Task 3. Algorithms for unconstrained nonlinear optimization. First- and second-order methods

Goal

The use of first- and second-order methods (Gradient Descent, Conjugate Gradient Descent, Newton's method and Levenberg-Marquardt algorithm) in the tasks of unconstrained nonlinear optimization (in particular, for curve fitting).

Problems and methods

Generate random numbers $\alpha \in (0,1)$ and $\beta \in (0,1)$. Furthermore, generate the noisy data $\{x_k, y_k\}$, where $k = 0, \dots, 1000$, according to the following rule:

$$y_k = \alpha x_k + \beta + \delta_k, \quad x_k = \frac{k}{1000},$$

where $\delta_k \sim N(0,1)$ are values of a random variable with standard normal distribution. For the data, find a solution to the linear and rational curve fitting problem by approximate (with precision $\varepsilon = 0.001$) minimization of the following function:

$$D(a,b) = \sum_{k=0}^{1000} (F(x_k, a, b) - y_k)^2,$$

where

1. $F(x,a,b) = ax + b$ (linear regression function);
2. $F(x,a,b) = \frac{a}{1+bx}$ (rational regression function).

To solve the minimization problem, use the methods of Gradient Descent, Conjugate Gradient Descent, Newton's method and Levenberg-Marquardt algorithm (you can use available implementations). If necessary, set the initial approximations and other parameters of the methods. Visualize the data and the regression lines obtained by the numerical optimization methods (for each type of regression). Analyze the results obtained in terms of the number of iterations required.

Comments

Use any programming language you want. The findings and the plots should be informative and correct.

The report should be a pdf-document containing

- Task number and its topic, your group name, your name and surname, the report date;
- code of your programs required values and graphs, as well as analysis of the results.

Reports must be sent to chunaev@itmo.ru no later than two weeks after the task is given. Use the following format for the email subject: Task #, Name Surname, Group.