Assignment 8 Justin Kim

1] m^2+nm+n^2 a perfect square?

$=(m+n)^2-nm \neq 1$

$\neq 4$

$\neq 9$

$1 \overset{}{\;} 4 \overset{5}{\;} 9 \overset{7}{\;} 16 \overset{9}{\;} 25 \overset{11}{\;} 36 \overset{13}{\;} 49 \overset{15}{\;} 64$

$\boxed{\text{Yes it is a perfect square for } n=5,\ m=3}$

2] $(\forall m)(\exists n)$

$(\forall m) \Rightarrow (\exists n)$ ($mn+1$ is perfect square

m	n	R
1	3	4
2	4	9
3	5	4^2
4	6	

Claim: for all positive integer m, $n=m+2$ (which is obviously always positive) will make $mn+1$ a perfect square. Proving this proves the original statement.

~~Proof by Induction~~

Specifically:

$m(m+2)+1 \stackrel{?}{=} (m+1)^2$

$(m+1)^2$ is a perfect sqr:

$m(m+2)+1 = m^2+2m+1$

$= (m+1)^2 = (m+1)^2$

$\therefore$ it is true. $\square$

3] $f(n)=n^2+bn+c$, $b,c \in \mathbb{Z}^+$

setting $b=2$, $c=1$, we have

$f(n)=n^2+2n+1=(n+1)^2$

$f(n)=(n+1)^2$. This implies that $\forall n \in \mathbb{Z}^+$, $f(n)$ is the product of two positive integers, namely $(n+1)$. $\therefore$, $f(n)$ is a composite number. $\therefore$ $\exists$ positive b,c such that $f(n)$ is composite $\forall n \in \mathbb{Z}^+$.

4] Domain is $\mathbb{N}$.

$\Big((\forall n)\left[n>2 \wedge (\exists m)(n=2m)\right]\Big) \Rightarrow \Big(\exists p,q \in \mathbb{P}\Big)(n=p+q)$

$n=4,\ 4=2+2,\ m=\cancel{7} = 2+2+3$

$n=6,\ 6=3+3,\ m=9=3+3+3$

$n=8,\ 8=3+5,\ m=11=3+8+3$

Claim:

$(\forall n \in \{4,6,8,10,\dots\})(\exists p,q \in \text{Prime})(n=p+q)$

$\Rightarrow m=n+3=p+q+3$ is combination of 3 primes. This is trivial b/c 3 is prime, so m is composed of 3 primes. and the domain of m is $\{7,9,11,13,\dots\}$ which is the domain of the problem.

$\therefore$ the claim is true.

5] Prove sum of first n odd # is equal to n^2.

Claim: $\forall n \in \mathbb{N}$, $1+3+\dots+n = n^2$.
$\wedge$ n = odd
will prove by induction.

for $n=1$, $1^2 = 1$ so claim holds.

Now suppose it is true for n.

~~Then $1+2+\dots+n+(n+1) = n^2+(n+1)$~~

Then next odd # is $n+2$.

So $1+\dots+n+(n+2) = n^2+(n+2)$

$$f(n) = \sum_{i=1}^{n}(2i-1) = (2n-1)^2$$

for $n=1$, $2-1=1 = (2\cdot1-1)^2 = 1$

Suppose true for n.

Then $\sum_{i=1}^{n+1}(2i-1) = \left(\sum_{i=1}^{n}(2i-1)\right) + 2n+1$

$$= (2n-1)^2 + 2n+1$$
$$= 4n^2 - 4n + 1 + 2n + 1$$
$$= 4n^2 - 2n + 2$$
$$= (2\cdot$$

$(2(n+1)-1)^2$

$(2n+1)^2$

Prove $f(n) = \sum_{i=1}^{n}(2i-1) = n^2$

true for $n=1$

$$f(n+1) = \sum_{i=1}^{n+1}(2i-1) = \sum_{i=1}^{n}(2i-1) + 2n-1$$
$$= n^2+2n+1 = (n+1)^2$$

$\therefore$ by principle of induction, true.

6] Prove $\forall n \in \mathbb{N}: \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

Proof by induction.

for $n=1$, $LHS = 1^2 = 1$

$RHS = \frac{1}{6}(1)(1+1)(2\cdot1+1) = \frac{1}{6}(2)(3) = 1$ ✓

Suppose its true for n.

Then $A(n+1) = \sum_{r=1}^{n+1} r^2 = \sum_{r=1}^{n} r^2 + (n+1)^2$

$$= \frac{1}{6}n(n+1)(2n+1) + (n+1)^2$$
$$= \frac{n(2n^2+3n+1)}{6} + n^2+2n+1$$
$$= \frac{2n^3+3n^2+n+6n^2+12n+6}{6}$$
$$= \frac{2n^3+9n^2+13n+6}{6}$$

$$\begin{array}{r|rrrr} -1 & 2 & 9 & 13 & 6 \\ & & -2 & -7 & -6 \\ \hline & 2 & 7 & 6 & \end{array}$$

$\frac{1}{2}$ $\frac{2}{3}$

$$= \frac{(n+1)(2n^2+7n+6)}{6}$$
$$= \frac{(n+1)(n+2)(2n+3)}{6}$$
$$= \frac{1}{6}\cdot(n+1)[(n+1)+1][2(n+1)+1]$$

$\therefore$ By proof of mathematical induction statement is true.