Problem 5.16)

$$x + 3y - 4z = 25$$
$$-2x + 5y + 7z = -66$$
$$3x - 2y + 3z = 7$$

(a) $x = -3y + 4z + 25$

Sustituimos en la segunda y tercera ecuación:

$$-2(-3y + 4z + 25) + 5y + 7z = -66$$
$$3(-3y + 4z + 25) - 2y + 3z = 7$$

$$
\begin{array}{rl}
 & 11y - z = -16 \\
+ & -11y + 15z = -68 \\
\hline
 & 14z = -84
\end{array}
$$

$$z = -\frac{84}{14}$$

$$z = -6$$

$$y = -2$$

$$x = -3(-2) + 4(-6) + 25 = 7$$

$$(x, y, z) = (7, -2, -6).$$

Problem 5.17)

$$(\text{I})\ 2x - 3y + 6z = -12 \xrightarrow{\times 2} 4x - 6y + 12z = -24$$
$$(\text{II})\ 5x + 2y - 8z = 29 \xrightarrow{\times 3} 15x + 6y - 24z = 87$$
$$(\text{III})\ 7x + 6y + 4z = 49 \qquad 7x + 6y + 4z = 49$$

Sumamos (I) y (II):

$$19x - 12z = 63$$

Sumamos (I) y (III):

$$11x + 16z = 25$$

$$\begin{aligned} 19x - 12z &= 63 \xrightarrow{\times 4} & 76x - 48z &= 252 \\ 11x + 16z &= 25 \xrightarrow{\times 3} & 33x + 48z &= 75 \\ & & \hline 109x &= 327 \\ & & x &= 3 \end{aligned}$$

$z = -1/2$

$y = 5$

$$(x, y, z) = (3, 5, -1/2)$$

Problem 5.18)

$$i + a + e = 6$$

$$u + a + e = 9 \rightarrow a = 2$$

$$o + a + e = 7$$

$$u + e = 7$$

$$2e + a + o = 8$$

$$2o + a + e$$

$$e + i = 4$$

$$e + u = 7$$

$$\boxed{\begin{aligned} e + o &= 5 \\ -2e + o &= 6 \end{aligned}}$$

$$-e = -1$$

$$e = 1$$

$$o = 4$$

$$2(4) + 2 + 1 = 8 + 2 + 1$$

$$= \boxed{11} \text{ USD}$$

Problem 5.19)

$$Ax + B = Cx + D$$

$x = -2 \implies -2A + B = -2C + D$

$x = -1 \implies -A + B = -C + D$

$x = 1 \implies A + C = B + D$

$x = 2 \implies 2A + C = 2B + D$

$x = 0 \implies B = D$

$A = C$ y $B = D$.

Importante: Si dos expresiones lineales son iguales para todos los valores de x, entonces sus constantes deben ser iguales y los coeficientes de los términos lineales también deben ser iguales.

Si A, B, C y D son constantes y

$$Ax + B = Cx + D$$

para todo x, entonces $A = C$ y $B = D$.

Ejercicios

5.6.1)

$$x + 3y + 2z = 6$$
$$-3x + y + 5z = 29$$
$$-2x - 3y + z = 14$$

$$\begin{array}{rrrrl} & x & +3y & +2z & = 6 \\ + & -2x & -3y & +z & = 14 \\ \hline & -x & & +3z & = 20 \end{array}$$

$$-9x + 3y + 15z = 87$$

$$+ \quad -2x - 3y + z = 14$$

$$-11x \quad + 16z = 101$$

$$11x - 33z = -220$$

$$+ \quad -11x + 16z = 101$$

$$-17z = -119$$

$$z = \frac{119}{17}$$

$$z = 7$$

$$-x = 20 - 3z$$

$$x = 3(7) - 20$$

$$x = 1$$

$$(x, y, z) = (1, -3, 7)$$

$$y = 29 - 5z + 3x$$

$$= 29 - 5(7) + 3$$

$$= 29 - 35 + 3$$

$$= -3$$

5.6.2)

$$2x - 5y + 3z = 25 \xrightarrow{\times 3} 6x - 15y + 9z = 75$$

$$-x - y + 4z = -6 \xrightarrow{\times 6} 6x + 6y - 24z = 36$$

$$3x + 3y - z = -4 \xrightarrow{\times 2} 6x + 6y - 2z = -8$$

$$-8 + 2z - 24z = 36$$

$$-22z = 44$$

$$z = -2$$

Sustituyendo z: $6x - 15y = 75 - 9(-2)$

$$6x - 15y = 93$$

$$-x - y = -6 - 4(-2)$$

$$-x - y = -6 + 8$$

$$-6x - 6y = 12$$

$$6x - 15y = 93$$

$$+ \quad -6x - 6y = 12$$

$$-21y = 105$$

$$y = -\frac{105}{21} = -5$$

$$-x = -6 + y - 4z$$

$$x = 6 - y + 4z$$

$$x = 6 + 5 - 8$$

$x = 3$

$(x,y,z) = (3, -5, -2)$

5.6.3)

$v + 18 + 25 = v + 24 + w$

$19 = w$

Cada grupo suma $25 + 22 + 19$
66

$19 + y + 21 = 18 + x + y$

$22 = x$

$66 = 25 + z + 21$

$66 - 25 - 21 = z$

$20 = z$

$66 = 19 + y + 21$

$26 = y$

$y + z = 26 + 20 = \boxed{46}$

5.6.4)

$2a + 3b - 4c = 7$

$a - b + 2c = 6$

(a)

Si $a = 0$

$$\begin{array}{r} 3b - 4c = 7 \\ + \; -2b + 4c = 6 \\ \hline b \quad = 13 \end{array}$$

$(a,b,c) = (0, 13, 8)$

$$4c = 6 + 26$$
$$4c = 32$$
$$c = 8$$

(b) Si $c = 0$

$$2a + 3b = 7$$
$$-\quad 2a - 2b = 12$$
$$5b = -5$$
$$b = -1$$

$$(a,b,c) = (5,-1,0)$$

$$2a = 7 - 3b$$
$$2a = 7 + 3$$
$$a = 5$$

(c)

$$2a + 3b - 4c = 7 \longrightarrow 2a + 3b - 4c = 7$$
$$a - b + 2c = 6 \xrightarrow{\times 2} + \ 2a - 2b + 4c = 12$$
$$4a + b = 19$$

$$b = 19 - 4a$$

Ahora sustituimos

- $2a + 3(19 - 4a) - 4c = 7$

$$2a + 57 - 12a - 4c = 7$$
$$-10a - 4c = 7 - 57$$
$$10a + 4c = 50$$
$$(I) \quad \underline{5a + 2c = 25}$$

- $a - (19 - 4a) + 2c = 6$

$$a - 19 + 4a + 2c = 6$$
$$(II) \quad \underline{5a + 2c = 25}$$

$$c = \frac{-5a + 25}{2}$$

(I) y (II) son iguales, por lo tanto toda solución de una ecuación será solución de la otra.

$$(a, b, c) = \left(a, 19-4a, \frac{-5a+25}{2}\right)$$

Para cualquier valor de a, el triple (a,b,c) satisface ambos ecuaciones.

Tiene infinitos soluciones .