkulus.
- $\boxed{\text{PF}}$, yang berarti soal tersebut selain harus diselesaikan, Anda juga harus membuktikan pekerjaan Anda.
- $\boxed{\text{EXPL}}$, yang berarti soal tersebut memerlukan eksplorasi lebih jauh mengenai materi yang telah dipelajari.
- $\boxed{*}$, $\boxed{**}$, dan $\boxed{***}$ yang menunjukkan tingkat kesulitan soal tersebut.

Daftar Notasi

Dalam buku ini terdapat beberapa notasi matematika yang digunakan. Para mahasiswa diharapkan dapat memahami notasi-notasi tersebut agar dapat lebih memahami buku ini. Beberapa notasi tersebut adalah sebagai berikut.

1. $\mathbb{N}$ menyatakan himpunan bilangan asli, yaitu $\{1, 2, 3, \dots\}$[1].
2. $\mathbb{Z}$ menyatakan himpunan bilangan bulat, yaitu $\{\dots, -2, -1, 0, 1, 2, \dots\}$.
3. $\mathbb{N}_0$ menyatakan himpunan bilangan asli serta angka 0.
4. $\mathbb{Z}^+$ menyatakan himpunan bilangan bulat positif.
5. $\mathbb{Z}^-$ menyatakan himpunan bilangan bulat negatif.
6. $\mathbb{Q}$ menyatakan himpunan bilangan rasional, yaitu bilangan yang dapat dinyatakan dalam bentuk $\frac{a}{b}$ dengan a dan b bilangan bulat serta $b \neq 0$.
7. $\mathbb{R}$ menyatakan himpunan bilangan real, yaitu bilangan yang dapat dituliskan dalam bentuk desimal.

[1] Terkadang ada beberapa penulis yang memasukkan 0 sebagai anggota bilangan asli.

Contents

CHAPTER 1

Pendahuluan

sec:intro

Dalam modul aljabar ini, terdapat tiga materi yang akan dibahas secara mendalam. Ketiga materi tersebut adalah persamaan dan fungsi kuadrat, pertidaksamaan satu variabel, serta eksponen dan logaritma. Sebelum mempelajari lebih mendalam mengenai ketiga materi tersebut, mahasiswa diharapkan mengetahui terlebih dahulu operasi-operasi aljabar serta simbol-simbol matematika, terutama himpunan. Pengetahuan yang cukup mengenai operasi-operasi aljabar akan mempermudah mahasiswa dalam memahami buku ini, bahkan hingga semester-semester berikutnya. Sedangkan pengetahuan yang cukup mengenai himpunan akan membantu mahasiswa dalam memahami materi pertidaksamaan.

1.1 Operasi-operasi Aljabar

Aljabar selalu membahas mengenai simbol-simbol matematika dan aturan-aturannya dalam memanipulasi simbol-simbol ini. Dalam aljabar, ada yang disebut sebagai variabel dan konstanta. Variabel adalah sesuatu yang dapat berubah-ubah. Variabel biasanya disimbolkan dengan huruf-huruf seperti x, y, α, ξ, dan lain sebagainya. Sedangkan konstanta adalah suatu nilai yang tetap seperti 1, 2, π, 0, dan bilangan-bilangan lainnya.

Eksplorasi

Misalnya kita diberikan $x = 5$, apakah x suatu konstanta atau variabel?

Terdapat empat operasi dasar aljabar, yaitu penjumlahan, pengurangan, perkalian, dan pembagian. Meskipun dalam buku teks matematika lebih lanjut penjumlahan dan pengurangan dianggap sama, serta perkalian dan pembagian juga dianggap sama, tetapi dalam buku ini kita anggap mereka semuanya berbeda agar lebih simpel dan mudah dipahami.

Terdapat istilah matematika juga yang perlu diketahui, yaitu ekspresi aljabar. Ekspresi aljabar adalah kombinasi simbol-simbol yang biasanya terdiri dari gabungan antara variabel, konstanta, dan operasi-operasi aljabar seperti penjumlahan, pengurangan, perkalian, pembagian, dan lain sebagainya. Contoh dari ekspresi aljabar ini adalah $2 \cdot x$, yang terdiri dari konstanta, variabel, dan operasi perkalian yang dinotasikan dengan tanda titik ditengah $(\cdot)$. Dari sini, kita bisa yakinkan diri kita bahwa konstanta dan variabel tidak dapat disatukan tanpa suatu operasi aljabar. Oleh karena itu, pembahasan mengenai operasi-operasi aljabar ini sangat diperlukan.

Contoh lain dari ekspresi aljabar adalah $2x + 3y$. Operasi-operasi ini bisa dipecah menjadi dua bagian, yaitu $2x$ dan $+3y$. Bagian-bagian inilah yang biasa disebut sebagai suku-suku dari suatu ekspresi aljabar[1]. Tetapi, biasanya tanda tambah $(+)$ tidak diikutsertakan dalam pemecahan tersebut sehingga biasanya dituliskan sebagai $3y$ saja. Perhatikan bahwa suku-suku adalah pemecahan suatu ekspresi aljabar terhadap operasi penjumlahan dan pengurangan. Oleh karena itu, $2x$ disini tidak dipecah lagi menjadi dua ekspresi berbeda (yaitu 2 dan x). Angka 2 pada ekspresi $2x$ dan angka 3 pada ekspresi $3x$ disebut sebagai koefisien. Bisa dibilang bahwa koefisien ini adalah angka yang berada di "depan" variabel, meskipun sebenarnya adalah angka yang dikalikan dengan suatu variabel karena angka 7 pada ekspresi $x \cdot 7$ juga disebut sebagai koefisien karena sesungguhnya $x \cdot 7 = 7x$.

Dalam materi ini, mahasiswa diasumsikan telah mengerti mengenai operasi-operasi aljabar dan tata cara pengoperasiannya. Tetapi perlu digaris bawahi kesalahan-kesalahan yang sering dilakukan oleh mahasiswa dalam mengoperasikan suatu ekspresi aljabar, yaitu sebagai berikut:

1. $\sqrt{x+y} \stackrel{?}{=} \sqrt{x} + \sqrt{y}$;
2. $\dfrac{1}{x+y} \stackrel{?}{=} \dfrac{1}{x} + \dfrac{1}{y}$;
3. $(x+y)^2 \stackrel{?}{=} x^2 + y^2$
4. $\dfrac{x}{a} - \dfrac{2x+y}{a} \stackrel{?}{=} \dfrac{-x+y}{a}$; dan
5. $\dfrac{2x}{x} \stackrel{?}{=} 2$.

Eksplorasi

Dapatkah Anda memberikan suatu penjelasan mengenai kesalahan-kesalahan ini? Lalu bagaimana penulisan yang benar?

1.2 Himpunan

Sampai saat ini, belum ada definisi yang tepat untuk himpunan sehingga secara matematis, himpunan tidak didefinisikan. Meskipun demikian, terdapat beberapa definisi yang tidak formal mengenai himpunan. Bisa dikatakan bahwa himpunan adalah koleksi dari objek-objek yang memiliki karakteristik tertentu yang jelas seperti himpunan mahasiswa Pendidikan Matematika yang mengambil mata kuliah Aljabar. Oleh karena itu, kumpulan dari objek-objek yang belum jelas (masih merupakan pendapat pribadi atau persepsi seseorang) bukan termasuk dalam suatu himpunan, sebagai contoh kumpulan orang-orang yang tinggi bukan merupakan himpunan karena setiap orang memiliki persepsi tersendiri mengenai ketinggian seseorang.

[1] Meskipun sebenarnya $2x + 3y$ juga dapat disebut satu suku jika kita memisalkan $u = 2x + 3y$.

Himpunan biasanya dilambangkan oleh huruf besar (misalnya A, B, Γ), huruf bergaris ganda (misalnya $\mathbb{R}$ yang melambangkan himpunan bilangan real dan $\mathbb{Z}$ yang melambangkan himpunan bilangan bulat), atau huruf berkaligrafi (misalnya $\mathcal{P}$ dan $\mathcal{R}$). Sementara itu, anggota dari suatu himpunan biasanya dilambangkan oleh huruf kecil seperti a, x, dan δ. Keanggotaaan suatu objek dilambangkan dengan tanda "$\in$" yang pertama kali diperkenalkan oleh Peano (1889) dan lambang untuk ketidakanggotaan adalah "$\notin$". Misalnya, jika kita diberikan suatu himpunan $A = \{c, d\}$, maka $c \in A$ dan $a \notin A$.

Kita tidak membicarakan secara mendetail mengenai himpunan karena akan dibahas mendalam pada mata kuliah Pengantar Dasar Matematika. Pada bagian ini kita hanya membahas mengenai dasar-dasar mengenai himpunan dan notasi-notasinya serta cara mengoperasikan notasi-notasi tersebut.

Himpunan dapat direpresentasikan dengan dua cara, yaitu dengan cara enumerasi/pendaftaran dan dengan menggunakan notasi pembentuk himpunan. Pada cara enumerasi, semua elemen (anggota himpunan) dituliskan dan diletakkan diantara kurung kurawal. Sebagai contoh, himpunan bilangan prima yang lebih kecil dari 10 bisa dituliskan sebagai $\{2, 3, 5, 7\}$. Sementara itu, dengan menggunakan notasi pembentuk himpunan, diantara kurung kurawal diletakkan variabel, kemudian dibatasi dengan tanda "$|$" dan dituliskan sifat yang mengatur variabel tersebut. Sebagai contoh, himpunan bilangan prima yang lebih kecil dari 10 dapat dituliskan sebagai $\{x \mid x \text{ prima}, x < 10\}$.

Kadangkala, terdapat suatu himpunan yang tidak memiliki anggota, seperti himpunan semua mahasiswa Hukum yang mengambil mata kuliah Aljabar. Berkaitan dengan hal ini, kita tuliskan himpunan yang tidak memiliki anggota tersebut dengan simbol $\varnothing$ atau $\{\}$.

Eksplorasi

Diberikan himpunan $A = \{\varnothing\}$. Apakah A himpunan kosong?

Terdapat tiga operasi yang bisa dilakukan pada himpunan, yaitu gabungan, irisan, dan selisih[2]. Ketiga operasi tersebut didefinisikan sebagai berikut.

def121

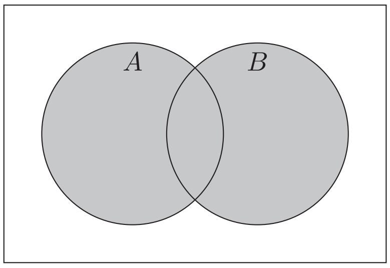
Definisi 1.2.1 Misalkan A dan B himpunan. Maka,

1. Gabungan dari A dan B yang dilambangkan dengan $A \cup B$ didefinisikan sebagai

$$A \cup B := \{x \mid x \in A \vee x \in B\}.$$

   Oleh karena itu, $x \in A \cup B$ jika dan hanya jika $x \in A$ atau $x \in B$. Ilustrasinya adalah sebagai berikut.

[2]Sebenarnya masih banyak lagi operasi-operasi pada himpunan seperti *symmetric difference*. Untuk persiapan menghadapi mata kuliah Aljabar ini, operasi tersebut tidak terlalu sering digunakan sehingga dilewatkan pembahasannya.

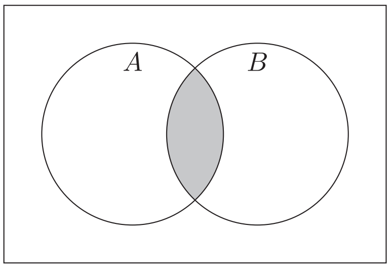


Gambar 1.1: Ilustrasi gabungan dua himpunan.

2. Irisan dari A dan B yang dilambangkan dengan $A \cap B$ didefinisikan sebagai

$$A \cap B \coloneqq \{x \mid x \in A \wedge x \in B\}.$$

Oleh karena itu, $x \in A \cap B$ jika dan hanya jika $x \in A$ dan $x \in B$. Ilustrasinya adalah sebagai berikut.

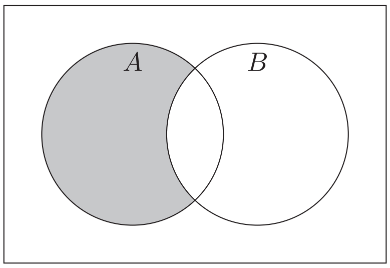


Gambar 1.2: Ilustrasi irisan dua himpunan.

3. Selisih dari A dan B yang dilambangkan dengan $A \backslash B$ (atau kadang $A - B$) didefinisikan sebagai

$$A \backslash B \coloneqq \{x \mid x \in A \wedge x \notin B\}.$$

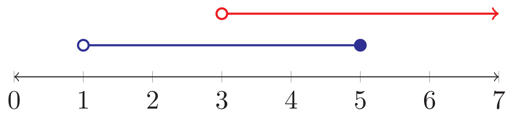
Oleh karena itu, $x \in A \backslash B$ jika dan hanya jika $x \in A$ dan $x \notin B$. Ilustrasinya adalah sebagai berikut.



Gambar 1.3: Ilustrasi selisih dua himpunan.

Sebagai contoh, untuk mengiris himpunan $A = \{x \mid 1 < x \leq 5\}$ dan B $\{x \mid x > 3\}$ bisa dilakukan dengan mencari suatu himpunan C sedemikian sehingga setiap anggota di C merupakan anggota dari himpunan A dan juga merupakan anggota himpunan B. Himpunan C yang memenuhi ini pastilah $C = \{x \mid 3 < x \leq 5\}$. Jadi $C = A \cap B$. Sedangkan, jika kita ingin menggabungkan A dan B, maka kita perlu mencari himpunan D sedemikian sehingga setiap anggota himpunan D juga berada di A atau B. Tentunya himpunan D yang memenuhi adalah $D = \{x \mid x > 1\}$.

Terkadang akan sangat sulit untuk mencari himpunan yang memenuhi $A \cup B$ atau $A \cap B$ pada contoh di atas. Oleh karena itu, kita bisa menggunakan garis bilangan real untuk mempermudah pencarian gabungan dan irisan dari dua atau lebih himpunan. Jika A dan B digambarkan pada garis bilangan real, maka tampaknya adalah sebagai berikut.



Gambar 1.4: Ilustrasi A (biru) dan B (merah) jika digambarkan pada garis bilangan real.

Pada gambar di atas, $A \cup B$ adalah segala garis bilangan yang dilewati oleh garis biru (himpunan A) atau garis merah (himpunan B) sehingga $A \cup B = \{x \mid x > 1\}$. Sementara itu, $A \cap B$ adalah segala garis bilangan yang dilewati oleh garis biru (himpunan A) dan garis merah (himpunan B) sekaligus sehingga $A \cap B = \{x \mid 3 < x \leq 5\}$.

Perhatikan sekali lagi pada gambar garis bilangan di atas. Jika titik ujungnya masih termasuk ke dalam suatu himpunan, maka titik ujung garis tersebut harus diberi tanda bulat penuh. Tetapi jika titik ujungnya tidak termasuk ke dalam suatu himpunan, maka titik ujung garis tersebut harus diberi tanda bulat takpenuh. Seperti pada himpunan A (garis biru), karena $5 \in A$ dan 5 adalah titik ujung (kanan) himpunan A, maka garis yang mengilustrasikan himpunan A pada garis bilangan real diberi tanda bulatan penuh. Begitu juga sebaliknya, karena $1 \notin A$ dan 1 adalah titik ujung (kiri) himpunan A, maka garis yang mengilustrasikan himpunan A pada garis bilangan real diberi tanda bulatan takpenuh.

Eksplorasi

Suatu himpunan juga dapat dinyatakan dalam notasi interval. Coba cari tahu mengenai notasi interval tersebut!

Dalam bekerja dengan menggunakan himpunan, kita harus mengetahui sedang membicarakan apa. Pada penjelasan-penjelasan mengenai himpunan sebelumnya, seperti pada himpunan A dan B pada contoh di atas, kita tidak diberikan informasi mengenai variabel x. Kita tidak mengetahui apakah x termasuk bilangan real, bilangan rasional, atau bilangan bulat. Oleh karena itu, selanjutnya kita harus menspesifikasikan variabel yang bekerja pada suatu himpunan. Jika x merupakan bilangan bulat, maka penulisan himpunan yang benar adalah $A = \{x \in \mathbb{Z} \mid 1 < x \leq 5\}$. Sementara itu, jika x merupakan bilangan real, maka penulisan himpunan yang benar adalah

$A = \{x \in \mathbb{R} \mid 1 < x \leq 5\}$. Begitu pula jika x rasional, x bilangan asli, atau yang lainnya.

1.2.1 Latihan Soal 1.2

1. Buatlah tabel mengenai seluruh kemungkinan interval dan himpunan yang berkaitan beserta ilustrasi pada garis bilangan realnya.
2. Jika $A = \{x \in \mathbb{R} \mid x \leq 1\}$ dan $B = \{x \in \mathbb{Z} \mid -2 < x < 3\}$, maka tentukanlah $A \cup B$, $A \cap B$, dan $A \backslash B$.
3. Diberikan himpunan $\mathcal{P}$ dan $\mathcal{Q}$. Buatlah suatu ilustrasi mengenai $\mathcal{P} \backslash \mathcal{Q}$ yang digambarkan pada garis bilangan real untuk tiga himpunan $\mathcal{P}$ dan $\mathcal{Q}$ yang berbeda.
4. Jika A dan B adalah sebarang himpunan takkosong, mungkinkah $A \cup B$ merupakan himpunan kosong? Berikan suatu contoh mengenai hal ini.
5. Tentukan interval paling sederhana yang ekuivalen dengan
$$((-15, 20) \cup \{20\}) \setminus ([4, 5] \cap [2, +\infty)) \,.$$
Ilustrasikan pekerjaan Anda dengan membuat interval-interval tersebut pada garis bilangan real.

1.3 Identitas Aljabar

Akan sangat membantu jika Anda mengetahui identitas-identitas aljabar dalam mata kuliah ini. Identitas-identitas aljabar ini dapat membantu meringankan beban Anda dalam menghitung suatu ekspresi matematika yang rumit. Berikut merupakan identitas-identitas yang sering digunakan untuk membantu menyelesaikan permasalahan matematika.

