

# Mediana a doi vectori sortați



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Se dau doi vectori  $a$  și  $b$  **de lungime  $n$** , cu elementele **ordonate crescător**. Să se determine mediana vectorului obținut prin interclasarea celor doi vectori.

# Mediana a doi vectori sortați

Exemplu:  $n = 5$

1 12 15 16 38

2 13 17 30 45

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Exemplu:  $n = 5$

1 12 15 16 38

2 13 17 30 45



1 2 12 13 15 16 17 30 38 45

Mediana  $(15+16)/2 = 15,5$

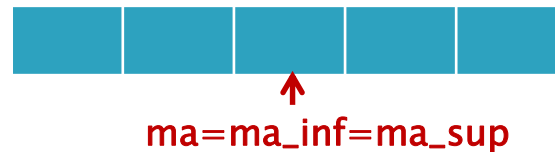
# Mediana a doi vectori sortați

- **Algoritm  $O(n)$**  – interclasăm vectorii și apoi aflăm mediana în timp constant (din elementele de la mijlocul vectorului, conform definiției)

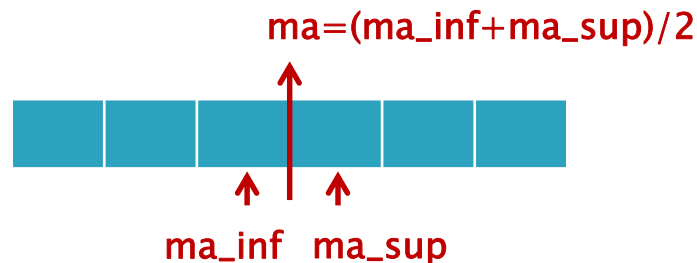
# Mediana a doi vectori sortați

- Algoritm  $O(\log(n))$

- ▶ Fie  $ma\_inf$ ,  $ma\_sup$ ,  $ma$  mediana inferioară, superioară, respectiv mediana vectorului  $a$
- ▶  $mb\_inf$ ,  $mb\_sup$ ,  $mb$  – similar pentru vectorul  $b$
- ▶  $c$  – vectorul obținut prin interclasare
- ▶  $n$  impar:



- ▶  $n$  par:





► Comparăm ma și mb



ma



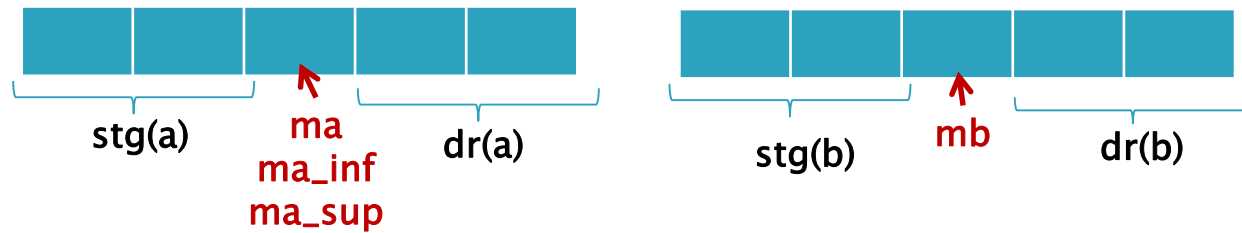
mb



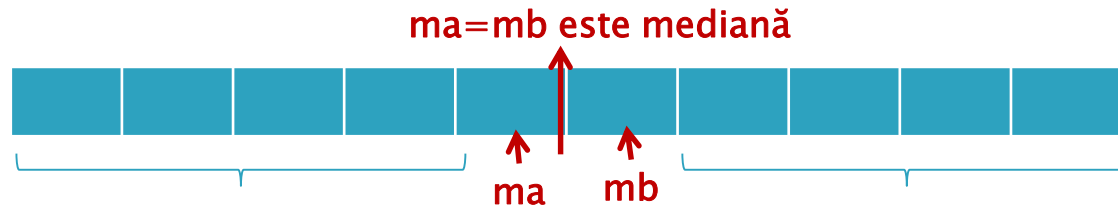
▶  $ma = mb \Rightarrow mc = ma = mb?$

►  $ma = mb \Rightarrow mc = ma = mb$

**n impar**

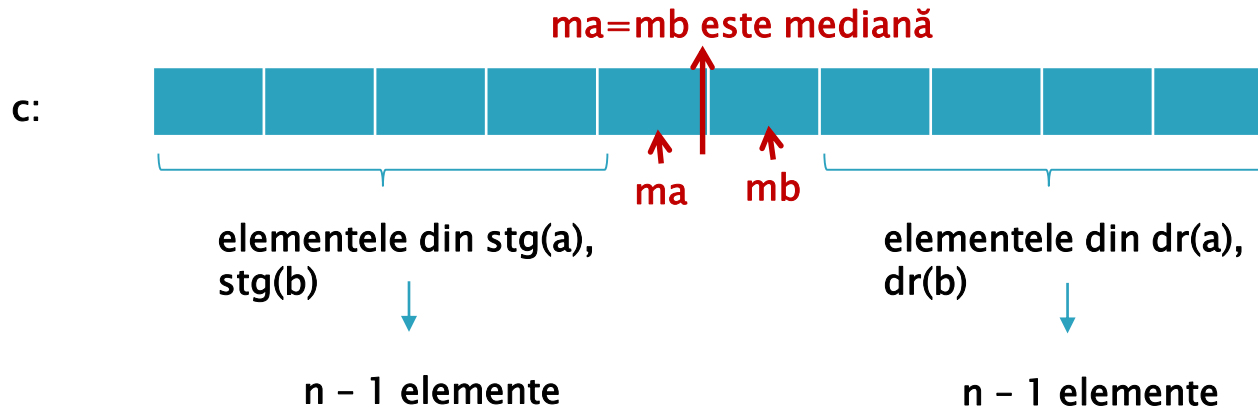
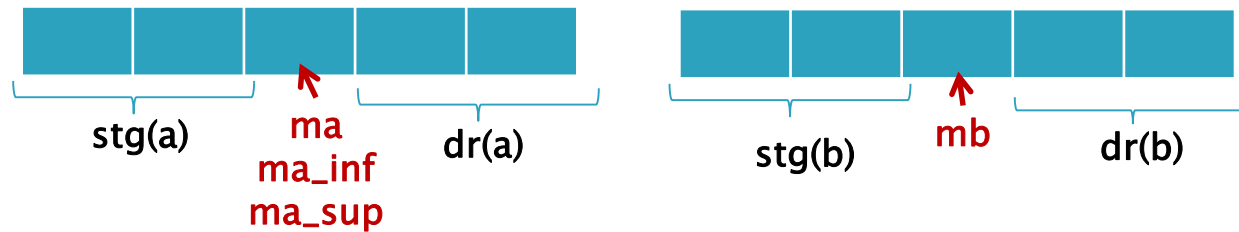


c:



