SAMPLING DISTRIBUTIONS

1. Consider a parent population with mean 75 and a standard deviation 7. The population doesn't appear to have extreme skewness or outliers.
    1. What are the mean and standard deviation of the distribution of sample means for $n = 40$?

       The mean is equivalent to the parent population mean: $\boxed{75}$

       The standard deviation is equivalent to the population standard deviation divided by the square root of the sample size:

       $$7/\sqrt{40} = \boxed{1.106}$$

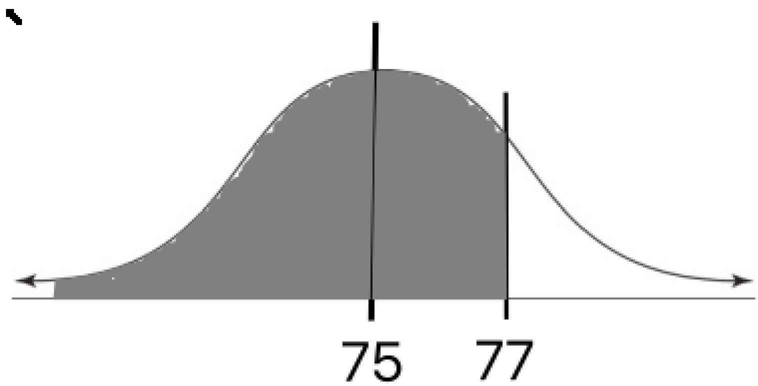
2. What's the shape of the distribution? Explain your answer in terms of the Central Limit Theorem.

   Because the sample size is big enough ($n > 30$) and as said in the problem the population distribution doesn't have extreme skewness or outliers, the distribution would bell-shaped and symmetrical -- a normal distribution.

3. What proportion of the sample means of size 40 would you expect to be 77 or less? If you use your calculator, show what you entered.

   $$P(Z < \frac{2}{1.107}) = P(Z < 1.807) = \boxed{0.965}$$

4. Draw a sketch of the probability you found in part C.



5. Suppose over several years of offering AP Statistics, a high school finds that final exam scores are normally distributed with a mean of 78 and a standard deviation of 6.
    1. What are the mean, standard deviation, and shape of the distribution of x-bar for $n = 50$?

       $$\text{Mean} = \boxed{78}$$

$$\text{Standard Deviation} = 6/\sqrt{50} = \boxed{0.84}$$

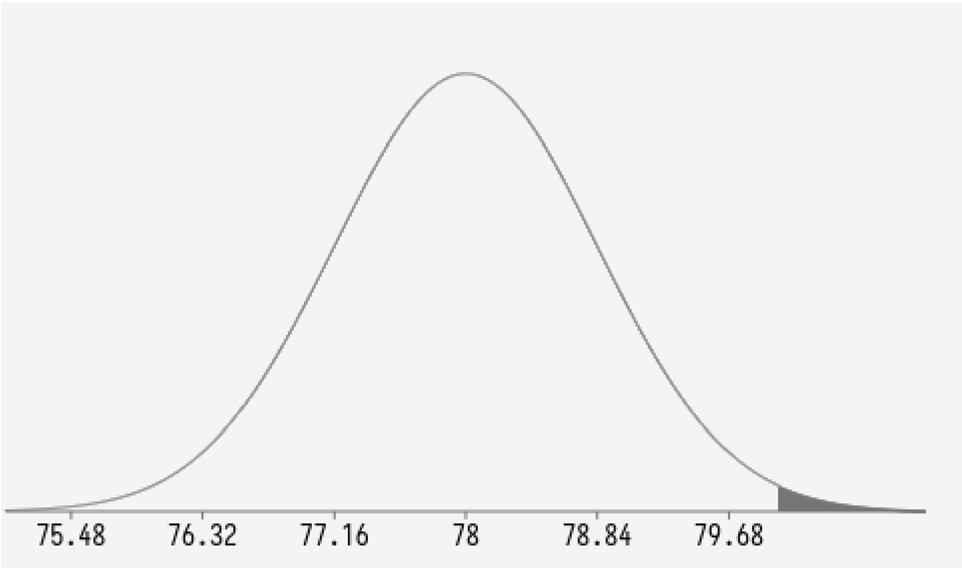
$$\boxed{\text{The distribution shape is normal.}}$$

2. What's the probability that a sample of scores will have a mean greater than 80?

$$\boxed{0.0086}$$

3.

4. Sketch the distribution curve for part B, showing that the area that represents the probability that you found.



6. Suppose college faculty members with the rank of professor at two-year institutions earn an average of $52,500 per year with a standard deviation of $4,000. In an attempt to verify this salary level, a random sample of 60 professors was selected from a personnel database for all two-year institutions in the United States.

   1. What are the mean and standard deviation of the sampling distribution for $n = 60$?

$$\boxed{\text{Mean} = \$52{,}500}$$

$$\boxed{\text{Standard Deviation} = 4,000/\sqrt{60} = \$516}$$

   2. What's the shape of the sampling distribution for $n = 60$?

      The shape of the sampling distribution is normal since $60 \geq 30$.

   3. Calculate the probability that the sample mean x-bar is greater than $55,000.

$$\boxed{0}$$

   4. If you drew a random sample with a mean of $55,000, would you consider this sample unusual? What conclusions might you draw?

This sample is very unusual since it is more than 3 standard deviations away from the mean. I might conclude that I am extremely lucky (????????????????????????????)

7. A manufacturer of paper used for packaging requires a minimum strength of 20 pounds per square inch. To check the quality of the paper, a random sample of ten pieces of paper is selected each hour from the previous hour's production and a strength measurement is recorded for each. The distribution of strengths is known to be normal and the standard deviation, computed from many samples, is known to equal 2 pounds per square inch. The mean is known to be 21 pounds per square inch.

    1. What are the mean and standard deviation of the sampling distribution for $n = 10$?

    $$\boxed{\text{Mean} = 21}$$

    $$\boxed{\text{Standard Deviation} = 2/\sqrt{10} = 0.632}$$

    2. What's the shape of the sampling distribution for $n = 10$?

    The distribution is normal.

    3. Write down all your sample values. What are the mean and sample standard deviation of this sample? Compare them with the mean and standard deviation of the parent population, and explain why they're different or the same.

    $[17.829, 18.339, 23.101, 20.926, 21.477, 19.751, 24.412, 21.828, 15.701, 22.681]$

    The mean is 20.61 and the standard deviation is 2.53. The mean of this distribution is similar to the mean of the sample distribution because this distribution is a sample of $n = 10$ from the parent distribution; therefore, it is likely that the mean would be the same. However, the standard deviation is different from the standard deviation of the sample distribution because the standard deviation depends on the sample itself, not the population. Therefore, it is expected that the value of a standard deviation of a sample has some statistical ambiguity.

    4. Using the mean and standard deviation of the sampling distribution, calculate the probability of getting a sample mean less than what you got. If you use a calculator, show what you entered.

    $$\boxed{0.2686}$$

8. Suppose a random sample of n=25 observations is selected from a normal population, with mean 106 and standard deviation 12.

    1. Find the probability that x-bar exceeds 110. Show the mean and standard deviation you used.

    The mean is 106, and the standard deviation is 2.4.

$\boxed{0.0478}$

2. Find the probability that the sample mean deviates from the population mean = 106 by no more than 4.

$\boxed{0.9044}$

3.