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Distributed adaptive consensus tracking control for non-linear multi-agent systems with time-varying delays

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Najmeh Zamani[1], Javad Askari[1] ✉, Marzieh Kamali[1], Amir Aghdam[2]

[1]Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran

[2]Electrical and Computer Engineering Department, Concordia University, Montreal, Canada

✉ *E-mail: j-askari@cc.iut.ac.ir*

Abstract: In this study, a novel distributed adaptive controller is provided for consensus control of high-order non-linear multi-agent systems with unknown time-varying delays. The system is subject to uncertain disturbances, and the agents' dynamics are not known. Unlike the existing literature, the proposed method does not require time-delay terms in system dynamics to be bounded. A neural network is used to model the unknown non-linear dynamics. Then, despite the destabilising effect of the unknown delays, some adaptive rules based on the dynamic surface control are designed to achieve the consensus objective. The semi-global uniform boundedness of the resultant closed-loop signals and the convergence of the tracking errors to a neighbourhood of the origin are shown mathematically. Simulations verify the effectiveness of the results.

1 Introduction

Multi-agent systems (MASs) have recently attracted many researchers in science and engineering due to their applications in systems biology, economics, smart grids, transportation and control of autonomous vehicles, to name only a few [1–5]. In particular, there has been an increasing interest in the control engineering community to develop effective distributed strategies to achieve some important global objectives in MASs such as consensus, formation, flocking and rendezvous [6–8]. In particular, consensus is a fundamental problem in a MAS which has attracted considerable attention due to its widespread applications [9, 10]. Different techniques are proposed in the literature for the consensus control of a MAS for agents with linear or non-linear dynamics [11–17].

A real-world MAS is, in general, subject to disturbances and uncertainties. Various methods have been developed to address these challenging problems. In particular, the backstepping adaptive control technique is proposed for a high-order non-linear MAS [18–21]. In [22, 23], the backstepping method is used to design a distributed adaptive tracking control for a strict feedback non-linear MAS with unknown parameters and uncertain disturbances. The backstepping method, however, suffers from the problem of 'explosion of complexity'. To overcome this drawback, dynamic surface control (DSC) method is introduced in [24, 25], where the derivatives of the virtual control signals are eliminated in the control signal, significantly reducing the computational burden of the distributed adaptive control scheme. In [26, 27], using DSC and a neural network (NN) approach, a consensus algorithm is developed for a MAS with unknown dynamics in a strict-feedback form.

In addition to the challenges noted in the previous paragraph, the input and state delays are known to impact the performance of this type of system [28, 29]. In [30, 31], an efficient protocol for distributed output consensus of a linear MAS with input and communication time-varying delays is introduced. Also, a leader tracking problem for a high-order non-linear Lipschitz MAS with multiple time-varying communication delays is investigated in [32]. In practical applications, such as a communication network, information is shared with multiple time-varying delays and the distributed adaptive approach should be capable of handling such a case. Neglecting these important phenomena can lead to performance degradation in the resultant closed-loop system, and may even cause instability. Moreover, considering the time delay and parameter uncertainties in a non-linear MAS simultaneously, often arising in practical applications is a challenging problem.

Different methods are introduced in the literature to tackle these challenging problems. A robust adaptive NN-based controller is proposed in [33] for a high-order non-linear MAS with unknown dynamics and time-varying delay in the leader dynamics. However, it is assumed in the above work that there is no time delay in the followers' dynamics. In [34–36], adaptive control schemes are provided for a non-linear MAS with unknown state delay, where the non-linearities of the system are modelled by a NN. Furthermore, the delay terms in the non-linear MAS are supposed to be bounded. It is also assumed that the time delay is constant, and that the control gain of the MAS is equal to '1'. In this paper, these assumptions are omitted completely; i.e. the approach proposed here does not impose any constraint on the time-varying delay terms in the MAS model and the time delay can be time-varying.

