$3^3 = 27$

$3^2 = 9$

$3^1 = 3$

$3^0 = 1$

$3^{-1} = \frac{1}{3}$

$3^{-2} = \frac{1}{9}$

$3^{-3} = \frac{1}{27}$

Importante señalar que la conexión entre a^n y a^{-n} es que son recíprocos.

Por tanto definimos las potencias negativas como el recíproco de la potencia

$$a^{-n} = \frac{1}{a^n}$$

Problemas

2.28)

(a) $1^{-5} = \frac{1}{1^5} = \frac{1}{1} = \boxed{1}$

(b) $10^{-4} = \frac{1}{10^4} = \frac{1}{10,000}$

(c) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(d) $56 \cdot 2^{-3} = 56 \cdot \frac{1}{2^3}$

$$= 56 \cdot \frac{1}{8} = \boxed{7}$$

$$\begin{array}{r} \overset{4}{56} \\ \times\ 8 \\ \hline 448 \end{array}$$

(e) $56 \div 2^{-3} = 56 \div \frac{1}{2^3}$

$$= 56 \cdot 2^3 = 56 \cdot 8 = \boxed{448}$$

2.29)

a) $3^5 \cdot 3^{-5} = 3^5 \cdot \frac{1}{3^5} = 1$

b) $3^6 \cdot 3^{-4} = 3^6 \div 3^4 = 3^2 = 9$

c) $3^{-1} \cdot 3^{-2} = \frac{1}{3} \cdot \frac{1}{9} = \boxed{\frac{1}{27}}$

d) $3^{15} \cdot 3^{-5} \cdot 3^{-4} \cdot 3^{-3}$

$$3^{15} \cdot \frac{1}{3^5} \cdot \frac{1}{3^4} \cdot \frac{1}{3^3} = 3^{15} \cdot \frac{1}{3^{12}}$$

$$= 3^3 = \boxed{27}$$

2.30)

a) $\dfrac{1}{2^{-3}} = 2^3 = 8$

b) $\dfrac{1}{5^{-2}} = 5^2 = 25$

c) $\dfrac{1}{a^{-n}}$ y a^n, $\boxed{\dfrac{1}{a^{-n}} = a^n}$

ya que a^{-n} es el recíproco de

a^n, el recíproco del recíproco

es a^n.

2.31)

a)
$\left(\dfrac{1}{2}\right)^{-1} = 2$

$\left(\dfrac{1}{2}\right)^{-2} = 2^2 = 4$

$\left(\dfrac{1}{2}\right)^{-3} = 2^3 = 8$

b)
$\boxed{\left(\dfrac{1}{a}\right)^{-n} = a^n}$

2.32)

(a) $-3^{-2} = \dfrac{1}{-3^2} = -\dfrac{1}{9}$

e) $\dfrac{1}{(-2)^{-3}} = (-2)^3 = -(2)^3$
$= -8$

(b) $(-3)^{-2} = \dfrac{1}{(-3)^2} = \dfrac{1}{9}$

(c) $(-2)^{-3} = \dfrac{1}{(-2)^3} = -\dfrac{1}{8}$

d) $\dfrac{1}{(-3)^{-2}} = (-3)^2 = 9$

2.33)

(a) 2^{-3} y $(2^{-1})^3$ $2^{-3} = (2^{-1})^3$

$2^{-3} = \dfrac{1}{2^3}$ $\left(\dfrac{1}{2}\right)^3 = \dfrac{1}{2^3} = \dfrac{1}{8}$

b) $2^{-n} = \dfrac{1}{2^n}$, $(2^{-1})^n$

$\left(\dfrac{1}{2}\right)^n = \dfrac{1}{2^n}$

$2^{-n} = (2^{-1})^n$

c) $a^{-n} = \dfrac{1}{a^n}$, $(a^{-1})^n$

$\left(\dfrac{1}{a}\right)^n = \dfrac{1}{a^n}$

$\boxed{a^{-n} = (a^{-1})^n}$

2.34)

