A toy model for simulating α particle emissions in $p +^{11} B \to 3\alpha$ reactions at $K_p \leq$10 MeV via Monte Carlo method

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I. MODEL DESCRIPTION

A. $p +^{11} B \to 3\alpha$ reaction

We present a toy model for simulating α particle emissions in $p +^{11} B \to 3\alpha$ reactions. We simulate this reaction in two steps, as it is illustrated in Fig. 1.

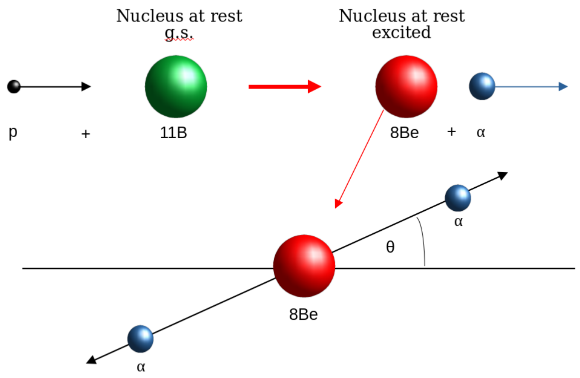

FIG. 1: (color online) Reaction scheme.

In the first step an incident proton with momentum $\boldsymbol{p}_p = (0, 0, p_z)$ collides with a ^{11}B nucleus at rest to produce a ^8Be excited nucleus at rest and an α particle moving in the same direction as incident proton. Thus, from the conservation laws, we can obtain:

$$\boldsymbol{p}_{\alpha 1} = \boldsymbol{p}_p = (0, 0, p_z) \, , \tag{1}$$

$$M_p + \frac{p_p^2}{2M_p} + M_{^{11}B} = M^*_{^8Be} + M_\alpha + \frac{p_{\alpha 1}^2}{2M_\alpha} \, . \tag{2}$$

Thus, from Eq. (1) the polar angle θ_1 at which this first α particle is emitted is constrained to two values: 0 and π, depending on if incident proton is moving forward or backward, respectively.

From Eq. (1) we can find the mass of excited ^8Be nucleus

$$M^*_{^8Be} = M_p + M_{^{11}B} - M_\alpha + \frac{p_p^2}{2M_p} - \frac{p_\alpha^2}{2M_\alpha} . \tag{3}$$

Later on, in the second step, ^8Be excited nucleus at rest breaks down in two α particles moving in opposite directions. The angles θ_2 and θ_3 at which these two α particles are emitted are randomly generated. Because of momentum conservation law, angle θ_3 is constrained to

$$\theta_3 = \pi - \theta_2 , \tag{4}$$

so, actually, we just have to randomly generate θ_2 angle. Similarly, the azimuthal angles are related by the following relation:

$$\phi_3 = \pi + \phi_2 . \tag{5}$$

Thus, in spherical coordinates the momentum of emitted $\alpha 2$ and $\alpha 3$ particles are

$$\boldsymbol{p_{\alpha 2}} = p_{\alpha 2} \left(sin(\theta_2) cos(\phi_2), sin(\theta_2) sin(\phi_2), cos(\theta_2) \right) , \tag{6}$$

$$\boldsymbol{p_{\alpha 3}} = -p_{\alpha 2} \left(sin(\theta_2) cos(\phi_2), sin(\theta_2) sin(\phi_2), cos(\theta_2) \right) , \tag{7}$$

where $p_{\alpha 2}$ is obtained from the energy conservation law:

$$p_{\alpha 2} = \sqrt{M_\alpha (M^*_{^8Be} - 2M_\alpha)} \, . \tag{8}$$

Please note that such a toy models gives maximal possible peaks of the final alpha distribution in the original proton direction: the first alpha always goes in the original proton directions, while the two α's from ^8Be decay move in opposite but random direction.