While the papers cited in the previous paragraphs are effective for a class of MASs, they either neglect some important phenomena such as state/input delay, disturbance, and uncertainties or impose some assumptions on the delay that may not be realistic in a practical setting. In this paper, a distributed adaptive control scheme is proposed to address some of the shortcomings of the existing techniques in the consensus control of MASs, noted above. To this end, it is assumed that the dynamics of each agent is non-linear and unknown, containing time-varying state delay, and is subject to disturbances. The DSC technique is employed to design an adaptive strategy with a NN to model non-linearities. Also, the Lyapunov–Krasovskii functional is used for the stability analysis of the MAS in the presence of state delay.

The main contributions of this paper are as follows: (i) The proposed method does not require the so-called 'strict assumption' on the time-delay terms in the MAS [34–36]. (ii) By the proposed approach, the singularity problem in the design of the controller is solved. Thus, the resultant controller is not segmented as in [34, 36]. (iii) In this paper, the time delay in the system is supposed to be time-varying, and the control gain of the MAS is a non-linear function. Therefore, a more general case is considered here in comparison with [34–36]. (iv) The problem of the explosion of complexity, reported in the previous literature [22, 35], is resolved by using the DSC approach.

The remainder of this paper is organised as follows. In Section 2, a brief background is given. Section 3 presents the problem statement, along with the dynamic model and some important assumptions. Then in Section 4, the distributed adaptive controller is provided, and its stability is analysed thoroughly. Simulation results are given in Section 5, confirming the efficacy of the proposed controller. Finally, some concluding remarks are presented in Section 6.

2 Preliminaries

2.1 Graph representation

Consider a MAS represented by a graph $\mathscr{G} \triangleq (V, \mathscr{E}, A)$, where the set of vertices $V = \{1, 2, \ldots, N\}$ denote the agents, the set of edges $\mathscr{E} \subseteq V \times V$ represent the communication links between agents, and $A = [a_{ij}]$ denotes a weighted adjacency matrix, representing the interaction of the links. In an undirected graph $\mathscr{G}$, a_{ij} is strictly positive if $(i, j) \in \mathscr{E}$, and is zero otherwise. In a directed graph $\mathscr{G}$, on the other hand, a_{ij} is strictly positive if $(i, j) \in \mathscr{E}$ and agent i can receive information from agent j, and is zero otherwise. The Laplacian matrix of $\mathscr{G}$ is defined as $L = D - A$, where $D = \mathrm{diag}(D_i) \in \mathfrak{R}^{N \times N}$ is a diagonal matrix with $D_i = \sum_{j=1}^{N} a_{ij}$ (i.e. the sum of the ith row of A). The Laplacian matrix is symmetric for an undirected graph but is not necessarily symmetric for a directed graph. Furthermore, corresponding to any agent $i \in V$, a binary parameter ξ_i is defined, which is equal to 1 if the leader is directly accessible to agent i, and 0 otherwise. A graph is connected if there is a path from each node to any other node. The directed graph has a spanning tree if there exists an agent with directed paths to all other agents.

2.2 Radial basis function NNs (RBFNN)

The RBFNN is an effective tool for approximating any smooth unknown function $h(Z): R^n \to R$. This approximation can be described as

$$h(Z) = w^T P(Z) + \varepsilon, \tag{1}$$

where $Z \in \Omega \subset R^n$ with Ω being a compact set over which the approximation is performed, $w = [w_1, w_2, \ldots, w_l]^{\mathrm{T}} \in R^l$ is the weight vector, and l is the number of neurons. Furthermore, $P(Z)$ is a vector of scalar basis functions $[P_1(Z), \ldots, P_l(Z)]^{\mathrm{T}}$ with

$$P_j(Z) = \exp\left[\frac{-(Z - \mu_j)^{\mathrm{T}}(Z - \mu_j)}{\sigma_j^2}\right], \quad j = 1, \ldots, l, \tag{2}$$

where $\mu_j \in R^n$ is the centre and $(\sqrt{2}\sigma_j)/2$ is the width of the above Gaussian function. Moreover, ε is the approximation error with $\| \varepsilon \| < \bar{\varepsilon}$, where $\bar{\varepsilon}$ is a positive real constant [37].

