

$$x_{n+1} = 4x_n - x_n^2$$

$$x_0 = \sin^2 \theta$$

3. Suppose $x_n = 4 \sin^2(2^{n-1} \theta)$

$$x_0 = \sin^2 \theta$$

$$x_1 = 4x_0 - x_0^2 = 4(\sin^2 \theta) - (\sin^2 \theta)^2 = 4\sin^2 \theta - \sin^4 \theta = \sin^2 \theta (4 - \sin^2 \theta) = 4\sin^2 \theta \cos^2 \theta$$

$$= 16 \left(\frac{\sin(2\theta)}{2} \right)^2 = \frac{16}{4} \sin^2(2\theta) = 4 \sin^2(2\theta)$$

$$x_2 = 4x_1 - x_1^2 = 4(4\sin^2(2\theta)) - (4\sin^2(2\theta))^2 = 16\sin^2(2\theta) - 16\sin^4(2\theta) =$$

$$16\sin^2(2\theta) (1 - \sin^2(2\theta)) = 16\sin^2(2\theta) \cos^2(2\theta) = 16 \left(\frac{\sin(4\theta)}{2} \right)^2 =$$

$$= \frac{16}{4} \sin^2(4\theta) = 4 \sin^2(4\theta)$$

$$x_{n+1} = 4(4\sin^2(2^n \theta)) - (4\sin^2(2^n \theta))^2 = 16\sin^2(2^n \theta) - 16\sin^4(2^n \theta) =$$

$$16\sin^2(2^n \theta) (1 - \sin^2(2^n \theta)) = 16\sin^2(2^n \theta) \cos^2(2^n \theta)$$

$$= 16 \left(\frac{\sin(2^{n+1} \theta)}{2} \right)^2 = \frac{16}{4} \sin^2(2^{n+1} \theta) = 4 \sin^2(2^{n+1} \theta)$$

$$x_{n+1} = 4x_n - x_n^2 \quad x_0 = \sin^2 \theta$$

Induce a $x_{n+1} = \sin^2(2^{n+1} \theta)$

$$= \sin^2 \theta$$

$$= 4\sin^2 \theta - 4\sin^4 \theta = 4\sin^2 \theta (1 - \sin^2 \theta) = 4\sin^2 \theta \cos^2 \theta$$

$$= 16 \left(\frac{\sin(2\theta)}{2} \right)^2 = \frac{16}{4} \sin^2(2\theta) = 4 \sin^2(2\theta)$$

$$4\sin^2(2\theta) - 4\sin^4(2\theta) = 4\sin^2(2\theta) (1 - \sin^2(2\theta)) = 4\sin^2(2\theta) \cos^2(2\theta)$$

$$= 16 \left(\frac{\sin(4\theta)}{2} \right)^2 = \frac{16}{4} \sin^2(4\theta) = 4 \sin^2(4\theta)$$

En general

$$x_{n+1} = 4 \sin^2(2^{n+1} \theta) - 4 \sin^2(2^n \theta) = 4 \sin^2(2^n \theta) (1 - \sin^2(2^n \theta)) \\ = 4 \sin^2(2^n \theta) \cos^2(2^n \theta) = 4 \left(\frac{\sin(2^n \cdot 2 \theta)}{2} \right)^2 = \frac{4}{4} \sin^2(2^{n+1} \theta)$$

$$= \sin^2(2^{n+1} \theta) \quad \square$$