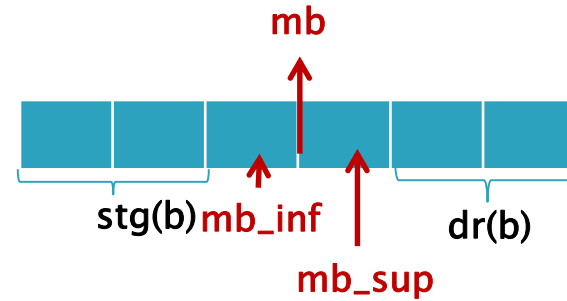
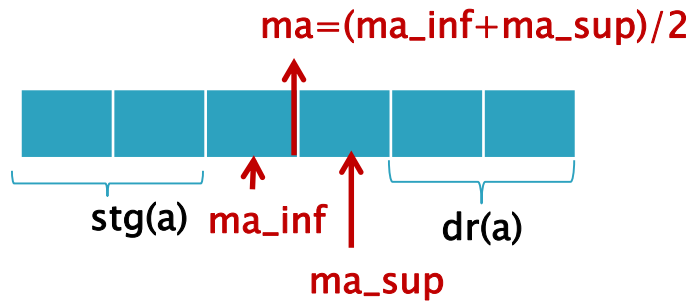
►  $ma = mb \Rightarrow mc = ma = mb$

**n impar**

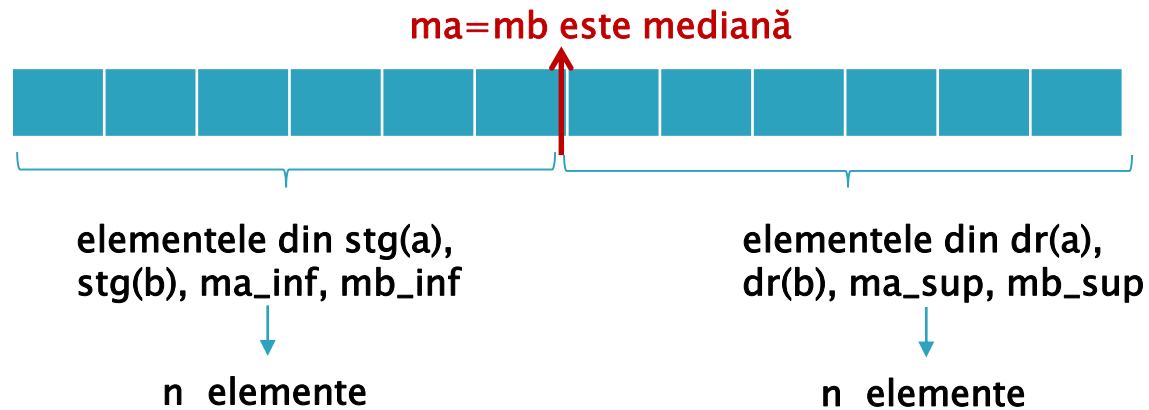


►  $ma = mb \Rightarrow mc = ma = mb$

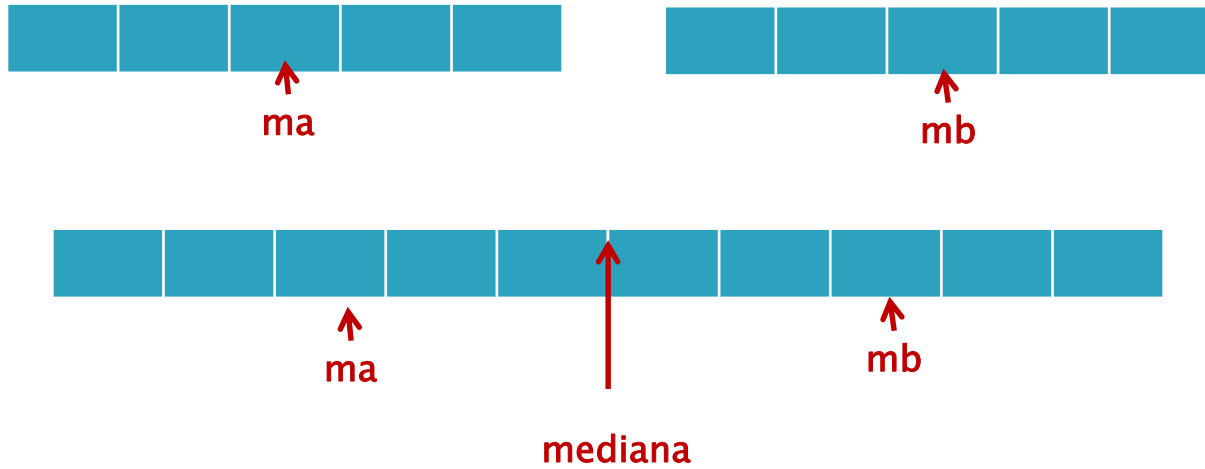
**n par**



c:

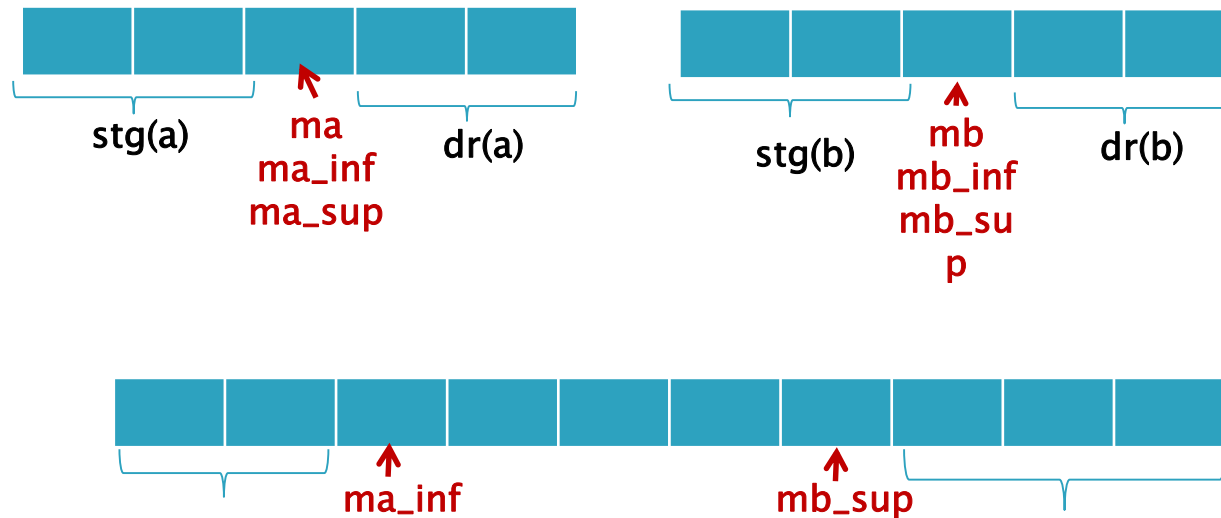


- ▶  $ma < mb \Rightarrow mc \in [ma, mb]$ ?



