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Name of the course : Probabilitv and Statistics

Q.1 (A) Let the missing frequencies be f_1 and f_2.

C.I	Frequency (f_i)	
0-10	14	
10-20	f_1	Pre-Modal class
20-30	27	Modal class
30-40	f_2	Post-Modal class
40-50	15	
Σ	$56+f_1+f_2$	

Given total number of families=100

$\therefore 56+f_1+f_2=100$ $\qquad\qquad$ $\therefore f_1+f_2=44$ $\qquad\qquad$ ------ (1)

Also given mode=24

$\therefore$ Modal class $((l_1-l_2)=(20\text{-}30)$

Frequency of modal class (f) = 27

$d_1=f\text{-}f1=27\text{-}f_1$, $\qquad d_2=f\text{-}f2=27\text{-}f_2$

$$Mode=l_1+\left(\frac{d_1}{d_1+d_2}\right)(l_2-l_1) \qquad\qquad \therefore 24=20+\left(\frac{27-f_1}{54-f_1-f_2}\right)(30-20)$$

$\therefore 2f_1\text{-}3f_2=-27$ $\qquad\qquad$ ------ (2)

Solving (1) and (2) we get $f_2=21$, $f_1=23$.

(B) We prepare the following table.

C.I	Frequency (f_i)	Cumulative Freq
00-10	5	5
10-20	7	12
20-30	20	32
30-40	12	44

40-50	6	50
Σ	50	

$N=\Sigma f_i=50$

<u>For the first quartile Q_1</u> $\quad \frac{N}{4}=12.5$

Q_1 class $((l_1-l_2)=(20\text{-}30)$

Frequency of the Q_1 class (f)= 20

Cumulative Frequency of the just previous to Q_1 class (F)=12

$$Q_1=l_1+\frac{(l_2-l_1)}{f}\left(\frac{N}{4}-F\right)=20+\frac{(30-20)}{20}(12.5-12)=20.25$$

<u>For the second quartile Q_2=Median</u> $\quad \frac{N}{2}=25$

Median class $((l_1-l_2)=(20\text{-}30)$

Frequency of the median class (f)= 20

Cumulative Frequency of the just previous to median class (F)=12

$$\text{Median } Q_2=l_1+\frac{(l_2-l_1)}{f}\left(\frac{N}{2}-F\right)=20+\frac{(30-20)}{20}(25-12)=26.5$$

<u>For the third quartile Q_3</u> $\quad \frac{3N}{4}=37.5$

Q_3 class $((l_1-l_2)=(30\text{-}40)$

Frequency of the Q_3 class (f)= 12

Cumulative Frequency of the just previous to Q_3 class (F)= 32

$$Q_3=l_1+\frac{(l_2-l_1)}{f}\left(\frac{3N}{4}-F\right)=30+\frac{(40-30)}{12}(37.5-32)=34.58$$

Bowley's Coefficient of skewness= $\frac{(Q_3+Q_1-2\times Median)}{(Q_3-Q_1)}$

$$=\frac{(34.58+20.25-2\times 26.5)}{(34.58-20.25)}=0.1277$$

<u>Q.2 (A)</u> (i) Solve given equation we get

5y+3x=52	3x+5y=52		3x+5y=52
2x+y=30	2x+ y=30	(-)	10x+5y=150

-7x = - 98

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$x=14$, $y=2$

Hence a.m. of $x=14$ and a.m. of $y=2$

(ii) The regression line of y on x is $5y+3x=52$. Hence $y=(-3/5)x+52/5 b_{yx}=-3/5$

The regression line of x on y is $2x+y=30$. Hence $x=(-1/2)y+15$ $\quad b_{xy}=-1/2$

But $r^2= b_{xy}\times b_{yx}$ Hence $r= - 0.5477$

(iii) The most probable value of y when $x=10$.
Use regression of y on x.

$$y=(-3/5)x+52/5 = (-3/5)\times 10+52/5 = 22/5 = 4.4$$

(B)n=10

Height of father (x)	Height of son (y)	Rank of x (r_1)	Rank of y (r_2)	$d=\lvert r_1-r_2\rvert$	d^2
68	62	4	5	1	1
64	58	6	7	1	1
75	68	2.5	3.5	1	1
50	45	9	10	1	1
64	81	6	1	5	25
80	60	1	6	5	25
75	68	2.5	3.5	1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
Σ					**72**

Spearman's rank correlation coefficient(R or ρ) is given by

(i) Rank 2.5 is repeated m=2 times. Hence the correction $\dfrac{m(m^2-1)}{12}=0.5$

(ii) Rank 3.5 is repeated m=2 times. Hence the correction $\dfrac{m(m^2-1)}{12}=0.5$

(iii) Rank 6 is repeated m=3 times. Hence the correction $\dfrac{m(m^2-1)}{12}=2$

Corrected $\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} d_i^2$ + correction factors = 72 +0.5 +0.5 + 2=75

$$R=1-\frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}=1-\frac{6\times 75}{10(10^2-1)}=0.5454$$

Q.3 (A) Computation for Chi-Square Test

Day	O_i	E_i	Oi-Ei	$(Oi-Ei)^2$	$(Oi-Ei)^2/E_i$
1	8	10	-2	4	0.4
2	8	10	-2	4	0.4
3	10	10	0	0	0.0
4	9	10	-1	1	0.1
5	12	10	2	4	0.4
6	8	10	-2	4	0.4
7	10	10	0	0	0.0
8	14	10	4	16	1.6
9	10	10	0	0	0.0
10	11	10	1	1	0.1
Total	**100**	**100**	**0**		**3.4**

Calculated $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$=3.4, critical $\chi^2_{0.05,9} = 16.9$.

