



Recommender Systems

Individual Software Project

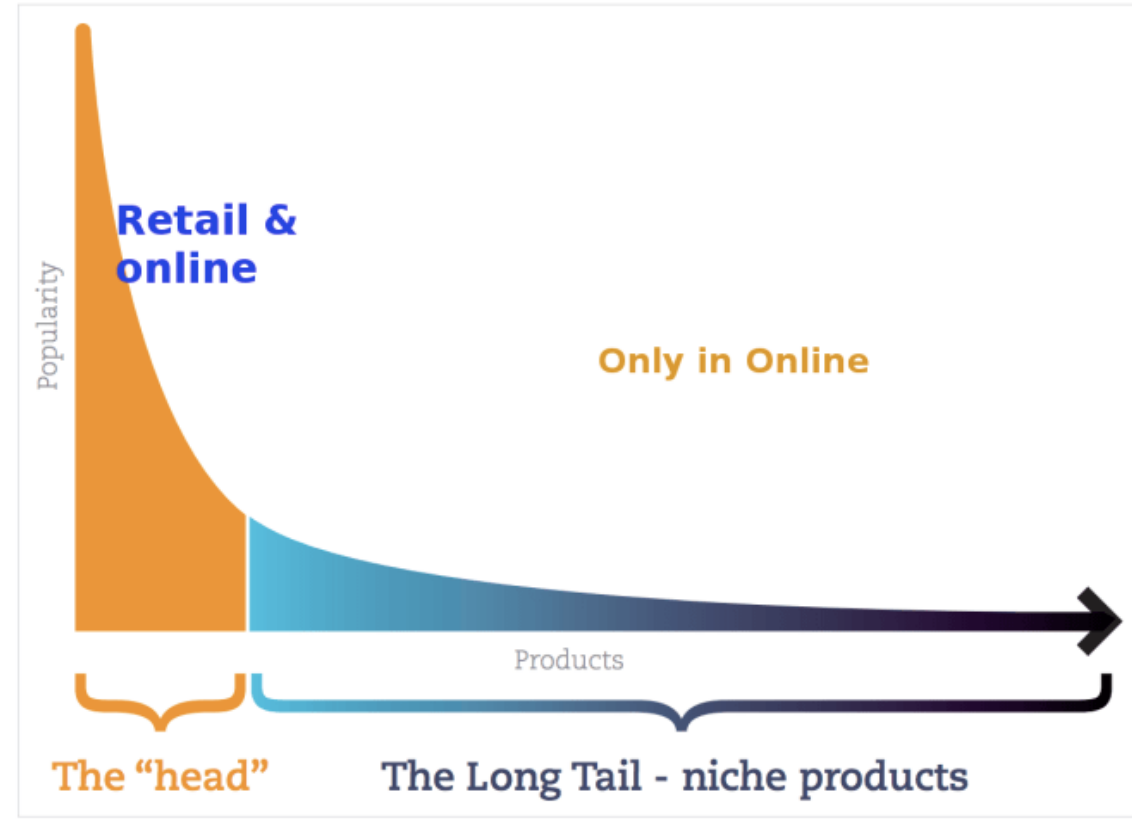
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Angie Granda

Introduction

❖ A **Recommendation System** is a subset of Artificial Intelligence that aims to learn patterns from data, which contains user-item interactions, and predicts how likely unknown items will be liked by the user

❖ ***Long Tail*** Phenomenon



Recommender System Model

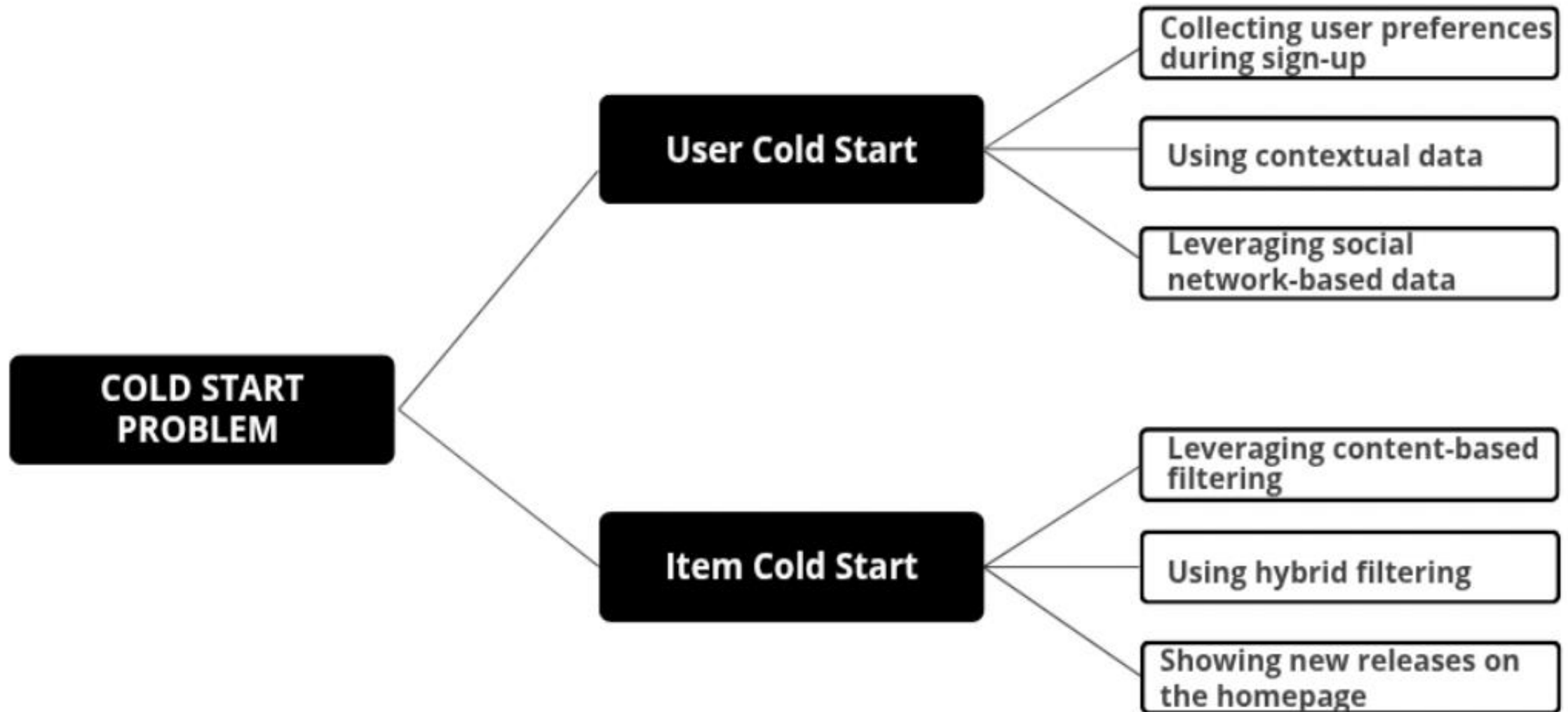
X = set of users, S = set of items, R = total ordering

