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"Solution"

Class: FYMCA
Course Code: MCA 25
Subject: **Probability and Statistics**

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Q. No.		Max Marks
1. (a)	Theory Define events as follows E_i= "Job is from MIDC$_i$, i=1 for Thane, =2 for Taloja and =3 for Andheri" and A= "Job requires set up" Then total probability P(A) is given by $P(A)= P(A/E_1).P(E_1)+ P(A/E_2).P(E_2)+ P(A/E_3).P(E_3)=0.029$ Now the second event of interest is $[E_2\|A]$ and from Bayes rule $P(E_2/A)=\dfrac{P(A/E_2).P(E_2)}{P(A)}=\dfrac{0.05\times 0.35}{0.029}=0.6034$ **OR** Karl Peareon's Coefficient of skewness= $\dfrac{Mean-Mode}{Standard\ Deviation}=\dfrac{37.25-36.67}{8.2878}$ $=0.06998$	
(b)	Consider an example of 2 dices. Let A=1st dice show 1,2,3. B=1st dice show 3,4,5. C= sum of 2 numbers is 9. $\therefore P(A)=\frac{18}{36}=\frac{1}{2}$, $P(B)=\frac{18}{36}=\frac{1}{2}$, $P(C)=\frac{4}{36}=\frac{1}{9}$. $A\cap B=\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\}$ $\therefore P(A\cap B)=\frac{6}{36}=\frac{1}{6}$ and $P(A)\times P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ $\Rightarrow P(A\cap B)\neq P(A)\times P(B)$ Now $A\cap C=\{(3,6)\}$ $\therefore P(A\cap C)=\frac{1}{36}$ and $P(A)\times P(C)=\frac{1}{2}\times\frac{1}{9}=\frac{1}{18}$ $\Rightarrow P(A\cap C)\neq P(A)\times P(C)$ Similarly $B\cap C=\{(3,6),(4,5),(5,4)\}$ $\therefore P(B\cap C)=\frac{3}{36}=\frac{1}{12}$ and $P(B)\times P(C)=\frac{1}{2}\times\frac{1}{9}=\frac{1}{18}$ $\Rightarrow P(B\cap C)\neq P(B)\times P(C)$ And $A\cap B\cap C=\{(3,6)\}$ $\therefore P(A\cap B\cap C)=\frac{1}{36}$ and $P(A)\times P(B)\times P(C)=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{9}=\frac{1}{36}$	

	$\Rightarrow P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$	
Q.2 (a)	The null hypothesis H_o is : average length of nail produced = 5 cm Alternate hypothesis H_a is : average length of nail produced ≠ 5 cm The test statistic $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{5.012 - 5}{0.05134/\sqrt{10-1}} = 0.7012$ Level of significance α=0.05 Critical value at 5% level of significance for 9 degrees of freedom is 1.833. Since the calculated value of $\mid t_\alpha \mid$= 0.7012 is less than the critical value, the null hypothesis is accepted. $\therefore$ The average height of the population is 5 cm. **OR** Calculated $\chi_0^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ =3.4, critical $\chi^2_{0.05,9} = 16.9$. Since the calculated value≤The critical value, the hypothesis that numbers are uniformly distributed can be accepted.	
(b)	(i) Given total number of families=100 $\therefore 56+f_1+f_2=100$ $\therefore f_1+f_2= 44$ Also given Median = 50 $\therefore$ Median class ($(l_1 - l_2)$ =(40-60) Frequency of median class(f) = 27 Cumulative Frequency of pre-median class(F) =14+ f_1 $\text{Median} = l_1 + \frac{(l_2 - l_1)}{f}\left(\frac{N}{2} - F\right)$ $\therefore f_1$=22.5 Substitute this value in (1) , we get f_2= 21.5 Hence missing frequencies are 22.5 and 21.5 (ii) Given E(X)=10, and V(X)=25 But Var(X)=E(X^2)-[E(X)]^2 Now Y=aX-b and E(Y)=0 E(aX-b)=0 aE(X)-b=0 a×10-b=0 $\therefore$ b=10a and V(Y)=1 $\therefore$ V(aX-b)=1 $\therefore a^2V(X)=1$ $\therefore a^2 \times 25=1$ $\therefore a=\pm\frac{1}{5}$ From (1) and (2), b=±2	

