Assignment 9 Justin Kim ①

1) $b|a \Rightarrow (\exists q \in \mathbb{Z})(a = qb)$

a/b is an amont, ie $K = a/b$

$\Longleftrightarrow Kb = a$

2 (a) $0|7$ $\boxed{\text{False.}}$

Suppose true. Then $(\exists q \in \mathbb{Z})(q \cdot 0 = 7)$

But any number times 0 is 0, a contradition

(b) $9|0$ $\boxed{\text{True.}}$

$9 \cdot 0 = 0$

(c) $0|0$ $\boxed{\text{True?}}$ $0 \cdot 0 = 0$ or any # for that matter

(d) $1|1$ $\boxed{\text{True.}}$ $1 \cdot 1 = 1$

(e) $7|44$ $\boxed{\text{False}}$ $44 = 7 \cdot 6 + 2$

(f) $7|(-42)$ $\boxed{\text{True}}$ $-42 = 7 \cdot (-6)$

(h) $(-7)|(-56)$ $\boxed{\text{True}}$ $-56 = (-7)(8)$

(i) $(\forall n \in \mathbb{Z})(1|n)$

True Suppose False. Then $(\exists n \in \mathbb{Z})(1 \nmid n)$ Then $\nexists q \in \mathbb{Z}$ such that $1 \cdot q = n$ but $q = n$ makes $1 \cdot n = n$ true, a contradicton.

So $\boxed{\text{True}}$

(j) $(\forall n \in \mathbb{N})(n|0)$ $\boxed{\text{True.}}$

Suppose false. Then $(\exists n \in \mathbb{N})(n \nmid 0)$

$\Longleftrightarrow (\nexists q \in \mathbb{N})(0 = nq)$ but if $q = 0$, then it holds, a contradicton.

(k) $(\forall n \in \mathbb{Z})(n|0)$

$\boxed{\text{True}}$

3 $a|0$

setting $q = 0$, we have $0 = q \cdot a$

$a|a$

setting $q = 1$, $a = 1a$

(b) $a|1 \Longleftrightarrow a = \pm 1$

Suppose $a|1$ Then $(\exists q \in \mathbb{Z})(1 = q \cdot a)$

prove $a|1 \Rightarrow a = \pm 1$

~~Supp~~ suppose $a \neq \pm 1$. Then $\nexists q \in \mathbb{Z}$ $1 = qa$ b/c $q = \frac{1}{a}$, and $\frac{1}{a} \notin \mathbb{N}$.

Now if $a = \pm 1$, then $q = \pm 1$.

(c) If $a|b \wedge c|d$, then $ac|bd$ (for $c \neq 0$)

If $a|b \wedge c|d$, $\exists q_1, q_2 \in \mathbb{Z}$ such that $b = q_1 a$ $d = q_2 c$

Then $bd = (q_1 q_2) ac$ since $q_1 q_2 \in \mathbb{Z}$, $ac | bd$. for $c \neq 0$.

(d) If $a|b \wedge b|c$ then $a|c$ $(b \neq 0)$

$b = q_1 a$ $c = q_2 b$

$\cancel{b} c = q_1 q_2 \cancel{b} a$ for $b \neq 0$

$c = (q_1 q_2) a$ so $a|c$

(e) $[a|b \wedge b|a] \Leftrightarrow a\pm b$

Suppose $a|b \wedge b|a$ $\exists q_1 q_2 \in \mathbb{Z}$
such that $b=q_1 a \wedge a=q_2 b$

$$\cancel{ab} = q_1 q_2 \cancel{ab}$$

$$1 = q_1 q_2 \text{ So } q_1 q_2 = \pm 1$$

So $a=\pm b$

Suppose $a=\pm b$. If $a=b$,
$a|b$ since $c|c\ \forall c$ If $a=-b$,
then $a|b$ since $q=-1$.

(f) If $a|b \wedge b\neq 0$, then $|a|\leq|b|$

$$b=qa.$$

$$|b|=|q||a| \text{ so}$$

$$|b| \geq |a|$$

(g) If $a|b \wedge a|c$ then
$a \mid (bx+cy)$

Suppose $a|b \wedge a|c$. Then

~~$a=q_1 b$~~ /

$$b=q_1 a \quad \wedge \quad c=q_2 a$$

$$bx=q_1 x a \qquad cy=q_2 y a$$

$$(bx+cy)=(q_1 x+q_2 y)a$$

So $a \mid bx+cy$