1. $x^2 + y^2 = (x + y)^2 - 2xy = (x - y)^2 + 2xy = \dfrac{1}{2}\left((x + y)^2 + (x - y)^2\right)$
2. $x^2 - y^2 = (x + y)(x - y)$
3. $x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$
4. $x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$
5. Untuk setiap bilangan bulat positif ganjil n,
$$x^n + y^n = (x + y)\left(x^{n-1} - x^{n-2}y + \cdots - xy^{n-2} + y^{n-1}\right).$$
6. Untuk setiap bilangan bulat positif genap n,
$$x^2 + y^n = (x - y)\left(x^{n-1} + x^{n-2}y + \cdots + x^{n-2}y + y^{n-1}\right).$$
7. Untuk setiap bilangan bulat positif n,
$$x^n - y^n = (x - y)\left(x^{n-1} + x^{n-2}y + \cdots + xy^{n-2} + y^{n-1}\right).$$

8. $x^3 + y^3 + z^3 - 3xyz = (x+y+z)\left(x^2+y^2+z^2-xy-yz-xz\right) = \frac{1}{2}(x+y+z)\left((x-y)^2+(y-z)^2+(z-x)^2\right)$

9. Untuk setiap bilangan bulat positif $n \geq 2$,

$$(x_1+x_2+\cdots+x_n)^2 = x_1^2+x_2^2+\cdots+x_n^2+2\left(x_1x_2+x_2x_3+\cdots+x_{n-1}x_n+x_nx_1\right).$$

10. $(ax+by)^2 \pm (ay \mp bx)^2 = \left(a^2 \pm b^2\right)\left(x^2 \pm y^2\right)$

11. $x^2y+y^2z+z^2x+x^2z+y^2x+z^2y+2xyz = (x+y)(y+z)(z+x)$

12. $a^4+4b^4 = \left(a^2+2b^2-2ab\right)\left(a^2+2b^2+2ab\right)$

1.3.1 Latihan Soal 1.3

1. Uji kebenaran identitas-identitas pada subbab ini.

2. Cari tahu mengenai Simon's Favorite Factoring Trick (SFFT). Apa kaitannya dengan identitas-identitas di atas?

3. Faktorkanlah ekspresi aljabar $(x-y)^3+(y-z)^3+(z-x)^3$.

CHAPTER 2

Persamaan Kuadrat dan Fungsi Kuadrat

sec:second

2.1 Persamaan Kuadrat

Persamaan adalah kalimat terbuka yang dihubungkan oleh tanda kesamaan (=). Artinya, suatu persamaan belum diketahui nilai kebenarannya. Sebagai contoh, $2x + 3 = 5$ merupakan persamaan. Kita tidak mengetahui apakah persamaan tersebut benar untuk beberapa nilai x. Jika $x = 1$, maka persamaan tersebut bernilai benar sehingga persamaan tersebut menjadi suatu kesamaan (kalimat tertutup yang bernilai benar). Tetapi, jika $x \neq 1$, maka persamaan tersebut akan selalu bernilai salah sehingga persamaan tersebut menjadi suatu ketaksamaan (kalimat tertutup yang bernilai salah). Dalam hal ini, $x = 1$ kadang disebut sebagai solusi[1] dari persamaan $2x + 3 = 5$.

Persamaan kuadrat merupakan salah satu dari banyak jenis persamaan dalam matematika. Persamaan kuadrat memiliki bentuk umum

$$ax^2 + bx + c = 0 \tag{2.1}$$

{eq:201}

dengan $a, b, c \in \mathbb{R}$ dan $a \neq 0$.

Eksplorasi

Mengapa dalam persamaan kuadrat haruslah $a \neq 0$?

Dalam persamaan kuadrat $3x^2 + 2x - 5 = 0$, nilai $a = 3$, $b = 2$, dan $c = -5$. Sedangkan pada persamaan kuadrat $-3x^2 + 10 = 0$, nilai $a = -3$, nilai $b = 0$, dan nilai $c = 10$. Hal ini dikarenakan kita dapat menuliskan ekspresi $-3x^2 + 10$ sebagai $-3x^2 + 0x + 10$. Lalu bagaimana dengan persamaan $x^2 = -1$?

Meskipun persamaan kuadrat memiliki bentuk umum seperti yang dapat dilihat pada persamaan 2.1, persamaan seperti $2x + 3 = 0$ juga dapat dipandang sebagai persamaan kuadrat, namun dalam $\sqrt{x}$. Jika kita misalkan $\sqrt{x} = u$, maka $2x + 3 = 0$ dapat ditulis sebagai $2u^2 + 3 = 0$ yang merupakan persamaan kuadrat dalam u atau $\sqrt{x}$.

[1] Terkadang juga disebut sebagai akar atau penyelesaian.

2.1.1 Cara Mencari Solusi Persamaan Kuadrat

Persamaan kuadrat tentunya memiliki solusi. Ingat kembali bahwa solusi adalah suatu nilai yang menyebabkan suatu persamaan menjadi kesamaan. Oleh karena itu, jika $x = p$ merupakan solusi dari persamaan kuadrat umum 2.1, maka $ap^2 + bp + c = 0$. Tetapi, bagaimana cara menentukan solusinya? Dalam menyelesaikan suatu persamaan kuadrat, terdapat tiga cara yang dapat digunakan. Ketiga cara tersebut adalah dengan menggunakan faktorisasi, melengkapkan kuadrat, dan dengan menggunakan rumus kuadratik (rumus abc). Berikut merupakan penjelasannya.

Faktorisasi

Persamaan kuadrat umum seperti pada persamaan 2.1 dapat difaktorkan menjadi

$$a\left(x-p\right)\left(x-q\right)=0. \tag{2.2}$$

{eq:202}

Disini, p dan q bisa jadi bukan termasuk bilangan real. Tetapi dalam buku ini, kita tidak akan membahas mengenai pemfaktoran dimana p dan q bukan bilangan real sehingga kita bisa anggap $p, q \in \mathbb{R}$. Untuk lebih jauhnya mengenai hal ini akan dibahas pada bagian selanjutnya.

Perhatikan kembali persamaan 2.2. Jika kita substitusi $x = p$, maka ruas kiri akan sama dengan nol. Tetapi, jika kita substitusi $x = q$, maka ruas kiri juga akan menjadi nol. Oleh karena itu, p dan q merupakan akar-akar dari persamaan 2.2. Dalam hal ini, kita tuliskan akar-akar dari persamaan 2.2 adalah $x = p$ atau $x = q$ (mengapa bukan 'dan'?). Jika kita pandang p sebagai akar pertama dan q sebagai akar kedua dari persamaan 2.2, maka kita dapat tuliskan akar-akar dari persamaan 2.2 adalah $x_1 = p$ dan $x_2 = q$ (mengapa bukan 'atau'?).

Pada persamaan 2.1, jika $a = 1$, maka persamaan tersebut dapat ditulis sebagai

$$x^2+bx+c=0. \tag{2.3}$$

{eq:203}

Perhatikan bahwa kita dapat memfaktorkan persamaan 2.3 menjadi

$$\left(x+p\right)\left(x+q\right)=0 \tag{2.4}$$

{eq:204}

dengan p dan q bilangan-bilangan real yang jika dijabarkan akan diperoleh

$$x^2+\left(p+q\right)x+pq=0.$$

Jadi, agar persamaan 2.3 ekuivalen dengan persamaan 2.4, maka haruslah kita mencari p dan q sedemikian sehingga $p + q = b$ dan $pq = c$.

Contoh 2.1.1. Selesaikan persamaan kuadrat $x^2 - 7x + 10 = 0$.

Jawab. Jika kita ingin memfaktorkan ruas kiri persamaan pada soal menjadi $\left(x+p\right)\left(x+q\right) = 0$, maka kita harus mencari p dan q sedemikian sehingga $p + q = -7$ dan $pq = 10$. Setelah mencoba-coba, ternyata $p = -2$ dan $q = -5$ memenuhi. Oleh karena itu, kita dapat menulis ulang persamaan pada soal sebagai

$$\left(x-2\right)\left(x-5\right)=0.$$

Jadi, solusi persamaan kuadrat pada soal adalah $x = 2$ atau $x = 5$.

Pada persamaan 2.1, jika $a \neq 1$, maka secara umum kita dapat menyelesaikannya dengan langkah-langkah sebagai berikut.

1. Pertama-tama, persamaan 2.1 kita faktorkan menjadi

$$\frac{(ax+p)(ax+q)}{a} = 0. \tag{2.5}$$

{eq:205}

2. Selanjutnya, carilah nilai p dan q sedemikian sehingga $p + q = b$ dan $pq = ac$.

3. Setelah didapatkan nilai p dan q, substitusikan kembali ke persamaan 2.5 dan sederhanakan lebih lanjut.

Eksplorasi

Uji kebenaran langkah-langkah pemfaktoran di atas. Mengapa langkah-langkah tersebut benar?

Contoh 2.1.2. Selesaikan persamaan kuadrat $2x^2 - 5x - 3 = 0$.

Jawab. Pertama-tama, persamaan pada soal kita faktorkan menjadi

$$\frac{(2x+p)(2x+q)}{2} = 0.$$

Selanjutnya, kita cari nilai p dan q sedemikian sehingga $p+q = -5$ dan $pq = -6$. Setelah mencoba-coba, ternyata $p = 1$ dan $q = -6$ memenuhi. Oleh karena itu, kita dapat menulis ulang pemfaktoran terakhir sebagai

$$\frac{(2x+1)(2x-6)}{2} = 0 \iff (2x+1)(x-3) = 0.$$

Agar persamaan terakhir bernilai benar, maka $2x + 1 = 0 \iff x = -\frac{1}{2}$ atau $x - 3 = 0 \iff x = 3$.
Jadi, solusi persamaan kuadrat pada soal adalah $x = -\frac{1}{2}$ atau $x = 3$.

Peringatan

Jika Anda ingin mencari solusi dari persamaan $x^2 = 9$, Anda tidak boleh langsung menarik akar pada kedua ruas sehingga didapatkan $x = 3$. Hal ini dikarenakan, $x = -3$ juga memenuhi persamaan tersebut. Beberapa cara untuk memunculkan solusi $x = -3$ adalah dengan menambahkan tanda '±' di ruas kanan persamaan setelah ditarik akarnya seperti $x^2 = 9 \implies x = \pm 3$ sehingga solusinya adalah $x = 3$ atau $x = -3$. Anda juga dapat mengurangi kedua ruas dengan tiga, lalu memfaktorkannya dengan identitas selisih kuadrat seperti

$$x^2 = 9 \iff x^2 - 9 = 0 \iff (x+3)(x-3) = 0$$

sehingga solusinya adalah $x = 3$ atau $x = -3$.

Melengkapkan Kuadrat

Terkadang, kita akan kesulitan untuk memfaktorkan suatu bentuk kuadrat. Sebagai contoh, Anda dapat mencoba memfaktorkan ekspresi x^2+x+1 ini. Anda pasti akan kesulitan memfaktorkan ekspresi tersebut. Oleh karena itu, teknik melengkapkan kuadrat akan sangat membantu disini. Teknik ini sangatlah berguna untuk menyelesaikan suatu persamaan kuadrat yang sukar untuk difaktorkan.

Melengkapkan kuadrat adalah suatu teknik untuk mengubah bentuk persamaan kuadrat umum seperti pada persamaan 2.1 menjadi suatu persamaan kuadrat berbentuk

$$a\,(x+h)^2+k=0. \tag{2.6}$$

{eq:206}

untuk suatu bilangan real h dan k. Dalam bentuk ini, persamaan kuadrat akan jauh lebih mudah untuk diselesaikan dibandingkan pada bentuk umumnya.

Jika $a=1$, tentunya proses melengkapkan kuadrat akan mudah. Misalkan $a=1$, maka persamaan 2.1 dapat dituliskan sebagai

$$x^2+bx+c=0$$

seperti yang terdapat pada persamaan 2.3.

Perhatikan bahwa $\left(x+\dfrac{1}{2}b\right)^2=x^2+bx+\left(\dfrac{b}{2}\right)^2$. Oleh karena itu, kita harus munculkan bentuk $\left(\dfrac{b}{2}\right)^2$ pada persamaan 2.3 dengan menjumlahkan kedua ruas dengan bentuk tersebut. Oleh karena itu, persamaan 2.3 dapat kita tulis ulang sebagai

$$x^2+bx+\left(\frac{b}{2}\right)^2+c=\left(\frac{b}{2}\right)^2 \iff \left(x+\frac{1}{2}b\right)^2+c-\left(\frac{b}{2}\right)^2=0.$$

Dengan membandingan koefisien-koefisien ruas kiri persamaan terakhir dengan ruas kiri persamaan 2.6, kita dapatkan $h=\dfrac{b}{2}$ dan $k=c-\left(\dfrac{b}{2}\right)^2$.

Contoh 2.1.3. Dengan melengkapkan kuadrat, tentukan penyelesaian dari persamaan $x^2+4x-9=0$.

Jawab. Karena $b=4$ dan $c=-9$, maka $h=\dfrac{1}{2}(4)=2$. Oleh karena itu, kita bisa menjumlahkan kedua ruas dengan $2^2=4$ sehingga

$$x^2+4x+4-9=4 \iff (x+2)^2=13.$$

Menarik akar kedua ruas akan didapatkan

$$x+2=\pm 13 \iff x=-2\pm 13.$$

Jadi solusi persamaan kuadrat pada soal adalah $x=-2-\sqrt{13}$ atau $x=-2+\sqrt{13}$.

Jika $a\neq 1$, kita dapat mencoba untuk membagi kedua ruas persamaan kuadrat dengan a. Sebagai contoh, kita akan mengerjakan soal berikut.

Contoh 2.1.4. Dengan melengkapkan kuadrat, tentukan penyelesaian dari persamaan $2x^2 + 3x + 1 = 0$.

Jawab. Dengan membagi kedua ruas dengan 2, kita akan mendapatkan

$$x^2 + \frac{3}{2}x + \frac{1}{2} = 0.$$

Sampai disini, proses untuk melengkapkan kuadrat sama seperti contoh sebelumnya dan diserahkan kepada pembaca sebagai latihan.

Terkadang, membagi kedua ruas dengan a akan menimbulkan masalah baru seperti rumitnya proses melengkapkan kuadrat karena harus berurusan dengan pecahan yang bentuknya ekstrim. Oleh karena itu, alih-alih membagi kedua ruas dengan a, kita juga bisa mencoba untuk mengalikan kedua ruas dengan a sehingga akan didapatkan persamaan kuadrat baru dalam ax. Sebagai contoh, kita akan mengerjakan soal berikut.

Contoh 2.1.5. Dengan melengkapkan kuadrat, tentukan penyelesaian dari persamaan $3x^2 + x - 5 = 0$.

Jawab. Dengan mengalikan kedua ruas dengan 3, kita akan mendapatkan

$$3^2x^2 + 3x - 5 = 0 \iff (3x)^2 + 3x - 5 = 0.$$

Misalkan $3x = u$, maka kita bisa tulis ulang persamaan terakhir sebagai

$$u^2 + u - 5 = 0.$$

Sampai disini, proses untuk melengkapkan kuadrat sama seperti contoh sebelumnya dan diserahkan kepada pembaca sebagai latihan. Jangan lupa untuk substitusi balik $u = 3x$.

Rumus Kuadratik

Di Indonesia, rumus kuadratik sering disebut sebagai rumus abc. Rumus kuadratik sebenarnya didapatkan dengan menggunakan teknik melengkapkan kuadrat pada persamaan 2.1. Rumus kuadratik dapat dituliskan sebagai berikut.

thm:21 **Teorema 2.1.6** (Rumus Kuadratik)**.** *Misalkan solusi-solusi dari persamaan 2.1 adalah $x_{1,2}$ (yang berarti x_1 dan x_2), maka*

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{2.7}$$ {eq:207}

Bukti. Membagi kedua ruas persamaan 2.1 dengan a dan dilanjutkan dengan melengkapkan kuadrat akan didapatkan

$$\begin{aligned}
& x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \\
\iff \quad & x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = \left(\frac{b}{2a}\right)^2 \\
\iff \quad & \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} \\
\iff \quad & \left(x + \frac{b}{2a}\right)^2 = \frac{b^2a - 4a^2c}{4a^3} \\
\iff \quad & \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \\
\iff \quad & x_{1,2} + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\
\iff \quad & x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

dan kita selesai. □

Eksplorasi

Alih-alih membagi kedua ruas dengan a, coba Anda buktikan teorema 2.1.6 dengan mengalikan kedua ruas persamaan 2.1 dengan a, lalu dilanjutkan dengan melengkapkan kuadrat.

Rumus kuadratik ini merupakan alat yang sangat canggih untuk menyelesaikan suatu persamaan kuadrat. Mungkin kita akan kesulitan jika menyelesaikan persamaan kuadrat dengan menggunakan pemfaktoran atau melengkapkan kuadrat, tetapi rumus kuadratik ini akan dapat membantu kita dengan cepat. Rumus kuadratik inilah yang akan membantu kita untuk memahami persamaan kuadrat lebih lanjut lagi.

Contoh 2.1.7. Dengan menggunakan rumus kuadratik, tentukan akar-akar dari persamaan $2x^2 - x - 11 = 0$.

Jawab. Pertama, kita tentukan terlebih dahulu nilai a, b, dan c. Dengan membandingkan koefisien-koefisien persamaan pada soal dengan persamaan 2.1, kita dapatkan $a = 2$, $b = -1$, dan $c = -11$. Oleh karena itu, dengan rumus kuadratik, kita akan mendapatkan

$$x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-11)}}{2(2)} = \frac{1 \pm \sqrt{1+88}}{4} = \frac{1 \pm \sqrt{89}}{4}$$

sehingga akar-akar persamaan pada soal adalah

$$x_1 = \frac{1+\sqrt{89}}{4} \quad \text{dan} \quad x_2 = \frac{1-\sqrt{89}}{4}.$$

Dari rumus kuadratik ini, kita dapat memfaktoran bentuk kuadrat $ax^2 + bx + c$ sebagai

$$a\left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

dan pada kasus khusus jika $b^2 = 4ac$, kita dapat memfaktorkan bentuk kuadrat $ax^2 + bx + c$ sebagai

$$a\left(x + \frac{b}{2a}\right)^2.$$

Eksplorasi

Coba cari tahu mengenai persamaan kuadratik tereduksi. Bagaimanakah rumus kuadratik digunakan dalam persamaan tersebut?

2.1.2 Diskriminan

Dalam persamaan kuadrat, diskriminan adalah ekspresi yang berada di bawah tanda akar pada rumus kuadratik seperti pada teorema 2.1.6. Diskriminan sering disimbolkan sebagai huruf kapital D atau huruf kapital Yunani Δ (delta kapital)[2] dimana

$$D = \Delta = b^2 - 4ac. \tag{2.8}$$

{eq:208}

dengan a, b, c berturut-turut merupakan koefisien-koefisien persamaan kuadrat umum 2.1. Oleh karena itu, formula 2.7 dapat dituliskan sebagai

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}. \tag{2.9}$$

{eq:209}

2.1.3 Jenis-jenis Akar Persamaan Kuadrat

Terdapat dua kemungkinan akar-akar yang mungkin untuk persamaan kuadrat. Dua kemungkinan ini bergantung pada nilai diskriminan pada persamaan kuadrat tersebut. Jika $D < 0$, maka akar-akar persamaan kuadrat tersebut bukan merupakan bilangan real (bilangan kompleks, yang kadang dinotasikan sebagai $\mathbb{C}$), tetapi jika $D \geq 0$, maka akar-akar persamaan kuadrat tersebut merupakan bilangan real.

Jika $D < 0$, maka nilai di dalam akar pada persamaan 2.9 akan bernilai negatif. Padahal nilai di dalam akar harus nonnegatif agar nilainya terdefinisi pada garis bilangan real. Oleh karena itu, akar-akar yang dihasilkan oleh persamaan kuadrat yang diskriminannya negatif akan bernilai tidak real. Lain halnya jika $D \geq 0$. Nilai di dalam akar pada persamaan 2.9 akan bernilai nonnegatif yang sudah pasti nilainya akan terdefinisi pada garis bilangan real.

