Assignment-3

Kmf

Name : K. Martin
Roll.no : 1P13C0R2

CFD

Q1) $\frac{dT}{dx} = \frac{UA}{\dot{m}c} \cdot \frac{T_0 - T}{L}$;

Let $\theta = \frac{T - T_{in}}{T_0 - T_{in}}$

any $y = x/L$

$\Rightarrow \frac{dT}{d\theta} = T_0 - T_{in}$

$\therefore \frac{dx}{dy} = L$

$$\frac{dT}{dx} = \frac{dT}{d\theta} \cdot \frac{d\theta}{dy} \cdot \frac{dy}{dx}$$

$$= \frac{T_0 - T_{in}}{L} \frac{d\theta}{dy} = \frac{UA}{\dot{m}c} \frac{T_0 - T}{L}$$

$$\frac{T_0 - T_{in}}{L}\left(\frac{d\theta}{dy}\right) = \frac{UA}{\dot{m}C} \frac{(T_0 - T)}{L}$$

$$\Rightarrow \left(\cancel{\frac{T_0 - T_{in}}{L}}\right)\left(\frac{d\theta}{dy}\right) = 2\left(\cancel{\frac{T_0 - T_{in}}{L}}\right)(1-\theta)$$

Boundary condition

$x = 0 \quad ; \quad \theta = 0$

$\theta = y \quad ; \quad \theta = 0$

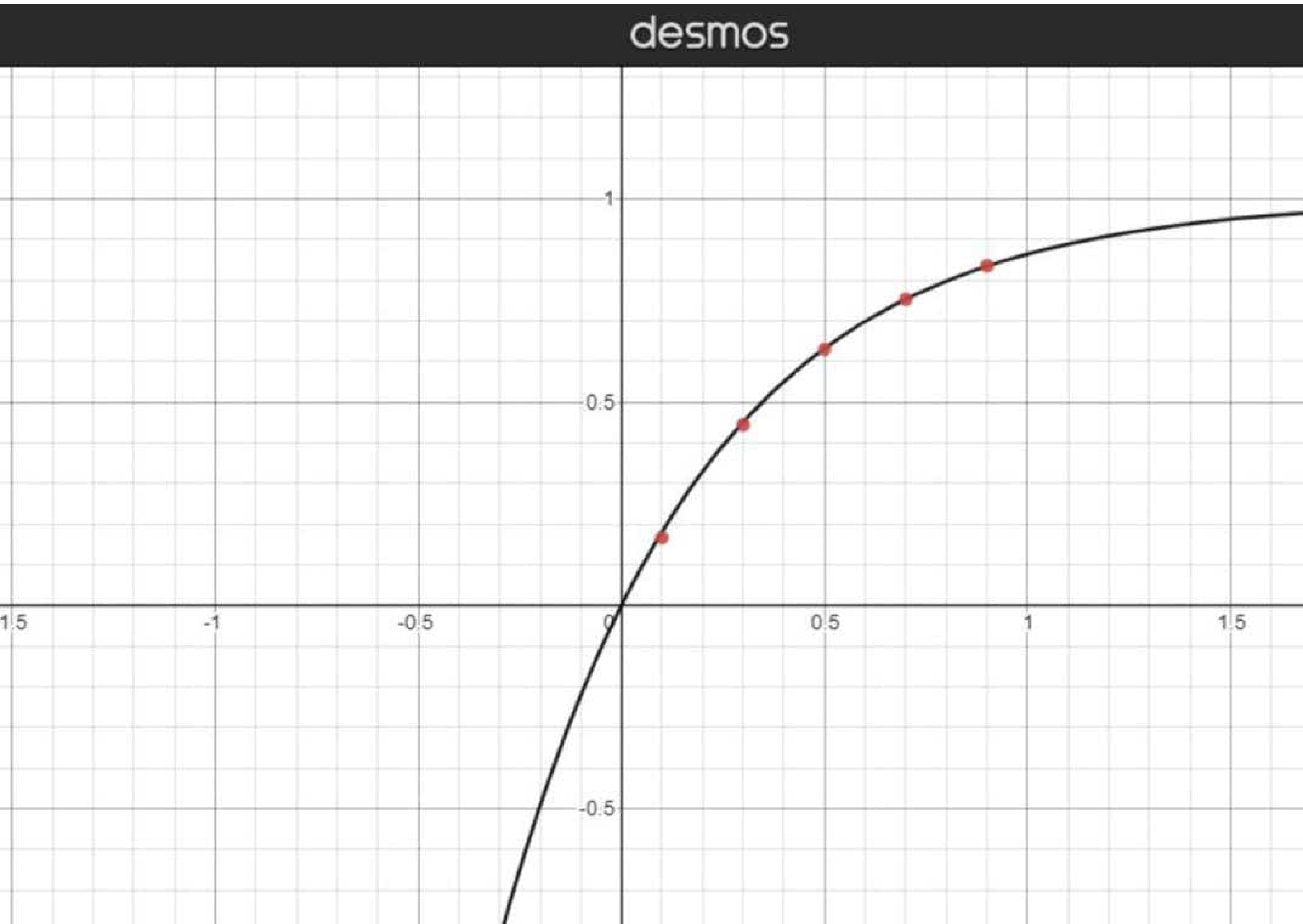
Analytical Soln:

