北京邮电大学 2019-—2020 学年第二学期补考

Discrete Mathematics – Supplementary Examination

考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学	考试时间	2020 年 8 月 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	5	10	10	10	10	5	6	6	6	6	6	6	10	4	
得分															
阅卷教师															

1. [5 points]

a) Which of these sentences are propositions? What are the truth values of those that are propositions?

i) The Medal of the Republic was conferred on renowned respiratory disease expert Zhong Nanshan in this month. ________

ii) The area of logic that deals with propositions is called the propositional calculus. ________

iii) $F=ma^2$. ________

iv) 2+2=3 unless 1+1=2. ________

v) Working together to defeat the COVID-19 outbreak. ________

b) Let S(x, y) be the statement “x+ y=0,” and M(x,y) be the statement “ x *y=0”, where the domain for both x and y are real number. Translate each of these (1),(2) nested quantifications into a statement. And use quantifiers to express each of these statements (3),(4),(5).

(1) $\neg\forall x\ \exists y\ M(x,y)$

(2) $\exists x\ \forall y S(x,y)\ \rightarrow \exists x\ \forall y M(x,y)$

(3) For every real number x there is a real number y such that M(x,y) or for any real number x and y , S(x,y).

(4) There is a real number x such that for every real number y, S(x,y).

(5) There is a real number y such that for every real number x, M(x,y) or S(x,y).

班级： 班内序号： 姓名：

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2. [10 points] Determine whether $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are logically equivalent.

3. [10 points] Find the principal conjunctive normal form of (a) and (b).

(a) $(p \wedge \neg q) \to (p \leftrightarrow q)$

(b) $(s \to (t \to w)) \wedge (\neg s \to (t \vee \neg w))$

4. [10 points] Put the statement (a) and (b) in prenex normal form.

(a) $\exists x Q(x) \vee (\forall y P(y) \to \exists z R(z))$

(b) $\forall y\, S(x,y) \to (\neg \exists z W(z) \wedge \exists x T(x))$

5. [10 points] Show that the premises "If you send me a bag of cookies, then I will finish homework on time," "If you do not send me a bag of cookies, then I will go out to buy food," and "If I go out to buy food, then I will go to the cinema" lead to the conclusion "If I do not finish homework on time, then I will go to the cinema."

6. [5 points] Prove that the product of two of the numbers $6^{510020} - 85^{29001} + 3^{310707}$, $70^{91212} - 98^{812399} + 23^{82001}$, and $24^{449223} - 58^{69192} + 7^{27901777}$ is nonnegative and explain the proof method you use.

7. [6 points] Find the Boolean product of A and B, where

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

8. [6 points]

a) a) Prove or disprove: if A, B, C and D are sets, then $A \cup (B \cap C \cap D) = (A \cup B \cup C) \cap D$.

b) Give an example of a function $f: \mathbf{R} \to \mathbf{N}$ that is onto $\mathbf{N}$ and not 1-1.

9. [6 points] Use the definition of "f (x) is O(g(x))" to show that $\sum_1^n (i^k + i)$ is $O(n^{k+1})$, where k is positive integer, and find the constant of k, C.

10. [6 points] There are 12 signs of the zodiac. How many people are needed to guarantee that at least six of these people have the same sign?

11. [6 points] Find all solutions, if any, to the system of congruences:

$$3x \equiv 5 (mod\ 7)$$
$$4x \equiv 7 (mod\ 11).$$

12. [6 points] Prove that 3 divides $n^3 + 2n$ whenever n is a positive integer.

13. [10 points]

(a) How many different strings can be made by reordering the letters of the word *GOOGOL*?

(b) Find the number of solutions to $x + y + z = 32$, where x, y, and z are nonnegative integers.

14. [4 points] A **routing transit number (RTN)** is a bank code used in the United States which appears on the bottom of checks.The most common form of an RTN has nine digits, where the last digit is a check digit. If $d_1d_2 \ldots d_9$ is a valid RTN, the congruence $3(d_1 + d_4 + d_7) + 7(d_2 + d_5 + d_8) + (d_3 + d_6 + d_9) \equiv 0 \pmod{10}$ must hold.

(a) Show that if $d_1d_2 \ldots d_9$ is a valid RTN, then

$d_9 = 7(d_1 + d_4 + d_7) + 3(d_2 + d_5 + d_8) + 9(d_3 + d_6)$ **mod** 10.

(b) Furthermore, use this formula to find the check digit that follows the eight digits 11100002 in a valid RTN.