**Question 8 [16 marks]:**

Prove by Mathematical Induction that for all n ≥ 1, the sum of the squares of the first 2n positive integers is given by the following formula:

12 + 22 + 32 + ⋯ + (2𝑛)2 = 𝑛(2𝑛 + 1)(4𝑛 + 1)

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Basic Step: when n = 1,

LHS = 12 + 22 = 5 RHS = = = 5

Thus, LHS = RHS, formula holds.

Inductive step: Assume formula holds when n = k

12 + 22+32 + …. + (2k)2 =

Thus, when n = k+1,

LHS = 12 + 22+32 + …. + (2(K+1))2

= 12 + 22+32 + …. + (2k +2)2

= 12 + 22+32 + …. +(2k)2 + (2k+1)2 + (2k+2)2

= (k)(2k+1) (4k+1) + (2k+1)2 + (2k+2)2

= [(8k3+6k2+k) +3(4k2+4k+1) +3(4k2+8k+4)]

= [8k3+6k2+k+12k2+12k+3+12k2+24k+12]

= [8k3+30k2+37k+15]

= (k+1) (8k2+22k+15)

= (k+1) (2k+3) (4k+5)

= (k+1) (2(k+1) +1) (4(k+1) +1)

=RHS

Hence, the formula holds.