Q.3 (a)	$\bar{x} = \frac{\sum x_i}{n} = 32$, $\bar{y} = \frac{\sum y_i}{n} = 38$, $b_{yx} = \frac{\frac{\sum x_i y_i}{n} - \frac{\sum x_i}{n} \times \frac{\sum y_i}{n}}{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = -0.66$ $b_{xy} = \frac{\frac{\sum x_i y_i}{n} - \frac{\sum x_i}{n} \times \frac{\sum y_i}{n}}{\frac{\sum y_i^2}{n} - \left(\frac{\sum y_i}{n}\right)^2} = -0.23$ Regression of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$ Hence $y = -0.66x + 59.26$ Regression of x on y is $x - \bar{x} = b_{xy}(y - \bar{y})$ Hence $x = -0.23y + 40.88$ **OR** $\int_{-\infty}^{\infty} f(x)dx = 1 \therefore \int_0^2 k(2-x)dx + \int_2^3 kx(x-2)dx = 1 \therefore k = \frac{3}{10}$ F(2)= $\int_0^2 k(2-x)dx = \int_0^2 \frac{3}{10}(2-x)dx = \frac{3}{5} > \frac{1}{2}$ $\therefore$ Median lies between 0 and 2. Let median be m. $\therefore \int_0^m f(x)dx = \frac{1}{2}$ $\therefore \int_0^m k(2-x)dx = \frac{1}{2}$ $\therefore \int_0^m \frac{3}{10}(2-x)dx = \frac{1}{2}$ $\therefore 3m^2 - 12m + 10 = 0.$ $\therefore m = \frac{6 \pm \sqrt{6}}{3}$ $\therefore$ median $= \frac{6-\sqrt{6}}{3}$ $(\because \textit{median lies between } 0 \textit{ and } 2.)$	
(b)	Coefficient of Variation = $\frac{S.D}{Mean} \times 100\ \% = \frac{28.43}{77.69} \times 100 = 36.59\%$	
Q.4 (a)	P(X>350) = P(Z>2)=0.5-P(0≤X≤2)=0.5-0.4772=0.0228 Where Z= $\frac{x-\mu}{\sigma} = \frac{x-300}{25}$ $\therefore$ 2.28% will have lifetime more than 350 hrs P(275≤X≤325) = P(-1≤Z≤1) = P(-1≤Z≤0)+ P(0≤Z≤1) = 2 P(0≤Z≤1) = 2×0.3413= 0.6826 $\therefore$ 68.26% will have lifetime between 275 and 325 hours. **OR** Theory	

(b)	X and Y are not uncorrelated. Also P(X=1,Y=1)= $\frac{2}{16} \neq P_X(1)P_Y(1) = \frac{5}{16} \times \frac{7}{16}$. Therefore X and Y are not independent.	
Q.5 (a)	(i) $Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{2500-2700}{\sqrt{\frac{200^2}{1000} + \frac{250^2}{1000}}} = \frac{-200}{32.0156} = -19.75$ Since $\|Z\| \leq Z_{critical}$, the null hypothesis is rejected at 1% level of significance. Hence there is a significant increase in the mean yield. (ii) P(X=x)= $^nC_x p^x q^{n-x} = {}^{10}C_x (0.65)^x (0.35)^{10-x}$ $\therefore$ P(at least 7 will live up to 70)= $\sum_{x=7}^{10} {}^{10}C_x (0.65)^x (0.35)^{10-x} = 0.5138$	
(b)	(i)E(X)=$1\times1/2+2\times1/4+3\times1/8+\ldots$ $\therefore$ E(X)=2 (ii) Sample space S = n jobs are send to n processor for execution. $\therefore$ n(S)=n^n Event A = One processor processes 2 jobs and (n-1) processors processes remaining (n-2) jobs. $\therefore$ n(A)= $\left({}^nC_2 + {}^nC_2 + \cdots + {}^nC_2 \quad (n\ times)\right) \times [(n-1)(n-2)\cdots 3\times 2]$ $= {}^nC_2 \times n \times (n-1)! = {}^nC_2 \times n!$ $\therefore$ P(A)= $\frac{{}^nC_2 \times n!}{n^n}$	