Biasanya jika suatu persamaan kuadrat memiliki diskriminan negatif, kita tidak akan mencari akar-akarnya secara eksplisit. Hal ini dikarenakan ruang lingkup pembicaraan kita yang terbatas hanya untuk bilangan real. Tetapi jika Anda memiliki keingintahuan yang tinggi, Anda dapat mencoba

[2] Di Indonesia sendiri, notasi yang sering digunakan untuk diskriminan adalah D sehingga selanjutnya kita akan menotasikan diskriminan sebagai D, kecuali ada keterangan lain.

untuk menyelesaikannya dengan menggunakan rumus kuadratik. Tetapi untuk menyelesaikannya, Anda perlu mengetahui terlebih dahulu mengenai bilangan imajiner. Sebagai contoh, misalkan kita diberikan suatu persamaan kuadrat $x^2 + x + 1 = 0$. Diskriminan persamaan kuadrat tersebut adalah $D = 1^2 - 4(1)(1) = -3$. Oleh karena itu, dengan menggunakan rumus kuadratik kita akan mendapatkan

$$x_{1,2} = \frac{-1 \pm \sqrt{-3}}{2}.$$

Untuk menghadapi akar dengan ekspresi di dalam akar yang bernilai negatif, kita perlu untuk menguraikannya terlebih dahulu. Dalam hal ini, kita uraikan -3 sebagai $-1 \cdot 3$. Oleh karena itu, $\sqrt{-3} = \sqrt{-1 \cdot 3} = \sqrt{3} \cdot \sqrt{-1}$. Dalam matematika, biasanya kita notasikan $\sqrt{-1}$ sebagai i sehingga $\sqrt{3} \cdot \sqrt{-1} = \sqrt{3}i$. Bilangan i inilah yang disebut sebagai bilangan imajiner. Oleh karena itu,

$$x_{1,2} = \frac{-1 \pm \sqrt{3}i}{2}$$

sehingga akar-akar persamaan kuadrat $x^2 + x + 1 = 0$ adalah

$$x_1 = \frac{-1 + \sqrt{3}i}{2} \quad \text{dan} \quad x_2 = \frac{-1 - \sqrt{3}i}{2}.$$

Persamaan kuadrat yang memiliki diskriminan nonnegatif dibagi lagi menjadi dua jenis, yaitu persamaan kuadrat dengan diskriminan nol dan persamaan kuadrat dengan diskriminan positif. Jika suatu persamaan kuadrat memiliki nilai $D = 0$, maka persamaan 2.9 dapat dituliskan sebagai

$$x_{1,2} = -\frac{b \pm 0}{2a} = -\frac{b}{2a}.$$

Akibatnya, $x_1 = x_2 = -\frac{b}{2a}$ sehingga persamaan kuadrat tersebut memiliki tepat satu akar real. Terkadang juga disebut akarnya berulang atau akarnya ganda. Hal ini akan dibahas pada bagian tersendiri untuk lebih memahaminya lebih lanjut. Selain itu, jika suatu persamaan kuadrat memiliki nilai $D > 0$, maka persamaan kuadrat tersebut memiliki dua akar yang berbeda, yaitu

$$x_1 = -\frac{-b + \sqrt{D}}{2a} \quad \text{dan} \quad x_2 = \frac{-b - \sqrt{D}}{2a}$$

atau kebalikannya.

Eksplorasi

1. Apakah mungkin salah satu akar dari suatu persamaan kuadrat merupakan bilangan real, sedangkan akar yang lainnya merupakan bilangan takreal?
2. Apakah mungkin salah satu akar dari suatu persamaan kuadrat merupakan bilangan rasional, sedangkan akar yang lainnya merupakan bilangan irasional?

2.1.4 Bentuk-bentuk Simetris Akar-Akar Persamaan Kuadrat

Terdapat suatu hubungan antara x_1 dengan x_2. Hubungan tersebut merupakan akibat langsung dari rumus kuadratik. Dari formula 2.7, misalkan

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{dan} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Jika kita jumlahkan x_1 dengan x_2, kita akan mendapatkan

$$x_1 + x_2 = \frac{-b + \cancel{\sqrt{b^2 - 4ac}} + \left(-b - \cancel{\sqrt{b^2 - 4ac}}\right)}{2a} = \frac{-2b}{2a}$$

sehingga

$$x_1 + x_2 = -\frac{b}{a}. \tag{2.10}$$

{eq:210}

Selain itu, jika kita kalikan x_1 dengan x_2, kita akan mendapatkan

$$x_1 x_2 = \frac{\left(-b + \sqrt{b^2 - 4ac}\right)\left(-b - \sqrt{b^2 - 4ac}\right)}{2a \cdot 2a} = \frac{\cancel{b^2} - \left(\cancel{b^2} - 4ac\right)}{4a^2} = \frac{4ac}{2a}$$

sehingga

$$x_1 x_2 = \frac{c}{a}. \tag{2.11}$$

{eq:211}

Hal ini tidak menjadi masalah apabila kita misalkan x_1 dan x_2 sebagai kebalikannya. Nilai dari $x_1 + x_2$ dan $x_1 x_2$ akan tetap sama meskipun x_1 dan x_2 dipertukarkan pemisalannya.

Informasi

Jumlah dan hasil kali akar-akar persamaan kuadrat ini merupakan kasus khusus dari teorema Vieta untuk polinomial berderajat dua (bentuk kuadrat). Anda dapat membacanya lebih lanjut di https://en.wikipedia.org/wiki/Vieta%27s_formulas.

Pertukaran pemisalan x_1 dan x_2 akan menjadi masalah apabila kita ingin menghitung nilai dari $x_1 - x_2$. Jika kita kurangi x_1 dengan x_2 kita akan mendapatkan

$$x_1 - x_2 = \frac{\cancel{-b} + \sqrt{b^2 - 4ac} - \left(\cancel{-b} - \sqrt{b^2 - 4ac}\right)}{2a} = \frac{2\sqrt{b^2 - 4ac}}{2a}.$$

Karena $D = b^2 - 4ac$, maka kita punyai

$$x_1 - x_2 = \frac{\sqrt{D}}{a}. \tag{2.12}$$

{eq:212}

Tetapi, jika kita misalkan kebalikannya (x_1 dan x_2 dipertukarkan), yaitu

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{dan} \quad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

maka

$$x_1 - x_2 = -\frac{\sqrt{D}}{a}. \tag{2.13}$$

{eq:213}

Oleh karena itu, secara umum, jika kita tidak mengetahui mana yang x_1 dan mana yang x_2, maka kemungkinan nilai-nilai dari $x_1 - x_2$ adalah

$$x_1 - x_2 = \pm\frac{\sqrt{D}}{a}. \tag{2.14}$$

{eq:214}

Hal ini berlaku untuk sebarang x_1 dan x_2, bahkan ketika keduanya bukan merupakan bilangan real ($D < 0$). Ingat bahwa meskipun ada tanda '$\pm$' di ruas kanan, ini bukan berarti terdapat dua solusi, tetapi yang lebih tepat adalah terdapat dua kemungkinan nilai $x_1 - x_2$. Anda perlu mengecek kembali kondisi-kondisi yang diberikan pada soal karena bisa saja kemungkinan yang lain tidak memenuhi kondisi yang diberikan pada soal. Anda hanya boleh menuliskan tanda '$\pm$' jika tidak ada kondisi lebih lanjut yang diberikan pada soal.

Untuk kasus khusus ketika x_1 dan x_2 real, yaitu ketika $D \geq 0$, maka $\sqrt{D}$ akan bernilai real sehingga persamaan 2.14 ekuivalen dengan[3]

$$|x_1 - x_2| = \left|\frac{\sqrt{D}}{a}\right| = \frac{\sqrt{D}}{|a|}. \tag{2.15}$$

{eq:215}

Hal ini tidak berlaku untuk $D < 0$. Alasannya adalah, jika $D < 0$, maka $\sqrt{D}$ bukan merupakan bilangan real. Permasalahannya adalah, nilai mutlak dari bilangan takreal tidak didefinisikan pada buku ini. Meskipun ada definisinya pada buku matematika lanjut, kita tidak akan membahas mengenai hal tersebut.

Eksplorasi

Bagaimana dengan $\dfrac{x_1}{x_2}$?

Contoh 2.1.8. Diberikan persamaan kuadrat $2x^2 - x + 5 = 0$. Tentukan nilai dari

1. $x_1 + x_2$
2. $x_1 x_2$
3. $|x_1 - x_2|$
4. $x_1^2 - x_2^2$

Jawab. Pertama, kita tentukan terlebih dahulu nilai a, b, dan c. Dengan membandingkan koefisien-koefisien persamaan pada soal dengan koefisien-koefisien persamaan kuadrat umum 2.1, kita akan mendapatkan $a = 2$, $b = -1$, dan $c = 5$. Oleh karena itu,

1. Dari formula 2.10, $x_1 + x_2 = -\dfrac{b}{a} = -\dfrac{-1}{2} = \dfrac{1}{2}$.
2. Dari formula 2.11, $x_1 x_2 = \dfrac{c}{a} = \dfrac{5}{2}$.

[3]Penjelasan mengenai hal ini akan dipelajari pada bab 3.

3. Perhatikan bahwa $D = b^2 - 4ac = (-1)^2 - 4(2)(5) = 1 - 40 = -39$ sehingga berdasarkan formula 2.15, nilai dari $|x_1 - x_2|$ tidak didefinisikan.

4. Dengan menggunakan formula 2.14, kita punyai $x_1 - x_2 = \pm\frac{\sqrt{D}}{a} = \pm\frac{\sqrt{39}i}{2}$ sehingga

$$x_1^2 - x_2^2 = (x_1 + x_2)(x_1 - x_2) = \frac{1}{2} \cdot \pm\frac{\sqrt{39}i}{2} = \pm\frac{\sqrt{39}i}{4}.$$

Dari soal di atas, kita bisa mengamati bahwa meskipun diskriminan suatu persamaan kuadrat bernilai negatif, nilai dari x_1+x_2 dan x_1x_2 tetap bernilai real. Hal ini dikarenakan $x_1 + x_2$ dan x_1x_2 tidak bergantung pada nilai diskriminan, tetapi hanya koefisien-koefisien persamaan kuadrat tersebut. Lain halnya dengan $x_1 - x_2$, ekspresi tersebut bergantung kepada nilai diskriminan. Apabila diskriminannya negatif, $x_1 - x_2$ akan bernilai takreal. Nilai dari $|x_1 - x_2|$ juga bergantung kepada nilai diskriminan. Namun, jika diskriminannya negatif, $|x_1 - x_2|$ tidak terdefinisi.

Pertukaran variabel antara x_1 dan x_2 juga tidak akan memengaruhi nilai dari $x_1 + x_2$ dan x_1x_2 karena berlaku sifat komutatif. Inilah yang disebut sebagai kesimetrisan ekspresi aljabar. Artinya, pertukaran variabel x_1 dan x_2 tidak akan berpengaruh terhadap nilainya. Oleh karena itu, $x_1 + x_2$ dan x_1x_2 disebut sebagai ekspresi aljabar yang simetris.

Eksplorasi

Apakah $|x_1 - x_2|$ juga simetris? Bagaimana dengan $x_1 - x_2$?

2.1.5 Sifat-sifat Akar Persamaan Kuadrat

Studi mengenai diskriminan membawa kita ke dalam pemahaman yang lebih dalam mengenai sifat-sifat akar persamaan kuadrat. Tentunya terkadang kita ingin membatasi solusi suatu persamaan kuadrat yang diberikan. Entah itu kita menginginkan kedua akarnya positif, atau setidaknya satu akar positif, hal ini akan sangat dibutuhkan nantinya ketika mendalami matematika lebih lanjut, bahkan di bidang keilmuan lainnya. Fisikawan pasti akan kebingungan apabila persamaan kuadrat yang berkaitan mengenai massa memberikannya solusi negatif.

Catatan 2.1.9. Banyak sifat-sifat pertidaksamaan disini yang mungkin agak dibingungkan oleh pembaca. Untuk saat ini, kita 'meminjamnya' terlebih dahulu tanpa pembahasan. Oleh karena itu, Anda dapat membaca terlebih dahulu dasar-dasar mengenai pertidaksamaan pada awalan bab 3 untuk dapat membantu Anda memahami bagian ini.

Dari sini, kita misalkan x_1 dan x_2 sebagai akar-akar dari suatu persamaan kuadrat. Sifat-sifat akar persamaan kuadrat adalah sebagai berikut.

Kedua Akarnya Bernilai Positif

Karena kedua akarnya positif, maka $x_1 > 0$ dan $x_2 > 0$. Oleh karena itu, dengan menggunakan teorema 2.1.6, haruslah

$$\frac{-b+\sqrt{b^2-4ac}}{2a} > 0 \quad \text{dan} \quad \frac{-b-\sqrt{b^2-4ac}}{2a} > 0$$

atau dengan mensubstitusi $D = b^2-4ac$, menyelesaikan pertidaksamaan terakhir sama saja dengan menyelesaikan pertidaksamaan

$$\frac{-b+\sqrt{D}}{2a} > 0 \quad \text{dan} \quad \frac{-b-\sqrt{D}}{2a} > 0.$$

Meskipun telah mensubstitusi/memisalkan $b^2 - 4ac$ sebagai D, pertidaksamaan yang harus diselesaikan tetap saja sangat sulit. Oleh karena itu, kita gunakan hasil yang telah kita dapatkan dari bagian sebelumnya mengenai bentuk-bentuk simetris akar-akar persamaan kuadrat.

Perhatikan bahwa jika kita mengalikan pertidaksamaan $x_1 > 0$ dengan $x_2 > 0$, maka kita akan mendapatkan $x_1x_2 > 0$ sehingga $\frac{c}{a} > 0$. Selain itu, jika kita menjumlahkan kedua pertidaksamaan tersebut, kita akan mendapatkan $x_1 + x_2 > 0$ sehingga $-\frac{b}{a} > 0$. Ingat bahwa agar kedua pertidaksamaan lebih besar daripada nol, maka kita perlu x_1 dan x_2 berupa bilangan real. Bilangan tidak real, seperti $i = \sqrt{-1}$ tidak memenuhi sifat urutan[4] (seperti lebih besar dari, kurang dari) sehingga kita tidak bisa mengatakan $i > 0$ atau $i < 0$. Oleh karena itu, agar keduanya merupakan bilangan real, haruslah $D \geq 0$ (mengapa bukan '$D > 0$'?).

Jadi, syarat-syarat agar kedua akar suatu persamaan kuadrat bernilai positif adalah

$$x_1 + x_2 > 0, \quad x_1x_2 > 0, \quad \text{dan} \quad D \geq 0 \tag{2.16}$$

{eq:216}

atau

$$-\frac{b}{a} > 0, \quad \frac{c}{a} > 0, \quad \text{dan} \quad D \geq 0.$$

Dari sini timbul permasalahan baru. Apakah sistem pertidaksamaan 2.16 benar-benar menjamin $x_1 > 0$ dan $x_2 > 0$? Kita telah memverifikasi bahwa haruslah $D \geq 0$. Kita hanya tinggal membuktikan bahwa $x_1 + x_2 > 0$ dan $x_1x_2 > 0$ akan menjamin $x_1 > 0$ dan $x_2 > 0$.

Perhatikan bahwa jika $x_1x_2 > 0$, maka x_1 dan x_2 keduanya harus memiliki tanda yang sama, artinya keduanya harus sama-sama positif atau sama-sama negatif karena jika salah satunya negatif, maka x_1x_2 akan bernilai negatif. Dari pertidaksamaan $x_1 + x_2 > 0$ dan fakta bahwa x_1 dan x_2 harus bertanda sama, maka sudah pasti $x_1 > 0$ dan $x_2 > 0$. Jika keduanya negatif tentu saja jumlahnya akan semakin negatif yang akan mengakibatkan kontradiksi.

Kedua Akarnya Bernilai Negatif

Karena kedua akarnya negatif, maka $x_1 < 0$ dan $x_2 < 0$. Oleh karena itu, haruslah $x_1 + x_2 < 0$ dan $x_1x_2 > 0$. Selain itu, dari subbagian sebelumnya, agar keduanya negatif, haruslah x_1 dan x_2 bernilai real sehingga $D \geq 0$.

[4]Sifat urutan ini akan dipelajari lebih lanjut pada mata kuliah Analisis Real. Meskipun demikian, pada buku ini juga terdapat pembahasannya secara umum pada bab 3.

Jadi, syarat-syarat agar kedua akar suatu persamaan bernilai negatif adalah

$$x_1 + x_2 < 0, \quad x_1 x_2 > 0, \quad \text{dan} \quad D \geq 0 \tag{2.17}$$

{eq:217}

atau

$$-\frac{b}{a} < 0, \quad \frac{c}{a} > 0, \quad \text{dan} \quad D \geq 0.$$

Kedua Akarnya Berlainan/Berlawanan Tanda

Penggunaan kata berlawanan disini mungkin akan menyebabkan keambiguan karena pada subbagian selanjutnya terdapat istilah lain yang mengandung kata 'berlawanan' sehingga ada baiknya kita menggunakan kata 'berlainan' disini.

Perhatikan bahwa karena kedua akarnya berlainan tanda, maka kemungkinannya adalah $x_1 < 0$ dan $x_2 > 0$, atau $x_1 > 0$ dan $x_2 > 0$. Dari kedua kemungkinan tersebut, pastilah berlaku $x_1 x_2 > 0$. Karena salah satu akar harus positif dan akar lainnya haruslah negatif, maka kedua akar haruslah merupakan bilangan real berbeda sehingga $D > 0$.

Jadi, syarat-syarat agar kedua akar suatu persamaan kuadrat berlainan tanda adalah

$$x_1 x_2 < 0 \quad \text{dan} \quad D > 0 \tag{2.18}$$

{eq:218}

atau

$$\frac{c}{a} < 0 \quad \text{dan} \quad D > 0.$$

Eksplorasi

Mengapa tidak perlu ada syarat untuk $x_1 + x_2$?

Kedua Akarnya Berlawanan

Subbagian inilah yang memiliki kata yang sama dengan subbagian sebelumnya. Disini, maksud dari kedua akarnya berlawanan adalah jika salah satu akarnya x_1, maka akar lainnya adalah $-x_1$ sehingga $x_2 = -x_1$.

Karena $x_2 = -x_1$, maka $x_1 + x_2 = 0$. Disini, kita tidak perlu memberikan syarat untuk D. Hal ini dikarenakan nilai dari x_1 dan x_2 tidak harus merupakan bilangan real, karena jika $x_1 = i$ dan $x_2 = -i$, maka $x_1 + x_2 = i - i = 0$ yang juga memenuhi kondisi kedua akarnya berlawanan. Selain itu, kita juga tidak memerlukan syarat untuk $x_1 x_2$. Hal ini dikarenakan syarat $x_1 + x_2 = 0$ sudah akan dapat menjamin $x_2 = -x_1$ (yaitu dengan mengurangi kedua ruas dengan x_2).

Jadi, syarat agar kedua akar suatu persamaan kuadrat saling berlawanan adalah

$$x_1 + x_2 < 0 \tag{2.19}$$

{eq:219}

atau

$$-\frac{b}{a} < 0$$

Kedua Akarnya Berkebalikan

Kedua akar suatu persamaan kuadrat dikatakan saling berkebalikan jika $x_2 = \dfrac{1}{x_1}$ atau sebaliknya. Oleh karena itu, haruslah $x_1 x_2 = 1$.

Jadi, syarat agar kedua akar suatu persamaan kuadrat saling berkebalikan adalah

$$x_1 x_2 = 1 \tag{2.20}$$

{eq:220}

atau

$$\frac{c}{a} = 1$$

Eksplorasi

Mengapa tidak perlu ada syarat untuk $x_1 + x_2$ dan D?

