1) $\mathbb{R}_\ell$ is Hausdorff because it is finer than $\mathbb{R}$.
2) Product of Hausdorff spaces is Hausdorff.
3) Therefore, $\mathbb{R}_\ell \times \mathbb{R}_\ell = \mathbb{R}_\ell^2$ is Hausdorff.
4) Subspaces of Hausdorff spaces is Hausdorff.
5) Therefore $L \subset \mathbb{R}_\ell^2$ is Hausdorff.
6) In Hausdorff spaces singletons are closed.
7) Then singletons in L are clopen.