- ▶  $ma < mb \Rightarrow mc, mc\_inf, mc\_sup \in [ma\_inf, mb\_sup]$

**n impar**

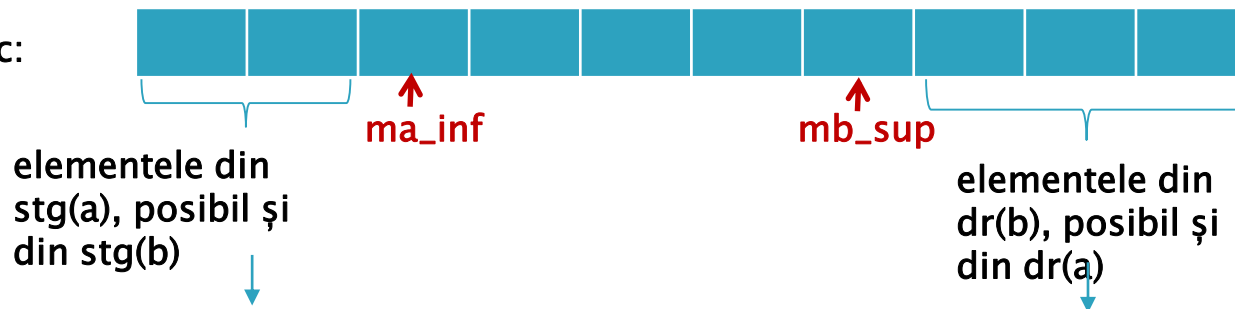


- ▶  $ma < mb \Rightarrow mc, mc\_inf, mc\_sup \in [ma\_inf, mb\_sup]$

**n impar**

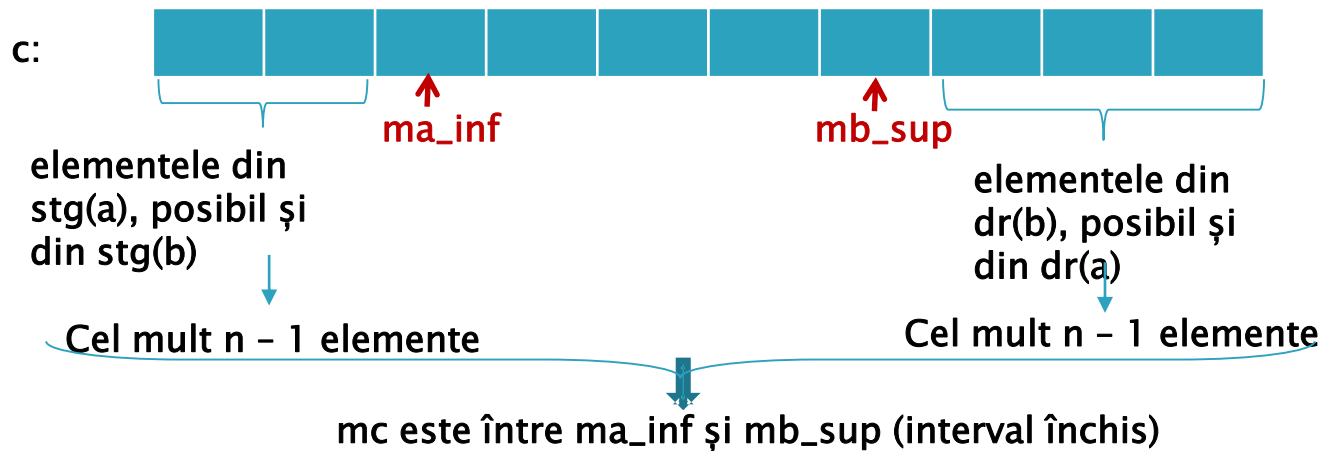
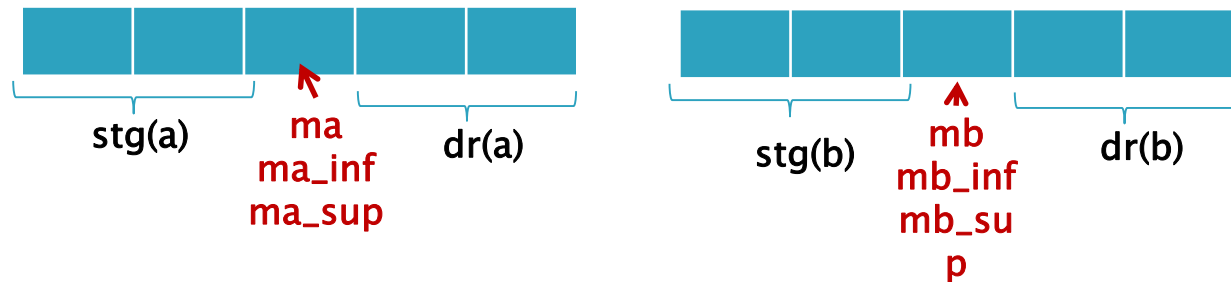


