24-677 Project 2

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1 Exercise 1

1.1 Controllability & Observability

Following the derivation on chapter 2 of Vehicle Dynamics and Control (Rajesh Rajamani) and setting a the time varying disturbance$\dot{\psi}_{des} = 0$, we get the linearized, error-based state space system for the vehicle:

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e_1} \\ e_2 \\ \dot{e_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_a}{mV_x} & \frac{4C_a}{m} & \frac{2C_a(l_r-l_f)}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_a(l_r-l_f)}{I_zV_x} & \frac{2C_a(l_f-l_r)}{I_z} & \frac{-2C_a(l_r^2+l_f^2)}{I_zV_x} \end{bmatrix} \begin{bmatrix} e_1 \\ \dot{e_1} \\ e_2 \\ \dot{e_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2C_a}{m} \\ 0 \\ \frac{2C_a l_f}{I_z} \end{bmatrix} \delta \tag{1}$$

Substituting $l_r = 1.7m, l_f = 1.1m, C_a = 15000N, I_z = 3344kgm^2, m = 2000kg$ and assuming all modes are controllable individually, we get the following state space matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-30}{V_x} & 30 & \frac{9}{V_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{5.3828}{V_x} & -5.3828 & \frac{-36.7823}{V_x} \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} 0 \\ 15 \\ 0 \\ 9.8684 \end{bmatrix} \tag{3}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$D = 0 \tag{5}$$

Plugging the values $V_x = 2\frac{m}{s}, V_x = 5\frac{m}{s}, V_x = 8\frac{m}{s}$ on python, we get the controllability $(P)$and obserbability (Q) matrices for each corresponding value of the velocity, where:

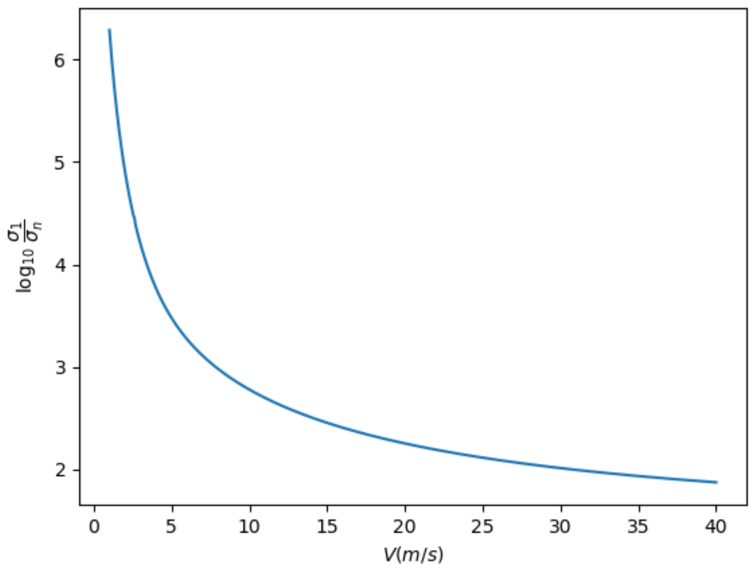
$$P = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix} \tag{6}$$

$$Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} \tag{7}$$

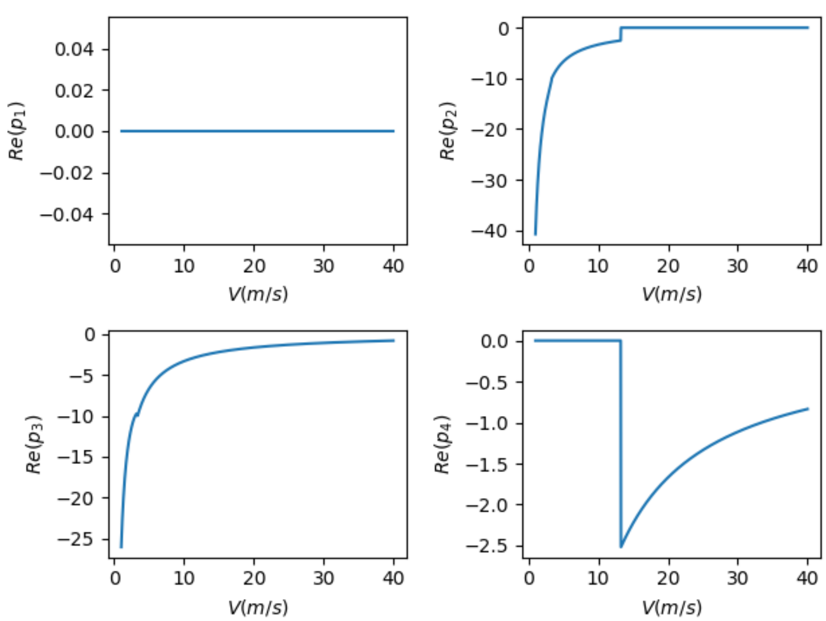
We calculate the rank of each controllability and observability matrix we derive and we observe that: $rank(P) = rank(Q) = 4 = n$ for all three different values of the velocity we plug in. Hence, we can conclude that the system is both **controllable** and **observable** for these values of V_x. (Code provided in angelosm_Q1.py)

