K. Martin

# Computational Fluid Dynamics

Name : K Martin

Roll no : 18135052

(i) (a) Gauss Law :

$$\oint E \cdot ds = \frac{Q_{in}}{\varepsilon_0}$$

$$\Downarrow$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\delta_t}{\varepsilon_0}$$

$\delta_t$ = Charge density (total)

$$-\nabla^2 \psi = \frac{\delta_T}{\varepsilon_0} \quad ; \quad \text{since } \vec{E} = -\nabla \psi$$

(b) for a given dielectric the gauss law

$$\vec{\nabla} \cdot (\epsilon \vec{E}) = \delta_f \quad ; \delta_f = \text{free charge density}$$

for Constant $\epsilon$

$$\vec{\nabla} \cdot \vec{E} = \frac{\delta_f}{\epsilon} \Rightarrow -\nabla^2 \psi = \frac{\delta_f}{\epsilon}$$

c) mass Conservation Eqn

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot [\rho \vec{v}] = 0$$

for ~~in crossenpble~~ in Compressible fluid

$$\vec{\nabla} \cdot \vec{v} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

momentum Conservation Eqn:

in the x- direction

$$\rho \left[ \frac{\partial u}{\partial t} + \frac{u \partial u}{\partial x} + \frac{v\, du}{\partial z} \right] =$$

$$-\frac{\partial p}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right]$$

$$-\rho_f \frac{\partial \phi}{\partial x}$$

in the $z$-direction

$$\delta\left[\frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} + v\frac{\partial v}{\partial z}\right] = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right] - \delta_f \frac{\partial \psi}{\partial x}$$

d) for steady state:

$$\frac{\partial v}{\partial t} = 0$$

for Low Reynolds number

Advection $= 0$

$\therefore$ hydrodynmic Eqn

momentum in $x$-direction:

$$-\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right] - \delta_f \frac{\partial \psi}{\partial x} = 0$$

momentum in $z$-direction

$$-\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right] - \delta_f \frac{\partial \psi}{\partial z} = 0$$

(ii) Conservation Equation (general form)

$$\frac{\partial(\delta\phi)}{\partial t} + \vec{\nabla}(\delta\phi\vec{v}) = \vec{\nabla}(\Gamma\vec{\nabla}\phi) + S_\phi$$

for mass

$$\frac{\partial\rho}{\partial t} + \left[\frac{\partial\rho u}{\partial x} + \frac{\partial\rho v}{\partial x}\right] = 0$$

$$\Rightarrow \phi = 1 \;;\; \Gamma = 0 \quad S_\phi = 0$$

for x-momentum:

$$\delta\left[\frac{\partial u}{\partial t} + \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial z}\right]\right] = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$- \delta_f\left(\frac{\partial \psi}{\partial x}\right)$$

$\phi = u$

$\Gamma = \mu$

$$S_\phi = -\frac{\partial p}{\partial u} - \delta_f\left(\frac{\partial \psi}{\partial x}\right)$$

for z - momentum

$$\rho \left[ \frac{\partial v}{\partial t} + u\frac{\partial u}{\partial x} + \frac{v \partial v}{\partial x} \right] =$$

$$- \frac{\partial p}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

$$- \delta_f \frac{\partial \psi}{\partial z}$$

$\phi = v$

$\Gamma = \mu$

$$S_\phi = - \frac{\partial p}{\partial z} - \rho_f \frac{\partial \psi}{\partial z}$$

For electro static equation:

$$\epsilon \left[ \frac{\partial \psi}{\partial x^2} + \frac{\partial \psi}{\partial y^2} \right) + \rho_f = 0$$

$\phi = \psi$ , $\Gamma = \epsilon$ , $S_\phi = \rho_f$

iii) order of each PDE:

1) mass = 1

2) $x$ - momentum - 2

3) $y$ momentum - 2

4) electrostatic equation - 2

iv) characteristics of PDE

for 2nd order PDE:

$$A\phi_{xx} + B\phi_{xy} + C\phi_{yy} + H = 0$$

$$\Rightarrow \frac{\partial y}{\partial x} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

and

$$\Delta = B^2 - 4AC$$

for $x$ - momentum

$$\mu(u_{xx} + u_{zz}) - \frac{\partial p}{\partial x} - \rho_f \frac{\partial \psi}{\partial x} = 0$$

$A = \mu$, $B = 0$, $C = \mu$

$\Delta = -4\mu^2 < 0$ it is a ellepticale eqn

$\Rightarrow$ no Characteristics

for z momentum:

$$\mu(V_{xx} + V_{zz}) - \frac{\partial p}{\partial x} - \rho_f \frac{\partial \varphi}{\partial x} = 0$$

$$A = \mu, B = 0, C = \mu$$

$$\Delta = -4\mu^2 < 0 \rightarrow \text{elliptical eqn}$$

$\Rightarrow$ no Characteristics

~~for the~~

for electrostatic eqn

$$\epsilon(\varphi_{xx} + \varphi_{zz}) + \rho_f = 0$$

$$A = \epsilon, B = 0, C = \epsilon$$

$$\Delta = -4\epsilon^2 > 0 \Rightarrow \text{Elliptical equation}$$

for Continuity eqn

It is a 1st order PDE hence Characteristics is not defined

v) non Conservative form of

1) Mass eq$\bar{n}$

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z}\right) = 0$$

2) x Momentum eq$\bar{n}$:

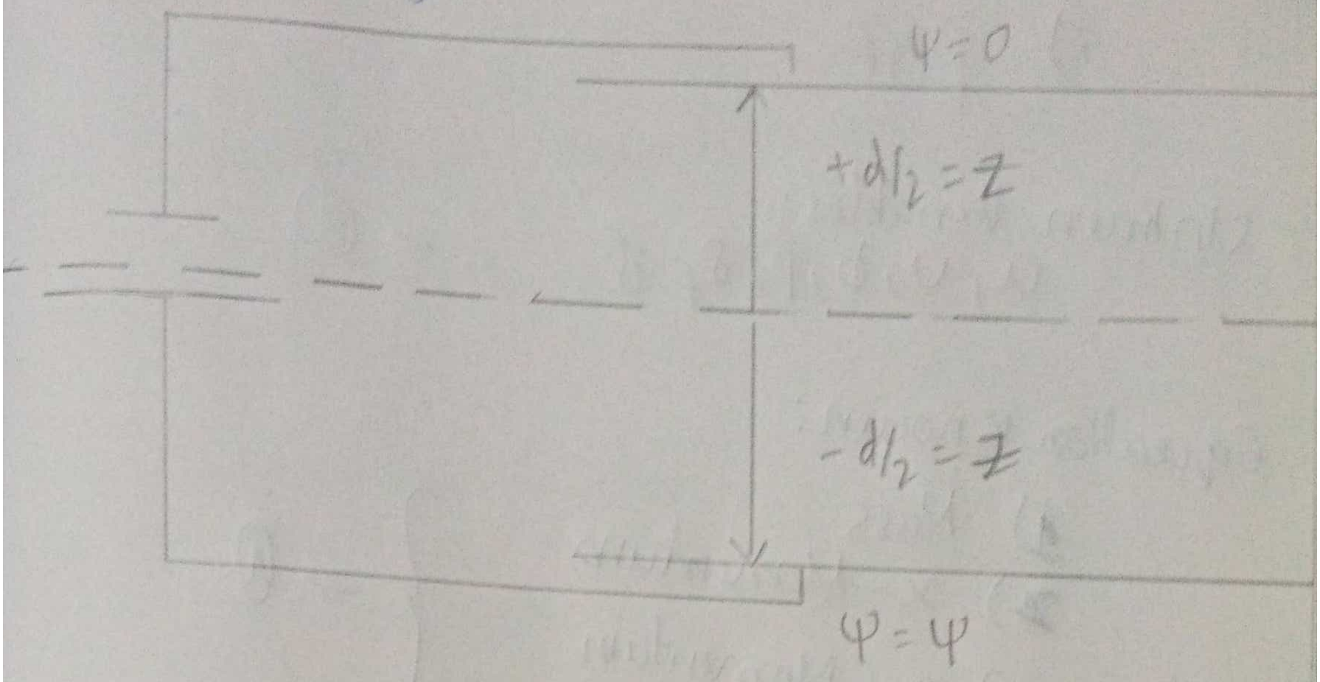
$$\rho \frac{Du}{Dt} = \frac{-\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

$$- \rho_f \frac{\partial \psi}{\partial x}$$

3) $$\frac{Dv}{Dt} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$- \rho_f \frac{\partial \psi}{\partial z}$$

vi) Boundary Conditions




→i) $z = \frac{d}{2}$ , $u = 0$

→ii) $z = \frac{-d}{2}$ , $u = 0$

→iii) $z = d/2$ , $\psi = 0$

→iv) $z = -d/2$ , $\psi = \psi$

→v) $z = d/2$ , $v = 0$

→vi) $z = -d/2$ , $v = 0$

vii)

a) DOF

Unknown Variables:

$u, v, b, P, \delta_f, \delta$ — 8

Equation Known:

1) Mass
2) x-Momentum
3) z-Momentum
4) Electronics

— 4

$DOF = 6 - 4 = 2$

the system is underdefined

for well defined system

we need to have 2 more defined variables

Let us define

Pressure = constant or 0 atm

and

density = constant

b) Number of B.C's = 4

x - momentum equation:

$\hookrightarrow$ BC's are at $z = \pm d/2$, $u = 0$

z - momentum Equation:

$\hookrightarrow$ BC's at $z = \pm d/2$, $v = 0$

Electrostatics:

order - 2

B.C's at $z = -d/2$, $\varphi = \varphi$

at $z = d/2$ $\varphi = 0$

for mass there is no Boundary condition since we have assumed density as constant

now there are enough boundary conditions to solve the PDE's.