BHARATIYA VIDYA BHAVAN'S

SARDAR PATEL INSTITUTE OF TECHNOLOGY

MUNSHI NAGAR, ANDHERI (WEST), MUMBAI – 400 058.

Class: F.Y.MCA **Course Code: MCA25** **Semester: II**

Date: 16/03/2018 **Subject: Probability & Statistics** **Time: 9.30 to 11 AM**

Synoptic

Q.1 (A) Theory

(B) Let event A= the film gets award for its story.

event B= the film gets award for its music.

event $A\cap B$= the film gets award for both.

(a) $P(A\cup B)= P(A)+P(B)-P(A\cap B) = 0.23+0.15-0.07 = 0.31$ 2 ½ marks

(b) $P(A\cap \overline{B})$ or $P(\overline{A}\cap B) = P(A\cap \overline{B}) + P(\overline{A}\cap B)$

$= P(A)- P(A\cap B)+P(B)- P(A\cap B)= 0.23-0.07+0.15-0.07 = 0.24$ 2 ½ marks

OR

(B) Sample Space S= total arrangements of the letters in the word 'MISSISSIPPI'.

$\therefore$ S=Arrangement of 4 letters S, 4 letters I, 2 letters P, 1 letter M

$$\therefore n(S)=\frac{11!}{4!\times 4!\times 2!\times 1!}$$ 1 mark

Event A= 4 S's come consecutively.

= Arrangement of (4'S) 1 group, 4 letters I, 2 letters P, 1 letter M

$$\therefore n(A)=\frac{8!}{4!\times 2!\times 1!\times 1!}$$ 1 mark

$$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{8!}{4!\times 2!\times 1!\times 1!}\div\frac{11!}{4!\times 4!\times 2!\times 1!}=\frac{4}{165}=0.02424$$ 1 mark

Q.2 (A) The joint pdf of the r.v.s (X,Y) is given by

$$f_{XY}(x,y)=\frac{\partial^2 F(x,y)}{\partial x\partial y}=\frac{\partial}{\partial x}\left[e^{-y}-e^{-(x+y)}\right]$$

$= e^{-(x+y)}$, $x \geq 0, y \geq 0$ 2 marks

$= 0$, otherwise

$\therefore$ We have $f_{XY}(x,y) = e^{-x} \times e^{-y}$

Marginal densities of X and Y are given by

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)dy = \int_{0}^{\infty} e^{-(x+y)}dy = e^{-x}$$

and $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)dx = \int_{0}^{\infty} e^{-(x+y)}dx = e^{-y}$ 2 marks

$\Rightarrow f_{XY}(x,y) = f_X(x) \times f_Y(y)$

$\therefore$ X and Y are independent. 1 mark

(B) $E(X)=\sum xP(x)=0\times1/3+1\times1/2+2\times0+3\times1/6=1$ 1 mark

$E(X^2)=\sum x^2P(x)=(0)^2\times1/3+(1)^2\times1/2+(2)^2\times0+(3)^2\times1/6=2$ 2 marks

$E(X-1)^2=E(X^2-2X+1)=E(x^2)-2E(x)+E(1)=2-2\times1+1=1$ 2 marks

Q3.(A) Given p=0.02, q=1-p=0.98, n=200

$\therefore$ mean $\lambda=np=4$ 1 mark

$P(X=x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, x=0,1,2,...

(i) $P(X=2) = \dfrac{e^{-4}4^2}{2!} = 8 \times e^{-4} = 0.146525$ 2 marks

(ii) $P(X\geq3) = 1-P(X<2) = 1-\sum_{X=0}^{2}\dfrac{e^{-4}4^x}{x!} = 1-e^{-4}\left(\dfrac{4^0}{0!}+\dfrac{4^1}{1!}+\dfrac{4^2}{2!}\right) = 1-13\times e^{-4}$

$=0.761897$ 2 marks

(B) Let p = probability that a thing is received by men = $\dfrac{a}{a+b}$

$\therefore q = 1-p = \dfrac{b}{a+b}$

$\therefore$ Probability that number of things received by men is odd is given by:

P = p(x=1)+p(x=3).....

$= {}^mC_1p^1q^{m-1} + {}^mC_3p^3q^{m-3} + {}^mC_5p^5q^{m-5} + \ldots$ 2 marks

Now,

$$(q+p)^m = q^m + {}^mC_1 p q^{m-1} + {}^mC_2 p^2 q^{m-2} + \dots$$

and $(q-p)^m = q^m - {}^mC_1 p q^{m-1} + {}^mC_2 p^2 q^{m-2} - \dots$

$\therefore (q+p)^m - (q-p)^m = 2[{}^mC_1 p^1 q^{m-1} + {}^mC_3 p^3 q^{m-3} + {}^mC_5 p^5 q^{m-5} + \dots]$ 1 mark

but $q+p=1$, and $q-p=\dfrac{b-a}{b+a}$

$$\therefore 1^m - \left(\frac{b-a}{b+a}\right)^m = 2P$$

$\therefore P = \dfrac{1}{2}\left[\dfrac{(b+a)^m - (b-a)^m}{(b+a)^m}\right]$ 2 marks

OR

(B) Theory