t-Intervals and Hypothesis Tests

1. Management at a radio station...

    1. Null and alternative hypotheses...

    $$H_o : \mu = 10$$

    $$H_a : \mu > 10$$

    2. What tests will you use...

    We can use one sample *t*-test to evaluate the alternative hypothesis. The conditions required for a *t*-test are:

    - The dependent variable must be continuous
    - The observations are independent
    - The dependent variable should be normally distributed
    - The dependent variable should not have any outliers.

    These conditions are met in this situation.

    3. Calculate your test statistic and your *P*-value. Show your work.

    $$n = 15$$

    $$\bar{x} = 10.33$$

    $$s^2 = 3.52$$

    $$s = 1.877$$

    $$t = \frac{0.33}{1.878/\sqrt{15}} = \boxed{0.6805}$$

    4. What's your critical t-value...

    The critical t-value is $\boxed{1.761}$ at the 0.05 level.

2. The station manager thinks...

    1. What are the null and alternative...

    $$H_o : \mu = 10$$

    $$H_a : \mu \neq 10$$

    2. What test will you use...

    We can use a *t*-test. For conditions, please see the above problem. The conditions are met.

    3. Calculate the test statistic and the *P*-value...

As seen in the previous problem, the t-value is 0.6805.

To calculate the P-value, we first need to calculate the degrees of freedom.

$$df = 15 - 1 = 14$$

$$p_v = 2 \cdot P(t_{14} > 0.6805) = 0.503$$

4. What is your critical t-value...

My critical values are

$$t_{\alpha/2} = \pm 2.145$$

Since the calculated values are less than the critical value we do not reject the null hypothesis.

Since the p-value is greater than the significance level given $\alpha = 0.05$ we do not reject the null hypothesis.

Either way, we do not reject the null hypothesis.

3. The same radio station wants to track the average number of commercial minutes it plays in an hour. A random sample over the past month yielded the following data in minutes: 15.5,10.2,8.2,16,14.4,9.5,8.9,11.4,16.3, 15.2.

   1. Compute a 99% confidence interval...

   $$\text{Degrees of Freedom} = 9$$

   $$\frac{1 - 0.99}{2} = 0.005$$

   According to the t-distribution table, the answer is 3.250.

   $$3.22/\sqrt{10} = 1.018$$

   $$1.018 \times 3.250 = 3.3085$$

   $$(9.2515, 15.86)$$

   2. The station manager is concerned that listeners will change the station if too many commercial are played per hour. The program manager says the show plays less than 15 minutes of commercial per hour. Set up the hypotheses to test the program manager's statement.

   $$H_o : \mu > 15$$

   $$H_a : \mu < 15$$

   3. What test will you use to evaluate the alternative hypothesis? What are the conditions on this test and are they met in this situation?

   We can use a one-sided t-test. The conditions include:

   - The sample is from a normal population

   This meets the condition.

   4. Calculate your test statistic and P-value. Show your work.

$$t = \frac{(12.56 - 15)}{3.22/\sqrt{10}} = -2.39$$

The p-value is 0.02.

5. **What's your critical t-value and conclusion at alpha=.05.**

According to the t-table, the critical t-value is **2.2621**. Since the calculated test statistic exceeds this, we reject the null hypothesis.