Task 2. Algorithms for unconstrained nonlinear optimization. Direct methods

Goal

The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss, Nelder-Mead) in the tasks of unconstrained nonlinear optimization (in particular, in applications to regression analysis).

Problems and methods

I. Code the one-dimensional methods of exhaustive search, dichotomy and golden section search to find an approximate (with precision $\varepsilon = 0.001$) solution $x: f(x) \to min$ for the following functions and domains:

1. $f(x) = x^3, x \in [0,1]$;
2. $f(x) = |x - 0.2|, x \in [0,1]$;
3. $f(x) = x \sin\frac{1}{x}, x \in [0.1,1]$.

Calculate the number of f calculations and the number of iterations performed in each method and analyze the results. Explain differences (if any) in the results obtained.

II. Generate random numbers $\alpha \in (0,1)$ and $\beta \in (0,1)$. Furthermore, generate the noisy data $\{x_k, y_k\}$, where $k = 0, \dots, 100$, according to the following rule:

$$y_k = \alpha x_k + \beta + \delta_k, \quad x_k = \frac{k}{100},$$

where $\delta_k \sim N(0,1)$ are values of a random variable with standard normal distribution. For the data, find a solution to the linear and rational regression problems by approximate (with precision $\varepsilon = 0.001$) minimization of the following function (related to the least squares method):

$$D(a,b) = \sum_{k=0}^{100} (F(x_k, a, b) - y_k)^2,$$

where

1. $F(x,a,b) = ax + b$ (linear regression function);
2. $F(x,a,b) = \frac{a}{1+bx}$ (rational regression function).

To solve the minimization problem, use the methods of exhaustive search, Gauss and Nelder-Mead (you can use available implementations). If necessary, set the initial approximations and other parameters of the methods. Visualize the data and the regression lines obtained by the numerical optimization methods (for each type of regression). Analyze the results obtained.

Comments

Use any programming language you want. The findings and the plots should be informative and correct.

The report should be a pdf-document containing

- Task number and its topic, your group name, your name and surname, the report date;
- code of your programs required values and graphs, as well as analysis of the results.

Reports must be sent to chunaev@itmo.ru no later than two weeks after the task is given. Use the following format for the email subject: Task #, Name Surname, Group.