Gravitational Forces and Fields

1. Satellite X orbits a planet with orbital radius R. Satellite Y orbits the same planets with orbital radius $2R$. Satellites X and Y have the same mass.

You can calculate centripetal acceleration with the equation

$$a_c = \frac{v^2}{r}$$

Plugging in the values, you can see that the centripetal acceleration of X is 1/2 that of Y. Therefore, the answer is $\boxed{\text{B}}$.

2. Two isolated point particles of mass 4M and 9M are separated by a distance 1 m. A point particle of mass M is placed a distance from the particle of mass 9M. The net gravitational force on M is zero. What is the distance from M to 9M?

Set the two equations equal to each other:

$$\frac{4}{d^2} = \frac{9}{(1-d)^2}$$

We find that d is 0.4, meaning that the answer is $\boxed{\text{C}}$.

3. On Mars, the gravitational field strength is about a fourth of that on Earth. The mass of Earth is approximately ten times that of Mars. What is the ratio of radius of Earth to the radius of Mars?

   $$g = \frac{GM}{r^2} \Rightarrow r = \sqrt{\frac{Gm}{g}}$$

   MARS

   $$g = \frac{G(4/10)M}{r^2} \Rightarrow r = 0.63\sqrt{\frac{Gm}{g}}$$

   $$\boxed{\text{C}}$$

4. $\boxed{\text{D}}$

5. The gravitational field strength at the surface of a certain planet is g. Which of the following is the gravitational field strength at the surface of a planet with twice the radius and twice the mass?

   $$g = \frac{Gm^2}{r^2} \quad \boxed{\text{A}}$$

6. A spacecraft travels away from Earth in a straight line with its motors shut down. At one instant the speed of the spacecraft is 5.4 km/s. After a time of 600 s, the speed is 5.1 km/s.

Find the acceleration: 0.5 m/s^2 $\boxed{\text{C}}$

7. Define *gravitational field strength* and state the SI Unit.

Gravitational field strength is the amount of force acting on a unit mass of a mass. Its SI Units are N/kg.

a. A planet orbits the Sun in a circular orbit with orbital period T and orbital radius R. The mass of the Sun is M. Show that

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}}$$

You can use the equation for orbital speed and substitute the definition of a period in for orbital speed:

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

The Earth's orbit around the Sun is almost circular with radius 1.5×10^{11} m. Estimate the mass of the Sun.

Find the velocity by dividing circumference over time:

$$\frac{3 \times 10^{11} \times 2\pi}{3.15 \times 10^7} = 59839\ m/s$$

$$F = \frac{mv^2}{r}$$

$$F = \frac{59,839^2 \times 5.972 \times 10^{24} j}{1.5 \times 10^{11}} = 1.4 \times 10^{23}\ N$$

$$1.4 \times 10^{23} = \frac{6.67 \times 10^{-11} \times 5.972 \times 10^{24} \times M}{1.5 \times 10^{11}} = \boxed{1.9 \times 10^{30}}$$

8. The two arrows in the diagram show the gravitational field strength vectors at the position of a planet due to each of two stars of equal mass M. Each star has Mass $M = 2.0 \times 10^{30}$ kg. The planet is at a distance 6.0×10^{11} m from each star.

Show that the gravitational field strength at the position of the planet due to one of the stars is $g = 3.7 \times 10^{-4}\ N/kg$.

You can calculate gravitational field strength with the equation

$$g(r) = \frac{Gm_E}{r^2}$$

$$g(r) = \frac{6.67 \times 10^{-11} \times 2.0 \times 10^{30}}{(6.0 \times 10^{11})^2} = \boxed{3.7 \times 10^{-4}\ N/kg}$$

Calculate the magnitude of the resultant gravitational field strength at the position of the planet.

$$\boxed{7.4 \times 10^{-4}\ N/kg}$$

9. **State, in words, Newton's Universal Law of Gravitation**

Newton's universal law of gravitation states that everything attracts everything with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between the two objects.

When the satellite is directly overhead, the microwave signal reaches the receiver 67ms after it leaves the satellite. State the order of magnitude of the wavelength of microwaves and calculate the height of the satellite above Earth.

$$\boxed{\text{1 cm}}$$

$$3 \times 10^9 \times 67 \times 10^{-3} = \boxed{2.0 \times 10^7\ m}$$

Explain why the satellite is accelerating, calculate the gravitational field strength due to Earth at the satellite position, determine orbital speed, and orbital period.

You need a constant force towards earth to maintain orbit – when there's a force, there's also an acceleration.

$$\frac{GM}{r^2} = \frac{6.7 \times 10^{-11} \times 6.9 \times 10^{24}}{[2.6 \times 10^7]^2} = \boxed{0.6\ N/kg}$$

$$9.8 = \frac{v^2}{r}$$

$$v = \sqrt{0.57 \times (2.0 \times 10^7 + 6.4 \times 10^6)} = \boxed{3900\ \text{m/s}}$$

$$T = \frac{2.6 \times 10^7 \times 2\pi}{3900} = \boxed{12\ \text{h}}$$

10.

$$g = \frac{F}{m}$$

$$g = \frac{GmM/R^2}{m}$$

$$g = \boxed{\tfrac{GM}{R^2}}$$

$$\frac{g_M}{g_E} = \frac{M_M}{M_E} = \frac{g_M}{g_E} \times [\frac{R_M}{R_E}]^2$$

$$M_M = \boxed{0.11 M_E}$$

11. I can't draw, but imagine Mars as a circle with arrows pointing inwards towards it all around the circle.

Acceleration isn't constant and the attraction between A and B isn't the same as the attraction at the surface.