

# A toy model for simulating $\alpha$ particle emissions in p + 11B reactions at $K_p = 10$ MeV via Monte Carlo method

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We present a toy model for simulating  $\alpha$  particle emissions in p+11B reactions at  $K_p = 10$  MeV via Monte Carlo method. Incident proton is accelerated along the z-axes by an oscillating electric field of frequency  $\omega$ , and its momentum is randomly generated at the moment of the collision. After collision, we get an  $\alpha$  particle, emitted in z-direction, and an excited 8Be nucleus at rest, which breaks down in two  $\alpha$  particles emitted in opposite direction. Pseudorandom number generator Mersenne twister engine algorithm is employed to randomly spawn the emission angles of  $\alpha$  particles.

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## I. MODEL DESCRIPTION

Incident proton is accelerated along the z-axes by an oscillating electric field of frequency  $\omega$ :

$$E_z = E_0 \cos(\omega t + \phi). \quad (1)$$

From Newton's third law we get proton momentum:

$$F_z = eE_0 \cos(\omega t + \phi) = \frac{dp_z}{dt}, \quad (2)$$

$$p_z = \frac{eE_0}{\omega} \sin(\omega t + \phi) = p_{z,max} \sin(\omega t + \phi), \quad (3)$$

where  $p_{z,max} = \sqrt{2M_p K_p}$ . Then  $\omega t + \phi$  angle is randomly spawned at the time of the collision via the pseudorandom number generator: Mersenne twister engine algorithm. Incident proton collides with a 11B nucleus at rest to produce a 8Be excited nucleus at rest and an  $\alpha$  particle moving in the same direction as incident proton. Thus, the polar angle  $\theta_1$  at which this  $\alpha$  particle is emitted is constrained to two values: 0 and  $\pi$ , depending on if incident proton is moving forward or backward, respectively.

Later on, 8Be excited nucleus breaks down in two  $\alpha$  particles moving in opposite direction. The angles  $\theta_2$  and  $\theta_3$  at which these two  $\alpha$  particles are emitted are randomly spawned. Because of momentum conservation law, angle  $\theta_3$  is constrained to

$$\theta_3 = \pi - \theta_2, \quad (4)$$

so, actually, we just have to randomly generate  $\theta_2$  angle. Similarly, the azimuthal angles are related by the following relation:

$$\phi_3 = \pi + \phi_2. \quad (5)$$

Thus, in spherical coordinates the momentum of emitted  $\alpha_1$  and  $\alpha_2$  particles are

$$\mathbf{p}_{\alpha 2} = p_{\alpha 2} (\sin(\theta_2) \cos(\phi_2), \sin(\theta_2) \sin(\phi_2), \cos(\theta_2)), \quad (6)$$

$$\mathbf{p}_{\alpha 3} = -p_{\alpha 2} (\sin(\theta_2) \cos(\phi_2), \sin(\theta_2) \sin(\phi_2), \cos(\theta_2)), \quad (7)$$

where  $p_{\alpha 2}$  is obtained from the energy conservation law,

$$M_p + \frac{p_p^2}{2M_p} + M_{11B} = M_{8Be}^* + M_{\alpha} + \frac{p_{\alpha 1}^2}{2M_{\alpha}}, \quad (8)$$

$$M_{8Be}^* = M_{\alpha} + \frac{p_{\alpha 2}^2}{2M_{\alpha}} + M_{\alpha} + \frac{p_{\alpha 3}^2}{2M_{\alpha}}, \quad (9)$$

$$M_p + \frac{p_p^2}{2M_p} + M_{11B} = 3M_{\alpha} + \frac{p_{\alpha 1}^2}{2M_{\alpha}} + \frac{p_{\alpha 2}^2}{2M_{\alpha}} + \frac{p_{\alpha 3}^2}{2M_{\alpha}}, \quad (10)$$

$$p_{\alpha 1}^2 + p_{\alpha 2}^2 + p_{\alpha 3}^2 = 2M_{\alpha} \left( M_p + \frac{p_p^2}{2M_p} + M_{11B} - 3M_{\alpha} \right), \quad (11)$$

Because of  $p_{\alpha 2} = p_{\alpha 3}$ ,

$$p_{\alpha 1}^2 + 2p_{\alpha 2}^2 = 2M_{\alpha} \left( M_p + \frac{p_p^2}{2M_p} + M_{11B} - 3M_{\alpha} \right) = x. \quad (12)$$

$$p_{\alpha 2}^2 = (x - p_{\alpha 1}^2)/2. \quad (13)$$

So, we just have to randomly generate  $p_{\alpha 1}$ , constrained to the interval  $[0, x]$ .

In Fig. 3 is illustrated the collision between p and 11B nucleus, and its subsequent decay products.

## II. RESULTS

We present the angular and the energy distributions of emitted  $\alpha$  particles.

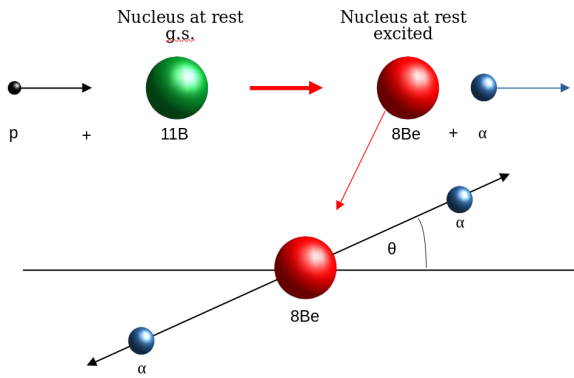


FIG. 1: (color online) Reaction scheme.

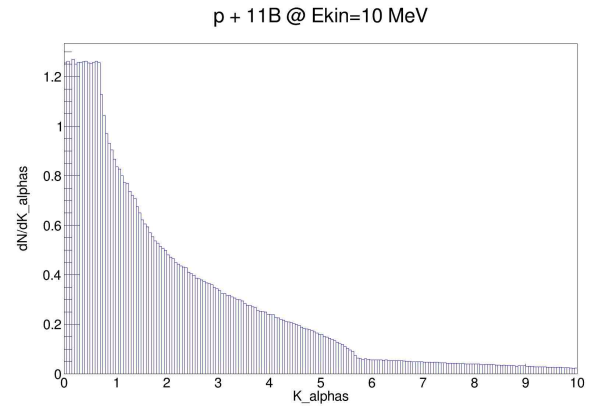


FIG. 3: (color online) Reaction scheme.

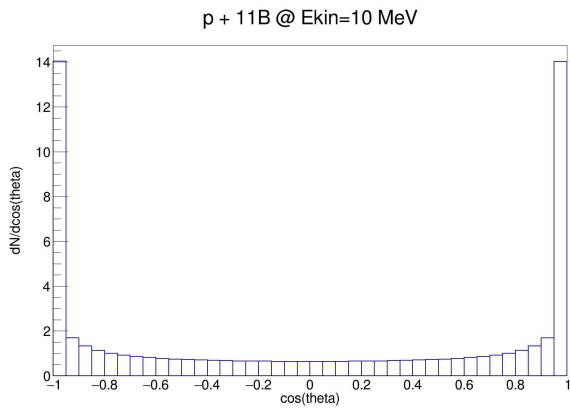


FIG. 2: (color online) Reaction scheme.

**A. Angular distribution of emitted  $\alpha$  particles**

**B. Energy distribution of emitted  $\alpha$  particles**

### III. REFERENCES

Appendix I