The following lemmas are borrowed from [38–42], and are used in the development of the main results.

Lemma 1: For any $z \in R$ and $\eta > 0$ the following inequality holds [38]:

$$0 \le |z| - \frac{z^2}{\sqrt{z^2 + \eta^2}} \le \eta. \tag{3}$$

Lemma 2: Let L be the Laplacian matrix for the undirected and connected graph $\mathscr{G}$ defined earlier and $\xi = \mathrm{diag}(\xi_i)$. Then $(L + \xi) > 0$ and this matrix is symmetric for at least one $\xi_i > 0, i = 1, \ldots, N$ [39].

Lemma 3: Consider the function $q_{i,m}(x): R \to R$ given by (4), (see (4)) , where $c_{qi} = \frac{[2(n-i)+1]!}{\lambda_{b_{i,m}}^{2(n-i)+1}[(n-i)!]^2}$, $\lambda_{a_{i,m}}, \lambda_{b_{i,m}} > 0$ are constant parameters, i and m are the positive integers (m will be defined in the next section) and n is the order of the system. The $(n-i)$th order derivative of this function exists and is bounded between 0 and 1 [40].

Lemma 4: Let $\underline{\sigma}(L + \xi)$ be the minimum singular value of $(L + \xi)$, where δ and $S_1 = [S_{1,1}, \ldots, S_{N,1}]^T$ will be defined in Section 4. Then [41]

$$\| \delta \| \leqslant \frac{\| S_1 \|}{\underline{\sigma}(L + \xi)}. \tag{5}$$

Lemma 5: For any $a, b \in \mathfrak{R}^+$, $\forall \epsilon > 0$ and $p, q > 0$ such that $(1/p) + (1/q) = 1$, the following inequality holds [42, Young's inequality]:

$$ab \le \frac{a^p}{\epsilon p} + \frac{\epsilon b^q}{q}. \tag{6}$$

3 Problem statement

Consider a MAS where the dynamics of each follower agent is described by:

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + \psi_{i,m}(\bar{x}_{i,m})^{\mathrm{T}}\theta_{i,m} + h_{i,m}(\bar{x}_{i,m}(t - \tau_{i,m}(t))) \\ &\quad + d_{i,m}(t) \\ \dot{x}_{i,n} &= \beta_i(x_i)u_i + \psi_{i,n}(x_i)^{\mathrm{T}}\theta_{i,n} + h_{i,n}(x_i(t - \tau_{i,n}(t))) \\ &\quad + d_{i,n}(t) \\ y_i &= x_{i,1}, \quad \text{for } i \in \{1, \ldots, N\} \text{ and } m \in \{1, \ldots, n-1\} \end{aligned} \tag{7}$$

where $\bar{x}_{i,m} = [x_{i,1}, \ldots, x_{i,m}]^{\mathrm{T}}$, $x_i = [x_{i,1}, \ldots, x_{i,n}]^{\mathrm{T}} \in \mathfrak{R}^n$ are the state vectors of the ith agent and $u_i \in \mathfrak{R}$ is the corresponding control input. Furthermore, $y_i \in \mathfrak{R}$ is the output of the ith agent, and for any $i \in \{1, \ldots, N\}$ and $m \in \{1, \ldots, n\}$, $\theta_{i,m} \in \mathfrak{R}^{p_i}$ is an unknown constant parameter vector, $\psi_{i,m}: \mathfrak{R}^m \to \mathfrak{R}^{p_i}$ and $\beta_i \neq 0$ are known smooth non-linear functions, $h_{i,m}(.)$ is an unknown smooth non-linear function, and $d_{i,m}(t) \in \mathfrak{R}$ is an uncertain external disturbance signal. For any $i \in \{1, \ldots, N\}$ and $m \in \{1, \ldots, n\}$, $\tau_{i,m}(t)$ is an unknown time-varying state delay.