**c:**



- ▶  $ma < mb \Rightarrow mc, mc\_inf, mc\_sup \in [ma\_inf, mb\_sup]$

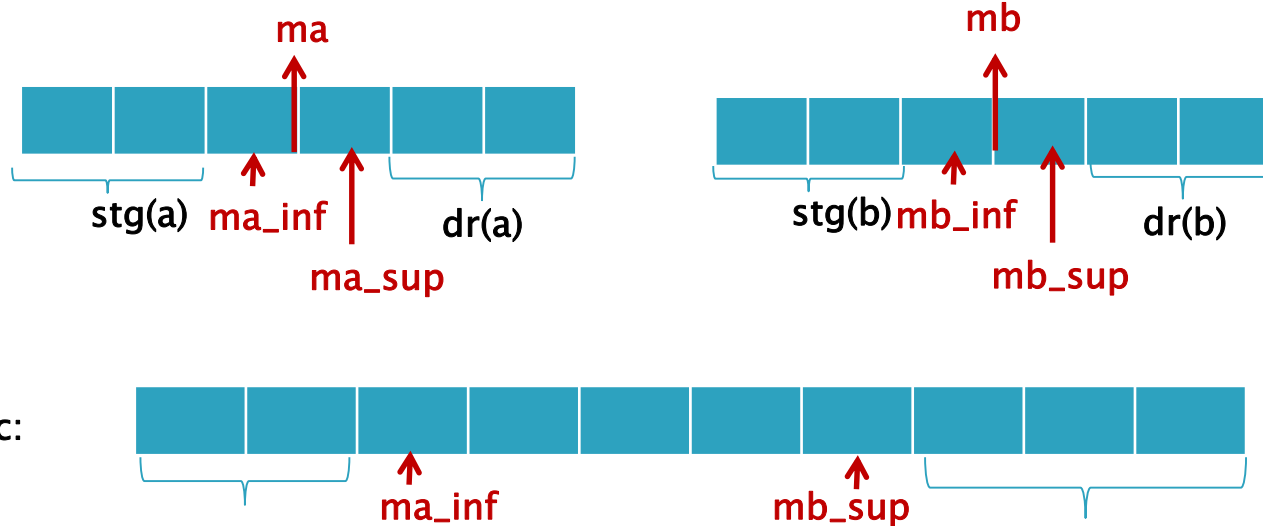
**n impar**





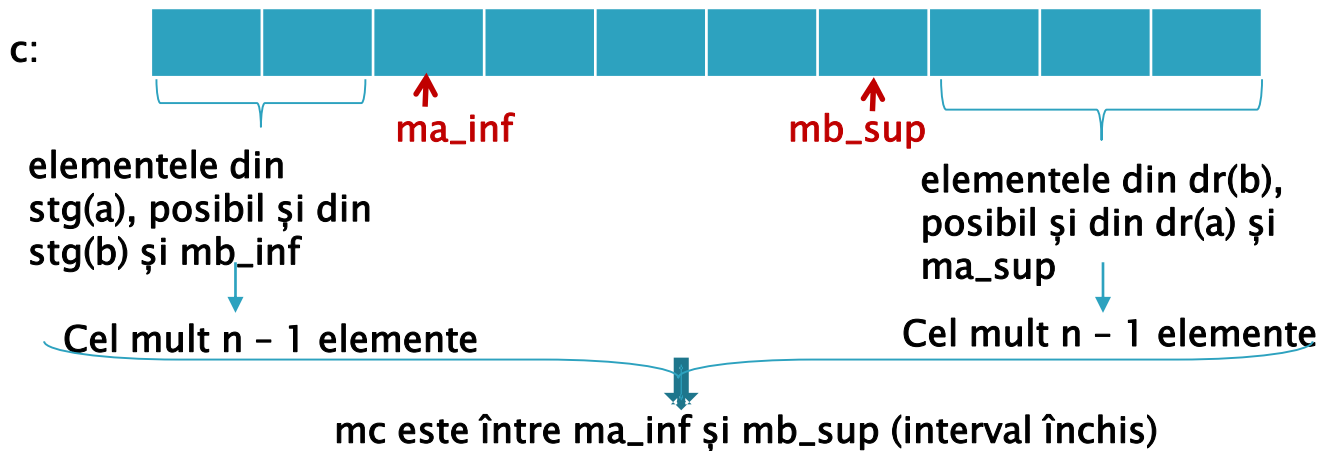
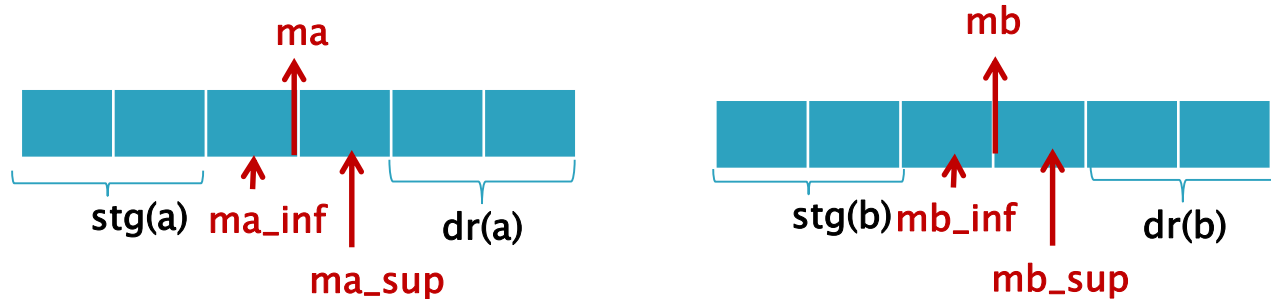
- ▶  $ma < mb \Rightarrow mc, mc\_inf, mc\_sup \in [ma\_inf, mb\_sup]$

**n par**



►  $ma < mb \Rightarrow mc, mc\_inf, mc\_sup \in [ma\_inf, mb\_sup]$

**n par**



- ▶  $ma < mb \Rightarrow mc, mc\_inf, mc\_sup \in [ma\_inf, mb\_sup]$

### **Rezultă:**

Pentru a determina mediana este suficient să considerăm:

- Subvectorul drept din primul vector ( inclusiv mediana inferioară)
- Subvectorul stâng din al doilea vector (inclusiv mediana superioară)

- ▶  $ma < mb \Rightarrow mc, mc\_inf, mc\_sup \in [ma\_inf, mb\_sup]$

### Rezultă:

Pentru a determina mediana este suficient să considerăm:

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Astfel

- din vectorul a renunțăm la  $[(n-1)/2]$  elemente care sunt înaintea lui  $ma\_inf$  (deci și a lui  $mc\_inf$ ) în c
- din vectorul b renunțăm **tot la**  $[(n-1)/2]$  elemente care sunt după lui  $mb\_sup$  (deci și după  $mc\_sup$ ) în c

deci media noilor vectori este egală cu mediana lui c

- ▶  $m_a > m_b$  – Similar

# Mediana a doi vectori sortați

Corectitudine:

**mediana noii probleme = mediana problemei inițiale**

# Pseudocod

- ▶ Fie  $m_a$  mediana vectorului  $a$  și  $m_b$  mediana vectorului  $b$ 
  - Dacă  $m_a = m_b$  atunci această valoare este mediana



