Assignment - 3

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CFD

Q1) $\frac{dT}{dx} = \frac{UA}{\dot{m}c} \cdot \frac{T_0 - T}{L}$;

Let $\theta = \frac{T - T_{in}}{T_0 - T_{in}}$

any $y = x/L$

$\Rightarrow \frac{dT}{d\theta} = T_0 - T_{in}$

$\therefore \frac{dx}{dy} = L$

$$\frac{dT}{dx} = \frac{dT}{d\theta} \cdot \frac{d\theta}{dy} \cdot \frac{dy}{dx}$$

$$= \frac{T_0 - T_{in}}{L} \frac{d\theta}{dy} = \frac{UA}{\dot{m}c} \frac{T_0 - T}{L}$$

$$\frac{T_0 - T_{in}}{L}\left(\frac{d\theta}{dy}\right) = \frac{UA}{\dot{m}C}\,\frac{(T_0 - T)}{L}$$

$$\Rightarrow \cancel{\left(\frac{T_0 - T_{in}}{L}\right)}\left(\frac{d\theta}{dy}\right) = 2\cancel{\left(\frac{T_0 - T_{in}}{L}\right)}(1-\theta)$$

Boundary condition

$x = 0$; $\theta = 0$

$\theta = y$; $\theta = 0$

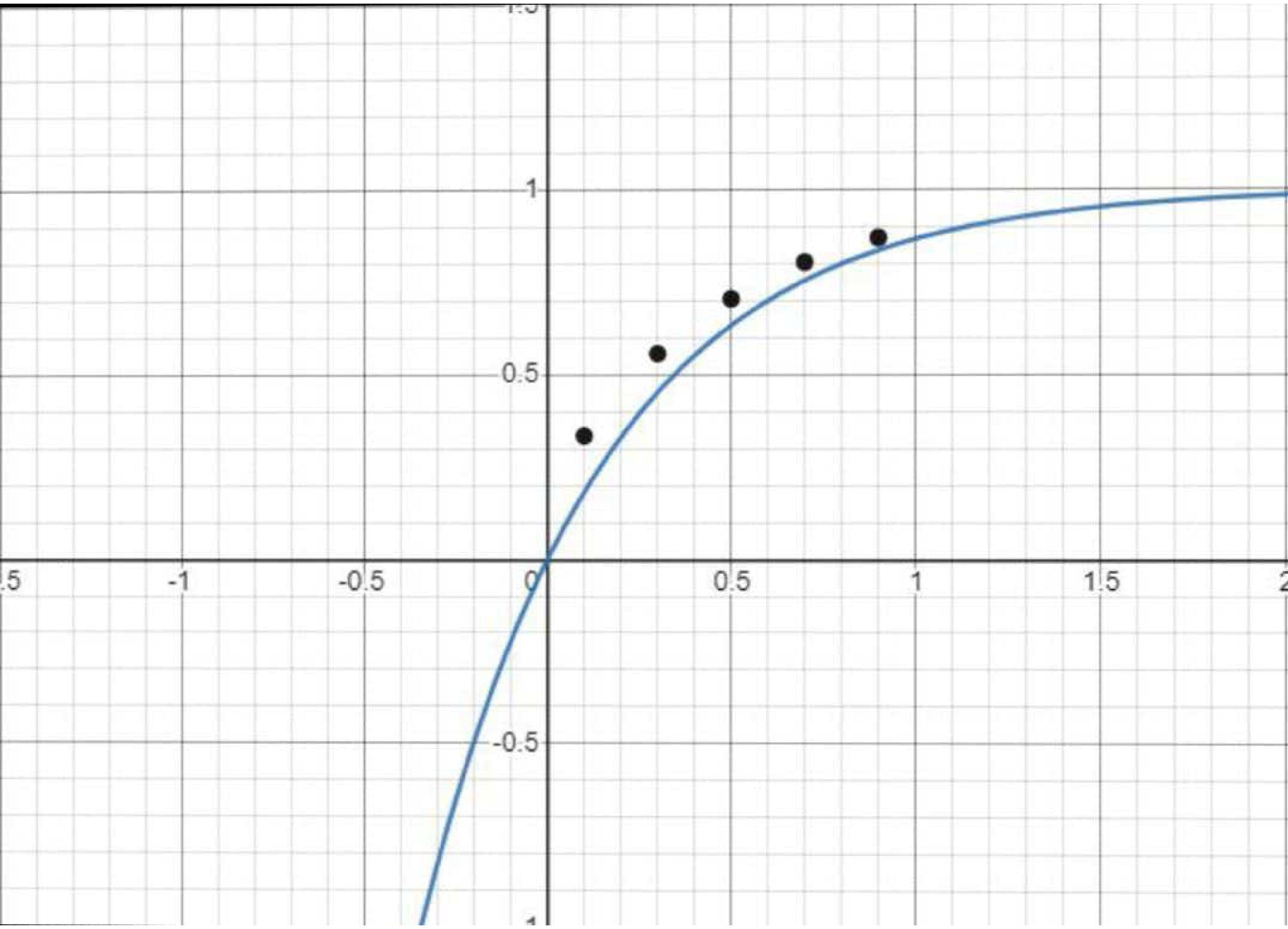
Analytical Soln:

$$d\theta = 2(1-\theta)\,dy$$

$$\Rightarrow \int_0^\theta \frac{d\theta}{1-\theta} = \int_0^y 2dy$$

$$\ln(1-\theta) = -2y$$

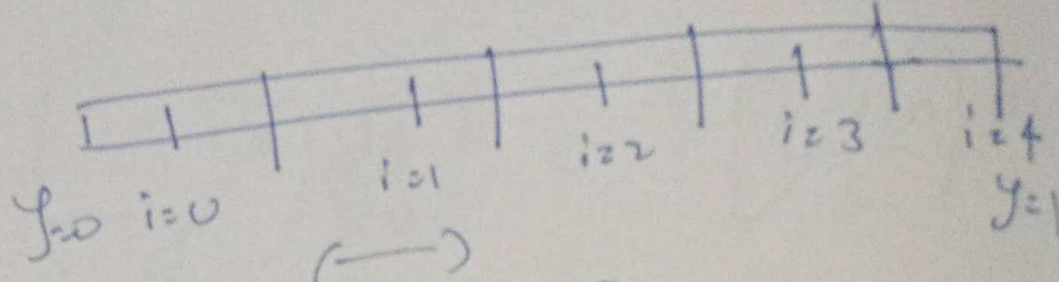
$$\theta = 1 - e^{-2y}$$



1
0.5
-0.5
-1
-0.5
0
0.5
1
1.5

Numerical solution:

(upwind scheme)

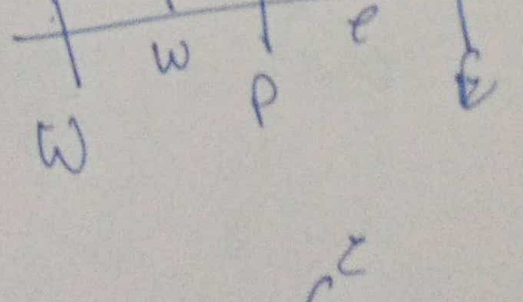


$\Delta y = 0.2$

$$\frac{d\theta}{dy} = 2(1-\theta)$$

$$d\theta = \int 2(1-\theta)\,dy$$

for the ith general cell having center P we have:



$$\int_w^e d\theta = \int_w^e 2(1-\theta)\,dy$$

$(\theta_c - \theta_\omega) =$

$2(1-\theta_\omega)\frac{\Delta y}{2} + 2(1-\theta_p)\frac{\Delta y}{2}$

$\theta_p - \theta_\omega = (2 - \theta_\omega - \theta_p)\Delta y$

$\Delta y = 0.2 = 1/5$

$6\theta_p - 4\theta_\omega = 2$

$3\theta_p - 2\theta_\omega = 1$

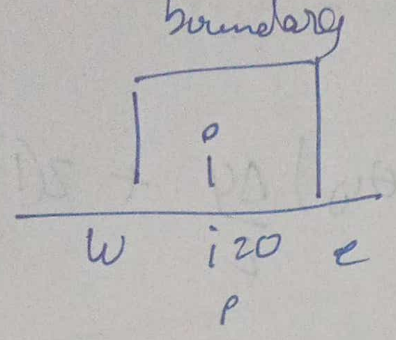
$\Rightarrow$ $i = 1$ $\quad 3\theta_1 - 2\theta_0 = 1$

$i = 2$ $\quad 3\theta_2 - 2\theta_1 = 1$

$i = 3$ $\quad 3\theta_3 - 2\theta_1 = 1$

$i = 4$ $\quad 3\theta_4 - 2\theta_3 = 1$

for $i = 0$



$Q_e = Q_p$ $\qquad$ $Q_w = 0$ due to B.C

$$\int_w^e d\theta = \int_w^e 2(1-\theta)\, dy$$

$$Q_{p} - 0 = \cancel{2}(1-\theta_p)\frac{\Delta y}{\cancel{2}}$$

$$\theta_p = 0.2 - 0.2\theta_p$$

$1.2\theta_p = 0.2$

or $\quad 1.2\theta_0 = 0.2$ for i

since $i = 0$

```python

import numpy as np
import sys

n = int(input('Enter number of unknowns: '))

a = np.zeros((n,n+1))

x = np.zeros(n)

print('Enter Augmented Matrix Coefficients:')
for i in range(n):
    for j in range(n+1):
        a[i][j] = float(input( 'a['+str(i)+']['+ str(j)+']='))

for i in range(n):
    if a[i][i] == 0.0:
        sys.exit('Divide by zero detected!')

    for j in range(i+1, n):
        ratio = a[j][i]/a[i][i]

        for k in range(n+1):
            a[j][k] = a[j][k] - ratio * a[i][k]

x[n-1] = a[n-1][n]/a[n-1][n-1]

for i in range(n-2,-1,-1):
    x[i] = a[i][n]

    for j in range(i+1,n):
        x[i] = x[i] - a[i][j]*x[j]

    x[i] = x[i]/a[i][i]

print('\nRequired solution is: ')
for i in range(n):
    print('X%d = %0.6f' %(i,x[i]), end = '\t')

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#CFD
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$\Rightarrow$

$$\begin{bmatrix} 1.2 & 0 & 0 & 0 & 0 \\ -0.8 & 1.2 & 0 & 0 & 0 \\ 0 & -0.8 & 1.2 & 0 & 0 \\ 0 & 0 & -0.8 & 1.2 & 0 \\ 0 & 0 & 0 & -0.8 & 1.2 \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \\ 0.4 \end{bmatrix}$$

y	θ
0.1	0.33
0.3	0.56
0.5	6.70
0.7	0.80
0.9	0.82

```
Enter number of unknowns: 5
Enter Augmented Matrix Coefficients:
a[0][0]=1.2
a[0][1]=0
a[0][2]=0
a[0][3]=0
a[0][4]=0
a[0][5]=0.4
a[1][0]=-0.8
a[1][1]=1.2
a[1][2]=0
a[1][3]=0
a[1][4]=0
a[1][5]=0.4
a[2][0]=0
a[2][1]=-0.8
a[2][2]=1.2
a[2][3]=0
a[2][4]=0
a[2][5]=0.4
a[3][0]=0
a[3][1]=0
a[3][2]=-0.8
a[3][3]=1.2
a[3][4]=0
a[3][5]=0.4
a[4][0]=0
a[4][1]=0
a[4][2]=0
a[4][3]=-0.8
a[4][4]=1.2
a[4][5]=0.4

Required solution is:
X0 = 0.333333   X1 = 0.555556   X2 = 0.703704   X3 = 0.802469   X4 = 0.868313
```