Să se calculeze următoarele limite:

✓ 1) $\lim_{n\to\infty} \sqrt{n+2} - \sqrt{n}$

✓ 2) $\lim_{n\to\infty} \sqrt[3]{n^3+n^2} - \sqrt[3]{n^3-n^2}$

✓ 3) $\lim_{n\to\infty} \sqrt[3]{6n^3+1} - \sqrt{6n^2+2}$

✓ 4) $\lim_{n\to\infty} \frac{a^n n!}{n^n}$ $a>0$

✓ 5) $\lim_{n\to\infty} \left(1+\sqrt{n+1}-\sqrt{n}\right)^{2\sqrt{n}}$

✓ 6) $\lim_{n\to\infty} \sqrt[n]{a^n+b^n+c^n}$ $a, b, c > 0$

✓ 7) $\lim_{n\to\infty} \left(\frac{a^{\frac{1}{n}}+b^{\frac{1}{n}}}{2}\right)^n$ $a, b>0$

✓ 8) $\lim_{n\to\infty} \sqrt[n]{n!}$

✓ 9) $\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{n}$

✓ 10) $\lim_{n\to\infty} \sqrt[n]{C_{2n}^n}$

11) $\lim_{n\to\infty} n\left(\sqrt[n]{n}-1\right)$

12) $\lim_{n\to\infty} \dfrac{1+\frac{1}{2}+\dots+\frac{1}{n}}{\ln n}$ ✓

13) $\lim_{n\to\infty} 1+\frac{1}{\sqrt{2}}+\dots+\frac{1}{\sqrt{n}}$ ✓

14) $\lim_{n\to\infty} \dfrac{1+\sqrt{2}+\dots+\sqrt{n}}{n\sqrt{n}}$ ✓

15) $\lim_{n\to\infty} \dfrac{1^p+2^p+\dots+n^p}{n^{p+1}}$ $p>1$ ✓

16) $\lim_{n\to\infty} \dfrac{10^n+n^2+1}{6^{n+1}+3} \cdot \dfrac{21^n+1}{14^n+7}$ ✓

17) $\lim_{n\to\infty} n\sqrt{n}\left(\sqrt{n+1}+\sqrt{n-1}-2\sqrt{n}\right)$ ✓

18) $\lim_{n\to\infty} \sqrt[n+1]{(n+1)!} - \sqrt[n]{n!}$ ✓

19) $\lim_{n\to\infty} \dfrac{n}{\ln n!}$ ✓

20) $\lim_{n\to\infty} \dfrac{\ln n!}{n \ln n}$ ✓

Să se calculeze limitele următoarelor șiruri definite prin recurență:

1) $x_{n+1} = a x_n + b \quad |a|<1 \quad x_0 \in \mathbb{R}$

2) $x_{n+1} = x_n - x_n^2 \quad x_0 \in (0,1) \quad \lim_{n\to\infty} n x_n = ?$

3) $x_{n+1} = x_n - x_n^3 \quad x_0 \in (0,1)$

4) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right) \quad a>0, \; x_0>0.$

5) $x_{n+1} = \sqrt{2+x_n} \quad x_0>0$

6) Arătați cu ajutorul definiției că

$\frac{n^2}{2n^2+1} \to \frac{1}{2}$, $\sqrt{n+1}-\sqrt{n} \to 0$, $\frac{n^3}{n^3+2n^2+7n+4} \to 1$.

7) Dacă $x_n \to a \in \mathbb{R} \Rightarrow$

$\frac{x_1+x_2+\dots+x_n}{n} \to a$ și $\frac{C_n^0 x_0 + C_n^1 x_1 + \dots + C_n^n x_n}{2^n} \to a$