CSIT113
Problem Solving

TUTORIAL 1 – FOR UNIT 1, 2, 3 AND 4
INTRODUCTION, BRUTE-FORCE VERSUS FINESSE, REASONING WITH LOGIC, INDUCTION AND WICKED PROBLEMS

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SIM GLOBAL EDUCATION

Question 1

Write an algorithm to find the sum of all the numbers in a sequence s of n numbers.

Answer:

```
sequence_Sum (s, n) {
    sum = 0
    for (i = 0 to (n-1) ) {
            sum = sum +s[i]
    }
    return sum
}
```


Question 2

Based on the notations designed discussed in Unit 2 Problem 2 for Jealous Husbands problem, calculate the total number of states.

Answer:

Note that the total number of states is the total number of possible values of the state notation used (including valid and invalid values).

Hence, total no of states = 4×4×4×4 = 256.

Question 3

Show that $p\vee(q\wedge r) \equiv (p\vee q)\wedge(p\vee r)$.

Answer:

We construct the truth table to prove it as follows:

p	q	r	$q\wedge r$	$p\vee(q\wedge r)$	$p\vee q$	$p\vee r$	$(p\vee q)\wedge(p\vee r)$	$p\vee(q\wedge r) \equiv (p\vee q)\wedge(p\vee r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

From the truth table, the truth value for $p\vee(q\wedge r) \equiv (p\vee q)\wedge(p\vee r)$ is always true, therefore, $p\vee(q\wedge r) \equiv (p\vee q)\wedge(p\vee r)$.

Question 4

Show that ~(p∧q) ≡ ~p∨~q.

Answer:

We construct the truth table to prove it as follows:

p	q	p∧q	~ (p∧q)	~p	~q	~p∨~q	~(p∧q) ≡ ~p∨~q
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

From the truth table, the truth value for ~(p∧q) ≡ ~p∨~q is always true, therefore, ~(p∧q) ≡ ~p∨~q.

Question 5

Prove or disprove that (p⇒q)∧q ⇒ p.

Answer:

We construct the truth table to determine the validity of (p⇒q)∧q ⇒ p as follows:

p	q	p⇒q	(p⇒q)∧q	(p⇒q)∧q ⇒ p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

In the truth table, the truth value of (p⇒q)∧q ⇒ p for the third row is false. Hence, (p⇒q)∧q ⇒ p is not always true. Therefore, (p⇒q)∧q ⇒ p is not valid.

Question 6

On Knights and Knave Island, all natives are either knights, who always tell the truth, or knaves, who always tell lies. You meet two islanders, John and Mary, who make the following statements:

John: "Two of us are both knights."
Mary: "John is a knave."

Identify the types of each of them.

Answer:

Let J represents the proposition, "John is a knight."
M represents the proposition, "Mary is a knight."

From John and Mary statements, we have:

J ≡ (J ∧ M)
M ≡ ~J

This implies J ≡ (J ∧ M) ≡ (J ∧ ~J) = false

And, M ≡ ~J ≡ ~false ≡ true

Therefore, Mary is a knight and John is a knave

Question 7

On Knights and Knave Island, all natives are either knights, who always tell the truth, or knaves, who always tell lies.

You meet two islanders, Alice and Bob, standing next to each other. Suddenly, Alice makes the following statement:
Alice: "At least one of us is a knave."

Identify the type of each of them.

Answer:
Solution 1: Using truth table
Let A represents the proposition, "Alice is a knight."
B represents the proposition, "Bob is a knight."
Then, from Alice statement, we have A ≡ ~A∨~B.

We draw the following truth table

A	B	~A	~B	~A∨~B	A ≡ ~A∨~B
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	T	F

Since from the natives statements, we conclude that A ≡ ~A∨~B and only the second row in the truth table A ≡ ~A∨~B is true, only the second case (second row) in the truth table is possible. In this row, A = T and B = F, therefore, Alice is a knight, Bob is a knave.

Question 7

On Knights and Knave Island, all natives are either knights, who always tell the truth, or knaves, who always tell lies.

You meet two islanders, Alice and Bob, standing next to each other. Suddenly, Alice makes the following statement:

Alice: "At least of us is a knave."

Identify the type of each of them.

Answer:

Solution 2: Using calculational logic

Let A represents the proposition, "Alice is a knight."
B represents the proposition, "Bob is a knight."
Then, from Alice statement, we have A ≡ ~A∨~B.

Clearly, A ≡ A ∨A
≡ A ∨ (~A∨~B) (substitute A ≡ ~A∨~B)
≡ (A ∨ ~A)∨~B
≡ true ∨~B = true

Substitute A = true into A ≡ ~A∨~B we get:
true = ~true∨~B
true = false∨~B = ~B
B = false

Therefore, Alice is a knight, Bob is a knave.

Question 8

Use inductive problem-solving strategy to design an recursive algorithm for computing the product of all the numbers in a sequence.

Answer:

Recursive Algorithm:

```
ProductR(A, n) {
   if n = 1
     product = A[0]
   else
     product = productR(A, n-1) ×A[n-1]
   return product
}
```


Question 9

Use mathematical induction to prove the following equation is true for every positive integer n:

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1$$

Proof:

Basis step: When n = 1,

LHS $=\sum_{i=1}^{1} i\,(i!)$ = 1×(1!) = 1×1 = 1

RHS = (1+1)! – 1 = 2! - 1 = 2 – 1 = 1

Thus, LHS = RHS

Hence, the formula holds

Question 9

Proof (Cont'd):

Inductive step: Assume that the formula holds when n = k. That is, we have:

$$\sum_{i=1}^{k} i(i!) = (k+1)! - 1$$

Then, when n = k +1,

LHS $= \sum_{i=1}^{k+1} i(i!) = 1(1!) + 2(2!) + 3(3!) + \ldots\ldots + k(k!) + (k+1)[(k+1)!]$

$= [1(1!) + 2(2!) + 3(3!) + \ldots\ldots + k(k!)] + (k+1)[(k+1)!]$

$= \sum_{i=1}^{k} i(i!) + (k+1)(k+1)!$

= *(k+1)! -1 + (k+1)(k+1)!* *from the assumption*

= *1.(k+1)! + (k+1)(k+1)! -1*

= *(k+1)!(1+ k + 1) -1*

= *(k+2)! -1*

= *((k+1) +1)! - 1*

= RHS

Hence, the formula holds.

Therefore, the formula holds.

Question 10

Use mathematical induction to prove the following equation is true for every positive integer n:

$$(1 + x)^n \geq 1 + nx$$

where x > -1

Proof:

Basis step: When n = 1,

LHS = $(1+ x)^1 = 1 + x$

RHS = $1 + 1 \cdot x = 1 + x$

Since $1+ x \geq 1 + x$, LHS ≥ RHS

Hence, the formula holds

Question 10

Proof (Cont'd):

Inductive step: Assume that the formula holds when n = k. That is, we have:

$$(1+x)^k \geq 1 + kx$$

Then, when n = k +1,

LHS = $(1+x)^{k+1} = (1+x)^k (1+x)$

$\geq (1 + kx) (1+x)$ from the assumption and x > -1

$= 1 + x + kx + kx^2$

$\geq 1 + x + kx$

$= 1 + (1+k)x$

$= 1+ (k+1)x$

= RHS

Hence, the formula holds.

Therefore, the formula holds.