- ▶ Fie  $m_a$  mediana vectorului  $a$  și  $m_b$  mediana vectorului  $b$ 
  - Dacă  $m_a = m_b$  atunci această valoare este mediana
  - Dacă  $m_a > m_b$  atunci mediana = mediana subvectorilor

$$a[0..\lfloor n/2 \rfloor], \quad b[\lfloor (n-1)/2 \rfloor..n-1]$$

- Dacă  $m_a < m_b$  atunci mediana = mediana subvectorilor

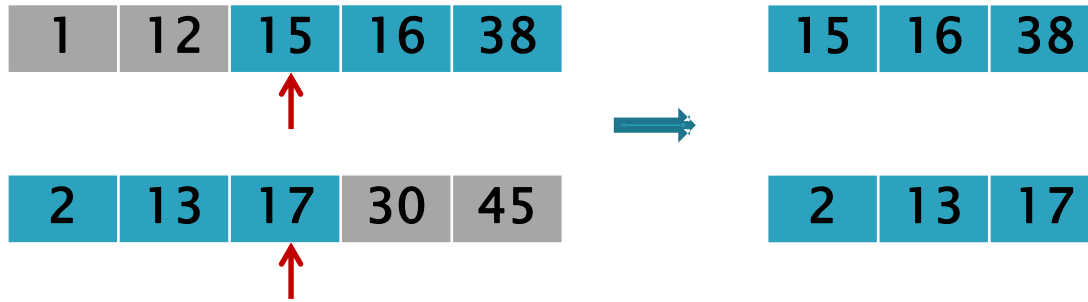
$$a[\lfloor (n-1)/2 \rfloor..n-1], \quad b[0..\lfloor n/2 \rfloor]$$

1	12	15	16	38
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2	13	17	30	45
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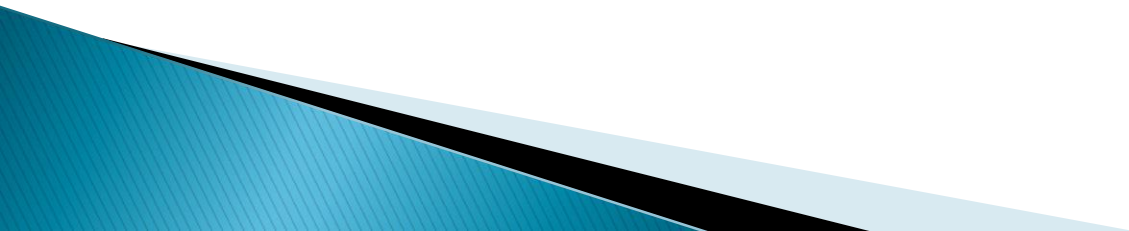
► **Știm să rezolvăm direct:**

- $n = 1: (a[1] + b[1]) / 2$

- $n = 2: \text{mediana lui } \{a[1], b[1], a[2], b[2]\}$

$$= (\max\{a[1], b[1]\} + \min\{a[2], b[2]\}) / 2$$

# Exemplu



1	12	15	16	38
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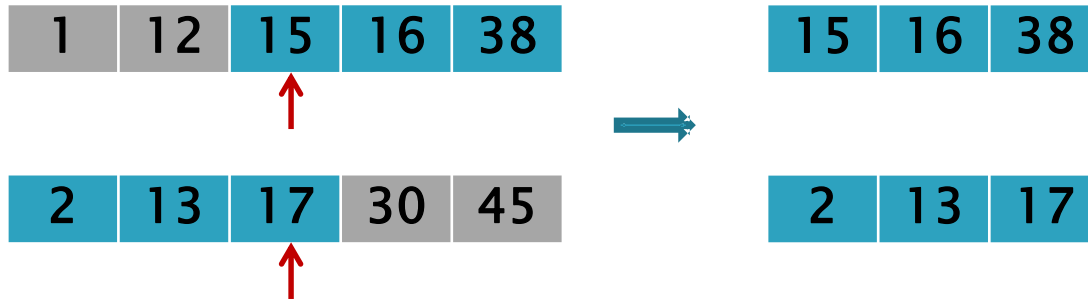
2	13	17	30	45
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1	12	15	16	38
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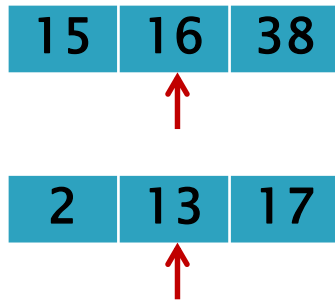
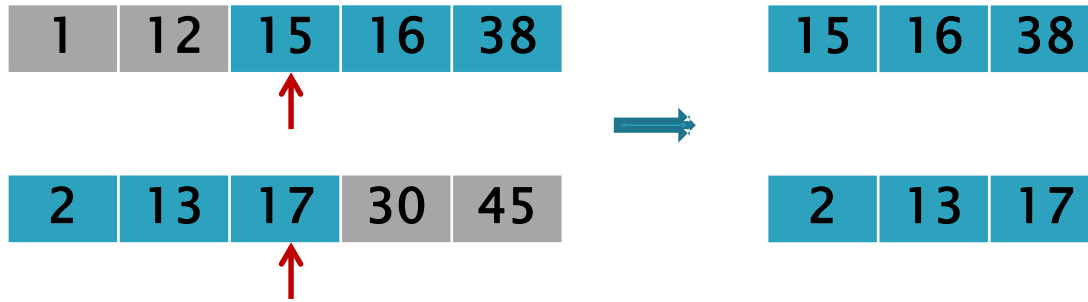