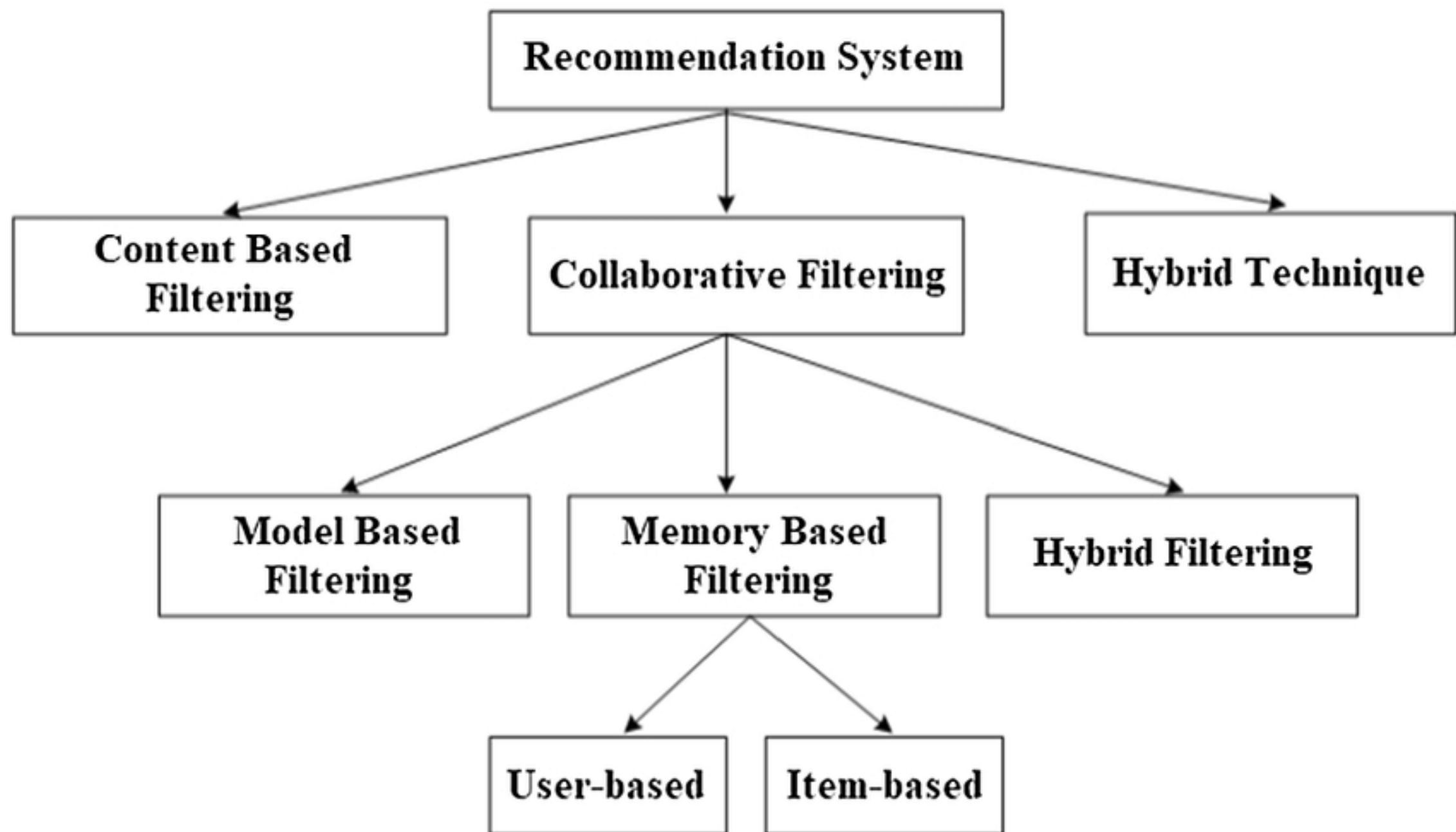
$F: X * S \rightarrow R$ is the utility function, F fills the **utility matrix**.

