Question 8 [16 marks]:

Prove by Mathematical Induction that for all n ≥ 1, the sum of the squares of the first 2n positive integers is given by the following formula:

$$1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$$

Basic Step: when n = 1,

LHS = $1^2 + 2^2 = 5$ RHS = $\frac{(1)(2+1)(3+1)}{3} = \frac{(1)(3)(5)}{3} = 5$

Thus, LHS = RHS, formula holds.

Inductive step: Assume formula holds when n = k

$1^2 + 2^2+3^2 + ... + (2k)^2 = \frac{(k)(2k+1)(4k+1)}{3}$

Thus, when n = k+1,

LHS $= 1^2 + 2^2+3^2 + ... + (2(K+1))^2$

$= 1^2 + 2^2+3^2 + ... + (2k+2)^2$

$= 1^2 + 2^2+3^2 + ... +(2k)^2 + (2k+1)^2 + (2k+2)^2$

$= \frac{1}{3}(k)(2k+1)(4k+1) + (2k+1)^2 + (2k+2)^2$

$= \frac{1}{3}[(8k^3+6k^2+k) +3(4k^2+4k+1) +3(4k^2+8k+4)]$

$= \frac{1}{3}[8k^3+6k^2+k+12k^2+12k+3+12k^2+24k+12]$

$= \frac{1}{3}[8k^3+30k^2+37k+15]$

$= \frac{1}{3}(k+1)(8k^2+22k+15)$

$= \frac{1}{3}(k+1)(2k+3)(4k+5)$

$= \frac{1}{3}(k+1)(2(k+1)+1)(4(k+1)+1)$

=RHS

Hence, the formula holds.