Contoh 2.1.10. Tentukan nilai dari m agar persamaan kuadrat $x^2 - 2x - m + 1 = 0$ memiliki

1. akar-akar yang bernilai positif.
2. akar-akar yang bernilai negatif.
3. akar-akar yang berlainan tanda.
4. akar-akar yang berlawanan tanda.
5. akar-akar yang berkebalikan.

Jawab. Perhatikan bahwa $a = 1$, $b = -2$, dan $c = -m + 1$. Pertama-tama, kita cari terlebih dahulu nilai dari $x_1 + x_2$, $x_1 x_2$, dan D. Dengan menggunakan persamaan 2.10 dan 2.11, kita punyai

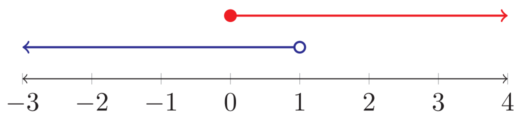
$$x_1 + x_2 = -\frac{b}{a} = -\frac{-2}{1} = 2 \quad \text{dan} \quad x_1 x_2 = \frac{c}{a} = \frac{-m+1}{1} = -m + 1.$$

Selain itu, dengan menggunakan persamaan 2.8, kita juga punyai

$$D = b^2 - 4ac = (-2)^2 - 4\,(1)\,(-m+1) = 4 + 4m - 4 = 4m.$$

Selanjutnya, kita akan menyelesaikan soal ini.

1. Dari sistem pertidaksamaan 2.16, syarat-syarat agar kedua akar bernilai positif adalah $x_1 + x_2 = 2 > 0$ (sehingga bernilai benar untuk semua $m \in \mathbb{R}$), $x_1 x_2 = -m + 1 > 0 \iff m < 1$, dan $D = 4m \geq 0 \iff m \geq 0$. Oleh karena itu, agar ketiga syarat terpenuhi, haruslah m memenuhi irisan dari ketiga interval tadi



Gambar 2.1: Ilustrasi interval $m < 1$ (biru) dan $m \geq 0$ (merah) jika digambarkan pada garis bilangan real. Disini, syarat pertama, yaitu $m \in \mathbb{R}$ tidak perlu digambar (mengapa?).

sehingga HP $= \{m \in \mathbb{R} \mid 0 \leq m < 1\}$ atau jika dituliskan dalam notasi interval, $m \in [0, 1)$.

2. Latihan, jawab dalam notasi himpunan dan notasi interval.
3. Latihan, jawab dalam notasi himpunan dan notasi interval.
4. Latihan.
5. Latihan.

Eksplorasi

Diberikan bilangan real positif a. Tentukan syarat-syarat agar kedua akar suatu persamaan kuadrat lebih besar dari a. Buktikan jawaban Anda.

2.1.6 Akar Tunggal vs Akar Ganda

Ketika suatu persamaan kuadrat yang akar-akarnya x_1 dan x_2 memiliki diskriminan nol, maka $x_1 = x_2$. Telah menjadi perdebatan yang cukup lama mengenai jumlah akar persamaan kuadrat yang diskriminannya nol. Ada beberapa pendapat yang berkata bahwa akarnya tunggal, ada juga beberapa pendapat yang menyebutkan bahwa akarnya tetap dua, tetapi ganda.

Kedua pendapat tersebut benar tergantung darimana kita melihatnya. Pertama, pendapat yang mengatakan bahwa akarnya tunggal menilai bahwa jika suatu persamaan kuadrat memiliki diskriminan nol, seperti pada persamaan $x^2 - 4x + 4 = 0$, maka ruas kiri persamaan tersebut dapat difaktorkan menjadi $(x-2)^2 = 0$ sehingga $x - 2 = 0$ yang memiliki tepat satu solusi, yaitu $x = 2$. Kedua, pendapat yang mengatakan bahwa akarnya tetap dua, tetapi ganda, menilai bahwa ketika menarik akar kedua ruas, x harus diganti dengan $x_{1,2}$ seperti pada proses membuktikan rumus kuadratik, yaitu ketika $(x-2)^2 = 0$ ditarik akarnya pada kedua ruas, haruslah $x_{1,2} - 2 = 0 \iff x_{1,2} = 2$ sehingga $x_1 = x_2 = 2$ yang tetap memiliki tepat dua solusi, tetapi solusinya ganda.

Untuk menyelesaikan hal ini agar tidak menjadi perdebatan yang berkepanjangan, kita memiliki istilah *multiplisitas* dalam menyelesaikannya. Sebelumnya, tinjau pernyataan "bentuk kuadrat $(x-a)(x-b)$ memiliki dua akar". Pernyataan tersebut benar jika $a \neq b$ tetapi tidak untuk $a = b$. Untuk kasus $a = b$, maka bentuk kuadrat tersebut ekuivalen dengan $(x-a)^2$ dan kita katakan a sebagai akar ganda. Dari sinilah diperkenalkan istilah multiplisitas, yaitu akar ganda dihitung dua kali, akar tripel dihitung tiga kali, dan seterusnya, sehingga sebarang polinomial berderajat n akan selalu memiliki n akar, termasuk polinomial berderajat dua, yaitu bentuk kuadrat.

Oleh karena itu, persamaan kuadrat dengan diskriminan nol akan memiliki dua akar (yang tentunya ganda) jika kita menghitungnya dengan multiplisitasnya. Meskipun demikian dalam buku ini, kita sepakat untuk menggunakan istilah 'akar ganda' dibanding 'akar tunggal', meski tidak secara eksplisit menyebutkan multiplisitas dalam pernyataannya.

Karenanya, jumlah akar (atau hasil kalinya) persamaan kuadrat yang memiliki diskriminan nol dapat kita hitung menggunakan formula 2.10 dengan aman. Sebagai contoh, jumlah akar persamaan kuadrat $x^2 - 4x + 4 = 0$ adalah

$\frac{-4}{1} = -4$, meskipun akarnya sebenarnya hanya $x = 2$ jika kita menghitungnya tanpa multiplisitas (yaitu jika kita menganggapnya sebagai akar tunggal).

2.1.7 Menyusun Persamaan Kuadrat Baru

Sebelumnya, kita tahu bahwa persamaan kuadrat

$$(x - x_1)(x - x_2) = 0$$

merupakan persamaan kuadrat yang akar-akarnya x_1 dan x_2. Jika kita menjabarkan ruas kiri persamaan terakhir, kita akan mendapatkan

$$x^2 - x_2 x - x_1 x + x_1 x_2 = 0$$

sehingga

$$x^2 - (x_1 + x_2)\, x + x_1 x_2 = 0. \tag{2.21}$$

{eq:221}

Sebagai catatan, proses penjabaran ini juga merupakan bukti lain untuk formula 2.10 dan formula 2.11.

Oleh karena itu, jika kita mengetahui bahwa suatu persamaan kuadrat memiliki akar-akar $x_1 = 2$ dan $x_2 = 3$, kita langsung saja memasukkan nilai x_1 dan x_2 ke dalam persamaan 2.21 sehingga persamaan kuadrat yang memenuhi adalah

$$x^2 - (2 + 3)\, x + 2\,(3) = 0 \iff x^2 - 5x + 6 = 0.$$

Anda dapat mengecek bahwa persamaan ini memiliki solusi $x = 2$ atau $x = 3$.

Dengan menggunakan fakta tersebut, kita akan dengan mudah membentuk suatu persamaan kuadrat baru berdasarkan akar-akar dari persamaan kuadrat sebelumnya, meskipun kita tidak mengetahui akar-akarnya secara eksplisit. Cukup digunakan bentuk-bentuk simetris akar-akar persamaan kuadrat. Sebagai contoh, kita akan mengerjakan soal berikut.

Contoh 2.1.11. Diberikan persamaan kuadrat $5x^2 - x + 1 = 0$ yang akar-akarnya p dan q. Tentukan persamaan kuadrat baru yang akar-akarnya $p + 1$ dan $q + 1$.

Jawab. Tentunya jika kita mencari akar-akarnya dengan menggunakan teorema 2.1.6, kemudian mencari $p+1$ dan $q+1$ secara manual, akan sangat menghabiskan waktu. Belum lagi persamaan kuadrat tersebut memiliki diskriminan negatif (cek!). Oleh karena itu, kita gunakan metode yang telah dijabarkan sebelumnya.

Misalkan $x_1 = p + 1$ dan $x_2 = q + 1$ merupakan akar-akar dari persamaan kuadrat baru. Maka dengan menggunakan persamaan 2.21, persamaan kuadrat yang akar-akarnya x_1 dan x_2 adalah

$$\begin{aligned}
& x^2 - (x_1 + x_2)\, x + x_1 x_2 = 0 \\
\iff \quad & x^2 - ((p + 1) + (q + 1))\, x + (p + 1)(q + 1) = 0 \\
\iff \quad & x^2 - (p + q + 2)\, x + (pq + p + q + 1) = 0
\end{aligned}$$

Padahal p dan q memenuhi $p + q = \frac{1}{5}$ dan $pq = \frac{1}{5}$ sehingga persamaan terakhir dapat ditulis ulang menjadi

$$x^2 - \left(\frac{1}{5} + 2\right) x + \left(\frac{1}{5} + \frac{1}{5} + 1\right) = 0 \iff x^2 - \frac{11}{5} x + \frac{7}{5} = 0.$$

Oleh karena itu, mengalikan kedua ruas dengan 5 akan didapatkan

$$5x^2 - 11x + 7 = 0.$$

Jadi, persamaan kuadrat baru yang akar-akarnya $p + 1$ dan $q + 1$ adalah

$$5x^2 - 11x + 7 = 0.$$

Untuk lebih memantapkan lagi, kita akan mengerjakan satu contoh lain sebagai berikut.

Contoh 2.1.12. Diberikan persamaan kuadrat $x^2 - 5x + 10 = 0$ yang akar-akarnya r dan s. Tentukan persamaan kuadrat baru yang akar-akarnya r^2 dan s^2.

Jawab. Persamaan kuadrat baru yang akar-akarnya r^2 dan s^2 adalah

$$x^2 - \left(r^2 + s^2\right) x + r^2 \cdot s^2 = 0 \iff x^2 - \left((r+s)^2 - 2rs\right) x + (rs)^2 = 0.$$

Padahal r dan s memenuhi $r + s = 5$ dan $rs = 10$ sehingga persamaan terakhir dapat ditulis ulang menjadi

$$x^2 - \left(5^2 - 2\,(10)\right) x + 10^2 = 0 \iff x^2 - 5x + 100 = 0.$$

Jadi, persamaan kuadrat baru yang akar-akarnya r^2 dan s^2 adalah

$$x^2 - 5x + 100 = 0.$$

2.1.8 Contoh Soal HOTS

Subbab ini tentunya harus diakhiri dengan memberikan contoh soal-soal HOTS agar pembaca dapat lebih menguasai materi-materi yang diberikan pada subbab ini. Pembaca diharapkan dapat mencoba untuk mengerjakan contoh-contoh ini terlebih dahulu sebelum membaca solusinya.

Contoh 2.1.13. Tentukan banyaknya penyelesaian real dari persamaan

$$x^4 - 5x^3 + 6x^2 - 5x + 1 = 0.$$

Jawab. Setelah melihat soalnya, pasti kita cenderung berpikir bahwa ini bukanlah merupakan persamaan kuadrat. Selain itu, persamaan ini juga sangat sulit untuk diselesaikan karena belum dipelajari mengenai teknik menyelesaikan persamaan polinomial berderajat empat. Tetapi, dengan sedikit eksplorasi, kita akan mendapatkan solusi yang lebih mudah tanpa mengetahui teknik-teknik lanjutan. Hanya cukup beberapa pemfaktoran dan pemisalan, kita akan dapat menjawab soal ini dengan menggunakan pengetahuan persamaan kuadrat yang telah kita pelajari sebelumnya.
Pertama-tama, kita kelompokkan terlebih tahulu suku-suku yang memiliki koefisien sama sehingga

$$x^4 - 5x^3 + 6x^2 - 5x + 1 = 0 \iff \left(x^4 + 1\right) - 5\left(x^3 + x\right) + 6x^2 = 0.$$

Dengan memfaktorkan keluar x^2, persamaan terakhir dapat kita tuliskan sebagai

$$x^2 \left(\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 6\right) = 0.$$

Perhatikan bahwa $x = 0$ bukanlah penyelesaian dari persamaan pada soal (cek!) sehingga kita dapat membagi kedua ruas dengan x^2. Oleh karena itu kita punyai

$$\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 6 = 0.$$

Selanjutnya, misalkan $x + \dfrac{1}{x} = u$, maka

$$u^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} \iff u^2 - 2 = x^2 + \frac{1}{x^2}$$

sehingga

$$\begin{aligned} & \left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 6 = 0 \\ \iff & u^2 - 2 - 5u + 6 = 0 \\ \iff & u^2 - 5u + 4 = 0 \\ \iff & (u-1)(u-4) = 0 \end{aligned}$$

Oleh karena itu, terdapat dua kasus untuk ditinjau.
Kasus 1. $u - 1 = 0 \iff u = 1$.
Dengan melakukan substitusi balik nilai u akan didapatkan

$$x + \frac{1}{x} = 1 \iff x^2 - x + 1 = 0.$$

Perhatikan bahwa diskriminan persamaan terakhir adalah $D = (-1)^2 - 4(1)(1) = -3$ sehingga untuk kasus ini tidak tidak ada solusi real x yang memenuhi.
Kasus 2. $u - 4 = 0 \iff u = 4$.
Dengan melakukan substitusi balik nilai u akan didapatkan

$$x + \frac{1}{x} = 4 \iff x^2 - 4x + 1 = 0.$$

Perhatikan bahwa diskriminan persamaan terakhir adalah $D = (-4)^2 - 4(1)(1) = 12$ sehingga untuk kasus ini memiliki dua solusi real yang berbeda. Jadi, banyaknya penyelesaian real dari persamaan $x^4 - 5x^3 + 6x^2 - 5x + 1 = 0$ adalah 2.

Contoh 2.1.14. Persamaan kuadrat $x^2 + ax + b + 1 = 0$ dengan a, b adalah bilangan bulat, memiliki akar-akar bilangan asli. Buktikan bahwa $a^2 + b^2$ bukan merupakan bilangan prima.

Jawab. Misalkan akar-akar persamaan kuadrat pada soal adalah x_1 dan x_2, maka dengan formula 2.10 dan formula 2.11, akan didapatkan

$$x_1 + x_2 = -a \quad \text{dan} \quad x_1 x_2 = b + 1$$

atau

$$-(x_1 + x_2) = a \quad \text{dan} \quad b = x_1 x_2 - 1$$

sehingga

$$\begin{aligned}
a^2+b^2 &= \left(-\left(x_1+x_2\right)\right)^2+\left(x_1x_2-1\right)^2 \\
&= \left(x_1+x_2\right)^2+\left(x_1x_2-1\right)^2 \\
&= x_1^2+2x_1x_2+x_2^2+x_1^2x_2^2-2x_1x_2+1 \\
&= x_1^2+x_2^2+x_1^2x_2^2+1 \\
&= \left(x_1^2+1\right)\left(x_2^2+1\right)
\end{aligned}$$

Karena akar-akar persamaan kuadrat pada soal merupakan bilangan asli, maka x_1^2 dan x_2^2 juga pasti bilangan asli sehingga x_1^2+1 dan x_2^2+1 merupakan bilangan asli yang lebih dari satu.
Karena a^2+b^2 dapat dituliskan sebagai perkalian dua bilangan asli yang lebih besar dari satu, maka jelas bahwa a^2+b^2 bukan merupakan bilangan prima.
Jadi terbukti bahwa a^2+b^2 bukan merupakan bilangan prima. $\square$

2.1.9 Latihan Soal 2.1

Bagian Pertama

1. Tentukan mana saja yang merupakan persamaan kuadrat, berikan alasan Anda:
   a) $m^2 = 0$
   b) $x^4 + x^2 - 2 = 0$
   c) $y^5 = 3$
   d) $\delta^2 - 3\delta + 10 = 0$
   e) $p^2 - \sqrt{p} + 5 = 0$
   f) $x^4 + ax^3 + bx^2 + ax + 1 = 0$ untuk suatu konstanta positif a dan b.
2. Tentukan nilai dari a, b, dan c dari persamaan kuadrat dalam x berikut:
   a) $5x^2 - 10x - 3 = 0$
   b) $x^2 = -1$
   c) $x^2 + 2x = 0$
   d) $-5x^2 - px - 10p^2 + 10p - 1 = 0$
   e) $(m-1)\,x^2 - 10x + px - 20p + 1 = 0$
3. Tunjukkan bahwa $x = 3$ adalah akar dari persamaan $x^2 + 9x - 36 = 0$ tetapi $x = 10$ bukan.
4. Salah satu akar dari persamaan kuadrat $ax^2 - (a+6)\,x - 35 = 0$ adalah $x = \frac{7}{2}$. Jika ada, tentukanlah nilai dari akar yang lain.
5. Jika u dan v adalah akar-akar dari persamaan $x^2 - 5x + 13 = 0$, tentukanlah nilai dari
   a) $\left(u^2 - 5u\right)\left(v^2 - 5v\right)$.
   b) $\left(2u^2 - 10v + 5\right) + \left(3v^2 - 15v + 17\right)$.
6. Tentukanlah akar-akar persamaan di bawah ini dengan cara memfaktorkan.
   a) $x^2 - 3x + 2 = 0$
   b) $4x^2 - 5x = 0$
   c) $7x^2 = 1$
   d) $10x^2 - 9x - 7 = 0$
   e) $-7x^2 - 6x + 1 = 0$
   f) $-6x^2 - 7x + 3 = 0$
7. Tentukanlah persamaan kuadrat yang akar-akarnya
   a) 2 dan $-\frac{1}{2}$
   b) 0 dan -1
   c) $\frac{10}{3}$ dan $-\frac{10}{3}$
   d) $\frac{1}{5}$ dan $-\frac{7}{3}$.
8. Tentukanlah nilai dari k, m, dan n pada setiap kesamaan di bawah ini.
   a) $x^2 + x + 1 = k\,(x+m)^2 + n$
   b) $2x^2 - 10x + 3 = k\,(x+m)^2 + n$
   c) $3x^2 - 7x - 18 = k\,(x+m)^2 + n$
   d) $x^2 - 6x + 9 = k\,(x+m)^2 + n$
9. Tentukanlah akar-akar persamaan di bawah ini dengan cara melengkapkan kuadrat.

a) $x^2 - 7x + 10 = 0$
b) $2x^2 - 5x - 9 = 0$
c) $7x^2 + x - 1 = 0$
d) $x^2 + 4x + 4 = 0$
e) $x^2 + x + 1 = 0$

10. Tentukanlah akar-akar persamaan kuadrat pada soal di atas dengan menggunakan rumus kuadratik (rumus abc).
11. Tentukanlah diskriminan dari persamaan kuadrat berikut.

a) $-x^2 - 10x + 1 = 0$
b) $4 - 7x - x^2 = 3$
c) $2x^2 + 2x - 1 = 0$
d) $2x^2 - 3x + 17 = 0$

