BHARATIYA VIDYA BHAVAN'S

SARDAR PATEL INSTITUTE OF TECHNOLOGY

MUNSHI NAGAR, ANDHERI (WEST), MUMBAI – 400 058, India

(Autonomous College Affiliated to University of Mumbai)

End Semester Examination April/May 2019

Max. Marks: 60	Duration: 3 hrs
Class: FYMCA	Semester: II
Course Code: MCA 25	Date: /05/2019
Subject: **Probability and Statistics**	Time: -

Instructions: (1) All questions are compulsory.
(2) Use of scientific calculator is allowed.
(3) Assume any necessary data but justify the same.

Q.1(a)

(i) $f_X(x) = \int_x^2 f(x,y)dy = \frac{4}{9}x(4-x^2)$; $\quad 1 \le x \le 2$

$= 0;$ $\quad$ otherwise

$f_Y(y) = \int_1^y f(x,y)dx = \frac{4}{9}y(y^2-1)$; $\quad 1 \le y \le 2$

$= 0;$ $\quad$ otherwise

(ii) $f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{\frac{8}{9}xy}{\frac{4}{9}y(y^2-1)} = \frac{2x}{y^2-1}$; $\quad 1 \le x \le y$

$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{\frac{8}{9}xy}{\frac{4}{9}x(4-x^2)} = \frac{2y}{4-x^2}$; $\quad x \le y \le 2$

(b) n=6

Year	X=Year-1995	y	x^2	y^2	xy
$\sum$	**15**	**561**	**55**	**53819**	**1519**

$$\bar{x} = \frac{\sum x_i}{n} = 2.5, \quad \bar{y} = \frac{\sum y_i}{n} = 93.5, \; b_{yx} = \frac{\frac{\sum x_i y_i}{n} - \frac{\sum x_i}{n} \times \frac{\sum y_i}{n}}{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = 6.657143$$

Regression of y on x is

$y - \bar{y} = b_{yx}(x - \bar{x})$ $\qquad$ Hence $y - 93.5 = 6.657143(x - 2.5)$

To estimate asset in the year 2002, x=2002-1995=7

Hence $y - 93.5 = 6.657143(7 - 2.5)$ $\therefore$ y=123.4571

Karl Pearson's coefficient of correlation is given by

$$r(x,y) = \frac{\frac{\sum x_i y_i}{n} - \frac{\sum x_i}{n}\frac{\sum y_i}{n}}{\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} \times \sqrt{\frac{\sum y_i^2}{n} - \left(\frac{\sum y_i}{n}\right)^2}} = 0.\ 753635$$

Q.2: Consider an example of 2 dices.

Let A=1st dice show 1,2,3. B=1st dice show 3,4,5. C= sum of 2 numbers is 9.

$\therefore P(A)=\frac{18}{36}=\frac{1}{2}$, $P(B)=\frac{18}{36}=\frac{1}{2}$, $P(C)=\frac{4}{36}=\frac{1}{9}$.

$\therefore P(A\cap B)=\frac{6}{36}=\frac{1}{6}$ and $P(A)\times P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ $\Rightarrow P(A\cap B) \neq P(A)\times P(B)$

Now $A\cap C=\{(3,6)\}$ $\therefore P(A\cap C)=\frac{1}{36}$ and $P(A)\times P(C)=\frac{1}{2}\times\frac{1}{9}=\frac{1}{18}$

$\Rightarrow P(A\cap C) \neq P(A)\times P(C)$

$\therefore P(B\cap C)=\frac{3}{36}=\frac{1}{12}$ and $P(B)\times P(C)=\frac{1}{2}\times\frac{1}{9}=\frac{1}{18}$ $\Rightarrow P(B\cap C) \neq P(B)\times P(C)$

$\therefore P(A\cap B\cap C)=\frac{1}{36}$ and $P(A)\times P(B)\times P(C)=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{9}=\frac{1}{36}$

$\Rightarrow P(A \cap B \cap C) = P(A)\times P(B)\times P(C)$

OR

Given that

The number of items produced by X : number of items produced by Y : number of items produced by Z = 3 : 1 :1

Let the event E_1 denotes that an item is produced by factory X
E_2 denotes that an item is produced by factory Y
E_3 denotes that an item is produced by factory Z
A denotes that an item is produced is defective

$\therefore P(E_1) = 0.6,\ P(E_2) = 0.2,\ P(E_3) = 0.2$

Given $P(A/E_1)=0.03$, $P(A/E_2)=0.05$, $P(A/E_3)=0.03$.

