More Hypothesis Tests for Means

1. Write a null and an alternative hypothesis for each of the followings. Be sure to state each hypothesis in the context of the problem.

    1. A forestry textbook claims that the mean age of pine trees in Colorado is 30 years. You think the age is greater than that.

        $H_o : \mu \leq 30$

        $H_a : \mu > 30$

    2. A brochure on anti-aging cream states that the means age of people in the United States is 75 years. You doubt the claim, and you think it's greater because the information is out of date and the population of the elderly is growing.

        $H_o : \mu \leq 75$

        $H_a : \mu > 75$

    3. A fisheries report states that the mean weight of trout in Lake Standarderror is 3.4 pounds. You think the information is out of date, and that the population may have changed.

        $H_o : \mu = 3.4$

        $H_a : \mu \neq 3.4$

    4. Acme Caterpillar Farm claims the mean length of their award-winning caterpillars is 8 centimeters. You've seen their caterpillars and you think the mean length must be shorter.

        $H_o : \mu \geq 8$

        $H_a : \mu < 8$

2. A theory predicts that the mean age of stars within a particular type of star cluster is 3.3 billion years, with a standard deviation of 0.4 billion years. (Their ages are approximately normally distributed.) You think the mean age is actually greater, and that this would lend support to an alternative theory about how the clusters were formed. You use a computer to randomly select the coordinates of 50 stars from the catalog of known stars of the type you're study and you estimate their ages. You find that the mean age of stars in your sample is 3.4 billion years.

1. To test whether the population mean is greater than 3.3 billion years, what would your null and alternate hypotheses be? Be sure to state the hypotheses in the context of the problem.

   $H_o : \mu \neq 3,300,000,000$

   $H_a > \mu = 3,300,000,000$

2. What test would you plan to use, how will the test work, and what are the conditions necessary to use the test? Does your situations meet those conditions?

   We can use a z-test to determine whether the population mean is actually equal to 3.3 billion. We know that the ages of the stars are normally distributed and we know the standard deviation. The conditions to use a z-test are that the sample size is greater than 30, data points are independent from each other, normally distributed, and should be randomly selected. This situation meets all the requirements.

   We can use a z-test to determine if the results from the sample size are valid -- in other words, if the mean age of the stars in the population is 3.4 billion years based on the random sample.

3. Calculate your test statistic and P-value. Show your work, including the formulas you use to calculate the statistic.

$$\bar{x} = 3.4$$

$$s = 0.4$$

$$\text{Standard error} = 0.4/\sqrt{50} = 0.0566$$

$$z = \frac{3.4 - 3.3}{0.0566} = 1.7678$$

$$P(z > 1.7678) = 0.0384$$

4. Since $p < \alpha$, we can reject the null hypothesis and conclude that the mean age is greater.

3. Suppose you doubt the assumption that the mean age of the stars is 3.3 billion years, but you don't know whether the true mean age is less or greater than 3.3 billion years.

   1. To test whether the population mean is 3.3 billion years, what would your null and alternative hypotheses be?

      $H_o = 3.3$

      $H_a \neq 3.3$

2. **What test will you use, how will the test work, and what are the conditions necessary to use the test? Does your situation meet those conditions?**

   We can use a z-test for calculating the mean, since this is the same situation as the previous problem.

3. **Using the population and sample from question 2, calculate your test statistic and P-value. Show your work, including the formulas you used to calculate the statistic.**

   The calculations are the same for finding the test statistic.

   $p = 2 \cdot 0.0384 = 0.0768$

4. **What's your conclusion, using α = .05? Compare your results to the previous conclusion, and explain the difference. Sketch the distribution of your sample mean, shading the area that represents your P-value. Label the value for the mean, and label the values for the bounds of the shaded area.**

   Since $p > \alpha$, we do not reject the null hypothesis. We don't have enough evidence to reject it.