

Dependency preservation Example

Decomposition $D = \{ R_1, R_2, R_3, \dots, R_m \}$ of R is said to be dependency-preserving with respect to F if the union of the projections of F on each R_i , in D is equivalent to F . In other words, $R \subset \text{join of } R_1, R_2 \text{ over } X$.

Example 1: Let a relation $R(A,B,C,D)$ and set of FDs $F = \{ A \rightarrow B, A \rightarrow C, C \rightarrow D \}$ are given.

A relation R is decomposed into -

$R_1 = (A, B, C)$ with FDs $F_1 = \{ A \rightarrow B, A \rightarrow C \}$, and

$R_2 = (C, D)$ with FDs $F_2 = \{ C \rightarrow D \}$.

$F' = F_1 \cup F_2 = \{ A \rightarrow B, A \rightarrow C, C \rightarrow D \}$

so, $F' = F$.

And so, $F'_+ = F_+$.

Example 2: $R(A,B,C)$ is decomposed into $R_1(A,B)$ and $R_2(B,C)$

The relation R contains following data.

A	B	C
1	1	1
2	1	2
3	2	1
4	2	2

Answer:

Here the FD's for R , $F = \{ A \rightarrow B, A \rightarrow C, BC \rightarrow A \}$

Now for $R_1(A,B)$ the relation would be

A	B
1	1
2	1
3	2
4	2

Hence, $F_1 = \{ A \rightarrow B \}$

Now for $R_2(B,C)$ the relation would be

B	C
1	1
1	2
2	1
2	2

Hence $F_2 = \{ \}$

Here, $F \neq F_1 \cup F_2$

Hence the relation is not dependency preserved.

Example 3: $R(A,B,C,D,E)$ and $F = \{ A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A \}$

and $R_1(A,B,C)$ and $R_2(C,D,E)$

Answer: **For $R_1(A,B,C)$ we will find F_1**

We need to find closure of A,B,C

$A^+ = \{ A, B, C, \emptyset \}$ hence FD will be $A \rightarrow BC$

$B^+ = \{ B, C, \emptyset, A \}$ hence FD will be $B \rightarrow CA$

$C^+ = \{ C, \emptyset, A, B \}$ hence FD will be $C \rightarrow AB$

$AB^+ = \{ A, B, C, \emptyset \}$ hence FD will be $AB \rightarrow C$ duplicate FD as $A \rightarrow C$ and definitely $AB \rightarrow C$. Hence we will discard it.

$AC^+ = \{ A, C, B, \emptyset \}$ hence FD will be $AC \rightarrow B$ hence duplicated

$BC^+ = \{ B, C, \emptyset, A \}$ hence FD will be $BC \rightarrow A$ hence duplicated.

ABC^+ no need to calculate as it is trivial dependency,

Hence $F_1 = \{ A \rightarrow BC, B \rightarrow CA, C \rightarrow AB \}$

For $R_2(C,D,E)$ we will find F_2

$C^+ = \{ C, D, A, B \}$ hence FD will be $C \rightarrow D$

$D^+ = \{ D, A, B, C \}$ hence FD will be $D \rightarrow C$

$E^+ = \{ E \}$

$CD^+ = \{ C, D, A, B \}$

$DE^+ = \{ D, E, A, B, C \}$ hence FD will be $DE \rightarrow C$ which is duplicated as $D \rightarrow C$.

$CE^+ = \{ C, E, D, A, B \}$ hence FD will be $CE \rightarrow D$ duplicated as $C \rightarrow D$

Hence $F_2 = \{ C \rightarrow D, D \rightarrow C \}$

For dependency preservation, $F = F_1 \cup F_2$

$F_1 \cup F_2 = \{ A \rightarrow BC, B \rightarrow CA, C \rightarrow AB, C \rightarrow D, D \rightarrow C \}$

Simplified $F_1 \cup F_2$ after applying Armstrong's axiom:

$F_1 \cup F_2 = \{ A \rightarrow B, A \rightarrow C, B \rightarrow C, B \rightarrow A, C \rightarrow A, C \rightarrow B, C \rightarrow D, D \rightarrow C \}$

$F = \{ A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A \}$

For, $A \rightarrow B, B \rightarrow C, C \rightarrow D$ they are part of $F_1 \cup F_2$ but $D \rightarrow A$ is not a part of $F_1 \cup F_2$, Hence find D^+ from $F_1 \cup F_2$

$D^+ = \{ D, C, A, B \}$ here A is in closure of D hence $D \rightarrow A$ is already preserved.

Hence This relation R is dependency preserved.