The **user-item value in the utility matrix** represent what is known about the degree of preference. Those values can be explicit or implicit.

	Hotel1	Hotel2	Hotel3	Hotel4	Hotel5
User 1	4	?	3	5	?
User 2	?	4	?	4	4
User 3	2	?	3	5	?
User4	3	?	?	?	3

The recommender system cannot accurately provide relevant suggestions to users when there is little or no historical information about them.





Prediction heuristic - Similarity metrics

➤ **Cosine Similarity:** Focus on direction, not in magnitudes $\cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|}$

➤ **Pearson Correlation:**

It measures linear correlation between two vectors. It is scale-invariant.

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

➤ **Jaccard Distance:**

Interest in shared presence of items only.

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

➤ **Euclidean Distance:**

Measures actual differences in feature values. Sensitive to rating bias.

Content-Based R.S.

- Relevant characteristics of an item are represented by an **item profile**
- **User profile** summarizes the preferences of the user, weighted average of rated item profiles.

$$\text{UserProfile} = \frac{\sum_{i=1}^N (\text{rating}_i \cdot \text{feature_vector}_i)}{\sum_{i=1}^N \text{rating}_i}$$

- **Prediction** is based on the similarity between user profile *i* and item profile *j*.
- **Pros:** No need for data on other users, no item cold-start problem, able to recommend to users with unique tastes.
- **Cons:** Find appropriate features, user-cold start problem, overspecialization.

Collaborative Filtering. User-Based Model

Task: Consider user x . Find set N of other users whose ratings are like x 's ratings. Estimate x 's ratings based on ratings of users in N .

Mean-Center Overlapping-Items Cosine Similarity

$$sim(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)(r_{ys} - \bar{r}_y)}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \bar{r}_y)^2}}$$

Prediction: Let R_x be the vector of user x 's ratings. Let N be the set of users most similar to x who have rated item i .

$$r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi} \quad r_{xi} = \frac{\sum_{y \in N} S_{xy} \cdot r_{yi}}{\sum_{y \in N} S_{xy}} \quad r_{ij} = \bar{r}_i + \frac{\sum_k Similarities(u_i, u_k)(r_{kj} - \bar{r}_k)}{\text{number of ratings}}$$

Collaborative Filtering. Item-Based Model

Task: For user \mathbf{x} and item \mathbf{y} , find other N similar items to \mathbf{y} , estimate rating for item \mathbf{y} based on ratings of the similar items given by \mathbf{x}

It can use same **similarity** metrics and **prediction** functions as in user-based model

Cons: If a user has rated few items, then most items can't be estimated

$N(i;\mathbf{x})$ set of items similar to item i ,
rated by \mathbf{x} .



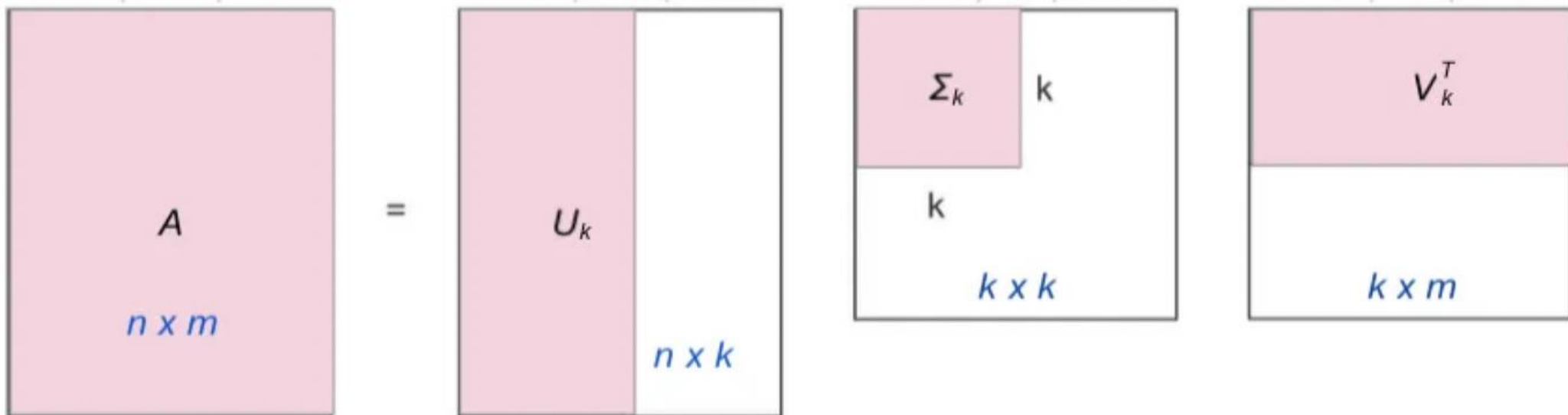
$$r_{xi} = \frac{\sum_{j \in N(i;\mathbf{x})} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;\mathbf{x})} s_{ij}}$$