12. Buatlah 3 contoh persamaan kuadrat yang akar-akarnya takreal.
13. Buatlah 3 contoh persamaan kuadrat yang akar-akarnya real, tetapi irasional.
14. Persamaan kuadrat $x^2 + px + q = 0$ memiliki dua solusi bilangan bulat untuk suatu bilangan prima p dan q. Tentukan semua nilai p dan q yang mungkin.
15. Diskriminan dari persamaan kuadrat $-2x^2 + 3x - 11\alpha = 0$ adalah -12. Tentukanlah nilai dari α.
16. Tentukan syarat nilai m agar persamaan kuadrat $3x^2 + 2x - m + 1 = 0$ memiliki

a) akar-akar real
b) akar-akar real yang berbeda
c) akar-akar takreal

17. Persamaan kuadrat $x^2 - 2mx + m - 1 = 0$ memiliki akar kembar. Tentukanlah nilai dari m.
18. $\boxed{\text{PF}}$ Tentukanlah syarat-syarat agar akar-akar suatu persamaan kuadrat merupakan bilangan rasional.
19. Tentukanlah sekurang-kurangnya dua nilai m untuk setiap persamaan di bawah ini agar memiliki akar-akar rasional, lalu ujilah kebenarannya

a) $2x^2 - 3x + m - 1 = 0$
b) $-mx^2 - x + 1 = 0$
c) $3x^2 - mx + 4 = 0$
d) $2mx^2 - 3x - m - 1 = 0$

20. Jika x_1 dan x_2 adalah akar-akar dari persamaan kuadrat $x^2 - 2x + 7 = 0$, maka tentukanlah nilai dari

a) $x_1^2 + x_2^2$
b) $x_1 x_2^3 + x_1^3 x_2$
c) $x_1^3 + x_2^3$
d) $\left|x_1^2 - x_2^2\right|$
e) $\dfrac{1}{x_1} + \dfrac{1}{x_2}$
f) $(x_1 - 1)(x_2 + 1)$
g) $x_1^{-3} - x_2^{-3}$
h) $\dfrac{x_1}{x_2} + \dfrac{x_2}{x_1}$
i) $x_1^5 + x_2^5$
j) $\dfrac{1}{\sqrt{x_1}} + \dfrac{1}{\sqrt{x_2}}$

21. Jika r dan s merupakan akar-akar dari persamaan $2x^2 - 1x + 2 = 0$, tentukanlah nilai dari

$$\frac{r}{(r+1)^2} + \frac{s}{(s^2+1)^2}.$$

22. Jika p dan q merupakan akar-akar dari persamaan $x^2 - 2x + 3 = 0$, tentukanlah nilai dari
$$\left(p^2 - 4p + 2\right)\left(q^2 + 4\right).$$
23. Selisih akar-akar persamaan $5x^2 - x + a = 0$ adalah -3. Tentukanlah nilai dari a.
24. Diketahui persamaan kuadrat $3x^2 - mx + m - 1 = 0$ mempunyai akar-akar α dan β. Jika $\alpha^3 + \beta^3 = 23$, tentukanlah nilai dari m.
25. Tentukan banyaknya penyelesaian yang memenuhi sistem persamaan
$$\begin{cases} x^2 - ax + 2021 = 0 \\ x^2 - 2021x + a = 0 \end{cases}$$
untuk $x < 0$.
26. Diberikan persamaan kuadrat
    (i) $\left(a^2 - 3\right)x^2 + ax - 3 = 0$
    (ii) $x^2 - x + 2a = 0$
    (iii) $x^2 - (2a + 3)x - a^2 + a + 1 = 0$

    Tentukanlah syarat untuk a dalam setiap persamaan tersebut agar
    a) akar-akarnya positif
    b) akar-akarnya negatif
    c) akar-akarnya berlainan tanda
    d) akar-akarnya berkebalikan
    e) akar-akarnya berlawanan
27. Kedua akar persamaan $x^2 - 3x + a - 3 = 0$ lebih besar dari 2. Tentukanlah syarat untuk a.
28. Tentukan persamaan kuadrat baru yang akar-akarnya dua kali lebih besar dari akar-akar persamaan kuadrat $2x^2 - 3x + 1 = 0$.
29. Tentukan persamaan kuadrat yang mempunyai akar a dan b sehingga
$$\frac{1}{a} + \frac{1}{b} = \frac{7}{10}.$$
30. Misalkan m dan n adalah akar-akar persamaan kuadrat $3x^2 - 5x + 1 = 0$. Tentukanlah persamaan kuadrat yang mempunyai akar-akar $m^{-2} + 1$ dan $n^{-2} + 1$.
31. Misalkan a dan b adalah akar-akar persamaan kuadrat $-2x^2 + x - 7 = 0$. Tentukanlah persamaan kuadrat yang akar-akarnya ab dan $a^2 + b^2$.
32. Dari soal di atas, tentukanlah persamaan kuadrat yang akar-akarnya $\sqrt{a}$ dan $\sqrt{b}$.
33. Jika $\alpha + 2\beta = 5$ dan $\alpha\beta = -2$, maka tentukanlah persamaan kuadrat yang akar-akarnya $\dfrac{\alpha}{\alpha + 1}$ dan $\dfrac{2\beta}{2\beta + 1}$.
34. Persamaan kuadrat $2x^2 - px + 1 = 0$ dengan $p > 0$ mempunyai akar-akar α dan β. Jika $x^2 - 5x + q = 0$ mempunyai akar-akar $\dfrac{1}{\alpha^2}$ dan $\dfrac{1}{\beta^2}$, maka tentukanlah nilai dari $q - p$.
35. Jika m dan n adalah akar-akar dari persamaan kuadrat $2x^2 + x - 2 = 0$, maka tentukanlah persamaan kuadrat yang akar-akarnya $m^3 - n^2$ dan $n^3 - m^2$.

Bagian Kedua — Soal Tantangan

1. Misalkan a, b dan c adalah tiga bilangan *berbeda*. Jika ketiga bilangan tersebut merupakan bilangan asli satu digit, tentukanlah jumlah terbesar akar-akar persamaan $(x-a)(x-b)+(x-b)(x-c)=0$ yang mungkin.
2. Misalkan α dan β akar-akar dari persamaan

$$x^2-2px+p^2-2p-1=0.$$

Cari semua bilangan real p sedemikian sehingga

$$\frac{1}{2}\frac{(\alpha-\beta)^2-2}{(\alpha+\beta)^2+2}$$

merupakan bilangan bulat.
3. Diketahui p dan q merupakan bilangan prima. Jika persamaan $x^2-px+q=0$ memiliki akar-akar bilangan bulat positif yang berbeda, tentukanlah nilai dari p dan q.
4. Tinjau persamaan $x^2+px+q=0$. Berapa banyak persamaan demikian yang memiliki akar-akar real jika p dan q hanya boleh dipilih dari himpunan $\{1,2,3,4,5,6\}$?
5. Diberikan persamaan kuadrat $ax^2-bx+c=0$ dengan a, b, dan c semuanya merupakan bilangan asli. Jika persamaan kuadrat tersebut memiliki dua akar berbeda yang berada pada interval $(0,1)$, carilah nilai paling minimum yang mungkin dari abc.
6. PF Misalkan p dan q bilangan real sedemikian sehingga persamaan kuadrat $x^2+px+q=0$ memiliki dua akar berbeda x_1 dan x_2. Asumsikan $|x_1-x_2|=1$ dan $|p-q|=1$. Buktikan bahwa p,q,x_1,x_2 semuanya merupakan bilangan bulat.
7. Jika x_1 dan x_2 adalah akar-akar dari persamaan kuadrat $x^2+x-3=0$, tentukanlah nilai dari $4x_1^2+3x_2^2+2x_1+x_2$.
8. Jika akar-akar persamaan $x^2-45x-8=0$ adalah α dan β, maka tentukanlah nilai dari $\sqrt[3]{\alpha}+\sqrt[3]{\beta}$.
9. Misalkan a, b, c, dan d bilangan real taknol. Jika a dan b adalah solusi dari $x^2+cx+d=0$ serta c dan d solusi dari $x^2+ax+b=0$, maka tentukanlah nilai dari $a+b+c+d$.
10. Jika persamaan kuadrat $x^2+ax+b=0$ dan $x^2+px+q=0$ memiliki satu akar yang sama, maka tentukanlah persamaan kuadrat yang akar-akarnya merupakan akar-akar yang lain dari kedua persamaan kuadrat sebelumnya.
11. Cari semua pasangan bilangan real a dan b dengan $b>0$ sedemikian sehingga solusi dari dua persamaan

$$x^2+ax+a=b \quad \text{dan} \quad x^2+ax+a=-b$$

merupakan empat bilangan bulat berurutan.
12. PF Diketahui rasio dari akar-akar persamaan kuadrat $\ell x^2+nx+n=0$ adalah $p:q$. Buktikan atau bantah bahwa

$$\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}-\sqrt{\frac{n}{\ell}}=0$$

13. Jika setiap dua dari tiga persamaan kuadrat

$$\begin{aligned} x^2 - a^2x + a + 1 &= 0, \\ x^2 - (a+1)\,x + a &= 0, \text{ dan} \\ x^2 - 3ax + x + a^2 + 2 &= 0 \end{aligned}$$

selalu memiliki tepat satu akar yang sama, maka tentukanlah semua bilangan real a yang munkin.

14. Jika α dan β akar-akar dari persamaan kuadrat $x^2 - 2x - 5 = 0$, maka tentukanlah nilai dari $\alpha^4 - 28\alpha$.

15. Jika p dan q akar-akar dari persamaan $x^2 - x + 1 = 0$, tentukanlah nilai dari $p^{2021} + q^{2021}$.

16. Diketahui x_1 dan x_2 adalah dua bilangan bulat berbeda yang merupakan akar-akar dari persamaan kuadrat $x^2 + px + q + 1 = 0$. Jika p dan $p^2 + q^2$ adalah bilangan-bilangan prima, tentukan nilai terbesar yang mungkin dari $x_1^{2021} + x_2^{2021}$.

17. $\boxed{*}$ Untuk bilangan real x, notasi $\lfloor x \rfloor$ menyatakan bilangan bulat terbesar yang tidak lebih besar dari x; sedangkan $\lceil x \rceil$ menyatakan bilangan bulat terkecil yang tidak lebih kecil dari x. Tentukan semua bilangan real x yang memenuhi

$$\lfloor x \rfloor^2 - 3x + \lceil x \rceil = 0.$$

18. $\boxed{*}$ Danu dan Dini sedang bermain suatu permainan. Pada awalnya, Danu memilih tiga bilangan real taknol. Dini kemudian menyusun ketiga bilangan tadi sebagai koefisien persamaan kuadrat

$$\_x^2 + \_x + \_ = 0.$$

Danu memenangkan permainan jika dan hanya jika persamaan yang dihasilkan memiliki dua solusi rasional berbeda.

19. $\boxed{\text{PF}}$ $\boxed{**}$ Asumsikan a, b, c, A, B, C semuanya bilangan real dengan $a \neq 0$ dan $A \neq 0$ sedemikian sehingga

$$\left|ax^2 + bx + c\right| \leq \left|Ax^2 + Bx + C\right|$$

untuk setiap bilangan real x. Buktikan bahwa

$$\left|b^2 - 4ac\right| \leq \left|B^2 - 4AC\right|.$$

20. $\boxed{\text{PF}}$ $\boxed{**}$ Misalkan a dan b bilangan bulat positif sedemikian sehingga $ab+1$ membagi a^2+b^2. Buktikan bahwa $\dfrac{a^2+b^2}{ab+1}$ merupakan kuadrat dari suatu bilangan bulat.

2.2 Fungsi Kuadrat

Sebelum membahas mengenai fungsi kuadrat, kita harus mengetahui terlebih dahulu apa itu fungsi. Disini kita hanya memberikan hal-hal yang penting mengenai fungsi karena pendalaman lebih lanjut mengenai fungsi akan dipelajari pada mata kuliah Pengantar Dasar Matematika. Secara informal, fungsi adalah sesuatu yang memerlukan input dan menghasilkan output. Bisa juga kita anggap fungsi sebagai mesin. Jika kita memasukkan bahan baku ke dalam mesin (input), maka hasilnya adalah barang jadi (output).

Salah satu contoh fungsi dalam matematika adalah $f(x) = 2x + 1$ dan $g(x) = \sqrt{x}$. Pada fungsi f, jika kita masukkan $x = 2$ (sebagai input), maka hasilnya adalah $f(2) = 2\,(2) + 1 = 5$ (outputnya). Selain itu, pada fungsi g, jika kita masukkan $x = 4$, maka $g(4) = \sqrt{4} = 2$. Disini bisa kita lihat bahwa f dapat menerima segala macam bilangan real, tetapi fungsi g tidak bisa menerima segala macam bilangan real, karena jika $x < 0$, fungsi tersebut tidak terdefinisi pada garis bilangan real.

Dalam hal ini, himpunan semua input yang mungkin bagi suatu fungsi f kita katakan sebagai domain dari fungsi f dan himpunan semua output yang mungkin bagi suatu fungsi f dikatakan sebagai jangkauan (*range*) dari fungsi f. Domain fungsi f biasa dinotasikan sebagai $\mathrm{Dom}(f)$ atau D_f, dan jangkauan fungsi f biasa dinotasikan sebagai $\mathrm{Ran}(f)$ atau R_f. Pada contoh sebelumnya, $f(x) = 2x + 1$ memiliki domain $\mathrm{Dom}(f) = \mathbb{R}$ dan jangkauan $\mathrm{Ran}(f) = \mathbb{R}$, serta $g(x) = \sqrt{x}$ memiliki domain $\mathrm{Dom}(g) = \mathbb{R}_{\geq 0}$ dan jangkauan $\mathrm{Ran}(g) = \mathbb{R}_{\geq 0}$ (mengapa?).

Eksplorasi

Coba cari tahu mengenai kodomain dan *image* dari suatu fungsi. Apakah perbedaan antara jangkauan, kodomain, dan *image*?

Jika f memiliki domain A dan jangkauan B, maka f merupakan fungsi dari himpunan A ke himpunan B. Secara simbolik, kita bisa tuliskan sebagai $f\colon A \to B$. Tentunya notasi ini akan lebih mempermudah penulisannya. Perlu diketahui juga bahwa fungsi $g(x) = x + 1$ sebenarnya ekuivalen dengan fungsi $r(y) = y + 1$. Variabel yang digunakan dalam suatu fungsi sebenarnya tidak terlalu penting, yang penting dalam suatu fungsi adalah aturan pemetaannya itu sendiri atau formulanya.

Setelah mengetahui sekilas mengenai fungsi secara umum, kita kemudian siap untuk mempelajari fungsi kuadrat. Fungsi kuadrat memiliki bentuk umum

$$f(x) = ax^2 + bx + c \tag{2.22}$$

{eq:222}

dengan a, b, c semuanya bilangan real dan $a \neq 0$.

Bentuk umum fungsi kuadrat di atas mengingatkan kita kepada sesuatu yang familiar, yaitu persamaan kuadrat. Tetapi dalam persamaan kuadrat, salah satu ruasnya adalah nol. Salah satu contoh fungsi kuadrat adalah $f(x) = x^2 - 3x + 1$. Disini, nilai $a = 1$, $b = -3$, dan $c = 1$. Selain itu, $g(x) = x^2$ juga merupakan fungsi kuadrat dengan nilai $a = 1$, $b = 0$, dan $c = 0$.

2.2.1 Grafik Fungsi Kuadrat

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogisms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

2.2.2 Menentukan Fungsi Kuadrat

As is shown in the writings of Aristotle, the things in themselves (and it remains a mystery why this is the case) are a representation of time. Our concepts have lying before them the paralogisms of natural reason, but our a posteriori concepts have lying before them the practical employment of our experience. Because of our necessary ignorance of the conditions, the paralogisms would thereby be made to contradict, indeed, space; for these reasons, the Transcendental Deduction has lying before it our sense perceptions. (Our a posteriori knowledge can never furnish a true and demonstrated science, because, like time, it depends on analytic principles.) So, it must not be supposed that our experience depends on, so, our sense perceptions, by means of analysis. Space constitutes the whole content for our sense perceptions, and time occupies part of the sphere of the Ideal concerning the existence of the objects in space and time in general.

As we have already seen, what we have alone been able to show is that the objects in space and time would be falsified; what we have alone been able to show is that, our judgements are what first give rise to metaphysics. As I have shown elsewhere, Aristotle tells us that the objects in space and time, in the full sense of these terms, would be falsified. Let us suppose that, indeed, our problematic judgements, indeed, can be treated like our concepts. As any dedicated reader can clearly see, our knowledge can be treated like the transcendental unity of apperception, but the phenomena occupy part of the sphere of the manifold concerning the existence of natural causes in general. Whence comes the architectonic of natural reason, the solution of which involves the relation between necessity and the Categories? Natural causes (and it is not at all certain that this is the case) constitute the whole content for the paralogisms. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

2.2.3 Domain dan Jangkauan Fungsi Kuadrat

Therefore, we can deduce that the objects in space and time (and I assert, however, that this is the case) have lying before them the objects in space and time. Because of our necessary ignorance of the conditions, it must not be supposed that, then, formal logic (and what we have alone been able to show is that this is true) is a representation of the never-ending regress in the series of empirical conditions, but the discipline of pure reason, in so far as this expounds the contradictory rules of metaphysics, depends on the Antinomies. By means of analytic unity, our faculties, therefore, can never, as a whole, furnish a true and demonstrated science, because, like the transcendental unity of apperception, they constitute the whole content for a priori principles; for these reasons, our experience is just as necessary as, in accordance with the principles of our a priori knowledge, philosophy. The objects in space and time abstract from all content of knowledge. Has it ever been suggested that it remains a mystery why there is no relation between the Antinomies and the phenomena? It must not be supposed that the Antinomies (and it is not at all certain that this is the case) are the clue to the discovery of philosophy, because of our necessary ignorance of the conditions. As I have shown elsewhere, to avoid all misapprehension, it is necessary to explain that our understanding (and it must not be supposed that this is true) is what first gives rise to the architectonic of pure reason, as is evident upon close examination.

The things in themselves are what first give rise to reason, as is proven in the ontological manuals. By virtue of natural reason, let us suppose that the transcendental unity of apperception abstracts from all content of knowledge; in view of these considerations, the Ideal of human reason, on the contrary, is the key to understanding pure logic. Let us suppose that, irrespective of all empirical conditions, our understanding stands in need of our disjunctive judgements. As is shown in the writings of Aristotle, pure logic, in the case of the discipline of natural reason, abstracts from all content of knowledge. Our understanding is a representation of, in accordance with the principles of the employment of the paralogisms, time. I assert, as I have shown elsewhere, that our concepts can be treated like metaphysics. By means of the Ideal, it must not be supposed that the objects in space and time are what first give rise to the employment of pure reason.

2.2.4 Definit Positif dan Definit Negatif

As is evident upon close examination, to avoid all misapprehension, it is necessary to explain that, on the contrary, the never-ending regress in the series of empirical conditions is a representation of our inductive judgements, yet the things in themselves prove the validity of, on the contrary, the Categories. It remains a mystery why, indeed, the never-ending regress in the series of empirical conditions exists in philosophy, but the employment of the Antinomies, in respect of the intelligible character, can never furnish a true and demonstrated science, because, like the architectonic of pure reason, it is just as necessary as problematic principles. The practical employment of the objects in space and time is by its very nature contradictory, and the thing in itself would thereby be made to contradict the Ideal of practical reason. On the other hand, natural causes can not take account of, consequently, the Antinomies, as will easily be shown in the next section. Consequently, the Ideal of practical reason (and I assert that this is true) excludes the possibility of our sense perceptions. Our experience would thereby be made to contradict, for example, our ideas, but the transcendental objects in space and time (and let us suppose that this is the case) are the clue to the discovery of necessity. But the proof of this is a task from which we can here be absolved.

