Circular Motion

Svadrut Kukunooru

September 14, 2020

0.1 Circular Motion Intro

- *Angular Displacement* (θ)
  - Angle which an object moves through in an arc
  - Measured in *radians*
  - *NOT A VECTOR QUANTITY* (unlike linear displacement)
  - $\theta = \frac{s}{r}$ is how you can relate arc length to radius, r, and distance traveled on the arc, s.
- *Angular Speed* is calculated with the equation $\omega = \frac{\Delta\theta}{\Delta t}$.
  - Units: rad $\cdot\, s^{-1}$
- *Period (T)*
  - The time required for an object to make one complete cycle
- *Frequency (f)*: The number of cycles per second (measured in **Hz**)
- Frequency can be related to period with the equation

$$f = \frac{1}{T}.$$

0.1.1 Practice Problems

1. A rope goes over a circular pulley with a radius of 6.50 cm. If the pulley makes exactly 4 revolutions without the rope slipping, what length of rope passes over the pulley?

   For this problem, all you need to do is calculate the circumference and multiply it by 4.

$$C = \pi(13) \times 4 = \boxed{52\pi}.$$

2. A ball with a radius of 15.0 cm rolls on a level surface. The translational speed of the center of mass is 0.250 $m \cdot s^{-1}$. What is the angular speed about the center of mass if the ball rolls without slipping?

   Angular speed can be related to translational speed with the equation

$$\omega = \frac{v_t}{r}.$$

   Therefore,

$$\frac{2 \times 0.250}{0.15} = \boxed{3.33\ \frac{rad}{s}}.$$

3. A bocce ball with a diameter of 6.00 cm rolls without slipping on a level lawn. It has an initial angular speed of 2.35 $\frac{rad}{s}$ and comes to rest after 2.50 m. Assuming uniform acceleration (deceleration), determine the following:

(a) The magnitude of its angular deceleration

You can find angular deceleration with the kinematic equation

$$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta).$$

You can find angular displacement with the equation

$$\theta = \frac{s}{\pi d}.$$

Plugging values in, you get

$$\frac{2.50}{\pi(0.06)} = 13.26 \text{ rotations}.$$

Converting to radians gives you

$$13.26 \times \frac{2\pi}{1 \text{ rotation}} = 83.6 \text{ rad}.$$

Plugging everything into the original equation:

$$0^2 = (2.35)^2 + 2\alpha(83.6).$$

$$\boxed{\alpha = -0.033 \text{ rad} \cdot s^{-2}}.$$

(b) The magnitude of its lineaer deceleration

With the equation

$$v = \omega r.$$

$$v = 2.35 \times 0.0300.$$

$$u = \boxed{0.0705\ m \cdot s^{-1}}.$$

0.2 Uniform Circular Motion Post-Lab Discussion

- Radius's effect on force is **decreasing exponential**
- Mass's effect on force is **increasing linear**
- Speed's effect on force is **increasing exponential**

From these conclusions, you can calculate centripetal force with the equation

$$\Sigma F_c = \frac{mv^2}{r}.$$

0.2.1 Practice Questions

What is the average speed of the Earth in its orbit around the Sun given the information below?

$$m_{earth} = 5.98 \times 10^{24}$$

$$r_orbit = 1.50 \times 10^{11}$$

$$F_{g(E+S)} = 3.53 \times 10^{22}.$$

You can calculate the speed of the earth with the equation

$$v = \sqrt{\frac{F_c r}{m}}.$$

$$v = \sqrt{\frac{3.53 \times 10^{22} \times 1.50 \times 10^{11}}{5.98 \times 10^{24}}}.$$

$$\boxed{v = 2.98 \times 10^4}.$$

0.2.2 Lecture, Continued

Since Newton's 2nd law is

$$F_{net} = ma.$$

Therefore, the centripetal acceleration can be calculated by

$$a_c = \frac{v^2}{r}.$$