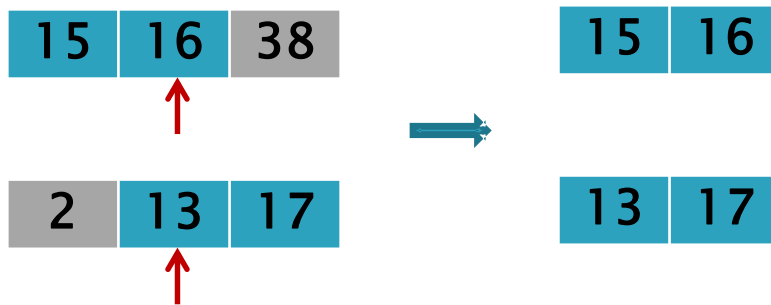
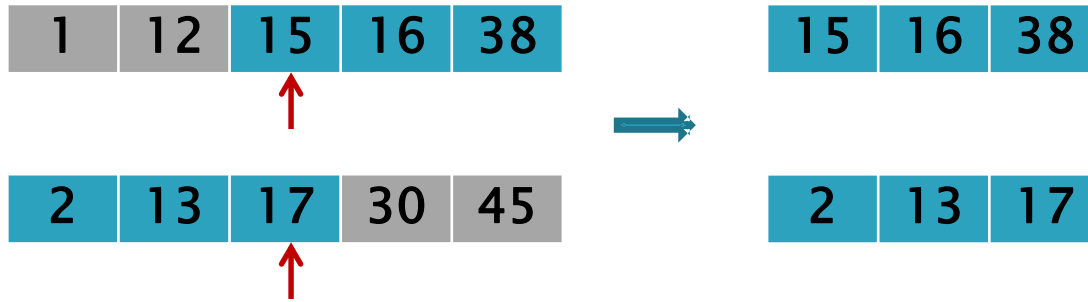
2	13	17	30	45
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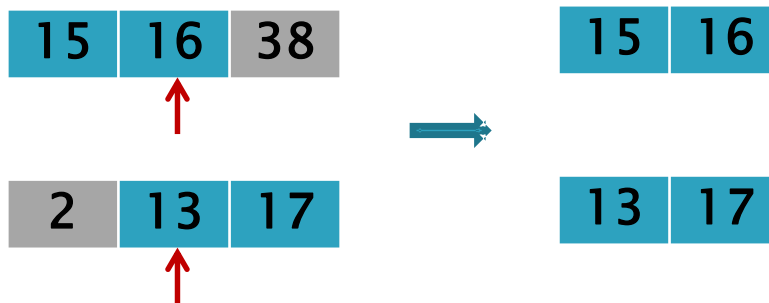
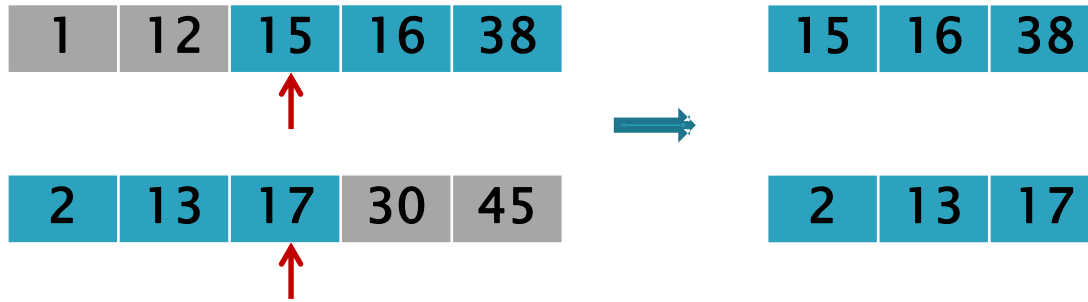






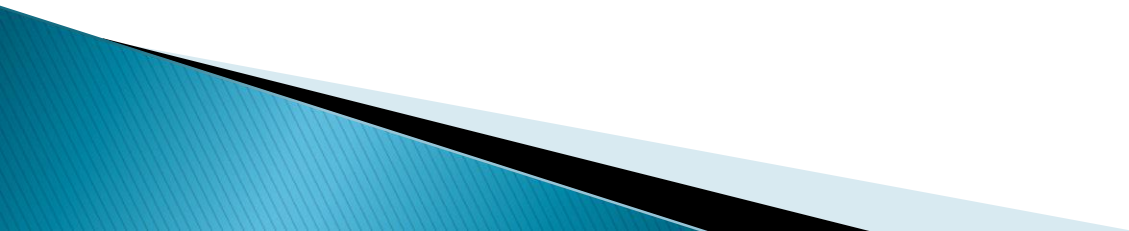






$$\begin{aligned} \text{Mediana} &= \frac{\max\{13,15\} + \min\{16,17\}}{2} = \frac{15+16}{2} \\ &= 15,5 \end{aligned}$$

# Exemplul 2



1	12	15	16	38	40
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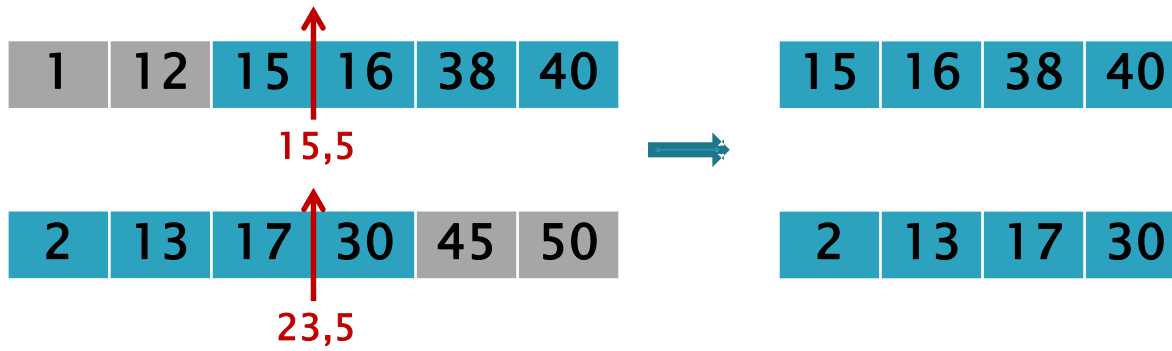
2	13	17	30	45	50
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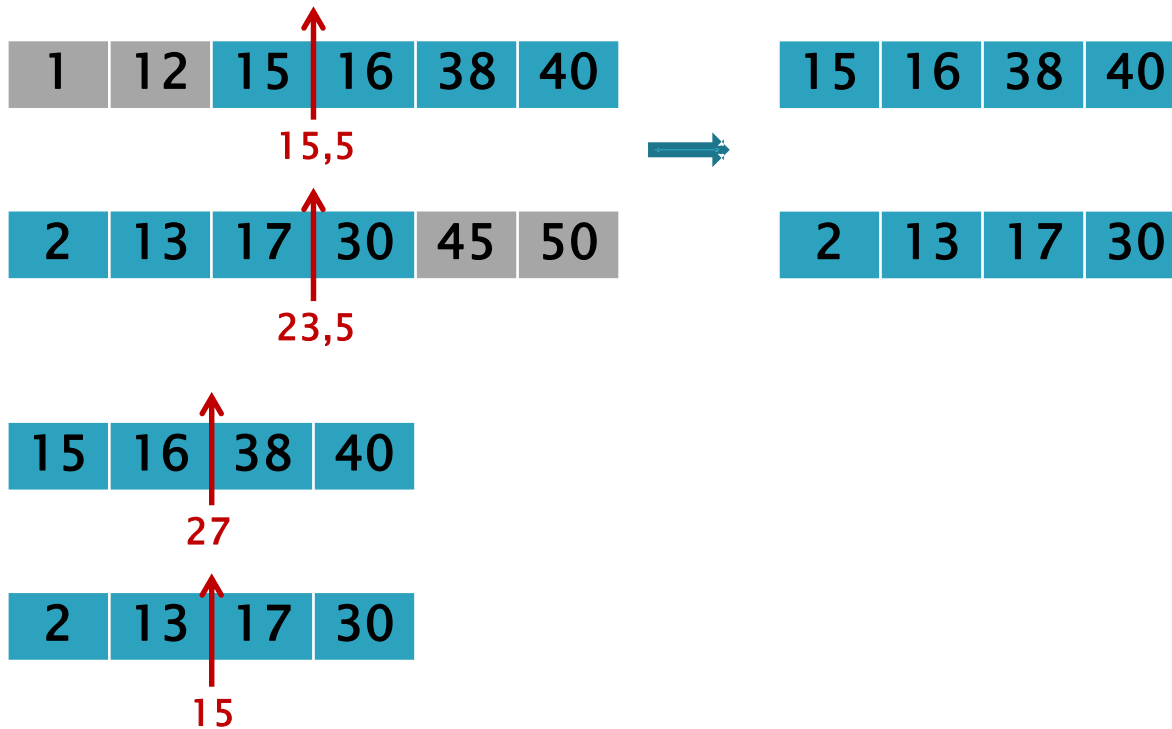
1	12	15	16	38	40
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15,5

