Usamos el sistema base 10 para representar números. Es decir que usamos 10 símbolos diferentes (0 al 9).

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

Si queremos seguir sumando a 9, usamos el 10.

$$10 = 1 \cdot 10^1 + 0 \cdot 10^0.$$

Eso quiere decir que la posición del dígito nos indica si contamos unidades, grupos de 10, grupos de 100, etc.

$$\begin{aligned} 572 &= 500 + 70 + 2 \\ &= (5 \cdot 100) + (7 \cdot 10) + (2 \cdot 1) \\ &= (5 \cdot 10^2) + (7 \cdot 10^1) + (2 \cdot 10^0). \end{aligned}$$

> **Idea:** los números son representados con potencias de 10.

los decimales usan la extensión de esta misma idea a la parte no entera del número:

$$29.17 = \underbrace{29}_{\text{Parte Entera}} + 0.17 \quad \text{Parte Decimal (entre 0 y 1)}$$

29.17 significa "2 dieces, 9 unos, 1 décimo, 7 centécimos."

$$\begin{aligned} 29.17 &= 20 + 9 + 0.1 + 0.07 \\ &= (2 \cdot 10^1) + (9 \cdot 10^0) + (1 \cdot 10^{-1}) + (7 \cdot 10^{-2}) \end{aligned}$$

Problemas (Personal)

5.1) Calcular las siguientes cantidades:

(a) $2.6 + 3.1$. La idea es la misma que al sumar enteros. Se suman las posiciones correspondientes.

$$\begin{aligned} 2.6 + 3.1 &= 2 \cdot 10^0 + 6 \cdot 10^{-1} + 3 \cdot 10^0 + 1 \cdot 10^{-1} \\ &= (2 \cdot 10^0 + 3 \cdot 10^0) + (6 \cdot 10^{-1} + 1 \cdot 10^{-1}) \\ &= 5 \cdot 10^0 + 7 \cdot 10^{-1} \\ &= \boxed{5.7} \end{aligned}$$

(b) $$\begin{aligned} 13.9 + 2.37 &= 1 \cdot 10^1 + 3 \cdot 10^0 + 9 \cdot 10^{-1} + 2 \cdot 10^0 + 3 \cdot 10^{-1} + 7 \cdot 10^{-2} \\ &= 1 \cdot 10^1 + 5 \cdot 10^0 + \underbrace{12 \cdot 10^{-1}} + 7 \cdot 10^{-2} \end{aligned}$$

Importante: Se desborda, hay una forma más directa de escribirlo: $12 \cdot 10^{-1} = (10+2) \cdot 10^{-1} = 10^0 + 2 \cdot 10^{-1}$

$$= 1 + 2 \cdot 10^{-1}$$

$= 1 \cdot 10^1 + 5 \cdot 10^0 + 1 \cdot 10^0 + 2 \cdot 10^{-1} + 7 \cdot 10^{-2}$

$= 1 \cdot 10^1 + 6 \cdot 10^0 + 2 \cdot 10^{-1} + 7 \cdot 10^{-2}$

$= \boxed{16.27}$

(c) $0.002 + 0.4 = \boxed{0.402}$

(d) $123.8 + 5.2 = \boxed{129}$

(e) $3 - 0.27 = \boxed{2.73}$

(f) $0.135 - 0.28 = 1 \times 10^{-1} + 3 \times 10^{-2} + 5 \times 10^{-3} - (2 \times 10^{-1} + 8 \times 10^{-2})$

$= -1 \times 10^{-1} - 5 \times 10^{-2} + 5 \times 10^{-3}$

$= -(1 \times 10^{-1} + 5 \times 10^{-2} - 5 \times 10^{-3})$

$= -(1 \times 10^{-1} + 4 \cdot 10^{-2} + 5 \times 10^{-3})$

$= \boxed{-0.145}$

6.2)

(a) $2.59 \cdot 100 = (2 \times 10^0 + 5 \times 10^{-1} + 9 \times 10^{-2}) \cdot 10^2$

$= 2 \times 10^2 + 5 \times 10^1 + 9 \times 10^0$

$= \boxed{259.}$

Idea: al multiplicar por una potencia de 10, se "corre" el punto.