Collaborative Filtering - Matrix Factorization

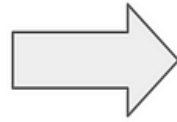
Singular Value Decomposition (SVD) is a matrix factorization technique

$$A = U\Sigma V^T \quad A \approx UV^T \text{ (when } \Sigma \text{ is absorbed into } U \text{ or } V\text{)}$$

Let $\mathbf{p}_u \in U$ and $\mathbf{q}_i \in V$ and rating of user u to item i be r_{ui} , then the prediction $\hat{r}_{ui} = \mathbf{q}_i^T \mathbf{p}_u$



	Items				
Users			5		
		5			
			1		3
	1				
			2		2
	2			4	
		2			5
Utility Matrix (m x n)					



Users	1.5	0.75
	3	1.25
	4	1.2
	3.6	4.1
	3.6	1.2
	1.1	0.8
	0.9	1.4
	3.6	5.1
m x k		

x

	Items				
	2.1	3.3	1.6	2.8	3
	1.3	4	1	2	0.7
k x n					

Matrix Factorization - Funk MF

The loss function is **MSE**

$$\min_{q^*, p^*} \sum_{(u,i) \in \kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2)$$

usually with **L2 regularization** to prevent overfitting.

$$\min_{p, q, b_u, b_i} \sum (r_{ui} - (p_u^T q_i + \mu + b_u + b_i))^2 + \lambda(\|p_u\|^2 + \|q_i\|^2 + b_u^2 + b_i^2)$$

After taking partial derivatives with respect to vector **p** and **q** we obtain the following updating rules:

$$q_i \leftarrow q_i + \gamma \cdot (e_{ui} \cdot p_u - \lambda \cdot q_i)$$

$$p_u \leftarrow p_u + \gamma \cdot (e_{ui} \cdot q_i - \lambda \cdot p_u)$$

Mini-Batch Stochastic Gradient Descent

Matrix Factorization – ALS

Alternating Least Squares (**ALS**) uses the Sum Square Error Function

Algorithm: Fix V, solve for U using least squares. Then fix U, solve for V. Alternate until convergence.

Suppose that you fix V, then we want to solve: $\mathcal{L}_i(p_i) = \sum_{j \in \mathcal{I}_i} (r_{ij} - p_i^T q_j)^2 + \lambda \|p_i\|^2$

$Q_i \in \mathbb{R}^{|\mathcal{I}_i| \times k}$: matrix whose rows are the q_j^T vectors for all $j \in \mathcal{I}_i$

$r_i \in \mathbb{R}^{|\mathcal{I}_i|}$: vector of the known ratings r_{ij}

$$\mathcal{L}_i(p_i) = \|r_i - Q_i p_i\|^2 + \lambda \|p_i\|^2$$

$$\nabla_{p_i} \mathcal{L}_i = -2Q_i^T (r_i - Q_i p_i) + 2\lambda p_i$$

$$-2Q_i^T (r_i - Q_i p_i) + 2\lambda p_i = 0$$

$$Q_i^T Q_i p_i + \lambda p_i = Q_i^T r_i$$

$$p_i = \left(\sum_{j \in \mathcal{I}_i} q_j q_j^T + \lambda I \right)^{-1} \sum_{j \in \mathcal{I}_i} r_{ij} q_j$$

$$q_j = \left(\sum_{i \in \mathcal{U}_j} p_i p_i^T + \lambda I \right)^{-1} \sum_{i \in \mathcal{U}_j} r_{ij} p_i$$

Hybrid Recommender Systems

- **Parallel use of several systems**

Multiple recommender systems operate independently and simultaneously on the same input. Their outputs are then combined.

- **Monolithic exploiting different features**

A single recommendation system that uses a wide variety of features — e.g., user profiles, item metadata, past interactions, contextual signals

- **Pipelined invocation of different systems**

Multiple recommenders are arranged in sequence, where the output of one system feeds into the next.

Data Split and Evaluation

Techniques to split the data:

- Leave One Last
- Temporal user-based
- Temporal Global

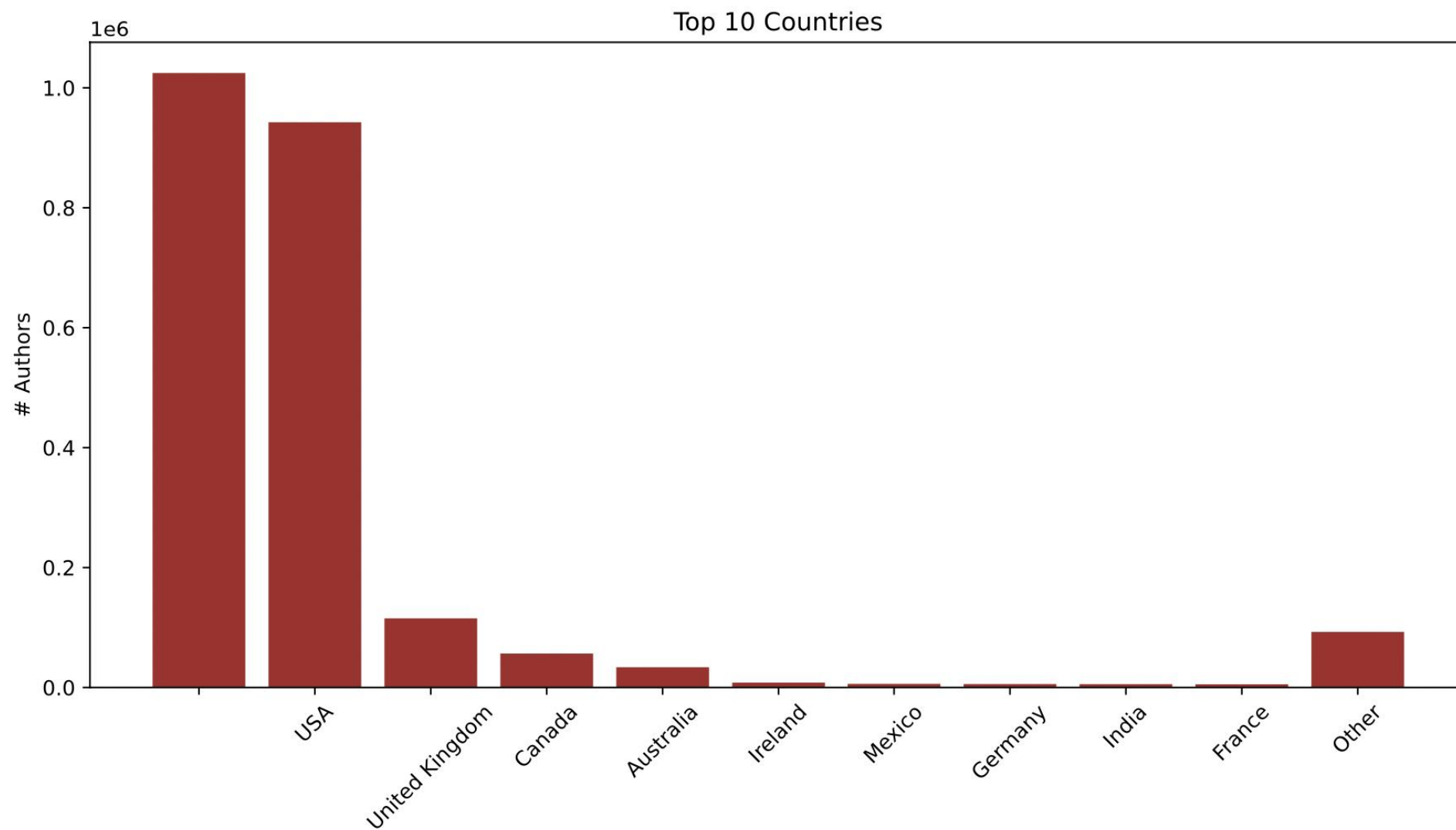
Evaluating predictions metrics:

- MAE, MSE, RMSE

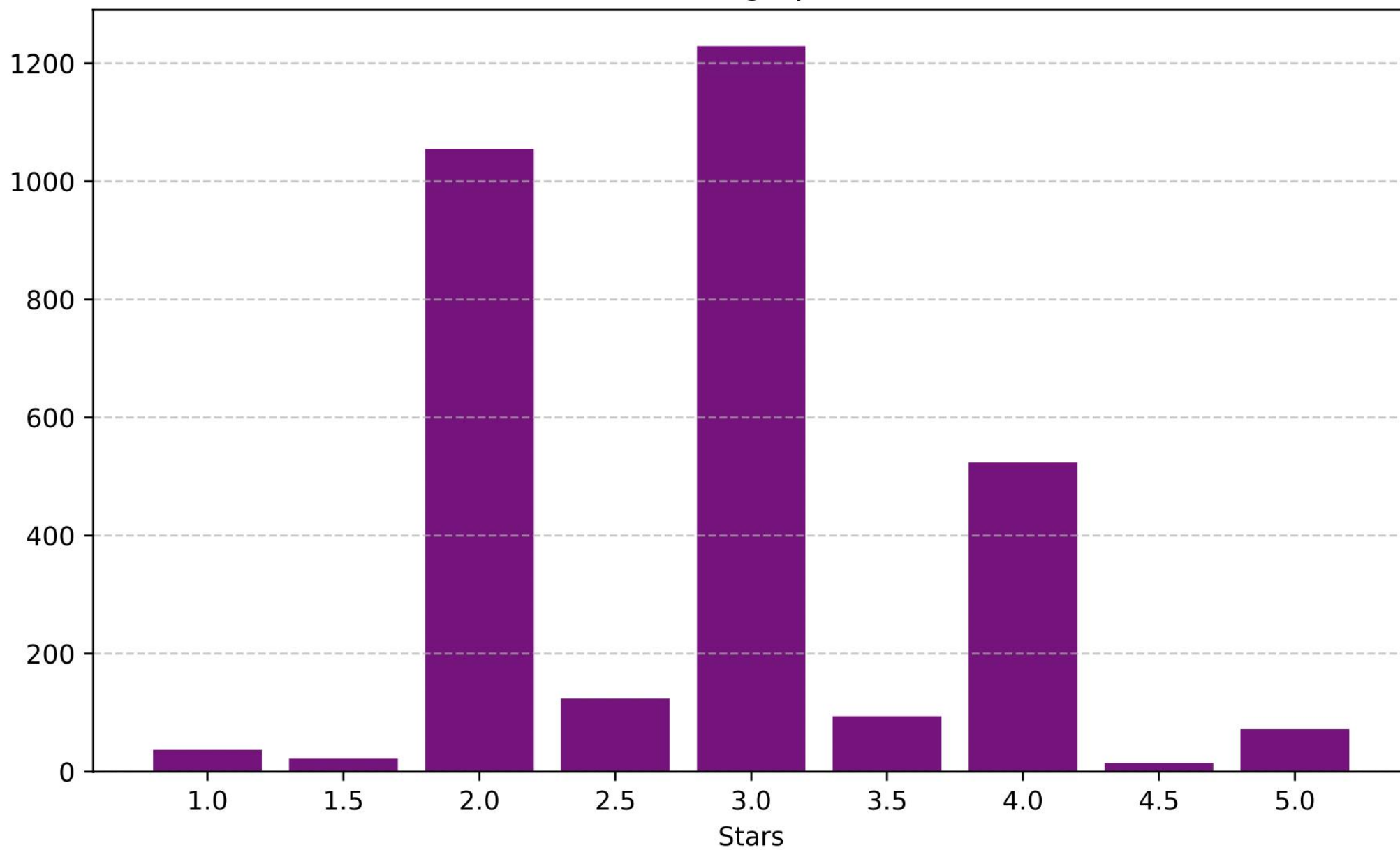
Evaluating lists of recommendation (based on relevancy levels):