Since the calculated value≤The critical value, the hypothesis that numbers are uniformly distributed can be accepted.

(B) Given population mean (μ) =60 kg, Sample size (n)= 100,

Sample mean ($\bar{x}$) = 58 kg, Sample s.d (s) = 4 kg

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{58 - 60}{4/\sqrt{100}} = -5$$

Level of significance (α)=0.01

Critical Value Z_α=2.58

Since $|Z| \leq / Z_\alpha$, the null hypothesis is rejected.

Q.4 (A) Let E_1, E_2 and E_3 denote the events that the box I, II and III is chosen, respectively and let A be the event that 'the first ball drawn is red and second is white'

Then $P(E_1)=P(E_2)=P(E_3)=1/3$

$P(A|E_1)=\frac{2\times 1}{^6C_2} = \frac{2}{15}$, $P(A|E_2)= \frac{3\times 2}{^6C_2} = \frac{6}{15} = \frac{2}{5}$, $P(A|E_3)= \frac{1\times 3}{^6C_2} = \frac{3}{15} = \frac{1}{5}$

To find $P(E_2/A)$ $P(E_2 | A) = \frac{P(E_2)\times P(A|E_2)}{\sum_{i=1}^{3} P(E_i)P(A|E_i)} == \frac{6}{11}$

(B) Let event A= the film gets award for its story.
event B= the film gets award for its music.
event $A\cap B$= the film gets award for both.

(a) $P(A\cup B)= P(A)+P(B)-P(A\cap B) = 0.23+0.15-0.07 = 0.31$

(b) $P(A\cap \bar{B})$or $P(\bar{A}\cap B) = P(A\cap \bar{B}) + P(\bar{A}\cap B)$

$= P(A)- P(A\cap B)+P(B)- P(A\cap B)= 0.23-0.07+0.15-0.07 = 0.24$

Q.5 (A) Let p= probability that a hen lays egg= $\frac{5}{7}$ $\therefore q=\frac{2}{7}$.

Then probability of getting at least 4 eggs with 5 hens = $P(X\geq 4)$

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$\therefore$ P(X=4)+P(X=5)= $\sum_{x=4}^{5} {}^5C_x p^x q^{5-x}$ =0.5578

Expectation =Np =100×0.5578 = 55.78 =56

(B) Let $E(X_i)$ = mean of i^{th} battery i=1,2
$V(X_i)$ = variance of i^{th} battery i=1,2
$\therefore$ For the total $X= X_1+X_2$
$\therefore E(X) = E(X_1)+E(X_2)=1.5+1.5 = 3.0$ V
and $V(X) = V(X_1)+V(X_2)=0.045+0.045=0.09$
$\therefore \mu=3$ and $\sigma^2=0.09$

Total voltage X~N(3,0.09).
The required probability is

$P(2.7\le X\le 3.30) = P(-1\le Z\le 1)$ Where $Z=\frac{x-\mu}{\sigma}=\frac{x-3}{0.3}$

$\therefore P(2.7\le X\le 3.30) = P(-1\le Z\le 0)+ P(0\le Z\le 1) = 2\,P(0\le Z\le 1) = 2\times 0.3413$

Q.6 (A)(i) To Find k.
We know, total probability = 1

$$\therefore \int_{x=0}^{2}\int_{y=0}^{2} f(x,y)dxdy=1 \qquad \therefore \int_{x=0}^{2}\int_{y=0}^{2} k(4-x-y)dydx=1$$

$$\therefore k\int_{x=0}^{2}\left[4y-xy-\frac{y^2}{2}\right]_{y=0}^{2} dx=1 \qquad \therefore k\int_{x=0}^{2}[6-2x]dx=1$$

$$\therefore k\left[6x-x^2\right]_0^2=1 \qquad \therefore k[12-4]=1 \qquad \therefore k=\frac{1}{8}$$

(ii) To find the marginal probability of x, f(x)

$$f(x)=\int_{y=-\infty}^{\infty} f(x,y)dy \qquad \therefore f(x)=\int_{y=0}^{2} k(4-x-y)dy$$

$$\therefore f(x)=\frac{1}{8}\int_{y=0}^{2}(4-x-y)dy \qquad \therefore f(x)=\frac{1}{8}\left[4y-xy-\frac{y^2}{2}\right]_{y=0}^{2}$$

$$\therefore f(x)=\frac{1}{8}[6-2x] \qquad \therefore f(x)=\frac{3-x}{4}$$

Similarly the marginal probability of y, f(y) is given by: $f(y)=\frac{3-y}{4}$

(B)<u>Given E(X)=24, and V(X)=16</u>

<u>But</u> Var(X)=$E(X^2)-[E(X)]^2$

Now Y=aX-b and E(Y)=0

E(aX-b)=0 aE(X)-b=0

a×24-b=0 $\therefore$ b=24a ------ (1)

and V(Y)=2 $\therefore$ V(aX-b)=2

$\therefore a^2V(X)= 2 \quad \therefore a^2\times 16= 2 \quad \therefore a=\pm\frac{1}{2\sqrt{2}}$ ------- (2)

From (1) and (2), $b=\pm 6\sqrt{2}$