CSIT113
Problem Solving

TUTORIAL 2– FOR UNIT 5 AND 6
INDUCTION, GREEDY APPROACH AND DIVIDE-AND-CONQUER AND REDUCE-AND-CONQUER

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Question 1

Consider the instance of discrete knapsack problem with the knapsack capacity 10 and the item information as follows:

Item	Weight	Value
1	7	\$42
2	3	\$12
3	4	\$40
4	5	\$25

Find the most valuable subset of the items that fits into the knapsack.

Question 1 - Answer

Strategy 1: The highest value-to-weight is the best

Compute the value-to-weight ratio and arrange the items in non-increasing order according to these ratios, we have the following:

Item	Weight	Value	Value/Weight
3	4	$40	10
1	7	$42	6
4	5	$25	5
2	3	$12	4

Using greedy approach based on this strategy 1, we select items as follows:

Step 1: Select item 3, so the accumulated weight = 4.

Step 2: Skip item 1 (too heavy) and select item 4 so the accumulated weight = 9.

Step 3: Skip item 2 (too heavy) and select nothing as there is nothing left

So, under strategy 1, we select item 3 and 4 which give a total weight of 9 and total value of 65.

Question 1 - Answer

Strategy 2: The highest value is the best

We arrange the items in non-increasing order according to their values, we have the following:

Item	Weight	Value
1	7	$42
3	4	$40
4	5	$25
2	3	$12

Using greedy approach based on this strategy 2, we will select items as follows:

Step 1: Select item 1, so the accumulated weight = 7.

Step 2: Skip item 3 and 4 (too heavy) and select item 2, so the accumulated weight = 10.

So, under strategy 2, we select item 1 and 2 which give a total weight of 10 and total value of 54.

As strategy 1 achieves a higher value, therefore, we follow the result of strategy 1. That is, we select item 3 and 4 which give a total weight of 9 and total value of 65.

Question 2

Consider the instance of continuous knapsack problem with the knapsack capacity 10 and the item information as follows:

Item	Weight	Value
1	7	$42
2	3	$12
3	4	$40
4	5	$25

Find the most valuable subset of the items that fits into the knapsack.

Question 2 - Answer

As it is a continuous knapsack, the strategy should be the highest value-to-weight is the best.

So, we compute the value-to-weight ratio and arrange the items in non-increasing order according to these ratios, we have the following:

Item	Weight	Value	Value/Weight
3	4	\$40	10
1	7	\$42	6
4	5	\$25	5
2	3	\$12	4

Using greedy approach based on this strategy, we select items as follows:

Step 1: Select item 3, so the accumulated weight = 4.

Step 2: Select part of item 1 - 6 out of 7 - which has a value = $6\times6 = 36$. So, the accumulated weight = 10.

Therefore, we select item 3 and part of item 1 - 6 out of 7. And, the total weight is 10 and the total value is 76.

Question 3

A factory produces custom widgets according to customer requirements. It has several orders, each for a single widget. For each order, we know the following information:

i)The profit that the factory will make from the order.
ii) The deadline – the last day on which the customer will accept delivery.

The factory can make exactly one widget per day. The following table shows all the orders for a 4-day period:

Order No.	1	2	3	4	5	6	7
Profit	200	400	1800	800	1000	9000	1400
Deadline	4	1	1	4	3	2	2

Use Greedy Approach to determine the sequence that the factory should accept and the resulting schedule for producing the widgets of the accepted orders. Explain the result for each step in using the approach.

Question 3 - Answer

First step is to sort the orders by their profits in non-increasing order as follows:

Order No.	6	3	7	5	4	2	1
Profit	9000	1800	1400	1000	800	400	200
Deadline	2	1	2	3	4	1	4

Next, consider Order No 6, since it is the first order considered, we fit in the following schedule as latest as possible:

Day.	1	2	3	4
Order No.		6		
Profit		9000		

Question 3 – Answer (cont'd)

Next, consider Order No. 3 and fit it in the following schedule as latest as possible:

Day.	1	2	3	4
Order No.	3	6		
Profit	1800	9000		

Next, consider Order No 7, but we cannot fit into the schedule.

Next, consider Order No 5 and fit it in the following schedule as latest as possible:

Day.	1	2	3	4
Order No.	3	6	5	
Profit	1800	9000	1000	

Order No.	6	3	7	5	4	2	1
Profit	9000	1800	1400	1000	800	400	200
Deadline	2	1	2	3	4	1	4

Question 3 – Answer (cont'd)

Next, consider Order No. 4 and fit it in the following schedule as latest as possible:

Day.	1	2	3	4
Order No.	3	6	5	4
Profit	1800	9000	1000	800

The schedule is now full, we stop. Hence, the total profit that the factory can achieve is 12,600. The above table shows the accepted orders and the schedule.

Note that the sequence that the orders are accepted is:

6, 3, 5, 4

And, the sequence that the orders are schedule is:

3, 6, 5, 4

If we do not schedule as late as possible, the total profit is only 12,200 (please work it works the details for this).

Order No.	6	3	7	5	4	2	1
Profit	9000	1800	1400	1000	800	400	200
Deadline	2	1	2	3	4	1	4

Question 4

Design a divide-and-conquer algorithm for finding the position (index) of the largest element in a sequence of n numbers.

Answer:

```
maxIndexDR(A, $f$, $l$) {
//Input: A portion of sequence A between indices $f$ and $l$ ($f \le l$)
//Output: The index of the largest element in the above portion

    if $f = l$
        return $l$
    else{
        temp1 = maxIndexDR(A, $f$, $\lfloor (f + l)/2 \rfloor$)
        temp2 = maxIndexDR(A, $\lfloor (f + l)/2 \rfloor + 1$, $l$)
      }
    if A[temp1] $\ge$ A[temp2]
        return temp1
    else
        return temp2
}
```

Question 5

Design a reduce-and-conquer algorithm for finding the position (index) of the largest element in a sequence of n numbers.

Answer:

```
maxIndexRR(A, f, l)  {
//Input: A portion of sequence A between indices f and l (f ≤ l)
//Output: The index of the largest element in the above portion


    if f = l
            return l
    else {
            temp = maxIndexRR(A, f, l-1)
            if A[temp] > A[l]
                    return temp
            else
                    return l
    }
}
```