1.2 Graphs

1.2.1 $\log_{10} \frac{\sigma_1}{\sigma_n}$ vs $V(\frac{m}{s})$



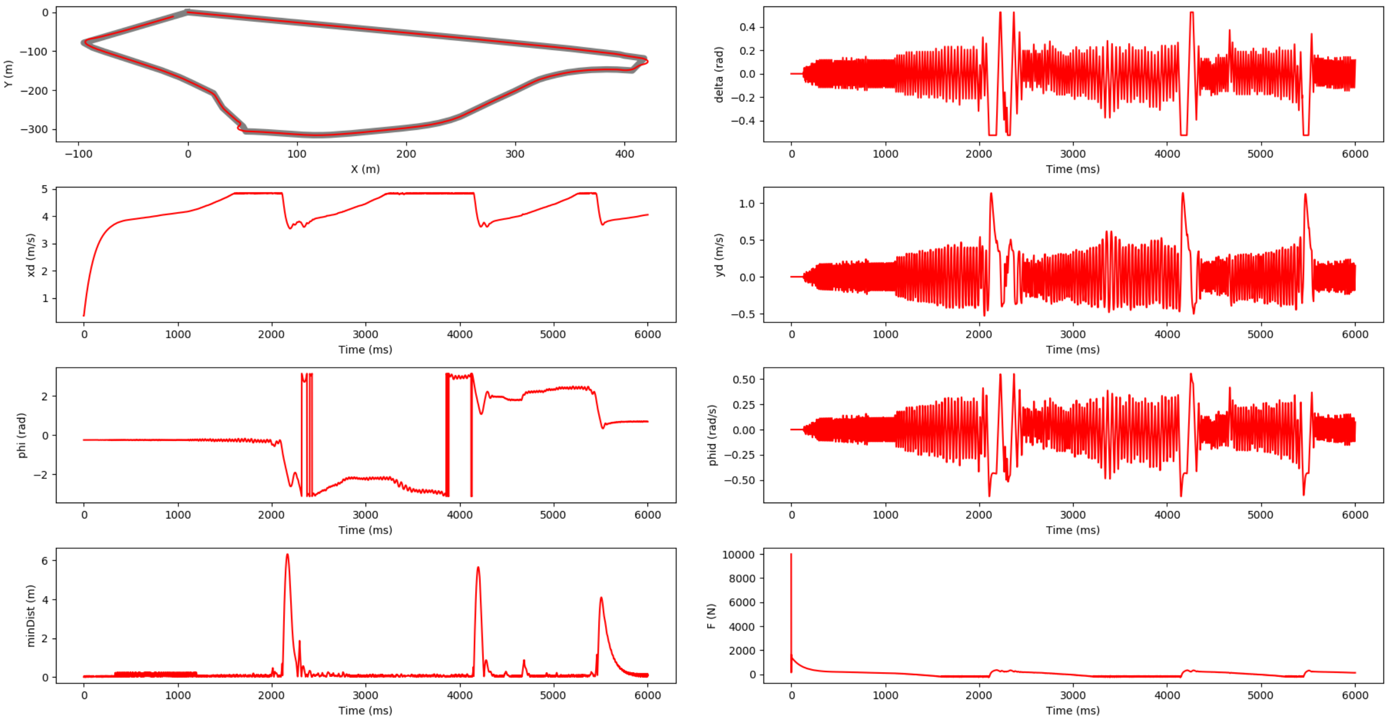
1.2.2 $Re(p_i)$ vs $V(\frac{m}{s})$



Looking at the figures, we can conclude that as we increase the velocity from $1\frac{m}{s}$ to $40\frac{m}{s}$ the poles of the system are moving from a negative range of values towards zero, which means that the system is becoming **less and less stable**. However, we see from the first plot that the logarithm of the ratio of the maximum singular value and the minimum singular value of the controllability matrix approaches zero as we increase the speed. This means that the system is becoming **more and more controllable**, since the minimum singular value is increasing, providing a higher probability of the controllability matrix being of full rank.

2 Exercise 2

Response plot:



(Code in angelosm_controller.py)