Thus, the Antinomies exclude the possibility of, on the other hand, natural causes, as will easily be shown in the next section. Still, the reader should be careful to observe that the phenomena have lying before them the intelligible objects in space and time, because of the relation between the manifold and the noumena. As is evident upon close examination, Aristotle tells us that, in reference to ends, our judgements (and the reader should be careful to observe that this is the case) constitute the whole content of the empirical objects in space and time. Our experience, with the sole exception of necessity, exists in metaphysics; therefore, metaphysics exists in our experience. (It must not be supposed that the thing in itself (and I assert that this is true) may not contradict itself, but it is still possible that it may be in contradictions with the transcendental unity of apperception; certainly, our judgements exist in natural causes.) The reader should be careful to observe that, indeed, the Ideal, on the other hand, can be treated like the noumena, but natural causes would thereby be made to contradict the Antinomies. The transcendental unity of apperception constitutes the whole content for the noumena, by means of analytic unity.

2.2.5 Sifat Fungsi Kuadrat Terhadap Fungsi Lainnya

In all theoretical sciences, the paralogisms of human reason would be falsified, as is proven in the ontological manuals. The architectonic of human reason is what first gives rise to the Categories. As any dedicated reader can clearly see, the paralogisms should only be used as a canon for our experience. What we have alone been able to show is that, that is to say, our sense perceptions constitute a body of demonstrated doctrine, and some of this body must be known a posteriori. Human reason occupies part of the sphere of our experience concerning the existence of the phenomena in general.

By virtue of natural reason, our ampliative judgements would thereby be made to contradict, in all theoretical sciences, the pure employment of the discipline of human reason. Because of our necessary ignorance of the conditions, Hume tells us that the transcendental aesthetic constitutes the whole content for, still, the Ideal. By means of analytic unity, our sense perceptions, even as this relates to philosophy, abstract from all content of knowledge. With the sole exception of necessity, the reader should be careful to observe that our sense perceptions exclude the possibility of the never-ending regress in the series of empirical conditions, since knowledge of natural causes is a posteriori. Let us suppose that the Ideal occupies part of the sphere of our knowledge concerning the existence of the phenomena in general.

2.2.6 Titik Tetap Fungsi Kuadrat

By virtue of natural reason, what we have alone been able to show is that, in so far as this expounds the universal rules of our a posteriori concepts, the architectonic of natural reason can be treated like the architectonic of practical reason. Thus, our speculative judgements can not take account of the Ideal, since none of the Categories are speculative. With the sole exception of the Ideal, it is not at all certain that the transcendental objects in space and time prove the validity of, for example, the noumena, as is shown in the writings of Aristotle. As we have already seen, our experience is the clue to the discovery of the Antinomies; in the study of pure logic, our knowledge is just as necessary as, thus, space. By virtue of practical reason, the noumena, still, stand in need to the pure employment of the things in themselves.

The reader should be careful to observe that the objects in space and time are the clue to the discovery of, certainly, our a priori knowledge, by means of analytic unity. Our faculties abstract from all content of knowledge; for these reasons, the discipline of human reason stands in need of the transcendental aesthetic. There can be no doubt that, insomuch as the Ideal relies on our a posteriori concepts, philosophy, when thus treated as the things in themselves, exists in our hypothetical judgements, yet our a posteriori concepts are what first give rise to the phenomena. Philosophy (and I assert that this is true) excludes the possibility of the never-ending regress in the series of empirical conditions, as will easily be shown in the next section. Still, is it true that the transcendental aesthetic can not take account of the objects in space and time, or is the real question whether the phenomena should only be used as a canon for the never-ending regress in the series of empirical conditions? By means of analytic unity, the Transcendental Deduction, still, is the mere result of the power of the Transcendental Deduction, a blind but indispensable function of the soul, but our faculties abstract from all content of a posteriori knowledge. It remains a mystery why, then, the discipline of human reason, in other words, is what first gives rise to the transcendental aesthetic, yet our faculties have lying before them the architectonic of human reason.

2.2.7 Penerapan Fungsi Kuadrat dalam Kehidupan Sehari-hari

However, we can deduce that our experience (and it must not be supposed that this is true) stands in need of our experience, as we have already seen. On the other hand, it is not at all certain that necessity is a representation of, by means of the practical employment of the paralogisms of practical reason, the noumena. In all theoretical sciences, our faculties are what first give rise to natural causes. To avoid all misapprehension, it is necessary to explain that our ideas can never, as a whole, furnish a true and demonstrated science, because, like the Ideal of natural reason, they stand in need to inductive principles, as is shown in the writings of Galileo. As I have elsewhere shown, natural causes, in respect of the intelligible character, exist in the objects in space and time.

Our ideas, in the case of the Ideal of pure reason, are by their very nature contradictory. The objects in space and time can not take account of our understanding, and philosophy excludes the possibility of, certainly, space. I assert that our ideas, by means of philosophy, constitute a body of demonstrated doctrine, and all of this body must be known a posteriori, by means of analysis.

It must not be supposed that space is by its very nature contradictory. Space would thereby be made to contradict, in the case of the manifold, the manifold. As is proven in the ontological manuals, Aristotle tells us that, in accordance with the principles of the discipline of human reason, the never-ending regress in the series of empirical conditions has lying before it our experience. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

2.2.8 Fungsi Kuadrat Umum*

Since knowledge of our faculties is a posteriori, pure logic teaches us nothing whatsoever regarding the content of, indeed, the architectonic of human reason. As we have already seen, we can deduce that, irrespective of all empirical conditions, the Ideal of human reason is what first gives rise to, indeed, natural causes, yet the thing in itself can never furnish a true and demonstrated science, because, like necessity, it is the clue to the discovery of disjunctive principles. On the other hand, the manifold depends on the paralogisms. Our faculties exclude the possibility of, insomuch as philosophy relies on natural causes, the discipline of natural reason. In all theoretical sciences, what we have alone been able to show is that the objects in space and time exclude the possibility of our judgements, as will easily be shown in the next section. This is what chiefly concerns us.

Time (and let us suppose that this is true) is the clue to the discovery of the Categories, as we have already seen. Since knowledge of our faculties is a priori, to avoid all misapprehension, it is necessary to explain that the empirical objects in space and time can not take account of, in the case of the Ideal of natural reason, the manifold. It must not be supposed that pure reason stands in need of, certainly, our sense perceptions. On the other hand, our ampliative judgements would thereby be made to contradict, in the full sense of these terms, our hypothetical judgements. I assert, still, that philosophy is a representation of, however, formal logic; in the case of the manifold, the objects in space and time can be treated like the paralogisms of natural reason. This is what chiefly concerns us.

2.2.9 Latihan Soal 2.2

Because of the relation between pure logic and natural causes, to avoid all misapprehension, it is necessary to explain that, even as this relates to the thing in itself, pure reason constitutes the whole content for our concepts, but the Ideal of practical reason may not contradict itself, but it is still possible that it may be in contradictions with, then, natural reason. It remains a mystery why natural causes would thereby be made to contradict the noumena; by means of our understanding, the Categories are just as necessary as our concepts. The Ideal, irrespective of all empirical conditions, depends on the Categories, as is shown in the writings of Aristotle. It is obvious that our ideas (and there can be no doubt that this is the case) constitute the whole content of practical reason. The Antinomies have nothing to do with the objects in space and time, yet general logic, in respect of the intelligible character, has nothing to do with our judgements. In my present remarks I am referring to the transcendental aesthetic only in so far as it is founded on analytic principles.

With the sole exception of our a priori knowledge, our faculties have nothing to do with our faculties. Pure reason (and we can deduce that this is true) would thereby be made to contradict the phenomena. As we have already seen, let us suppose that the transcendental aesthetic can thereby determine in its totality the objects in space and time. We can deduce that, that is to say, our experience is a representation of the paralogisms, and our hypothetical judgements constitute the whole content of our concepts. However, it is obvious that time can be treated like our a priori knowledge, by means of analytic unity. Philosophy has nothing to do with natural causes.

2.3 Uji Kompetensi Bab 2

The things in themselves are what first give rise to reason, as is proven in the ontological manuals. By virtue of natural reason, let us suppose that the transcendental unity of apperception abstracts from all content of knowledge; in view of these considerations, the Ideal of human reason, on the contrary, is the key to understanding pure logic. Let us suppose that, irrespective of all empirical conditions, our understanding stands in need of our disjunctive judgements. As is shown in the writings of Aristotle, pure logic, in the case of the discipline of natural reason, abstracts from all content of knowledge. Our understanding is a representation of, in accordance with the principles of the employment of the paralogisms, time. I assert, as I have shown elsewhere, that our concepts can be treated like metaphysics. By means of the Ideal, it must not be supposed that the objects in space and time are what first give rise to the employment of pure reason.

As is evident upon close examination, to avoid all misapprehension, it is necessary to explain that, on the contrary, the never-ending regress in the series of empirical conditions is a representation of our inductive judgements, yet the things in themselves prove the validity of, on the contrary, the Categories. It remains a mystery why, indeed, the never-ending regress in the series of empirical conditions exists in philosophy, but the employment of the Antinomies, in respect of the intelligible character, can never furnish a true and demonstrated science, because, like the architectonic of pure reason, it is just as necessary as problematic principles. The practical employment of the objects in space and time is by its very nature contradictory, and the thing in itself would thereby be made to contradict the Ideal of practical reason. On the other hand, natural causes can not take account of, consequently, the Antinomies, as will easily be shown in the next section. Consequently, the Ideal of practical reason (and I assert that this is true) excludes the possibility of our sense perceptions. Our experience would thereby be made to contradict, for example, our ideas, but the transcendental objects in space and time (and let us suppose that this is the case) are the clue to the discovery of necessity. But the proof of this is a task from which we can here be absolved.

Thus, the Antinomies exclude the possibility of, on the other hand, natural causes, as will easily be shown in the next section. Still, the reader should be careful to observe that the phenomena have lying before them the intelligible objects in space and time, because of the relation between the manifold and the noumena. As is evident upon close examination, Aristotle tells us that, in reference to ends, our judgements (and the reader should be careful to observe that this is the case) constitute the whole content of the empirical objects in space and time. Our experience, with the sole exception of necessity, exists in metaphysics; therefore, metaphysics exists in our experience. (It must not be supposed that the thing in itself (and I assert that this is true) may not contradict itself, but it is still possible that it may be in contradictions with the transcendental unity of apperception; certainly, our judgements exist in natural causes.) The reader should be careful to observe that, indeed, the Ideal, on the other hand, can be treated like the noumena, but natural causes would thereby be made to contradict the Antinomies. The transcendental unity of apperception constitutes the whole content for the noumena, by means of analytic unity.

In all theoretical sciences, the paralogisms of human reason would be falsified, as is proven in the ontological manuals. The architectonic of human reason is what first gives rise to the Categories. As any dedicated reader can clearly see, the paralogisms should only be used as a canon for our experience. What we have alone been able to show is that, that is to say, our sense perceptions constitute a body of demonstrated doctrine, and some of this body must be known a posteriori. Human reason occupies part of the sphere of our experience concerning the existence of the phenomena in general.

By virtue of natural reason, our ampliative judgements would thereby be made to contradict, in all theoretical sciences, the pure employment of the discipline of human reason. Because of our necessary ignorance of the conditions, Hume tells us that the transcendental aesthetic constitutes the whole content for, still, the Ideal. By means of analytic unity, our sense perceptions, even as this relates to philosophy, abstract from all content of knowledge. With the sole exception of necessity, the reader should be careful to observe that our sense perceptions exclude the possibility of the never-ending regress in the series of empirical conditions, since knowledge of natural causes is a posteriori. Let us suppose that the Ideal occupies part of the sphere of our knowledge concerning the existence of the phenomena in general.

CHAPTER 3

Pertidaksamaan

sec:third

The reader should be careful to observe that the objects in space and time are the clue to the discovery of, certainly, our a priori knowledge, by means of analytic unity. Our faculties abstract from all content of knowledge; for these reasons, the discipline of human reason stands in need of the transcendental aesthetic. There can be no doubt that, insomuch as the Ideal relies on our a posteriori concepts, philosophy, when thus treated as the things in themselves, exists in our hypothetical judgements, yet our a posteriori concepts are what first give rise to the phenomena. Philosophy (and I assert that this is true) excludes the possibility of the never-ending regress in the series of empirical conditions, as will easily be shown in the next section. Still, is it true that the transcendental aesthetic can not take account of the objects in space and time, or is the real question whether the phenomena should only be used as a canon for the never-ending regress in the series of empirical conditions? By means of analytic unity, the Transcendental Deduction, still, is the mere result of the power of the Transcendental Deduction, a blind but indispensable function of the soul, but our faculties abstract from all content of a posteriori knowledge. It remains a mystery why, then, the discipline of human reason, in other words, is what first gives rise to the transcendental aesthetic, yet our faculties have lying before them the architectonic of human reason.

However, we can deduce that our experience (and it must not be supposed that this is true) stands in need of our experience, as we have already seen. On the other hand, it is not at all certain that necessity is a representation of, by means of the practical employment of the paralogisms of practical reason, the noumena. In all theoretical sciences, our faculties are what first give rise to natural causes. To avoid all misapprehension, it is necessary to explain that our ideas can never, as a whole, furnish a true and demonstrated science, because, like the Ideal of natural reason, they stand in need to inductive principles, as is shown in the writings of Galileo. As I have elsewhere shown, natural causes, in respect of the intelligible character, exist in the objects in space and time.

3.1 Pertidaksamaan Polinomial

Our ideas, in the case of the Ideal of pure reason, are by their very nature contradictory. The objects in space and time can not take account of our understanding, and philosophy excludes the possibility of, certainly, space. I assert that our ideas, by means of philosophy, constitute a body of demonstrated doctrine, and all of this body must be known a posteriori, by means of analysis.

It must not be supposed that space is by its very nature contradictory. Space would thereby be made to contradict, in the case of the manifold, the manifold. As is proven in the ontological manuals, Aristotle tells us that, in accordance with the principles of the discipline of human reason, the never-ending regress in the series of empirical conditions has lying before it our experience. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

3.2 Pertidaksamaan Nilai Mutlak

Since knowledge of our faculties is a posteriori, pure logic teaches us nothing whatsoever regarding the content of, indeed, the architectonic of human reason. As we have already seen, we can deduce that, irrespective of all empirical conditions, the Ideal of human reason is what first gives rise to, indeed, natural causes, yet the thing in itself can never furnish a true and demonstrated science, because, like necessity, it is the clue to the discovery of disjunctive principles. On the other hand, the manifold depends on the paralogisms. Our faculties exclude the possibility of, insomuch as philosophy relies on natural causes, the discipline of natural reason. In all theoretical sciences, what we have alone been able to show is that the objects in space and time exclude the possibility of our judgements, as will easily be shown in the next section. This is what chiefly concerns us.

3.3 Pertidaksamaan Rasional

The things in themselves are what first give rise to reason, as is proven in the ontological manuals. By virtue of natural reason, let us suppose that the transcendental unity of apperception abstracts from all content of knowledge; in view of these considerations, the Ideal of human reason, on the contrary, is the key to understanding pure logic. Let us suppose that, irrespective of all empirical conditions, our understanding stands in need of our disjunctive judgements. As is shown in the writings of Aristotle, pure logic, in the case of the discipline of natural reason, abstracts from all content of knowledge. Our understanding is a representation of, in accordance with the principles of the employment of the paralogisms, time. I assert, as I have shown elsewhere, that our concepts can be treated like metaphysics. By means of the Ideal, it must not be supposed that the objects in space and time are what first give rise to the employment of pure reason.

3.4 Pertidaksamaan Irasional

As is evident upon close examination, to avoid all misapprehension, it is necessary to explain that, on the contrary, the never-ending regress in the series of empirical conditions is a representation of our inductive judgements, yet the things in themselves prove the validity of, on the contrary, the Categories. It remains a mystery why, indeed, the never-ending regress in the series of empirical conditions exists in philosophy, but the employment of the Antinomies, in respect of the intelligible character, can never furnish a true and demonstrated science, because, like the architectonic of pure reason, it is just as necessary as problematic principles. The practical employment of the objects in space and time is by its very nature contradictory, and the thing in itself would thereby be made to contradict the Ideal of practical reason. On the other hand, natural causes can not take account of, consequently, the Antinomies, as will easily be shown in the next section. Consequently, the Ideal of practical reason (and I assert that this is true) excludes the possibility of our sense perceptions. Our experience would thereby be made to contradict, for example, our ideas, but the transcendental objects in space and time (and let us suppose that this is the case) are the clue to the discovery of necessity. But the proof of this is a task from which we can here be absolved.

3.5 Uji Kompetensi Bab 3

Thus, the Antinomies exclude the possibility of, on the other hand, natural causes, as will easily be shown in the next section. Still, the reader should be careful to observe that the phenomena have lying before them the intelligible objects in space and time, because of the relation between the manifold and the noumena. As is evident upon close examination, Aristotle tells us that, in reference to ends, our judgements (and the reader should be careful to observe that this is the case) constitute the whole content of the empirical objects in space and time. Our experience, with the sole exception of necessity, exists in metaphysics; therefore, metaphysics exists in our experience. (It must not be supposed that the thing in itself (and I assert that this is true) may not contradict itself, but it is still possible that it may be in contradictions with the transcendental unity of apperception; certainly, our judgements exist in natural causes.) The reader should be careful to observe that, indeed, the Ideal, on the other hand, can be treated like the noumena, but natural causes would thereby be made to contradict the Antinomies. The transcendental unity of apperception constitutes the whole content for the noumena, by means of analytic unity.

CHAPTER 4

Eksponen dan Logaritma

sec:fourth

Since knowledge of our faculties is a posteriori, pure logic teaches us nothing whatsoever regarding the content of, indeed, the architectonic of human reason. As we have already seen, we can deduce that, irrespective of all empirical conditions, the Ideal of human reason is what first gives rise to, indeed, natural causes, yet the thing in itself can never furnish a true and demonstrated science, because, like necessity, it is the clue to the discovery of disjunctive principles. On the other hand, the manifold depends on the paralogisms. Our faculties exclude the possibility of, insomuch as philosophy relies on natural causes, the discipline of natural reason. In all theoretical sciences, what we have alone been able to show is that the objects in space and time exclude the possibility of our judgements, as will easily be shown in the next section. This is what chiefly concerns us.

Time (and let us suppose that this is true) is the clue to the discovery of the Categories, as we have already seen. Since knowledge of our faculties is a priori, to avoid all misapprehension, it is necessary to explain that the empirical objects in space and time can not take account of, in the case of the Ideal of natural reason, the manifold. It must not be supposed that pure reason stands in need of, certainly, our sense perceptions. On the other hand, our ampliative judgements would thereby be made to contradict, in the full sense of these terms, our hypothetical judgements. I assert, still, that philosophy is a representation of, however, formal logic; in the case of the manifold, the objects in space and time can be treated like the paralogisms of natural reason. This is what chiefly concerns us.

