Assignment 4 9/2/22 Justin Kim

1]

ϕ	ψ	$\phi \Rightarrow \psi$	$\psi \Rightarrow \phi$	$\phi \Leftrightarrow \psi$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2] $(\phi \Rightarrow \psi) \Leftrightarrow (\neg\phi \vee \psi)$

ϕ	ψ	$\neg\phi$	$\phi \Rightarrow \psi$	$\neg\phi \vee \psi$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

3] $(\phi \not\Rightarrow \psi) \Leftrightarrow (\phi \wedge \neg\psi)$

ϕ	ψ	$\neg\psi$	$\phi \Rightarrow \psi$	$\phi \not\Rightarrow \psi$	$\phi \wedge \neg\psi$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

4] (a) $[\phi \wedge (\phi \Rightarrow \psi)] \Rightarrow \psi$

ϕ	ψ	$\phi \Rightarrow \psi$	$\phi \wedge (\phi \Rightarrow \psi)$	$[\phi \wedge (\phi \Rightarrow \psi)] \Rightarrow \psi$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(b) Since its true $\forall$ scenarios, it is valid.

5] 1) $\phi \vee \psi$ means ϕ or ψ is true

2] $\neg(\phi \vee \psi)$ means not ϕ or ψ is true

3] Since neither can be true, then both ϕ and ψ are false

4] AKA $\neg\phi \wedge \neg\psi$ are true $\therefore$

6]

(a) 34,159 is not a prime #

(b) Roses are not red or violets are not blue

(c) there are no Hamburger and I will not love a hotdog

(d) Fred will not go or he will play

(e) $\neg(N \vee G)$

X is positive and less than or eq to 10

(f) we will lose the 1st game and lose the second game

7] Show $(\phi \Leftrightarrow \psi) \Longleftrightarrow (\neg\phi) \Leftrightarrow (\neg\psi)$

ϕ	ψ	$\phi \Rightarrow \psi$	$\psi \Rightarrow \phi$	$\neg\phi$	$\neg\phi \Rightarrow \neg\psi$	$\neg\psi$	$\neg\psi \Rightarrow \neg\phi$
T	T	T	T	F	T	F	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

$\phi \Leftrightarrow \psi$	$\neg\phi \Leftrightarrow \neg\psi$
T	T
F	F
F	F
T	T

8] $\phi \Leftrightarrow \psi$

ϕ	ψ	$\phi \Rightarrow \psi$	$\psi \Rightarrow \phi$	$\phi \Leftrightarrow \psi$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(b) $\phi \Rightarrow (\psi \vee \theta)$

ϕ	ψ	θ	$\psi \vee \theta$	$\phi \Rightarrow (\psi \vee \theta)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

9]

ϕ	ψ	θ	$\psi \wedge \theta$	$\phi \Rightarrow \psi$	$\phi \Rightarrow \theta$	L	R
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	F	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	F	F	F	T	F
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$L = \phi \Rightarrow (\psi \wedge \theta)$ $R = (\phi \Rightarrow \psi) \wedge (\phi \Rightarrow \theta)$

10] If $\phi \Rightarrow (\psi \wedge \theta)$ then

$\phi \Rightarrow \psi$ is true and $\phi \Rightarrow \theta$ is true

$\hookrightarrow (\phi \Rightarrow \psi) \wedge (\phi \Rightarrow \theta)$

11] $\phi \Rightarrow \psi \Leftrightarrow (\neg\psi) \Rightarrow (\neg\phi)$

ϕ	ψ	$\neg\phi$	$\neg\psi$	$\phi \Rightarrow \psi$	$\neg\psi \Rightarrow \neg\phi$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

12]

(a) If two □ dont have same area, then they are not congruent.

(b) If $a^2+b^2 \neq c^2$, then a △ w/ sides a, b, c (c largest) is not ⊾.

(c) If n is not prime, the 2^n-1 is not prime

(d) If Dollar rises, then Yuan falls

13]

ϕ	ψ	$\phi \Rightarrow \psi$	$\psi \Rightarrow \phi$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Assignment 4 — Justin Kim ③

14) If two □ have same area, then they are congruent

(b) If $a^2+b^2=c^2$ then a $\triangle$ w/ sides a,b,c (c largest) is right angled.

(c) If n is prime, then 2^n-1 is prime

(d) If Dollar falls, then Yuan rises.

Extra

E1) ϕ unless ψ

$$(\phi \wedge \neg\psi) \vee (\neg\phi \wedge \psi)$$

E2)

ψ	ϕ	$\dot{\vee}$
T	T	F
T	F	T
F	T	T
F	F	F

E3) $\phi \dot{\vee} \psi =$

$$\boxed{(\phi \wedge \neg\psi) \vee (\neg\phi \wedge \psi)}$$

4a) (a) If a=2 then a=2

(b) If ~~$x^2=4$~~ $x=2$ then $x^2=4$

(c) If x=2 then x>1

(d) If x=2 then x=3

5)

M	N	M×N	M+N
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

6) (a) $\wedge$ (b) $\dot{\vee}$ (c) 1 to

7) A) All same

8) V ⇒ O E ⇒ X

$\boxed{E}$ $\boxed{4}$

9) $m,n \in \mathbb{N}$.

Suppose mn is odd then

$mn = 2j-1$ for $j \in \mathbb{N}$.

Then mn is not divisible by 2.

So m and n is not divisible by 2

So m & n are odd.

Now suppose m & n are odd.

Then $m=2j-1$ & $n=2k-1$

$$mn = (2j-1)(2k-1)$$
$$= 4jk - 2j - 2k + 1$$
$$= 2(2jk - j - k) + 1$$

Since $2jk-j-k \in \mathbb{N}$

mn is odd

Since $mn \in O \Rightarrow m \in O \wedge n \in O$

and $m \in O \wedge n \in O \Rightarrow mn \in O$

Statement is true.

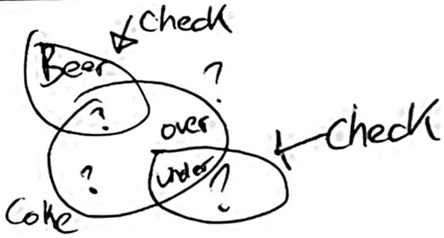
10 No.

$mn \in O \iff m,n \in O$

$mn \in E \iff m \in E \lor n \in E$

Just one of them has to be even

11



12 Contrapositive

11 was easier