$L(x_1, x_2, \ldots, x_n) \quad A \sim N(\mu, \sigma)$

Se sabe que

$$L(\mu) = \prod_{i=1}^{n} f(x_i|\mu) = \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2\sigma^2} \cdot (x_i - \mu)^2} \right]$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} \right)^n \cdot e^{\frac{1}{2\sigma^2} \cdot \sum_{i=1}^{n} (x_i - \mu)^2}$$

$$= (2\pi\sigma^2)^{-n/2} \cdot e^{\frac{1}{2\sigma^2} \cdot \sum_{i=1}^{n} (x_i - \mu)^2}$$

Se sabe que

$$l(\mu) = \ln L(\mu) = \ln (2\pi\sigma^2)^{-n/2} + \ln \left(e^{\frac{1}{2\sigma^2}} \cdot (x_i - \mu)^2 \right)$$

$$= -\frac{n}{2} \ln (2\pi\sigma^2) + \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \cdot \cancel{\ln(e)}$$

Para hallar el maximo se deriva en función del parametro a hallar

$$\frac{\partial l(\mu)}{\partial \mu} = -\frac{n}{2} \cancel{\ln (2\pi\sigma^2)} - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 = 0$$

$$\frac{\partial l(\mu)}{\partial \mu} = \frac{1}{\cancel{2}\sigma^2} \cdot \cancel{2} \sum_{i=1}^{n} x_i - \mu \cdot 1 = 0$$

$$\frac{\sum_{i=1}^{n} (x_i - \mu)}{\sigma^2} = 0$$

$$\sum_{i=1}^{n} (x_i - \mu) = 0$$

$$\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \mu \Rightarrow \sum_{n=1}^{n} x_i - n\mu = 0$$

$$\mu = \frac{1}{n} \sum_{n=1}^{n} x_i = \bar{x}$$

Ahora para σ se tiene:

$$\frac{\partial l(\sigma)}{\partial \sigma} = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(xi-\mu) = 0$$

$$= -\frac{n}{2}\frac{1}{\cancel{2\pi\sigma^2}} \cdot 4\pi\sigma - \frac{n}{2\sigma^3}\sum_{i=1}^{n}(xi-\mu)^2 = 0$$

$$= -\frac{n}{\cancel{2}}\frac{\cancel{2}}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(xi-\mu) = 0$$

$$= \frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(xi-\mu)^2 = 0$$

Si multiplicamos por sigma en ambos lados se tiene

$$-n + \frac{1}{\sigma^2}\sum_{i=1}^{n}(xi-\mu)^2 = 0$$

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}(xi-\mu)^2 = n$$

$$n\sigma^2 = \sum_{i=1}^{n}(xi-\mu)^2$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(xi-\mu)^2$$