***9.** Let A be the set of all points of $\mathbb{R}_\ell^2$ of the form $x \times (-x)$, for x rational; let B be the set of all points of this form for x irrational. If V is an open set of $\mathbb{R}_\ell^2$ containing B, show there exists no open set U containing A that is disjoint from V, as follows:

(a) Let K_n consist of all irrational numbers x in [0, 1] such that $[x, x + 1/n) \times [-x, -x + 1/n)$ is contained in V. Show [0, 1] is the union of the sets K_n and countably many one-point sets.

(b) Use Exercise 5 of §27 to show that some set $\bar{K}_n$ contains an open interval (a, b) of $\mathbb{R}$.

(c) Show that V contains the open parallelogram consisting of all points of the form $x \times (-x + \epsilon)$ for which $a < x < b$ and $0 < \epsilon < 1/n$.

(d) Conclude that if q is a rational number with $a < q < b$, then the point $q \times (-q)$ of $\mathbb{R}_\ell^2$ is a limit point of V.

$A = \{(x, -x) \mid x \in \mathbb{Q}\}$ $\quad$ V open in $\mathbb{R}_\ell^2$

$B = \{(x, -x) \mid x \in \mathbb{I}\}$ $\quad$ $B \subset V$

$\nexists\, U$ open in $\mathbb{R}_\ell^2$ s.t $A \subset U$ and U disjoint from V.

(a) Let K_n consist of all irrational numbers x in [0, 1] such that $[x, x + 1/n) \times [-x, -x + 1/n)$ is contained in V. Show [0, 1] is the union of the sets K_n and countably many one-point sets.

$K_n = \{x \in \mathbb{I} \mid x \in [0,1],\ [x, x+\frac{1}{n}) \times [-x, -x+\frac{1}{n}) \subset V\}$

Show $[0,1] = \bigcup_n K_n \cup \{\ \}$

$\sqrt{2} \ldots\ldots \sqrt{2}+1$

$-\sqrt{2}+1 \ldots\ldots -\sqrt{2}$