Parte teórica

- Ejercicios: Derivación

8. a) $\Omega = \{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))\}$

$a_0 = f(x_0)$

$a_1 = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{f(x_1) - f(x_0)}{h}$

$a_2 = \dfrac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - a_1}{x_2 - x_0} = \dfrac{\dfrac{f(x_2) - f(x_1)}{h} - \dfrac{f(x_1) - f(x_0)}{h}}{2h}$

$a_2 = \dfrac{f(x_2) - f(x_1) - (f(x_1) - f(x_0))}{2h^2}$

$p(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$

$p(x) = f(x_0) + \dfrac{f(x_1) - f(x_0)}{h}(x - x_0) + \dfrac{f(x_2) - f(x_1) - [f(x_1) - f(x_0)]}{2h^2}[(x - x_0)(x - x_1)]$

$p'(x) = \dfrac{f(x_1) - f(x_0)}{h} + \dfrac{f(x_2) - f(x_1) - f(x_1) + f(x_0)}{2h^2}(x - x_1 + x - x_0)$

$p'(x_0) = \dfrac{f(x_1) - f(x_0)}{h} + \dfrac{f(x_2) - f(x_1) - f(x_1) + f(x_0)}{2h^{\cancel{2}}}(-\cancel{h})$

$= \dfrac{f(x_1) - f(x_0)}{h} + \dfrac{2f(x_1) - f(x_0) - f(x_2)}{2h}$

$= \dfrac{\cancel{h}}{2h^{\cancel{2}}}\left(2f(x_1) - 2f(x_0) + 2f(x_1) - f(x_0) - f(x_2)\right)$

$p'(x_0) = \dfrac{1}{2h}\left(-3f(x_0) + 4f(x_1) - f(x_2)\right)$

e) $f(x) = \sqrt{\tan(x)}$

$$f'(x) = \frac{1}{2}(\tan(x))^{-1/2} \sec^2(x)$$

$$f'(x) = \frac{\sec^2(x)}{2\sqrt{\tan(x)}}$$

5) $$D^4 f(x_j) \simeq \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4}$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h^2) \quad (2)$$

reemplazo (1) en (2)
siendo $f''(x_j) = f(x)$

$$f''(x_j) \approx \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2} \quad (1)$$

$$f''''(x) = \frac{\frac{f(x_{j+2}) - 2f(x_{j+1}) + f(x_j)}{h^2} - 2\left[\frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2}\right] + \frac{f(x_j) - 2f(x_{j-1}) + f(x_{j-2})}{h^2}}{h^2}$$

$$= \frac{1}{h^4}\left[f(x_{j+2}) - 2f(x_{j+1}) + f(x_j) - 2f(x_{j+1}) + 4f(x_j) - 2f(x_{j-1}) + f(x_j) - 2f(x_{j-1}) + f(x_{j-2})\right]$$

$$D^4 f(x_j) = \frac{1}{h^4}\left[f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})\right]$$

Orden: $h^4 \longrightarrow O(h^4)$