(b) $36.7 \div 1000 = (3 \times 10^1 + 6 \times 10^0 + 7 \times 10^{-1}) \cdot 10^{-3}$

$= 3 \times 10^{-2} + 6 \times 10^{-3} + 7 \times 10^{-4}$

$= \boxed{0.0367.}$

(c) $0.0028 \cdot 1000 = \boxed{2.8}$

6.3)

(a) $3.1 \times 5 = 15.5$

$$\begin{array}{r} 3.1 \\ \times\ 5 \\ \hline 15.5 \end{array}$$

(b) $2.9 \cdot 1.3$ $\quad 3.77$

$$\begin{array}{r} {\scriptstyle 2} \\ 2.9 \\ \times 1.3 \\ \hline 87 \\ 29 \\ \hline 3.77 \end{array}$$

(c) $0.002 \cdot 0.003$

$$2 \times 10^{-3} \cdot 3 \times 10^{-3} = 6 \cdot 10^{-6}$$
$$= 0.000006$$

0.000006

(d) $0.11 \cdot 0.15 = (1 \times 10^{-1} + 1 \times 10^{-2}) \cdot (1 \times 10^{-1} + 5 \times 10^{-2})$

$$\begin{array}{r} 0.11 \\ \times 0.15 \\ \hline 055 \\ 011 \\ 000 \\ \hline 0.0165 \end{array}$$

$$= 1 \times 10^{-2} + 1 \times 10^{-3} + 5 \times 10^{-3} + 5 \times 10^{-4}$$
$$= 1 \times 10^{-2} + 6 \times 10^{-3} + 5 \times 10^{-4}$$
$$= 0.0165 \qquad 0.0165$$

> Idea: Para revisar si la multiplicación tiene sentido, podemos comparar que $m \cdot n$ esté entre m^2 y n^2 (si $n > m$).

(e) $0.48 \div 0.06 = 48 \times 10^{-2} \div 6 \times 10^{-2}$

$$= 48 \times \frac{1}{10^2} \cdot \frac{1}{6} \cdot 10^2 = 8$$

(f) $0.48 \div 0.6 = 48 \times 10^{-2} \div 6 \times 10^{-1}$

$$= 48 \cdot \frac{1}{100} \div \left(6 \cdot \frac{1}{10}\right) = \frac{\cancel{48}^{\,4}}{\cancel{100}_{\,5}} \cdot \frac{\cancel{10}}{\cancel{6}_{\,1}}$$

$$= \frac{4}{5} = \frac{8}{10} = 8 \times 10^{-1}$$
$$= 0.8$$

(g) $0.001 \div 0.0001 = 1 \times 10^{-3} \div 1 \times 10^{-4}$

$$= \frac{1}{1000} \div \frac{1}{10000} = \frac{1}{\cancel{1000}} \cdot \cancel{10000}$$
$$= 10 \qquad 10$$

(h) $100 \div 0.25 = 100 \div \frac{1}{4} = 100 \cdot 4 = 400$

$$\begin{array}{r} {\scriptstyle 4} \\ 17 \\ \times 17 \\ \hline 119 \\ 17 \\ \hline 289 \end{array}$$

6.4)

0.5, 0.505, 0.55, 0.555, 0.06, ~~0.6~~, ~~0.65~~, ~~0.56~~, 0.005

De mayor a menor: 0.65, 0.6, 0.56, 0.555, 0.55, 0.505, 0.5, 0.06, 0.005

6.5) (a) $(0.2)^2 = \left(2 \times 10^{-1}\right)^2 = 4 \times 10^{-2} = 0.04$ $\quad 0.04$

(b) $(1.7)^2 = \left(17 \times 10^{-1}\right)^2 = 289 \times 10^{-2} = 2.89$ $\quad 2.89$

(c) $\frac{1}{(0.2)} = 1 \div \frac{1}{5} \approx 1 \cdot 5 = 5$

(d) $(0.03)^3 = (3 \times 10^{-2})^3$

$= 27 \times 10^{-6}$

$= 0,000027$

Ejercicios

6.1.1) 0.99 , 0.9099 , 0.9 , 0.909 , 0.9009

5 4 1 3 2

6.1.2) 0.54321 . 5

6.1.3)

(a) $0.4 + 0.02 + 0.006 = 0.426$

(b) $0.92 + 0.093 = 1.013$

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 0.920 \\ +\ 0.093 \\ \hline 1.013 \end{array}$$

(c) $1.28 - 0.377$

$$\begin{array}{r} {\scriptstyle 0\ 1\ 7\ 1\ \ } \\ 1.280 \\ -\ 0.377 \\ \hline 0.903 \end{array}$$

0.903

(d) $8 - 1.001$

$$\begin{array}{r} {\scriptstyle 7\ 9\ 9\ 1\ \ } \\ 8.000 \\ -\ 1.001 \\ \hline 6.999 \end{array}$$