$a^{-6}\, b^{-6} = (ab)^{-6}$

$a^{-6}\, b^{-6} = \dfrac{1}{a^6} \cdot \dfrac{1}{b^6} = \dfrac{1}{a^6 \cdot b^6}$

$(ab)^{-6} = \dfrac{1}{(ab)^6} = \dfrac{1}{a^6 \cdot b^6}$

$a^{-6} \cdot b^{-6} = (ab)^{-6}$

2.35)

(a) $-4^{16} = (2 \cdot 2)^{16} = 2^{16} \cdot 2^{16}$
$= 2^{32}$

$(-2)^{34} = 2^{34}$

$16^8 = (2^4)^8 = 2^{32}$

(b) $\left(\dfrac{1}{8}\right)^{-11} = 8^{11} = (2^3)^{11} = 2^{33}$

$(2^{-4})^{-8} = 2^{32}$

c) $\left(\dfrac{1}{8}\right)^{-11} = 2^{33}$

$4^{16} = 2^{32}$

$\boxed{(-2)^{34}} = 2^{34}$

$16^8 = (2^4)^8 = 2^{32}$

$(2^{-4})^{-8} = 2^{32}$

2.36)

$(x^4 y^{-2})(x^{-1} y^5)$

$(x^4 \cdot x^{-1})(y^{-2} \cdot y^5)$

$x^3 \cdot y^3 = (x \cdot y)^3$

Exercises

2.4.1)

(a) $2^{(-1)^{11}} = 2^{-1} = \frac{1}{2}$

(b) $3^7 \cdot 3^{-4} = 3^7 \cdot \frac{1}{3^4} = 3^3 = \boxed{27}$

(c)
$$2^3 \div 2^{-4} = 2^3 \div \frac{1}{2^4}$$
$$= 2^3 \cdot 2^4 = 2^7 = \boxed{128}$$

(d)
$$1 \div 5^{-2} = 1 \div \frac{1}{5^2}$$
$$= 1 \cdot 5^2 = \boxed{25}$$

(e)
$$(-3)^{-5} \cdot 3^3 = \frac{1}{(-3)^5} \cdot 3^3 = -\frac{1}{3^5} \cdot 3^3$$
$$= -\left(3^3 \div 3^5\right)$$
$$= -\left(3^{-2}\right) = -\frac{1}{9}$$

(f)
$$\left(\frac{1}{4}\right)^{-3} \cdot 8^{-2} = 4^3 \cdot \frac{1}{8^2}$$
$$= 4^3 \cdot \frac{1}{4^2 \cdot 2^2}$$
$$= 4 \cdot \frac{1}{2^2} = \boxed{1}$$

2.4.2)
$$2 \div 2^{-4} = 2 \div \frac{1}{2^4}$$
$$= 2 \cdot 2^4 = 2^5 = \boxed{32}$$

2.4.4) $243 = 3^5$

$$3^3 + 3^3 + 3^3 = 243 \cdot 3^k$$
$$3^3(3) = 3^4$$
$$3^4 = 3^5 \cdot 3^{-1}$$
$$k = \boxed{-1}$$

2.4.3) $15(1)^2(-1)^{-3} + 18(-1)(1)^{-1} + 27(1)(-1)^4$

$15 \cdot \frac{1}{(-1)^3} + 18(-1)\left(\frac{1}{1}\right) + 27(1)(1) = -15 - 18 + 27$
$$= \boxed{-6}$$

$$2^{12} = \left(2^3\right)^4 = 8^4 = \left(\frac{1}{8}\right)^4$$

$$2^{12} = \left(\frac{1}{8}\right)^{-4}$$

$$\left(6a^2 b\right)^2 \div 3a^2 b^3 = 6^2 a^4 b^2 \div 3a^2 b^3$$

$$= 36 a^4 b^2 \div 3a^2 b^3$$

$$= \boxed{12 a^2 b^{-1}}$$