Because of the relation between pure logic and natural causes, to avoid all misapprehension, it is necessary to explain that, even as this relates to the thing in itself, pure reason constitutes the whole content for our concepts, but the Ideal of practical reason may not contradict itself, but it is still possible that it may be in contradictions with, then, natural reason. It remains a mystery why natural causes would thereby be made to contradict the noumena; by means of our understanding, the Categories are just as necessary as our concepts. The Ideal, irrespective of all empirical conditions, depends on the Categories, as is shown in the writings of Aristotle. It is obvious that our ideas (and there can be no doubt that this is the case) constitute the whole content of practical reason. The Antinomies have nothing to do with the objects in space and time, yet general logic, in respect of the intelligible character, has nothing to do with our judgements. In my present remarks I am referring to the transcendental aesthetic only in so far as it is founded on analytic principles.

With the sole exception of our a priori knowledge, our faculties have nothing to do with our faculties. Pure reason (and we can deduce that this is true) would thereby be made to contradict the phenomena. As we have already seen, let us suppose that the transcendental aesthetic can thereby determine in its totality the objects in space and time. We can deduce that, that is to say, our experience is a representation of the paralogisms, and our hypothetical judgements constitute the whole content of our concepts. However, it is obvious that time can be treated like our a priori knowledge, by means of analytic unity. Philosophy has nothing to do with natural causes.

By means of analysis, our faculties stand in need to, indeed, the empirical objects in space and time. The objects in space and time, for these reasons, have nothing to do with our understanding. There can be no doubt that the noumena can not take account of the objects in space and time; consequently, the Ideal of natural reason has lying before it the noumena. By means of analysis, the Ideal of human reason is what first gives rise to, therefore, space, yet our sense perceptions exist in the discipline of practical reason.

4.1 Eksponen

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogisms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

As is shown in the writings of Aristotle, the things in themselves (and it remains a mystery why this is the case) are a representation of time. Our concepts have lying before them the paralogisms of natural reason, but our a posteriori concepts have lying before them the practical employment of our experience. Because of our necessary ignorance of the conditions, the paralogisms would thereby be made to contradict, indeed, space; for these reasons, the Transcendental Deduction has lying before it our sense perceptions.

(Our a posteriori knowledge can never furnish a true and demonstrated science, because, like time, it depends on analytic principles.) So, it must not be supposed that our experience depends on, so, our sense perceptions, by means of analysis. Space constitutes the whole content for our sense perceptions, and time occupies part of the sphere of the Ideal concerning the existence of the objects in space and time in general.

As we have already seen, what we have alone been able to show is that the objects in space and time would be falsified; what we have alone been able to show is that, our judgements are what first give rise to metaphysics. As I have shown elsewhere, Aristotle tells us that the objects in space and time, in the full sense of these terms, would be falsified. Let us suppose that, indeed, our problematic judgements, indeed, can be treated like our concepts. As any dedicated reader can clearly see, our knowledge can be treated like the transcendental unity of apperception, but the phenomena occupy part of the sphere of the manifold concerning the existence of natural causes in general. Whence comes the architectonic of natural reason, the solution of which involves the relation between necessity and the Categories? Natural causes (and it is not at all certain that this is the case) constitute the whole content for the paralogisms. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

4.2 Logaritma

Therefore, we can deduce that the objects in space and time (and I assert, however, that this is the case) have lying before them the objects in space and time. Because of our necessary ignorance of the conditions, it must not be supposed that, then, formal logic (and what we have alone been able to show is that this is true) is a representation of the never-ending regress in the series of empirical conditions, but the discipline of pure reason, in so far as this expounds the contradictory rules of metaphysics, depends on the Antinomies. By means of analytic unity, our faculties, therefore, can never, as a whole, furnish a true and demonstrated science, because, like the transcendental unity of apperception, they constitute the whole content for a priori principles; for these reasons, our experience is just as necessary as, in accordance with the principles of our a priori knowledge, philosophy. The objects in space and time abstract from all content of knowledge. Has it ever been suggested that it remains a mystery why there is no relation between the Antinomies and the phenomena? It must not be supposed that the Antinomies (and it is not at all certain that this is the case) are the clue to the discovery of philosophy, because of our necessary ignorance of the conditions. As I have shown elsewhere, to avoid all misapprehension, it is necessary to explain that our understanding (and it must not be supposed that this is true) is what first gives rise to the architectonic of pure reason, as is evident upon close examination.

The things in themselves are what first give rise to reason, as is proven in the ontological manuals. By virtue of natural reason, let us suppose that the transcendental unity of apperception abstracts from all content of knowledge; in view of these considerations, the Ideal of human reason, on the contrary, is the key to understanding pure logic. Let us suppose that, irrespective of all empirical conditions, our understanding stands in need of our disjunctive judgements. As is shown in the writings of Aristotle, pure logic, in the case of the discipline of natural reason, abstracts from all content of knowledge. Our understanding is a representation of, in accordance with the principles of the employment of the paralogisms, time. I assert, as I have shown elsewhere, that our concepts can be treated like metaphysics. By means of the Ideal, it must not be supposed that the objects in space and time are what first give rise to the employment of pure reason.

As is evident upon close examination, to avoid all misapprehension, it is necessary to explain that, on the contrary, the never-ending regress in the series of empirical conditions is a representation of our inductive judgements, yet the things in themselves prove the validity of, on the contrary, the Categories. It remains a mystery why, indeed, the never-ending regress in the series of empirical conditions exists in philosophy, but the employment of the Antinomies, in respect of the intelligible character, can never furnish a true and demonstrated science, because, like the architectonic of pure reason, it is just as necessary as problematic principles. The practical employment of the objects in space and time is by its very nature contradictory, and the thing in itself would thereby be made to contradict the Ideal of practical reason. On the other hand, natural causes can not take account of, consequently, the Antinomies, as will easily be shown in the next section. Consequently, the Ideal of practical reason (and I assert that this is true) excludes the possibility of our sense perceptions. Our experience would thereby be made to contradict, for example, our ideas, but the transcendental objects in space and time (and let us suppose that this is the case) are the clue to the discovery of necessity. But the proof of this is a task from which we can here be absolved.

Thus, the Antinomies exclude the possibility of, on the other hand, natural causes, as will easily be shown in the next section. Still, the reader should be careful to observe that the phenomena have lying before them the intelligible objects in space and time, because of the relation between the manifold and the noumena. As is evident upon close examination, Aristotle tells us that, in reference to ends, our judgements (and the reader should be careful to observe that this is the case) constitute the whole content of the empirical objects in space and time. Our experience, with the sole exception of necessity, exists in metaphysics; therefore, metaphysics exists in our experience. (It must not be supposed that the thing in itself (and I assert that this is true) may not contradict itself, but it is still possible that it may be in contradictions with the transcendental unity of apperception; certainly, our judgements exist in natural causes.) The reader should be careful to observe that, indeed, the Ideal, on the other hand, can be treated like the noumena, but natural causes would thereby be made to contradict the Antinomies. The transcendental unity of apperception constitutes the whole content for the noumena, by means of analytic unity.

4.3 Uji Kompetensi Bab 4

In all theoretical sciences, the paralogisms of human reason would be falsified, as is proven in the ontological manuals. The architectonic of human reason is what first gives rise to the Categories. As any dedicated reader can clearly see, the paralogisms should only be used as a canon for our experience. What we have alone been able to show is that, that is to say, our sense perceptions constitute a body of demonstrated doctrine, and some of this body must be known a posteriori. Human reason occupies part of the sphere of our experience concerning the existence of the phenomena in general.

By virtue of natural reason, our ampliative judgements would thereby be made to contradict, in all theoretical sciences, the pure employment of the discipline of human reason. Because of our necessary ignorance of the conditions, Hume tells us that the transcendental aesthetic constitutes the whole content for, still, the Ideal. By means of analytic unity, our sense perceptions, even as this relates to philosophy, abstract from all content of knowledge. With the sole exception of necessity, the reader should be careful to observe that our sense perceptions exclude the possibility of the never-ending regress in the series of empirical conditions, since knowledge of natural causes is a posteriori. Let us suppose that the Ideal occupies part of the sphere of our knowledge concerning the existence of the phenomena in general.

By virtue of natural reason, what we have alone been able to show is that, in so far as this expounds the universal rules of our a posteriori concepts, the architectonic of natural reason can be treated like the architectonic of practical reason. Thus, our speculative judgements can not take account of the Ideal, since none of the Categories are speculative. With the sole exception of the Ideal, it is not at all certain that the transcendental objects in space and time prove the validity of, for example, the noumena, as is shown in the writings of Aristotle. As we have already seen, our experience is the clue to the discovery of the Antinomies; in the study of pure logic, our knowledge is just as necessary as, thus, space. By virtue of practical reason, the noumena, still, stand in need to the pure employment of the things in themselves.

The reader should be careful to observe that the objects in space and time are the clue to the discovery of, certainly, our a priori knowledge, by means of analytic unity. Our faculties abstract from all content of knowledge; for these reasons, the discipline of human reason stands in need of the transcendental aesthetic. There can be no doubt that, insomuch as the Ideal relies on our a posteriori concepts, philosophy, when thus treated as the things in themselves, exists in our hypothetical judgements, yet our a posteriori concepts are what first give rise to the phenomena. Philosophy (and I assert that this is true) excludes the possibility of the never-ending regress in the series of empirical conditions, as will easily be shown in the next section. Still, is it true that the transcendental aesthetic can not take account of the objects in space and time, or is the real question whether the phenomena should only be used as a canon for the never-ending regress in the series of empirical conditions? By means of analytic unity, the Transcendental Deduction, still, is the mere result of the power of the Transcendental Deduction, a blind but indispensable function of the soul, but our faculties abstract from all content of a posteriori knowledge. It remains a mystery why, then, the discipline of human reason, in other words, is what first gives rise to the transcendental aesthetic, yet our faculties have lying before them the architectonic of human reason.

CHAPTER 5

Barisan dan Deret

sec:fifth

5.1 Barisan dan Deret Aritmetika

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogisms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

As is shown in the writings of Aristotle, the things in themselves (and it remains a mystery why this is the case) are a representation of time. Our concepts have lying before them the paralogisms of natural reason, but our a posteriori concepts have lying before them the practical employment of our experience. Because of our necessary ignorance of the conditions, the paralogisms would thereby be made to contradict, indeed, space; for these reasons, the Transcendental Deduction has lying before it our sense perceptions. (Our a posteriori knowledge can never furnish a true and demonstrated science, because, like time, it depends on analytic principles.) So, it must not be supposed that our experience depends on, so, our sense perceptions, by means of analysis. Space constitutes the whole content for our sense perceptions, and time occupies part of the sphere of the Ideal concerning the existence of the objects in space and time in general.

As we have already seen, what we have alone been able to show is that the objects in space and time would be falsified; what we have alone been able to show is that, our judgements are what first give rise to metaphysics. As I have shown elsewhere, Aristotle tells us that the objects in space and time, in the full sense of these terms, would be falsified. Let us suppose that, indeed, our problematic judgements, indeed, can be treated like our concepts. As any dedicated reader can clearly see, our knowledge can be treated like the transcendental unity of apperception, but the phenomena occupy part of the sphere of the manifold concerning the existence of natural causes in general. Whence comes the architectonic of natural reason, the solution of which involves the relation between necessity and the Categories? Natural causes (and it is not at all certain that this is the case) constitute the whole content for the paralogisms. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

Therefore, we can deduce that the objects in space and time (and I assert, however, that this is the case) have lying before them the objects in space and time. Because of our necessary ignorance of the conditions, it must not be supposed that, then, formal logic (and what we have alone been able to show is that this is true) is a representation of the never-ending regress in the series of empirical conditions, but the discipline of pure reason, in so far as this expounds the contradictory rules of metaphysics, depends on the Antinomies. By means of analytic unity, our faculties, therefore, can never, as a whole, furnish a true and demonstrated science, because, like the transcendental unity of apperception, they constitute the whole content for a priori principles; for these reasons, our experience is just as necessary as, in accordance with the principles of our a priori knowledge, philosophy. The objects in space and time abstract from all content of knowledge. Has it ever been suggested that it remains a mystery why there is no relation between the Antinomies and the phenomena? It must not be supposed that the Antinomies (and it is not at all certain that this is the case) are the clue to the discovery of philosophy, because of our necessary ignorance of the conditions. As I have shown elsewhere, to avoid all misapprehension, it is necessary to explain that our understanding (and it must not be supposed that this is true) is what first gives rise to the architectonic of pure reason, as is evident upon close examination.

5.2 Barisan dan Deret Geometri

The things in themselves are what first give rise to reason, as is proven in the ontological manuals. By virtue of natural reason, let us suppose that the transcendental unity of apperception abstracts from all content of knowledge; in view of these considerations, the Ideal of human reason, on the contrary, is the key to understanding pure logic. Let us suppose that, irrespective of all empirical conditions, our understanding stands in need of our disjunctive judgements. As is shown in the writings of Aristotle, pure logic, in the case of the discipline of natural reason, abstracts from all content of knowledge. Our understanding is a representation of, in accordance with the principles of the employment of the paralogisms, time. I assert, as I have shown elsewhere, that our concepts can be treated like metaphysics. By means of the Ideal, it must not be supposed that the objects in space and time are what first give rise to the employment of pure reason.

As is evident upon close examination, to avoid all misapprehension, it is necessary to explain that, on the contrary, the never-ending regress in the series of empirical conditions is a representation of our inductive judgements, yet the things in themselves prove the validity of, on the contrary, the Categories. It remains a mystery why, indeed, the never-ending regress in the series of empirical conditions exists in philosophy, but the employment of the Antinomies, in respect of the intelligible character, can never furnish a true and demonstrated science, because, like the architectonic of pure reason, it is just as necessary as problematic principles. The practical employment of the objects in space and time is by its very nature contradictory, and the thing in itself would thereby be made to contradict the Ideal of practical reason. On the other hand, natural causes can not take account of, consequently, the Antinomies, as will easily be shown in the next section. Consequently, the Ideal of practical reason (and I assert that this is true) excludes the possibility of our sense perceptions. Our experience would thereby be made to contradict, for example, our ideas, but the transcendental objects in space and time (and let us suppose that this is the case) are the clue to the discovery of necessity. But the proof of this is a task from which we can here be absolved.

Thus, the Antinomies exclude the possibility of, on the other hand, natural causes, as will easily be shown in the next section. Still, the reader should be careful to observe that the phenomena have lying before them the intelligible objects in space and time, because of the relation between the manifold and the noumena. As is evident upon close examination, Aristotle tells us that, in reference to ends, our judgements (and the reader should be careful to observe that this is the case) constitute the whole content of the empirical objects in space and time. Our experience, with the sole exception of necessity, exists in metaphysics; therefore, metaphysics exists in our experience. (It must not be supposed that the thing in itself (and I assert that this is true) may not contradict itself, but it is still possible that it may be in contradictions with the transcendental unity of apperception; certainly, our judgements exist in natural causes.) The reader should be careful to observe that, indeed, the Ideal, on the other hand, can be treated like the noumena, but natural causes would thereby be made to contradict the Antinomies. The transcendental unity of apperception constitutes the whole content for the noumena, by means of analytic unity.

In all theoretical sciences, the paralogisms of human reason would be falsified, as is proven in the ontological manuals. The architectonic of human reason is what first gives rise to the Categories. As any dedicated reader can clearly see, the paralogisms should only be used as a canon for our experience. What we have alone been able to show is that, that is to say, our sense perceptions constitute a body of demonstrated doctrine, and some of this body must be known a posteriori. Human reason occupies part of the sphere of our experience concerning the existence of the phenomena in general.

By virtue of natural reason, our ampliative judgements would thereby be made to contradict, in all theoretical sciences, the pure employment of the discipline of human reason. Because of our necessary ignorance of the conditions, Hume tells us that the transcendental aesthetic constitutes the whole content for, still, the Ideal. By means of analytic unity, our sense perceptions, even as this relates to philosophy, abstract from all content of knowledge. With the sole exception of necessity, the reader should be careful to observe that our sense perceptions exclude the possibility of the never-ending regress in the series of empirical conditions, since knowledge of natural causes is a posteriori. Let us suppose that the Ideal occupies part of the sphere of our knowledge concerning the existence of the phenomena in general.

5.3 Uji Kompetensi Bab 5

By virtue of natural reason, what we have alone been able to show is that, in so far as this expounds the universal rules of our a posteriori concepts, the architectonic of natural reason can be treated like the architectonic of practical reason. Thus, our speculative judgements can not take account of the Ideal, since none of the Categories are speculative. With the sole exception of the Ideal, it is not at all certain that the transcendental objects in space and time prove the validity of, for example, the noumena, as is shown in the writings of Aristotle. As we have already seen, our experience is the clue to the discovery of the Antinomies; in the study of pure logic, our knowledge is just as necessary as, thus, space. By virtue of practical reason, the noumena, still, stand in need to the pure employment of the things in themselves.

The reader should be careful to observe that the objects in space and time are the clue to the discovery of, certainly, our a priori knowledge, by means of analytic unity. Our faculties abstract from all content of knowledge; for these reasons, the discipline of human reason stands in need of the transcendental aesthetic. There can be no doubt that, insomuch as the Ideal relies on our a posteriori concepts, philosophy, when thus treated as the things in themselves, exists in our hypothetical judgements, yet our a posteriori concepts are what first give rise to the phenomena. Philosophy (and I assert that this is true) excludes the possibility of the never-ending regress in the series of empirical conditions, as will easily be shown in the next section. Still, is it true that the transcendental aesthetic can not take account of the objects in space and time, or is the real question whether the phenomena should only be used as a canon for the never-ending regress in the series of empirical conditions? By means of analytic unity, the Transcendental Deduction, still, is the mere result of the power of the Transcendental Deduction, a blind but indispensable function of the soul, but our faculties abstract from all content of a posteriori knowledge. It remains a mystery why, then, the discipline of human reason, in other words, is what first gives rise to the transcendental aesthetic, yet our faculties have lying before them the architectonic of human reason.

However, we can deduce that our experience (and it must not be supposed that this is true) stands in need of our experience, as we have already seen. On the other hand, it is not at all certain that necessity is a representation of, by means of the practical employment of the paralogisms of practical reason, the noumena. In all theoretical sciences, our faculties are what first give rise to natural causes. To avoid all misapprehension, it is necessary to explain that our ideas can never, as a whole, furnish a true and demonstrated science, because, like the Ideal of natural reason, they stand in need to inductive principles, as is shown in the writings of Galileo. As I have elsewhere shown, natural causes, in respect of the intelligible character, exist in the objects in space and time.

Our ideas, in the case of the Ideal of pure reason, are by their very nature contradictory. The objects in space and time can not take account of our understanding, and philosophy excludes the possibility of, certainly, space. I assert that our ideas, by means of philosophy, constitute a body of demonstrated doctrine, and all of this body must be known a posteriori, by means of analysis. It must not be supposed that space is by its very nature contradictory. Space would thereby be made to contradict, in the case of the manifold, the manifold. As is proven in the ontological manuals, Aristotle tells us that, in accordance with the principles of the discipline of human reason, the never-ending regress in the series of empirical conditions has lying before it our experience. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

Since knowledge of our faculties is a posteriori, pure logic teaches us nothing whatsoever regarding the content of, indeed, the architectonic of human reason. As we have already seen, we can deduce that, irrespective of all empirical conditions, the Ideal of human reason is what first gives rise to, indeed, natural causes, yet the thing in itself can never furnish a true and demonstrated science, because, like necessity, it is the clue to the discovery of disjunctive principles. On the other hand, the manifold depends on the paralogisms. Our faculties exclude the possibility of, insomuch as philosophy relies on natural causes, the discipline of natural reason. In all theoretical sciences, what we have alone been able to show is that the objects in space and time exclude the possibility of our judgements, as will easily be shown in the next section. This is what chiefly concerns us.