- Precision, Mean Average Precision
- Recall

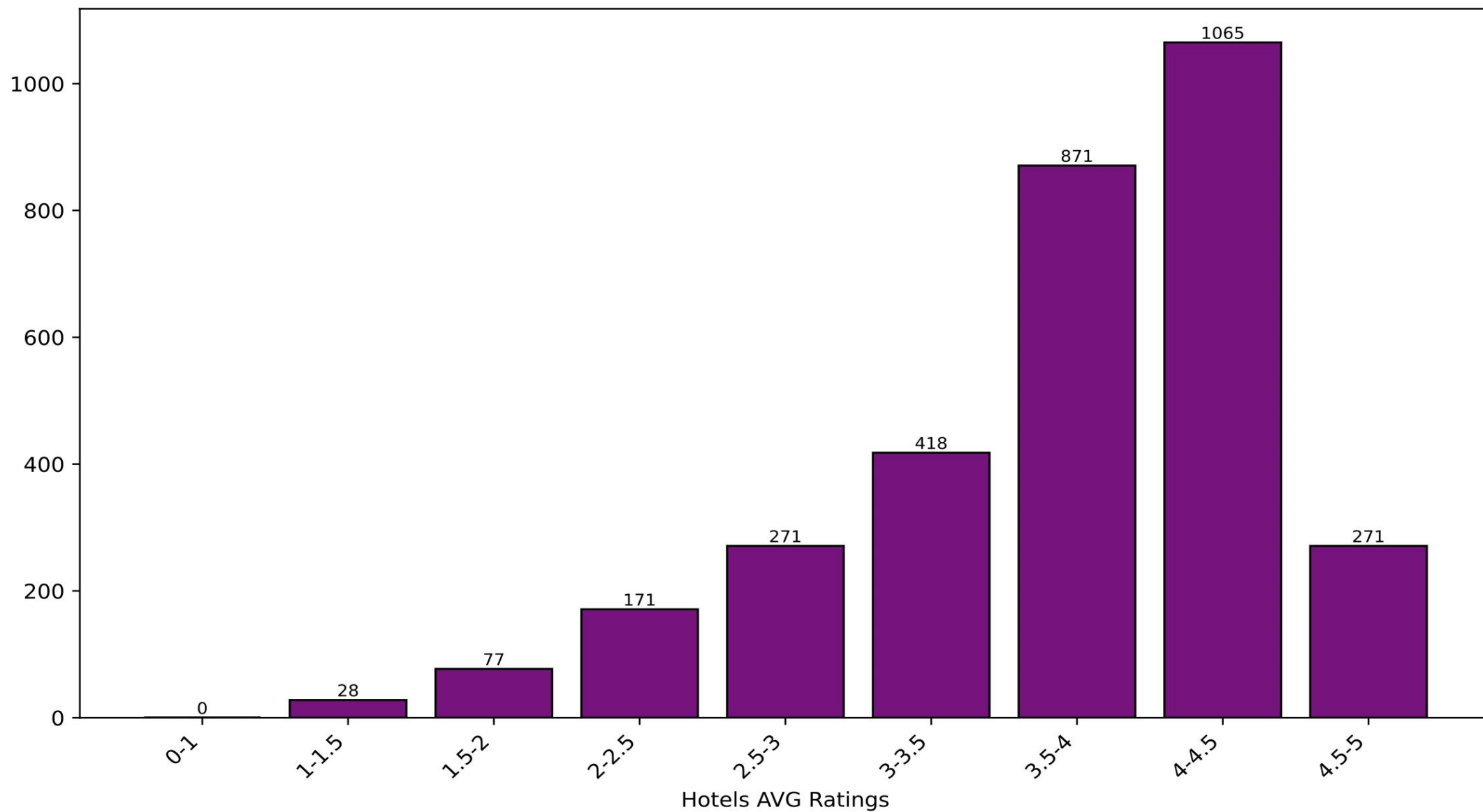
Dataset - Hotels



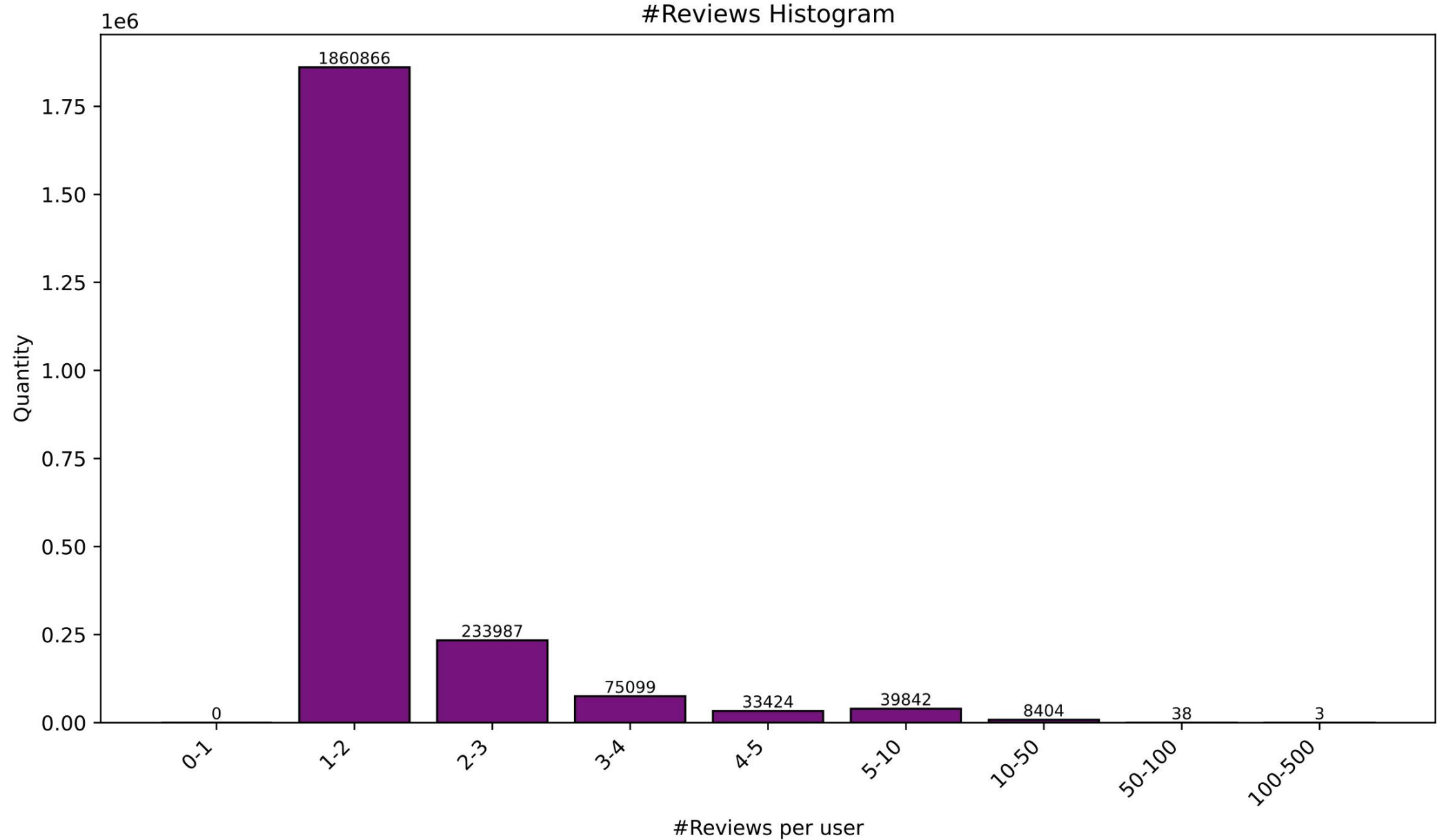
Dataset - Hotels



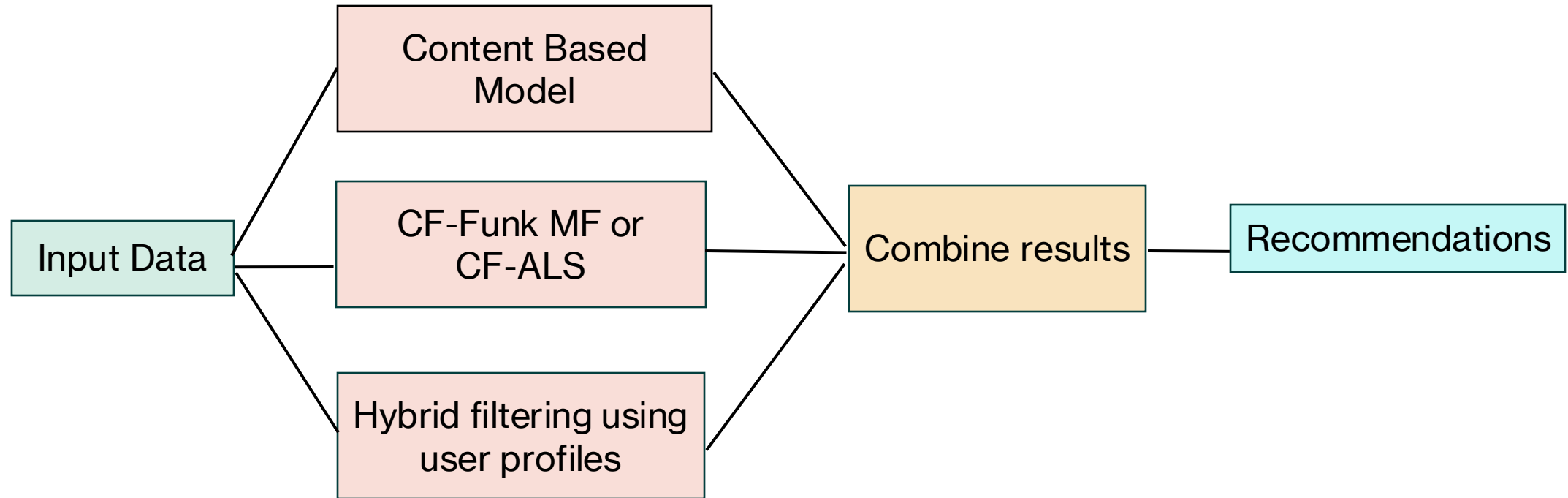
Dataset - Hotels



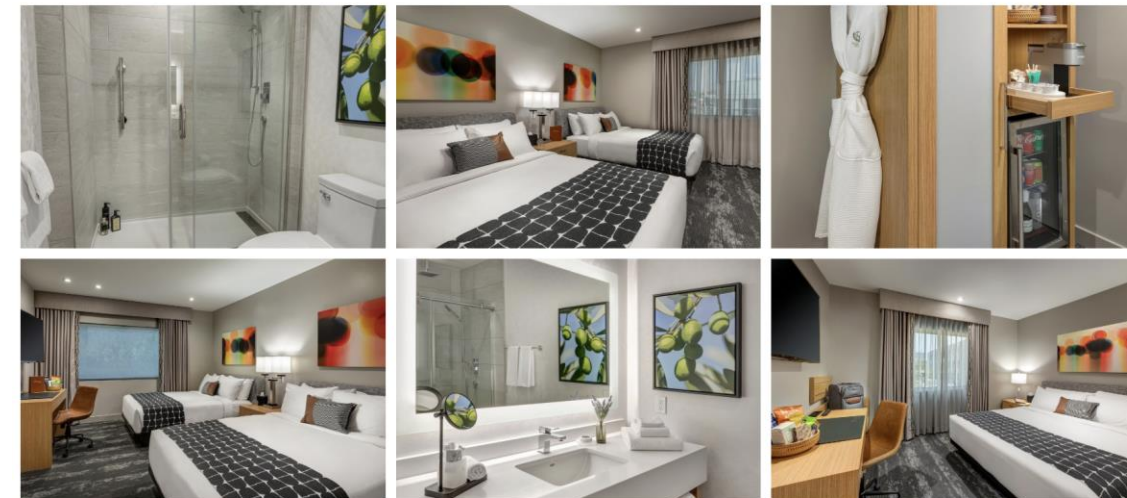
