# The Noetherian property of invariant rings

Annie Giokas
Advisor: Anurag Singh

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A Noetherian ring is a ring that has the ascending chain condition, or the maximal condition, which means that any ascending chain of ideals of the ring must stabilize after a finite number of steps. The concept of Noetherian rings came to be after the German mathematician, Emmy Noether, discovered that primary decomposition of ideals is a consequence of the ascending chain condition in 1921. It is known that for a graded ring over a field, the Noetherian property is equivalent to the ring being finitely generated. Noether proved in 1926 that the ring of invariants for the action of a finite group via k-algebra automorphisms of finitely generated algebras over a field k are in fact, Noetherian. In a related direction, a Noetherian ring containing the field of rational numbers, will have also a Noetherian invariant ring under a finite group action. The proof depends on the ring containing a characteristic 0 field, because it uses an especially useful formula, the Reynold's operator, that involves the inverse of the order of the given finite group to define the map between the ring and its invariant ring that turns out to be a projection. This however is not necessarily true in other cases. There exists a class of rings of characteristic p for each prime integer p such that each ring in the class is Noetherian with a finite group G acting on it such that the ring of invariants under this group action is not Noetherian. This class of rings is generalized from the counterexample $\mathbb{Z}/2\mathbb{Z}(a_1, b_1, a_2, b_2, ...)[[x, y]]$ power series ring over a field of fractions of a polynomial ring in infinitely many variables.