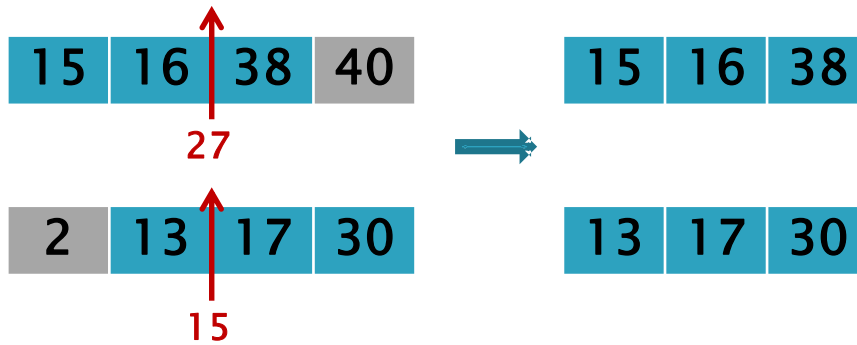
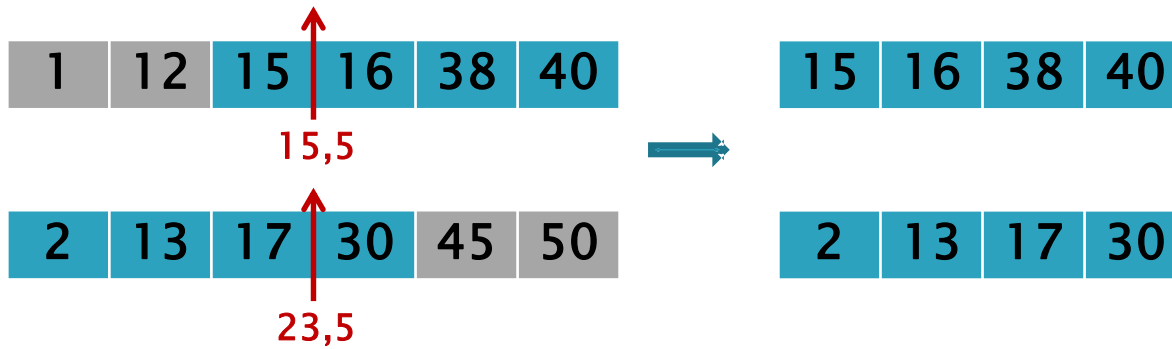
2	13	17	30	45	50
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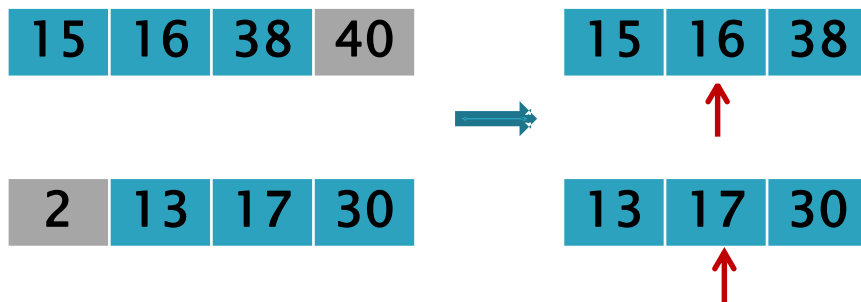
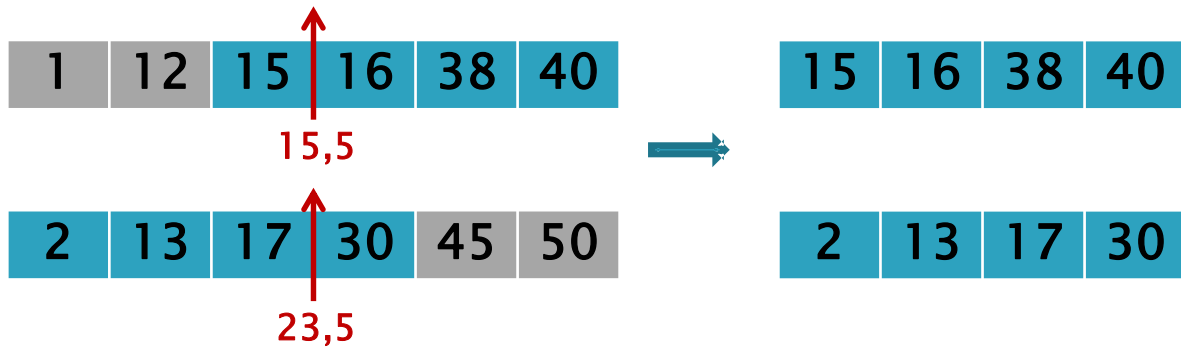
23,5

