Assignment 6 — Justin Kim (1)

1] Show $\neg[\exists x A(x)] \iff \forall x[\neg A(x)]$

Suppose $\neg[\exists x A(x)]$ is true. Then there does not exist an x such that $A(x)$ is true. That means for all x, $A(x)$ is false. Hence $\forall x[\neg A(x)]$.

2] Suppose ~~$\exists (x \in N)$~~ $(\exists x \in \mathbb{N})(P(x) \land x > 2)$.
Then $x = 2m$ for some $m \in \mathbb{N}$ by definition of even. Then x is divisible by 2. Since $x \neq 2$, that means that x is divisible by a # other than 1 or itself, hence it's a composite #, which is a contradiction. $\therefore$ the statement "There is an even prime # bigger than 2" is false.

3] (a) $\forall x[\text{student}(x) \Rightarrow \text{likesPizza}(x)]$

(b) $\exists x(\text{myfriend}(x) \land \neg \text{haveCar}(x))$ — $\exists x[\text{myfr}(x) \Rightarrow \neg \text{Car}(x)]$

(c) $\exists x[\text{Elephant}(x) \land \neg \text{LikesMuffins}(x)]$

(d) $\forall x[\Delta(x) \Rightarrow \text{isoseceles}(x)]$

(e) $\exists x[\text{student}(x) \land \neg \text{Heretoday}(x)]$

(f) ~~$\forall x \forall y\, \text{Loves}(x,y)$~~
$\forall x \exists y\, \text{Loves}(x,y)$

(g) $\forall x \exists y(\neg \text{loves}(x,y))$

(h) $\forall x \forall y[\text{Man}(x) \land \text{Comes}(x) \land \text{woman}(y) \Rightarrow \cancel{\text{woman}(y)} \land \text{leave}(y)]$

(i) $\forall x[\text{Short}(x) \lor \text{Tall}(x)]$

(j) $\forall x\, \text{Tall}(x) \lor \forall x\, \text{Short}(x)$

k] $\exists x[\text{precious}(x) \Rightarrow \text{beautiful}(x)]$

l] $\forall x\, \neg \text{lovesMe}(x)$

m] $\exists x(\text{American}(x) \Rightarrow \text{poisonous}(x))$

n] $\exists x(\text{American}(x) \land \text{Snorkle}(x) \Rightarrow \text{poisons}(x))$

4] (a) $\exists x(\text{student}(x) \land \neg \text{likesPizza}(x))$
there is a student that doesnt like pizza.

(b) ~~$\forall x[\neg \text{myfriend}(x) \lor \text{hasCar}(x)]$~~
$\boxed{\forall x[\text{myFriend}(x) \land \text{hasCar}(x)]}$
all of my friends has a car

(c) $\boxed{\forall x[\text{Elephant}(x) \land \text{LikesMuffin}(x)]}$
All elephants like muffins

(d) $\exists x[\Delta(x) \land \neg \text{isoseles}(x)]$
There is a triangle that is not isosceles.

(e) $\forall x[\text{Student}(x) \land \text{Heretoday}(x)]$
All students in the class are here today.

(f) $\exists x \forall y\, \neg \text{loves}(x,y)$
There exists someone that does not love everyone

(g) $\exists x \forall y(\text{loves}(x,y))$
There exists someone that loves everyone.

(h) $\exists x \exists y[\text{Man}(x) \land \text{comes}(x) \land \text{woman}(y) \not\Rightarrow \land \neg \text{man'll}$ [illegible] $(y)]$
There exists a man and a woman such that if the man comes, that woman will stay.

(i) $\exists x [\cancel{Tall(x)}\ \neg Short(x) \wedge \cancel{Short(x)}\ \neg tall(x)]$

There is a person that is not short and not tall.

(j) $\exists x \neg Tall(x) \wedge \exists x \neg Short(x)$

There is a person that is not tall and there is a person that is not Short.

K) $\forall x [precious(x) \Rightarrow \neg beautiful(x)]$

All precious stones are beautiful

l) $\exists x$ loves me(x)

There is someone that loves me.

m) $\forall x (American(x) \Rightarrow \neg poisonous(x))$

all American snakes are not poisonous.

n) $\forall x [American(x) \wedge snake(x) \Rightarrow \neg poisonus(x)]$

all american snakes are not poisonus.

5 a) False
b) False
c) True
d) True
e) true
(f) true ?
g) false
h) true

6 (a) $\forall x (2x+3 \neq 5x+1)$

~~(b) $\forall x (x^2 \neq 2$~~
$\forall x (x \neq \frac{2}{3})$?

(b) $\forall x (x^2 \neq 2)$

(c) $\exists x \forall y (y \neq x^2)$

(d) $\exists x \forall y (y \neq x^2)$

(e) $\exists x \forall y \exists z (xy \neq xz)$

(f) $\exists x \forall y \exists z (xy \neq xz)$

(g) ~~$\exists x [x<0 \Rightarrow$~~
$\exists x [x<0 \wedge \forall y (y^2 \neq x)]$

(h) $\exists x [x<0 \wedge \forall y (y \neq x)]$

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(a) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})(x+y \neq 1)$

(b) $(\exists x > 0)(\forall y < 0)(x+y \notin 0)$

(c) $\forall x (\exists \epsilon > 0)(-\epsilon \geq x \geq \epsilon)$

$\forall x (\exists \epsilon > 0)(-\epsilon \geq x > \epsilon)$

(d) $(\exists x \in \mathbb{N})(\exists y \in \mathbb{N})(\forall z \in \mathbb{N})(xy \neq z^2)$

8 $\forall x \exists t f(x,t) \wedge \exists x \forall t f(x,t) \wedge \forall x \forall t f(x,t)$

$\exists x \forall t \neg f(x,t) \vee \forall x \exists t \neg f(x,t) \vee \exists x \exists t \neg f(x,t)$

You may not fool some people all the time, or you may not fool all people some times, or you may not fool some people some of the time

9 $(\exists \epsilon > 0)(\forall \delta > 0)(\exists x)$
$[\,|x-a| < \delta \wedge |f(x)-f(c)| \geq \epsilon\,]$