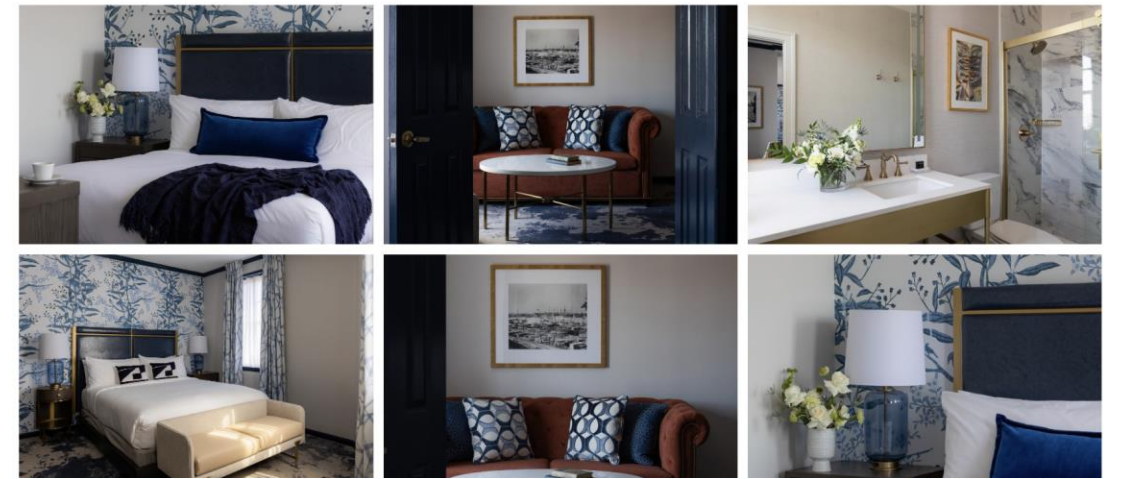
Dataset - Authors



Project Model



User rating hotels 3-4 overall and with a budget of 150-500\$



Remaining Work

- Finalize and evaluate each model thoroughly
- Test hybrid filtering by integrating user-based KNN using user profiles.
- Assess regional influence on user preferences:
Given users with a substantial number of ratings, we'll evaluate how effectively their preferences can be predicted using region-based average user profiles. Are regional-based predictions actually meaningful?

Thank you for your attention!

Questions?