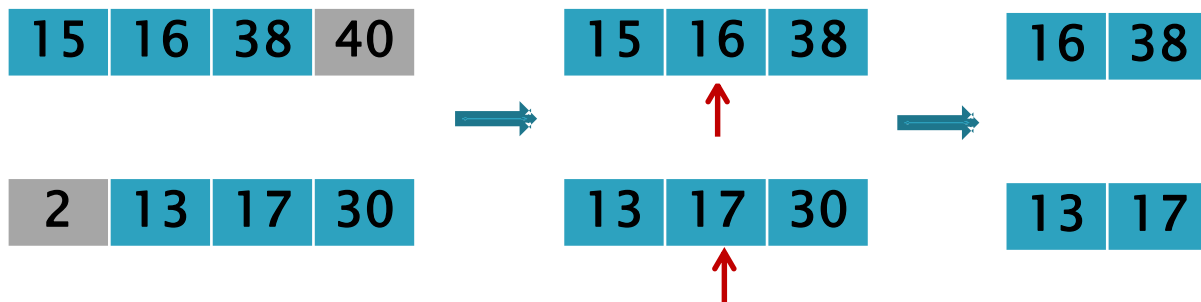
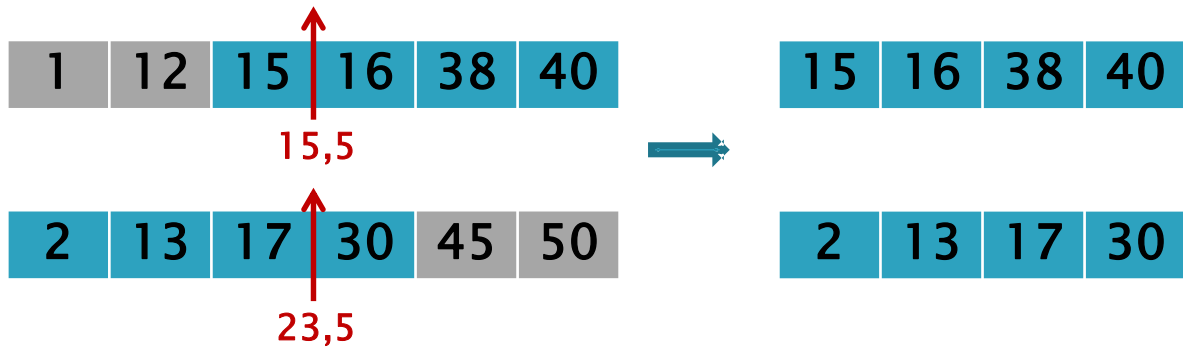


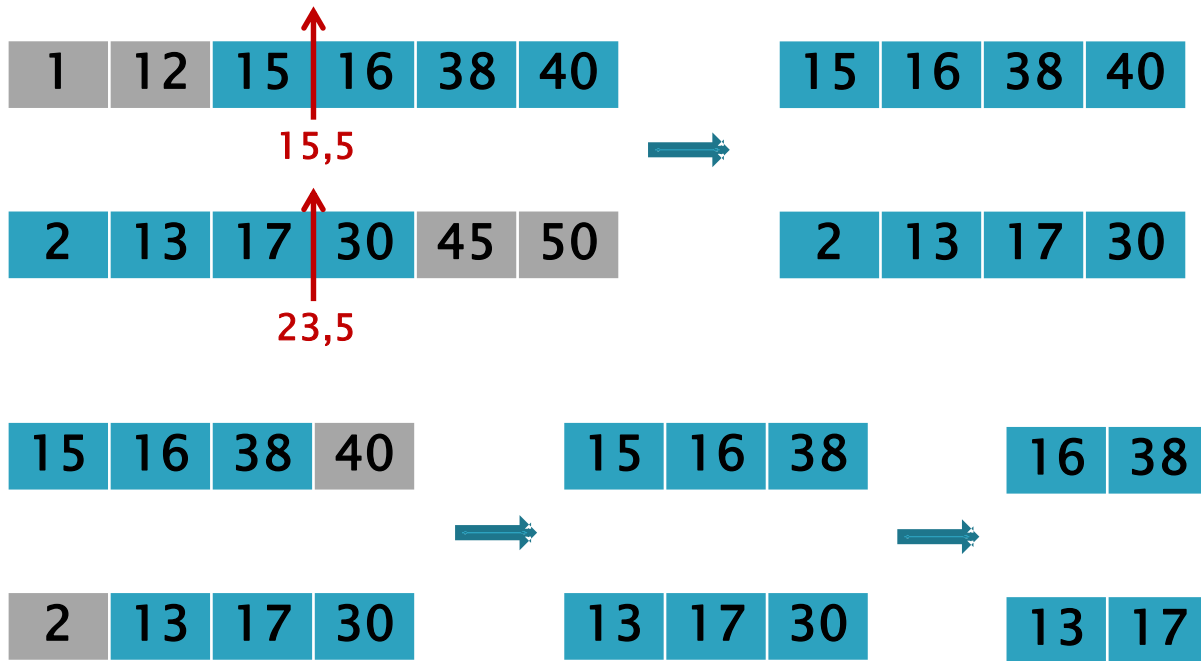












$$\begin{aligned} \text{Mediana} &= \frac{\max\{13, 16\} + \min\{17, 38\}}{2} = \frac{16 + 17}{2} \\ &= 16,5 \end{aligned}$$

# Algoritm

```
mediana(v, pv, uv){  
    n = uv-pv+1  
    m =(uv+pv)/2  
    if (n%2==0)  
        return (v[m]+v[m+1])/2  
    else  
        return v[m]  
}
```

```
calculMediana(pa, ua, pb, ub){  
    n = ua-pa+1  
    if (n<=2) //rezolv direct  
        return (max(a[pa],b[pb])+min(a[ua],b[ub]))/2
```

```
calculMediana(pa, ua, pb, ub){  
    n = ua-pa+1  
    if (n<=2) //rezolv direct  
        return (max(a[pa],b[pb])+min(a[ua],b[ub]))/2  
  
    ma = mediana(a,pa,ua); //mediana lui a[pa..ua]  
    mb = mediana(b,pb,ub); //mediana lui b[pb..ub]  
  
    if(ma == mb) return ma
```



```

calculMediana(pa, ua, pb, ub){
    n = ua-pa+1
    if (n<=2) //rezolv direct
        return (max(a[pa],b[pb])+min(a[ua],b[ub]))/2
    ma = mediana(a,pa,ua); //mediana lui a[pa..ua]
    mb = mediana(b,pb,ub); //mediana lui b[pb..ub]
    if(ma == mb) return ma
    if (ma>mb)
        return calculMediana(pa, pa+n/2, pb+(n-1)/2,ub)
    else
        return calculMediana(pa+(n-1)/2, ua, pb,pb+n/2)
}

```

```

calculMediana(){
    return calculMediana(0,n-1,0,n-1)
}

```

# Mediana a doi vectori sortați

- ▶ Complexitate:  $O(\log n)$

# Mediana a doi vectori sortați



Mai este valabilă ideea pentru vectori de lungimi diferite (reducem problema la o problemă de același tip păstrând jumătate din fiecare vector)?