6.999

(e) $0.0006 - 0.002$

$-(0.002 - 0.0006)$

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ } \\ 0.0020 \\ -\ 0.0006 \\ \hline 0.0014 \end{array}$$

-0.0014

(f) $1.1 - 0.11 + 0.011$

$1.1 + 0.011 - 0.11$

$1.111 - 0.11 =$ $\begin{array}{r} 1.111 \\ -\ 0.110 \\ \hline 1.001 \end{array}$

1.001

6.1.4)

(a) $23.879 \cdot 100 = 2387.9$

(b) $2 \div 10^5 = 2 \cdot 10^{-5} = 0.00002$

(c) $1.6 \div 400 = 16 \times 10^{-1} \cdot \frac{1}{4 \cdot 10^2}$

$= 4 \times 10^{-1} \cdot 10^{-2}$

$= 4 \times 10^{-3} = 0.004$

0.004

(d) $0.0031 \cdot 10^6 = 3100$

3100

(e) $3.6 \div 0.04 = 36 \times 10^{-1} \div (9 \times 10^{-2})$

$= 36 \times 10^{-1} \cdot \frac{1}{9} \cdot 10^2$

$= 4 \times 10^1 = 40$

(f) $1.01 \cdot 3.03 = 101 \times 10^{-2} \cdot (303 \times 10^{-2})$

$= 30603 \times 10^{-4}$

$= 3.0603$

$$\begin{array}{r} 303 \\ \times\ 101 \\ \hline 303 \\ 060 \\ 303 \\ \hline 30603 \end{array}$$

6.1.5) $(250+25+2.5+0.25+0.025) \div (50+5+0.5+0.05+0.005)$

$25\times10^{1} + 25\times10^{0} + 25\times10^{-1} + 25\times10^{-2} + 25\times10^{-3} \div \left(5\times10^{1} + 5\times10^{0} + 5\times10^{-1} + 5\times10^{-2} + 5\times10^{-3}\right)$

$= 25\left(10^{1} + 10^{0} + 10^{-1} + 10^{-2} + 10^{-3}\right) \div 5\left(10^{1} + 10^{0} + 10^{-1} + 10^{-2} + 10^{-3}\right)$

$= 5$

6.1.6) $X = 0.\underbrace{00\ldots001}_{5000 \text{ zeros}}$ $\qquad 2+x,\ 2-x,\ 2x,\ \frac{2}{x},\ \frac{x}{2}$

$x = 10^{-5001} = \frac{1}{10^{5001}}$

(5) $\cdot \frac{2}{x} = 2\cdot10^{5001}$

(1) $\cdot \frac{x}{2} = \frac{1}{10^{5001}}\cdot\frac{1}{2} = \frac{1}{2\times10^{5001}}$

(2) $\cdot 2x = \frac{2}{10^{5001}}$

(4) $\cdot 2+x = 2+\frac{1}{10^{5001}}$

(3) $\cdot 2-x = 2-\frac{1}{10^{5001}}$

$\frac{x}{2},\ 2x,\ 2-x,\ 2+x,\ \frac{2}{x}$

6.1.7) $0.075 \cdot 2.56 = 75\cdot256\cdot10^{-3}\cdot10^{-2} = 19200\cdot10^{-5}$

$75\cdot256 = 19200$

$= 0.192$

6.1.8) $1000\cdot1993\cdot0.1993\cdot10 = 1993\cdot0.1993\times10^{4}$

$= 1993\cdot1993\times10^{-4}\times10^{4}$

$= 1993^{2}$

- $1.993\cdot10^{3} = 1993$ No
- 1993.1993 No
- $(199.3)^{2}$ No
- $(1993)^{2}$
- No

6.1.9) $0.0481\cdot10^{-4} = 4.81\cdot N$

$0.0481\cdot10^{-4} = 10^{-4}\cdot10^{-2}\cdot4.81$

$= 4.81\cdot10^{-6}$ $\qquad N = 10^{-6}$

6.1.10) $X\cdot10^{1} = (X\cdot10^{-1}) + 33.66$

$10X = \frac{X}{10} + 3366\times10^{-2}$

$X\left(10-\frac{1}{10}\right) = 3366\times10^{-2}$

$X\left(\frac{99}{10}\right) = 3366\cdot10^{-2}$

$X = \frac{3366\cdot10^{-2}\cdot10^{1}}{99}$

$X = 34\cdot10^{-1}$

$X = 3.4$