

Trapezo simple:

$$I = \int_a^b f(x) dx \rightarrow f(x) \approx f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\int_a^b \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b) dx = \int_a^b \left[\frac{x-b}{a-b} f(a) + \frac{x-a}{-(a-b)} f(b) \right] \frac{1}{a-b} = \text{cte}$$

$$\rightarrow \int_a^b (x-b) f(a) dx - \int_a^b (x-a) f(b) dx = f(a) \left[\frac{x^2}{2} - bx \right]_a^b - f(b) \left[\frac{x^2}{2} - ax \right]_a^b$$

$$= f(a) \left(\frac{b^2}{2} - b^2 + ab - \frac{a^2}{2} \right) + f(b) \left(\frac{b^2}{2} + ab + \frac{a^2}{2} - a^2 \right) \left\{ \text{com } \frac{1}{a-b} \text{ queda...} \right.$$

$$\int_a^b f_1(x) = \frac{1}{a-b} \left[(f(a) + f(b)) \left(-\frac{b^2}{2} - \frac{a^2}{2} + ab \right) \right] = \frac{-1}{2(a-b)} (b^2 - a^2 - 2ab) (f(a) + f(b))$$

$$= \frac{-(a-b)^2}{2(a-b)} [f(a) + f(b)] = \frac{b-a}{2} [f(a) + f(b)]$$

$$\int_a^b f(x) dx \approx \int_a^b f_1(x) dx = \frac{b-a}{2} [f(a) + f(b)]$$