1. Design a reduce-and-conquer algorithm for finding the position (index) of the smallest element in an array of n numbers.

2. Design a divide-and-conquer algorithm for finding the position (index) of the smallest element in an array of n numbers.

3. Parts a and b both refer to the following sequence:

   7, 2, 5, 8, 6, 1, 4, 3

   a) Sort the sequence from smallest to largest using selection sort. Show each step on a new line, underline the sorted part of the array.

   b) Sort the sequence from smallest to largest using quicksort. Show each step on a new line.

4. Prove by induction that $1+4+16+\ldots+4^{n-1} = (4^n-1)/3$

5. On Knights and Knaves Island, all natives are either knights, who always tell the truth, or knaves, who always tell lies.

   You meet three islanders, Alice, Bob and Carol, who make the following statements:

   Alice: "Bob and Carol are of the same type."
   Bob: "Carol is a knave."

   a) Is Alice a knight or a knave?

   b) What question can you ask Carol to identify exactly what Bob is?

6. The following numbers are inserted, in order, into a binary search tree:

   34, 44, 50, 22, 11, 19, 7, 12, 31

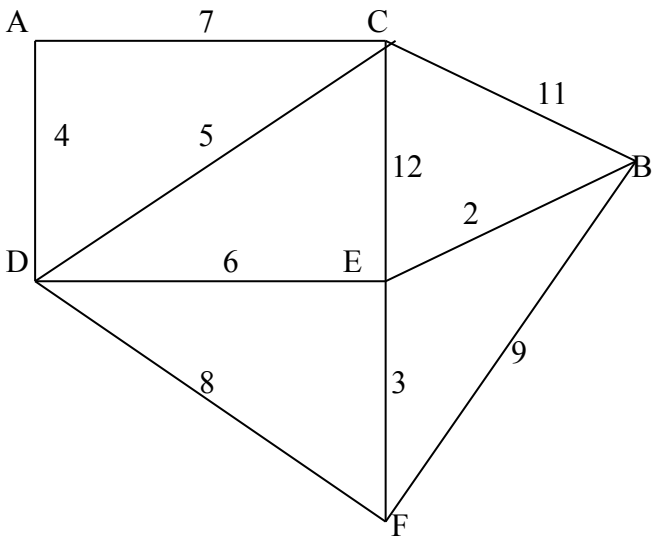
   a) Draw the resulting tree.

   b) What is the depth of the node containing the value 11?

   c) Draw the tree after the deletion of the node with value 22.

   d) Someone suggests that it would be a good idea to sort the numbers before building the tree. Is this really a good idea? Explain your answer.

7. Consider the following weighted graph.



a) Is the graph connected?

b) Is the graph directed?

c) List, in order, the edges added to the solution if we use the Prim's algorithm to construct the minimal spanning tree for this graph (specify an edge by providing the nodes at each end – e.g. A–D is the edge from node A to node D). Start with node A.

8. A factory produces widgets. It has several orders, each for a single widget. For each order, we know the following information:

i) The profit that this widget will make for the factory.
ii) The deadline – the last day on which the customer will accept delivery.

The factory can make exactly one widget per day.

The following table shows all the orders for a 5-day period.

Order#	1	2	3	4	5	6	7	8	9	10
Profit	42	59	54	95	84	66	33	80	64	75
Deadline	2	5	3	4	5	3	2	4	1	2

a) What sequence of widgets should be made to allow the company to make the greatest profit?

b) Explain the strategy you used to get this answer?

9. Three coins are arranged, from left to right as follows, where H means the coin is showing heads and T means the coin is showing tails:

H T H

We can turn coins over according to the following rules:

i) The centre coin can be turned at any time.
ii) An end coin can only be turned if the other two coins are both showing the same face as each other.

We want to turn coins, following these rules, to get the pattern:

T H T

a) Construct the game graph for this problem.

b) List a sequence of valid moves that will lead from the start to the end position.

10. A factory has four workers, Anne, Bob, Carla and Dave. It also has 4 tasks which must be carried out. Each worker can carry out all the tasks but with varying efficiency. The cost for each worker to carry out each task is given in the following table:

	Task 1	Task 2	Task 3	Task 4
Anne	$5.00	$3.00	$2.00	$8.00
Bob	$6.00	$7.00	$5.00	$9.00
Carla	$3.00	$2.00	$3.00	$6.00
Dave	$4.00	$5.00	$4.00	$7.00

Each task must be completed by a different worker.

a) Calculate a lower bound on the cost of completing the four tasks.

b) Is this a feasible solution?

c) Calculate an upper bound on the cost of completing the four tasks.

d) Is this solution optimal?

e) Explain how a branch and bound strategy could be used to find the most cost efficient allocation of tasks to workers. Do not attempt to solve the problem.