B. Proton momentum

Incident proton is accelerated along the z-axes by an oscillating electric field of frequency ω:

$$E_z = E_0 cos(\omega t + \phi) \, , \tag{9}$$

what gives an oscillating force acting on protons

$$F_z = eE_0 cos(\omega t + \phi) = \frac{dp_z}{dt} \, , \tag{10}$$

what finally leads to forced oscillations of the protons. In particular the proton z-momentum oscillates as

$$p_z = \frac{eE_0}{\omega} sin(\omega t + \phi) = p_{z,max} sin(\omega t + \phi) \, , \tag{11}$$

where its amplitude, $p_{z,max} = \sqrt{2M_p K_p}$, is calculated assuming maximal kinetic energy of the proton to be $K_p = 10$ MeV.

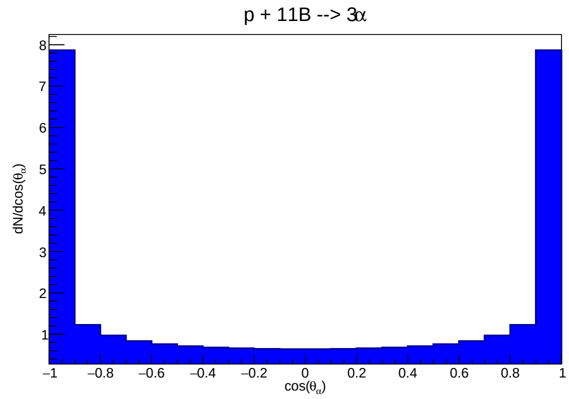

FIG. 2: The angular distribution of the produced α's, $dN/d\cos(\theta_\alpha)$.

Since the time of the collision is not known and can't be controlled the phase of the proton momentum oscillation is randomly chosen via the pseudorandom number generator: Mersenne twister engine algorithm:

$$p_z = \sqrt{2M_pK_p} sin(\phi_p)\,, \quad \phi_p = 2\pi * rand\,. \tag{12}$$

Please note, that if we generate $p_z < p_p^{min}$ from eq. (14) this can not generate the studied reaction and just such an event is just ignored.

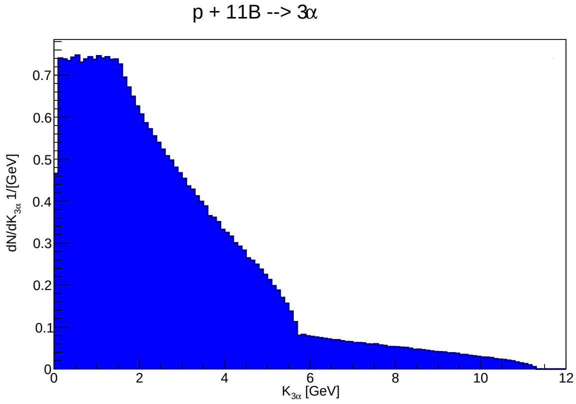

FIG. 3: the energy distributions of emitted α particles, dN/dK_α.

C. Reaction threshold

Eq. (3) shows that the mass of excited ^8Be nucleus, produced in the first step of our reaction, is always bigger that its rest mass. This is because our reaction is exothermic, $M_p + M_{^{11}B} > M_\alpha + M_{^8Be}$, and, thus, from the point of view of the strong interaction our reaction has no threshold and can be activated even by a stopped proton. On the other hand in order to approach a ^{11}B nucleus our proton has to overcome the Coulomb barrier, which is

$$U_{p+B} = 5 * \frac{\alpha * \hbar c}{2 * R_p + R_{^{11}B}} \simeq 1.61\text{MeV}, \tag{13}$$

where α is the fine-structure constant. Out of this equation we can find the minimal proton momentum which can generate the above reaction:

$$p_p^{min} = \sqrt{2M_pU_{p+B}} \simeq 55.04 \text{ MeV}. \tag{14}$$

II. RESULTS

In this section we present the results of our study, namely the angular distribution of the produced α's, $dN/d\cos\theta_\alpha$, in Fig. 2, and the energy distributions of emitted α particles, dN/dK_α in Fig. 3. Since the final α's are indistinguishable at the moment we take into account all of these, although the experimental setup may require to apply some cuts in angles and energy.