CSIT113 Problem Solving

TUTORIAL 5 – FOR UNIT 9 AND 10 BACKTRACKING AND BRANCH AND BOUND

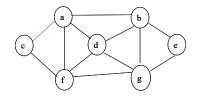
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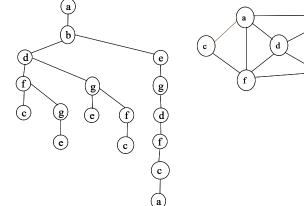
Question 1

Apply the backtracking problem-solving technique to find a Hamiltonian circuit in the following graph:



If multiple vertices are suitable, they are to be selected in alphabetical order,

Question 1 - Answer



The Hamiltonian circuit found is (a, b, e, g, d, f, c, a)

Question 2

Use backtracking to find all the subsets of numbers in the following sequence with sum =12:

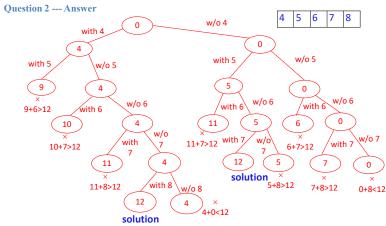
4 5 6 7 8

The elements in the sequence are sorted in non-decreasing order.

Answer:

We shall name a state using the sum of the elements in the subset formed.

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From the state-space tree, the required subsets are: {4, 8} and {5, 7}

Question 3 - Answer 2 3 start $UB = \frac{19}{9}$ 9 2 3 7 Mary Lb = 3+2+2 = 7Lucy 3 4 2 Kelly 5 3 8 $M \rightarrow 3$ M**→**1 $M \rightarrow 2$ Lb = 7 + 3 + 2 = 12Lb = 2 + 3 + 2 = 7Lb = 3 + 3 + 3 = 9* (Lb > UB (9)) $L \rightarrow 1, K \rightarrow 2$ $L\rightarrow 2, K\rightarrow 1$ $L\rightarrow 1, K\rightarrow 3$ $L\rightarrow 3, K\rightarrow 1$ Lb=3+4+5=12Lb=2+3+8=13Lb = 3 + 3 + 3 = 9Lb = 2 + 2 + 5 = 9* (Lb > UB (9)) Solution * (Lb > UB (9)) **Solution**

Both node 4 and 6 yields the lowest cost. There are two possible solutions in this case and they are:

- Assigning task 2 to Mary, task 3 to Lucy and task 1 to Kelly with a total cost of 9.
- Assigning task 3 to Mary, task 1 to Lucy and task 2 to Kelly with a total cost of 9.

Question 3

We need to assign to three (3) persons three (3) tasks so that the total cost of assignments is as small as possible. We are allowed to assign one and only one task to each person. The table below shows the cost of a person completing a particular task.

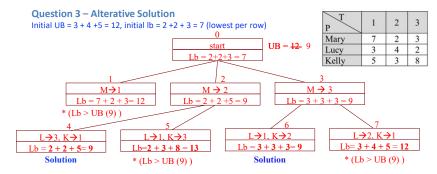
Task Person	1	2	3
Mary	7	2	3
Lucy	3	4	2
Kelly	5	3	8

Find all the possible assignments using the "branch and bound" strategy such that the total cost is minimum. For each node, clearly state what the lower bounds are and clearly mark with "*" any node that is to be ruled out.

Answer:

Initial UB = 7 + 4 + 8 = 19, initial lb = 3 + 2 + 2 = 7 (lowest per column)

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Question 4

The following tables show the weights and values of four items. The capacity of the knapsack is 16:

Item	Weight	Value
1	10	\$100
2	7	\$63
3	8	\$56
4	4	\$12

Find the most valuable subset of the items that fit in the knapsack using branch-and-bound.

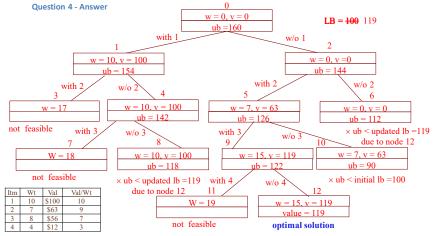
Answer:

First, we order the item in non-increasing value/weight ratios:

Item	Weight	Value	Value/Weight
1	10	\$100	10
2	7	\$63	9
3	8	\$56	7
4	4	\$12	3

Initially, the lower bound is set to 100 just include the most valuable (one) that is within the capacity – item 1. Upper bound is set to $16 \times 10 = 160$.

A node is "not feasible" if it exceeds the knapsack capacity.



Therefore, the optimal solution is the subset that contains item 2 and 3.