$$d\theta = 2(1-\theta)\,dy$$

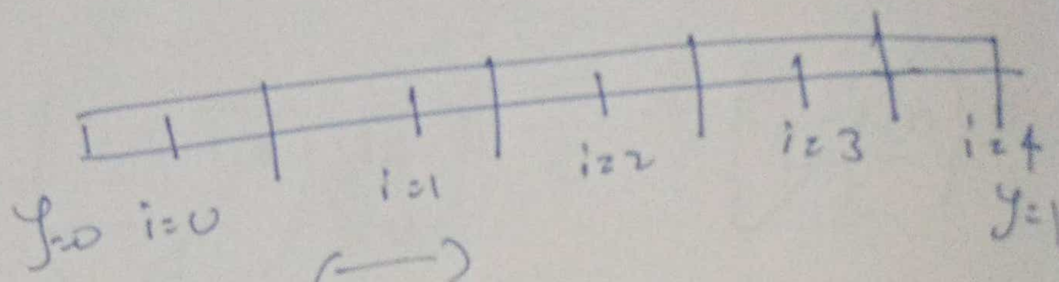
$$\Rightarrow \int_0^{\theta} \frac{d\theta}{1-\theta} = \int_0^{y} 2\,dy$$

$$\ln(1-\theta) = -2y$$

$$\theta = 1 - e^{-2y}$$


desmos
1
0.5
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5

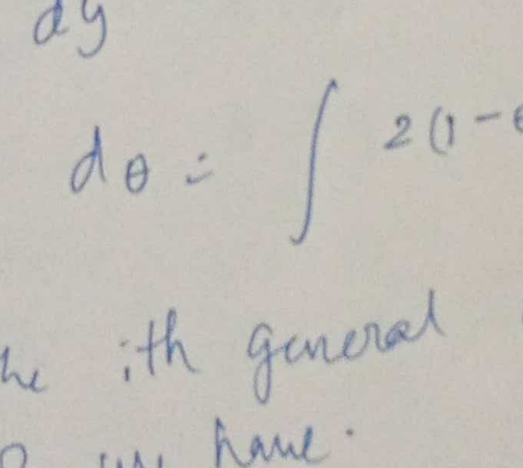
Numerical solution :
(upwind Scheme)



$$\frac{d\theta}{dy} = 2(1-\theta)$$

$$d\theta = \int 2(1-\theta)\,dy$$

for the ith general cell having center P we have :



$$\int_w^e d\theta = \int_w^e 2(1-\theta)\,dy$$

$(\theta_c - \theta_w) =$

$2(1-\theta_w)\frac{\Delta y}{2} + 2(1-\theta_p)\frac{\Delta y}{2}$

$\theta_p - \theta_w = (2 - \theta_w - \theta_p)\Delta y$

$\Delta y = 0.2 = 1/5$

$6\theta_p - 4\theta_w = 2$

$3\theta_p - 2\theta_w = 1$

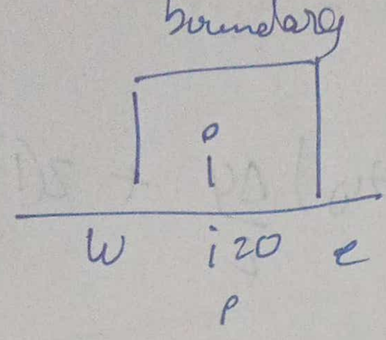
$\Rightarrow$ $i=1$ $\quad 3\theta_1 - 2\theta_0 = 1$

$i=2$ $\quad 3\theta_2 - 2\theta_1 = 1$

$i=3$ $\quad 3\theta_3 - 2\theta_1 = 1$

$i=4$ $\quad 3\theta_4 - 2\theta_3 = 1$

for $i = 0$



$Q_e = Q_p$

$\underline{Q_w = 0}$
due to B.C

$$\int_w^e d\theta = \int_w^e 2(1-\theta)\, dy$$

$$Q_p - 0 = \cancel{2}(1-\theta_p)\frac{\Delta y}{\cancel{2}}$$

$$\theta_p = 0.2 - 0.2\theta_p$$

$$1.2\theta_p = 0.2$$

or $1.2\,\theta_0 = 0.2$ for i

since $i = 0$

```python
import numpy as np
import sys

n = int(input('Enter number of unknowns: '))

a = np.zeros((n,n+1))

x = np.zeros(n)

print('Enter Augmented Matrix Coefficients:')
for i in range(n):
    for j in range(n+1):
        a[i][j] = float(input( 'a['+str(i)+']['+ str(j)+']='))

for i in range(n):
    if a[i][i] == 0.0:
        sys.exit('Divide by zero detected!')

    for j in range(i+1, n):
        ratio = a[j][i]/a[i][i]

        for k in range(n+1):
            a[j][k] = a[j][k] - ratio * a[i][k]

x[n-1] = a[n-1][n]/a[n-1][n-1]

for i in range(n-2,-1,-1):
    x[i] = a[i][n]

    for j in range(i+1,n):
        x[i] = x[i] - a[i][j]*x[j]

    x[i] = x[i]/a[i][i]

print('\nRequired solution is: ')
for i in range(n):
    print('X%d = %0.2f' %(i,x[i]), end = '\t')

#K . Martin
#18135052
#CFD
```

$\Rightarrow$

$$\begin{pmatrix} 1.2 & 0 & 0 & 0 & 0 \\ -0.8 & 1.2 & 0 & 0 & 0 \\ 0 & -0.8 & 1.2 & 0 & 0 \\ 0 & 0 & -0.8 & 1.2 & 0 \\ 0 & 0 & 0 & -0.8 & 1.2 \end{pmatrix} \begin{pmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix}$$

$$= \begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \end{bmatrix}$$

y	Q
0.1	1/6
0.3	4/9
0.5	19/27
0.7	61/81
0.9	203/243