(i) $P(A)= P(A/E_1)P(E_1)+P(A/E_2)P(E_2)+ P(A/E_3)P(E_3)$

$= 0.03\times0.6+0.05\times0.2+0.03\times0.2 = 0.018+0.010+0.006 = 0.034$

(ii) $P(E_1/A)=\frac{P(A/E_1)\times P(E_1)}{P(A)}=\frac{0.03\times 0.6}{0.0.34}=\frac{9}{17}=0.5294$

$$P(E_2/A)=\frac{P(A/E_2)\times P(E_2)}{P(A)}=\frac{0.05\times 0.2}{0.0.34}=\frac{5}{17}=0.2941$$

$$P(E_3/A)=\frac{P(A/E_3)\times P(E_3)}{P(A)}=\frac{0.03\times 0.2}{0.0.34}=\frac{3}{17}=0.1765$$

Q.3 Given population mean μ= 5 cm

Sample size n =10

	x	x^2
$\sum$	**50.12**	**251.2278**

Sample mean $\bar{x}=\frac{\sum x}{n}=5.012$ cm

Sample standard deviation $s=\sqrt{\frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2}=\sqrt{\frac{251.2278}{10}-\left(\frac{5.012}{10}\right)^2}=0.05134$

The null hypothesis H_o is : average length of nail produced = 5 cm

Alternate hypothesis H_a is : average length of nail produced $\neq$ 5 cm

The test statistic $t=\frac{\bar{x}-\mu}{s/\sqrt{n-1}}=\frac{5.012-5}{0.05134/\sqrt{10-1}}=0.7012$

Level of significance α=0.05

Critical value at 5% level of significance for 9 degrees of freedom is 1.833.

Since the calculated value of $|t_\alpha|$= 0.7012 is less than the critical value, the null hypothesis is accepted. $\therefore$ The average height of the population is 5 cm.

OR

: Given n=100, $\bar{x}$ =40, σ_x=10 $\therefore \Sigma x=\bar{x}\times n=4000$

and $\sigma_x=\sqrt{\frac{\Sigma x_i^2}{n}-(\frac{\Sigma x_i}{n})^2}$ $\therefore 10=\sqrt{\frac{\Sigma x_i^2}{100}-(40)^2}$ This gives $\therefore \Sigma x^2$ =170000

For the correct data Σx = 4000-(30+72)+(3+27)= 3928

and Σx^2 =170000-(30^2+72^2)+(3^2+27^2)= 164654

Hence correct mean $\bar{x}=\frac{\sum x_i}{n}=\frac{3928}{100}=39.28$

and correct s.d. $\sigma_x=\sqrt{\frac{\Sigma x_i^2}{n}-(\frac{\Sigma x_i}{n})^2}=\sqrt{\frac{164654}{100}-\left(\frac{3928}{100}\right)^2}$ =10.18

Q. 4 Theory

OR

Given p= 0.001 = probability that an individual suffers a bad reaction

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n= 2000 = Sample size
q=1-p=0.999
Mean λ=np=2000×0.001=2

$$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}, \qquad x=0,1,2----$$

$$P(X>4)=1-P(X\le 4)=1-\sum_{x=0}^{4}\frac{e^{-2}\times 2^x}{x!}=1-e^{-2}\left(\frac{2^0}{0!}+\frac{2^1}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^3}{3!}+\frac{2^4}{4!}\right)$$

$$=1-0.94735=0.05265$$

$$P(X<3)=P(X\le 2)=\sum_{x=0}^{2}\frac{e^{-2}\times 2^x}{x!}=e^{-2}\left(\frac{2^0}{0!}+\frac{2^1}{1!}+\frac{2^2}{2!}\right)=0.6767$$

Q. 5 (a) Assumed mean (a)=37.5, class width (c) = 5

Age (years)	Class Marks(x_i)	Frequency f_i	Cumulative Freq	$u_i=\frac{x_i-a}{c}$	f_iu_i	$f_iu_i^2$
Σ		100			-5	275

The A.M. is given by $\bar{x}=a+c\times\frac{\sum f_iu_i}{\sum f_i}=37.5+5\times\frac{(-5)}{100}=37.25$

$$\sigma_x=c\sqrt{\frac{\sum(f_fu_i^2)}{N}-\left(\frac{\sum f_iu_i}{N}\right)^2}=5\sqrt{\frac{275}{100}-\left(\frac{-5}{100}\right)^2}=8.2878$$

$$\text{Mode}=l_1+\left(\frac{d_1}{d_1+d_2}\right)(l_2-l_1)=35+\left(\frac{5}{5+10}\right)(40-35)=36.67$$

Karl Peareon's Coefficient of skewness= $\frac{Mean-Mode}{S\tan dard\ Deviation}=\frac{37.25-36.67}{8.2878}$

= 0.06998

(b) $E(X)=\sum_{n=1}^{n}xP(x)=\sum_{n=1}^{n}x\times\frac{1}{n}=\frac{1}{n}\sum_{n=1}^{n}x=\frac{1}{n}\times\frac{n(n+1)}{2}=\frac{n+1}{2}$