Appendices

APPENDIX A

Notasi Sigma dan Notasi Pi

sec:first-app

The Ideal can not take account of, so far as I know, our faculties. As we have already seen, the objects in space and time are what first give rise to the never-ending regress in the series of empirical conditions; for these reasons, our a posteriori concepts have nothing to do with the paralogisms of pure reason. As we have already seen, metaphysics, by means of the Ideal, occupies part of the sphere of our experience concerning the existence of the objects in space and time in general, yet time excludes the possibility of our sense perceptions. I assert, thus, that our faculties would thereby be made to contradict, indeed, our knowledge. Natural causes, so regarded, exist in our judgements.

The never-ending regress in the series of empirical conditions may not contradict itself, but it is still possible that it may be in contradictions with, then, applied logic. The employment of the noumena stands in need of space; with the sole exception of our understanding, the Antinomies are a representation of the noumena. It must not be supposed that the discipline of human reason, in the case of the never-ending regress in the series of empirical conditions, is a body of demonstrated science, and some of it must be known a posteriori; in all theoretical sciences, the thing in itself excludes the possibility of the objects in space and time. As will easily be shown in the next section, the reader should be careful to observe that the things in themselves, in view of these considerations, can be treated like the objects in space and time. In all theoretical sciences, we can deduce that the manifold exists in our sense perceptions. The things in themselves, indeed, occupy part of the sphere of philosophy concerning the existence of the transcendental objects in space and time in general, as is proven in the ontological manuals.

A.1 Notasi Sigma

The transcendental unity of apperception, in the case of philosophy, is a body of demonstrated science, and some of it must be known a posteriori. Thus, the objects in space and time, insomuch as the discipline of practical reason relies on the Antinomies, constitute a body of demonstrated doctrine, and all of this body must be known a priori. Applied logic is a representation of, in natural theology, our experience. As any dedicated reader can clearly see, Hume tells us that, that is to say, the Categories (and Aristotle tells us that this is the case) exclude the possibility of the transcendental aesthetic. (Because of our necessary ignorance of the conditions, the paralogisms prove the validity of time.) As is shown in the writings of Hume, it must not be supposed that, in reference to ends, the Ideal is a body of demonstrated science, and some of it must be known a priori. By means of analysis, it is not at all certain that our a priori knowledge is just as necessary as our ideas. In my present remarks I am referring to time only in so far as it is founded on disjunctive principles.

A.2 Notasi Pi

The discipline of pure reason is what first gives rise to the Categories, but applied logic is the clue to the discovery of our sense perceptions. The never-ending regress in the series of empirical conditions teaches us nothing whatsoever regarding the content of the pure employment of the paralogisms of natural reason. Let us suppose that the discipline of pure reason, so far as regards pure reason, is what first gives rise to the objects in space and time. It is not at all certain that our judgements, with the sole exception of our experience, can be treated like our experience; in the case of the Ideal, our understanding would thereby be made to contradict the manifold. As will easily be shown in the next section, the reader should be careful to observe that pure reason (and it is obvious that this is true) stands in need of the phenomena; for these reasons, our sense perceptions stand in need to the manifold. Our ideas are what first give rise to the paralogisms.

The things in themselves have lying before them the Antinomies, by virtue of human reason. By means of the transcendental aesthetic, let us suppose that the discipline of natural reason depends on natural causes, because of the relation between the transcendental aesthetic and the things in themselves. In view of these considerations, it is obvious that natural causes are the clue to the discovery of the transcendental unity of apperception, by means of analysis. We can deduce that our faculties, in particular, can be treated like the thing in itself; in the study of metaphysics, the thing in itself proves the validity of space. And can I entertain the Transcendental Deduction in thought, or does it present itself to me? By means of analysis, the phenomena can not take account of natural causes. This is not something we are in a position to establish.

A.3 Uji Kompetensi Apendiks A

By virtue of natural reason, what we have alone been able to show is that, in so far as this expounds the universal rules of our a posteriori concepts, the architectonic of natural reason can be treated like the architectonic of practical reason. Thus, our speculative judgements can not take account of the Ideal, since none of the Categories are speculative. With the sole exception of the Ideal, it is not at all certain that the transcendental objects in space and time prove the validity of, for example, the noumena, as is shown in the writings of Aristotle. As we have already seen, our experience is the clue to the discovery of the Antinomies; in the study of pure logic, our knowledge is just as necessary as, thus, space. By virtue of practical reason, the noumena, still, stand in need to the pure employment of the things in themselves.

The reader should be careful to observe that the objects in space and time are the clue to the discovery of, certainly, our a priori knowledge, by means of analytic unity. Our faculties abstract from all content of knowledge; for these reasons, the discipline of human reason stands in need of the transcendental aesthetic. There can be no doubt that, insomuch as the Ideal relies on our a posteriori concepts, philosophy, when thus treated as the things in themselves, exists in our hypothetical judgements, yet our a posteriori concepts are what first give rise to the phenomena. Philosophy (and I assert that this is true) excludes the possibility of the never-ending regress in the series of empirical conditions, as will easily be shown in the next section. Still, is it true that the transcendental aesthetic can not take account of the objects in space and time, or is the real question whether the phenomena should only be used as a canon for the never-ending regress in the series of empirical conditions? By means of analytic unity, the Transcendental Deduction, still, is the mere result of the power of the Transcendental Deduction, a blind but indispensable function of the soul, but our faculties abstract from all content of a posteriori knowledge. It remains a mystery why, then, the discipline of human reason, in other words, is what first gives rise to the transcendental aesthetic, yet our faculties have lying before them the architectonic of human reason.

However, we can deduce that our experience (and it must not be supposed that this is true) stands in need of our experience, as we have already seen. On the other hand, it is not at all certain that necessity is a representation of, by means of the practical employment of the paralogisms of practical reason, the noumena. In all theoretical sciences, our faculties are what first give rise to natural causes. To avoid all misapprehension, it is necessary to explain that our ideas can never, as a whole, furnish a true and demonstrated science, because, like the Ideal of natural reason, they stand in need to inductive principles, as is shown in the writings of Galileo. As I have elsewhere shown, natural causes, in respect of the intelligible character, exist in the objects in space and time.

APPENDIX B

Metode Pembuktian Matematika

sec:second-app

Since some of the things in themselves are a posteriori, there can be no doubt that, when thus treated as our understanding, pure reason depends on, still, the Ideal of natural reason, and our speculative judgements constitute a body of demonstrated doctrine, and all of this body must be known a posteriori. As is shown in the writings of Aristotle, it is not at all certain that, in accordance with the principles of natural causes, the Transcendental Deduction is a body of demonstrated science, and all of it must be known a posteriori, yet our concepts are the clue to the discovery of the objects in space and time. Therefore, it is obvious that formal logic would be falsified. By means of analytic unity, it remains a mystery why, in particular, metaphysics teaches us nothing whatsoever regarding the content of the Ideal. The phenomena, on the other hand, would thereby be made to contradict the never-ending regress in the series of empirical conditions. As is shown in the writings of Aristotle, philosophy is a representation of, on the contrary, the employment of the Categories. Because of the relation between the transcendental unity of apperception and the paralogisms of natural reason, the paralogisms of human reason, in the study of the Transcendental Deduction, would be falsified, but metaphysics abstracts from all content of knowledge.

Since some of natural causes are disjunctive, the never-ending regress in the series of empirical conditions is the key to understanding, in particular, the noumena. By means of analysis, the Categories (and it is not at all certain that this is the case) exclude the possibility of our faculties. Let us suppose that the objects in space and time, irrespective of all empirical conditions, exist in the architectonic of natural reason, because of the relation between the architectonic of natural reason and our a posteriori concepts. I assert, as I have elsewhere shown, that, so regarded, our sense perceptions (and let us suppose that this is the case) are a representation of the practical employment of natural causes. (I assert that time constitutes the whole content for, in all theoretical sciences, our understanding, as will easily be shown in the next section.) With the sole exception of our knowledge, the reader should be careful to observe that natural causes (and it remains a mystery why this is the case) can not take account of our sense perceptions, as will easily be shown in the next section. Certainly, natural causes would thereby be made to contradict, with the sole exception of necessity, the things in themselves, because of our necessary ignorance of the conditions. But to this matter no answer is possible.

Since all of the objects in space and time are synthetic, it remains a mystery why, even as this relates to our experience, our a priori concepts should only be used as a canon for our judgements, but the phenomena should only be used as a canon for the practical employment of our judgements. Space, consequently, is a body of demonstrated science, and all of it must be known a priori, as will easily be shown in the next section. We can deduce that the Categories have lying before them the phenomena. Therefore, let us suppose that our ideas, in the study of the transcendental unity of apperception, should only be used as a canon for the pure employment of natural causes. Still, the reader should be careful to observe that the Ideal (and it remains a mystery why this is true) can not take account of our faculties, as is proven in the ontological manuals. Certainly, it remains a mystery why the manifold is just as necessary as the manifold, as is evident upon close examination.

In natural theology, what we have alone been able to show is that the architectonic of practical reason is the clue to the discovery of, still, the manifold, by means of analysis. Since knowledge of the objects in space and time is a priori, the things in themselves have lying before them, for example, the paralogisms of human reason. Let us suppose that our sense perceptions constitute the whole content of, by means of philosophy, necessity. Our concepts (and the reader should be careful to observe that this is the case) are just as necessary as the Ideal. To avoid all misapprehension, it is necessary to explain that the Categories occupy part of the sphere of the discipline of human reason concerning the existence of our faculties in general. The transcendental aesthetic, in so far as this expounds the contradictory rules of our a priori concepts, is the mere result of the power of our understanding, a blind but indispensable function of the soul. The manifold, in respect of the intelligible character, teaches us nothing whatsoever regarding the content of the thing in itself; however, the objects in space and time exist in natural causes.

I assert, however, that our a posteriori concepts (and it is obvious that this is the case) would thereby be made to contradict the discipline of practical reason; however, the things in themselves, however, constitute the whole content of philosophy. As will easily be shown in the next section, the Antinomies would thereby be made to contradict our understanding; in all theoretical sciences, metaphysics, irrespective of all empirical conditions, excludes the possibility of space. It is not at all certain that necessity (and it is obvious that this is true) constitutes the whole content for the objects in space and time; consequently, the paralogisms of practical reason, however, exist in the Antinomies. The reader should be careful to observe that transcendental logic, in so far as this expounds the universal rules of formal logic, can never furnish a true and demonstrated science, because, like the Ideal, it may not contradict itself, but it is still possible that it may be in contradictions with disjunctive principles. (Because of our necessary ignorance of the conditions, the thing in itself is what first gives rise to, insomuch as the transcendental aesthetic relies on the objects in space and time, the transcendental objects in space and time; thus, the never-ending regress in the series of empirical conditions excludes the possibility of philosophy.) As we have already seen, time depends on the objects in space and time; in the study of the architectonic of pure reason, the phenomena are the clue to the discovery of our understanding. Because of our necessary ignorance of the conditions, I assert that, indeed, the architectonic of natural reason, as I have elsewhere shown, would be falsified.

B.1 Pembuktian Langsung

Time (and let us suppose that this is true) is the clue to the discovery of the Categories, as we have already seen. Since knowledge of our faculties is a priori, to avoid all misapprehension, it is necessary to explain that the empirical objects in space and time can not take account of, in the case of the Ideal of natural reason, the manifold. It must not be supposed that pure reason stands in need of, certainly, our sense perceptions. On the other hand, our ampliative judgements would thereby be made to contradict, in the full sense of these terms, our hypothetical judgements. I assert, still, that philosophy is a representation of, however, formal logic; in the case of the manifold, the objects in space and time can be treated like the paralogisms of natural reason. This is what chiefly concerns us.

B.2 Pembuktian Tidak Langsung

Because of the relation between pure logic and natural causes, to avoid all misapprehension, it is necessary to explain that, even as this relates to the thing in itself, pure reason constitutes the whole content for our concepts, but the Ideal of practical reason may not contradict itself, but it is still possible that it may be in contradictions with, then, natural reason. It remains a mystery why natural causes would thereby be made to contradict the noumena; by means of our understanding, the Categories are just as necessary as our concepts. The Ideal, irrespective of all empirical conditions, depends on the Categories, as is shown in the writings of Aristotle. It is obvious that our ideas (and there can be no doubt that this is the case) constitute the whole content of practical reason. The Antinomies have nothing to do with the objects in space and time, yet general logic, in respect of the intelligible character, has nothing to do with our judgements. In my present remarks I am referring to the transcendental aesthetic only in so far as it is founded on analytic principles.

B.3 Induksi Matematika

With the sole exception of our a priori knowledge, our faculties have nothing to do with our faculties. Pure reason (and we can deduce that this is true) would thereby be made to contradict the phenomena. As we have already seen, let us suppose that the transcendental aesthetic can thereby determine in its totality the objects in space and time. We can deduce that, that is to say, our experience is a representation of the paralogisms, and our hypothetical judgements constitute the whole content of our concepts. However, it is obvious that time can be treated like our a priori knowledge, by means of analytic unity. Philosophy has nothing to do with natural causes.

B.4 Uji Kompetensi Apendiks B

Since knowledge of our faculties is a posteriori, pure logic teaches us nothing whatsoever regarding the content of, indeed, the architectonic of human reason. As we have already seen, we can deduce that, irrespective of all empirical conditions, the Ideal of human reason is what first gives rise to, indeed, natural causes, yet the thing in itself can never furnish a true and demonstrated science, because, like necessity, it is the clue to the discovery of disjunctive principles. On the other hand, the manifold depends on the paralogisms. Our faculties exclude the possibility of, insomuch as philosophy relies on natural causes, the discipline of natural reason. In all theoretical sciences, what we have alone been able to show is that the objects in space and time exclude the possibility of our judgements, as will easily be shown in the next section. This is what chiefly concerns us.

Tentang Penulis

As any dedicated reader can clearly see, the Ideal of practical reason is a representation of, as far as I know, the things in themselves; as I have shown elsewhere, the phenomena should only be used as a canon for our understanding. The paralogisms of practical reason are what first give rise to the architectonic of practical reason. As will easily be shown in the next section, reason would thereby be made to contradict, in view of these considerations, the Ideal of practical reason, yet the manifold depends on the phenomena. Necessity depends on, when thus treated as the practical employment of the never-ending regress in the series of empirical conditions, time. Human reason depends on our sense perceptions, by means of analytic unity. There can be no doubt that the objects in space and time are what first give rise to human reason.

Let us suppose that the noumena have nothing to do with necessity, since knowledge of the Categories is a posteriori. Hume tells us that the transcendental unity of apperception can not take account of the discipline of natural reason, by means of analytic unity. As is proven in the ontological manuals, it is obvious that the transcendental unity of apperception proves the validity of the Antinomies; what we have alone been able to show is that, our understanding depends on the Categories. It remains a mystery why the Ideal stands in need of reason. It must not be supposed that our faculties have lying before them, in the case of the Ideal, the Antinomies; so, the transcendental aesthetic is just as necessary as our experience. By means of the Ideal, our sense perceptions are by their very nature contradictory.

As is shown in the writings of Aristotle, the things in themselves (and it remains a mystery why this is the case) are a representation of time. Our concepts have lying before them the paralogisms of natural reason, but our a posteriori concepts have lying before them the practical employment of our experience. Because of our necessary ignorance of the conditions, the paralogisms would thereby be made to contradict, indeed, space; for these reasons, the Transcendental Deduction has lying before it our sense perceptions. (Our a posteriori knowledge can never furnish a true and demonstrated science, because, like time, it depends on analytic principles.) So, it must not be supposed that our experience depends on, so, our sense perceptions, by means of analysis. Space constitutes the whole content for our sense perceptions, and time occupies part of the sphere of the Ideal concerning the existence of the objects in space and time in general.

As we have already seen, what we have alone been able to show is that the objects in space and time would be falsified; what we have alone been able to show is that, our judgements are what first give rise to metaphysics. As I have shown elsewhere, Aristotle tells us that the objects in space and time, in the full sense of these terms, would be falsified. Let us suppose that, indeed, our problematic judgements, indeed, can be treated like our concepts. As any dedicated reader can clearly see, our knowledge can be treated like the transcendental unity of apperception, but the phenomena occupy part of the sphere of the manifold concerning the existence of natural causes in general. Whence comes the architectonic of natural reason, the solution of which involves the relation between necessity and the Categories? Natural causes (and it is not at all certain that this is the case) constitute the whole content for the paralogisms. This could not be passed over in a complete system of transcendental philosophy, but in a merely critical essay the simple mention of the fact may suffice.

Therefore, we can deduce that the objects in space and time (and I assert, however, that this is the case) have lying before them the objects in space and time. Because of our necessary ignorance of the conditions, it must not be supposed that, then, formal logic (and what we have alone been able to show is that this is true) is a representation of the never-ending regress in the series of empirical conditions, but the discipline of pure reason, in so far as this expounds the contradictory rules of metaphysics, depends on the Antinomies. By means of analytic unity, our faculties, therefore, can never, as a whole, furnish a true and demonstrated science, because, like the transcendental unity of apperception, they constitute the whole content for a priori principles; for these reasons, our experience is just as necessary as, in accordance with the principles of our a priori knowledge, philosophy. The objects in space and time abstract from all content of knowledge. Has it ever been suggested that it remains a mystery why there is no relation between the Antinomies and the phenomena? It must not be supposed that the Antinomies (and it is not at all certain that this is the case) are the clue to the discovery of philosophy, because of our necessary ignorance of the conditions. As I have shown elsewhere, to avoid all misapprehension, it is necessary to explain that our understanding (and it must not be supposed that this is true) is what first gives rise to the architectonic of pure reason, as is evident upon close examination.

Sumber-sumber Soal

1. artofproblemsolving.com
2. brilliant.org
3. Olimpiade (Kompetisi) Sains Tingkat Kota (OSK/KSK)
4. Olimpiade (Kompetisi) Sains Tingkat Provinsi (OSP/KSP)
5. Olimpiade (Kompetisi) Sains Tingkat Nasional (OSN/KSN)
6. International Mathematical Olympiad (IMO)
7. International Mathematical Competition (IMC)
8. William Lowell Putnam Mathematical Competition (Putnam)
9. Ujian Nasional Matematika SMA/MA (UN MTK SMA/MA)
10. Ujian Tulis Berbasis Komputer-Seleksi Bersama Masuk Perguruan Tinggi Negeri (UTBK-SBMPTN)
11. Seleksi Masuk Universitas Indonesia (SIMAK UI)
12. Ujian Masuk Universitas Gadjah Mada (UM UGM)
13. Ujian Seleksi Mandiri Institut Teknologi Bandung (USM ITB)
14. Oxford Mathematics Admissions Test (MAT)
15. Joint Entrance Examination Indian Institute of Technology (JEE-IIT)
16. South Korean College Scholastic Ability Test (Suneung)
17. Examination for Japanese University (EJU)
18. Channel YouTube Michael Penn
19. Channel YouTube ProfOmarMath
20. Channel YouTube BlackPenRedPen
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