Assumption 1: The communication graph $\mathscr{G}$ is fixed and connected.

Assumption 2: The leader trajectory y_r has a continuous, second-order derivative, and there is a positive constant D_0 such that [43]

$$\prod := \{(y_r, \dot{y}_r, \ddot{y}_r): y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \le D_0\}. \tag{8}$$

$$q_{i,m}(x) = \begin{cases} 1, & |x| \ge \lambda_{a_{i,m}} + \lambda_{b_{i,m}} \\ c_{qi}\displaystyle\int_{\lambda_{a_{i,m}}}^{x} \left[(\lambda_{a_{i,m}} + \lambda_{b_{i,m}} - \sigma)(\sigma - \lambda_{a_{i,m}})\right]^{(n-i)} \mathrm{d}\sigma, & \lambda_{a_{i,m}} < x < \lambda_{a_{i,m}} + \lambda_{b_{i,m}} \\ c_{qi}\displaystyle\int_{x}^{-\lambda_{a_{i,m}}} \left[-(\lambda_{a_{i,m}} + \lambda_{b_{i,m}} + \sigma)(\sigma + \lambda_{a_{i,m}})\right]^{(n-i)} \mathrm{d}\sigma, & -(\lambda_{a_{i,m}} + \lambda_{b_{i,m}}) < x < -\lambda_{a_{i,m}} \\ 0 & |x| \le \lambda_{a_{i,m}} \end{cases} \tag{4}$$

Assumption 3: The first derivative of the state of the leader is only known by agent i with $\xi_i = 1$. Furthermore, for agents with $\xi_i = 0$, $|\dot{y}_r| \leqslant H_{r,1}$, where $H_{r,1}$ is an unknown positive number [22].

Assumption 4: The disturbance signal $d_{i,m}$ is upper-bounded with an unknown positive constant bound $D_{i,m}$, i.e. $|d_{i,m}(t)| \leq D_{i,m}$, $\forall i, m \in V$ [22].

Assumption 5: The unknown time-varying state delay satisfies the inequalities $\tau_{i,m}(t) \leq \bar{\tau}_{i,m}$ and $\dot{\tau}_{i,m}(t) \leq \hbar < 1$, where $\bar{\tau}_{i,m}$ and $\hbar$ are unknown positive constants [44].

The control objective is to design a distributed adaptive dynamic surface controller $u_i(t) \in R, i = 1, \ldots, N$, such that the output of every agent tracks the leader's trajectory y_r, and that the states of the closed-loop system are semi-globally uniformly bounded.

4 Main results

In this section, an adaptive control scheme based on the DSC method for system (7) is developed. To this end, the following change of variable is introduced first

$$S_{i,1} = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + \xi_i(y_i - y_r), \quad i \in \{1, 2, \ldots, N\}. \tag{9}$$

Define also,

$$S_1 = (L + \xi)\delta, \tag{10}$$

where $\delta = \bar{y} - \bar{y}_r = [\delta_{1,1}, \ldots, \delta_{N,1}]^{\mathrm{T}} \in \mathfrak{R}^N$, $\bar{y} = [y_1, \ldots, y_N]^{\mathrm{T}} \in \mathfrak{R}^N$, $\bar{y}_r = [y_r, \ldots, y_r]^{\mathrm{T}} \in \mathfrak{R}^N$, and $S_1 = [S_{1,1}, \ldots, S_{N,1}]^{\mathrm{T}}$. Also

$$S_{i,m} = x_{i,m} - z_{i,m}, \quad i \in \{1, 2, \ldots, N\}, \, m \in \{2, 3, \ldots, n\}, \tag{11}$$

where $S_{i,m}$ is an error surface and $z_{i,m}$ is generated by the following first-order filter:

$$\zeta_{i,m}\dot{z}_{i,m} + z_{i,m} = \alpha_{i,m-1} \quad z_{i,m}(0) = \alpha_{i,m-1}(0), \tag{12}$$

wherein $\alpha_{i,m-1}$ is a virtual control signal (to be chosen later) and $\zeta_{i,m}$ is a constant. Let $X_{i,m}$ be the difference between the virtual control signal and its filtered version as

$$X_{i,m} = z_{i,m} - \alpha_{i,m-1}, \tag{13}$$

for any $i \in \{1, 2, \ldots, N\}$, $m \in \{2, 3, \ldots, n\}$. A three-step procedure is presented in the sequel to design a distributed adaptive controller based on the dynamic surface control method.

Step 1. Taking the derivative of S_1, one arrives at

$$\dot{S}_1 = (L + \xi)[\dot{\delta}_{1,1}, \ldots, \dot{\delta}_{N,1}]^{\mathrm{T}}, \tag{14}$$

where

$$\begin{aligned}\dot{\delta}_{i,1} = \alpha_{i,1} + S_{i,2} + X_{i,2} + \psi_{i,1}^T\theta_{i,1} + h_{i,1}(\bar{x}_{i,1}(t - \tau_{i,1})) \\ + d_{i,1} - \dot{y}_r.\end{aligned} \tag{15}$$

The Lyapunov function candidate is defined as

$$V_1 = V_{S_1} + V_{\theta_1} + V_{H_1} + V_{w_1} + V_{h_1}, \tag{16}$$

with

$$\begin{aligned}
V_{S_1} &= \frac{\delta^T(L+\xi)\delta}{2}, \quad V_{\theta_1} = \sum_{i=1}^{N} V_{\theta_{i,1}} = \frac{\sum_{i=1}^{N}\tilde{\theta}_{i,1}^T\Gamma_{i,1}^{-1}\tilde{\theta}_{i,1}}{2}, \\
V_{H_1} &= \sum_{i=1}^{N} V_{H_{i,1}} = \sum_{i=1}^{N}\frac{\tilde{H}_{i,1}^2}{2\gamma_{i,1}}, \\
V_{w_1} &= \sum_{i=1}^{N} V_{w_{i,1}} = \frac{1}{2}\sum_{i=1}^{N}\tilde{w}_{i,1}^T\Lambda_{i,1}^{-1}\tilde{w}_{i,1}, \\
V_{h_1} &= \frac{1}{2(1-\hbar)}\sum_{i=1}^{N}\int_{t-\tau_{i,1}(t)}^{t} e^{\Upsilon_{i,1}(\varrho - t)}h_{i,1}^2(\bar{x}_{i,1}(\varrho))\mathrm{d}\varrho \\
&= \sum_{i=1}^{N} V_{h_{i,1}}.
\end{aligned} \tag{17}$$

Note that $\tilde{\theta}_{i,1} = \hat{\theta}_{i,1} - \theta_{i,1}$ and $\tilde{H}_{i,1} = \hat{H}_{i,1} - H_{i,1}$ are errors of the corresponding parameter estimations, $\tilde{w}_{i,1} = \hat{w}_{i,1} - w_{i,1}$ is the error of the corresponding weight vector, $H_{i,1} = D_{i,1} + (1 - \xi_i)H_{r,1}$, where $D_{i,1}$ is an unknown upper bound of the disturbance as introduced in Assumption 4 (the hat symbol on a parameter represents its estimate). Also, $\Gamma_{i,1}$ and $\Lambda_{i,1}$ are positive definite matrices with appropriate dimensions, and $\gamma_{i,1}$ and $\Upsilon_{i,1}$ are strictly positive real numbers. From (16) and (17), the time derivative of V_1 is obtained as

$$\begin{aligned}
\dot{V}_1 = \sum_{i=1}^{N}\dot{V}_{i,1} = \sum_{i=1}^{N}\Big[S_{i,1}\dot{\delta}_{i,1} + \frac{1}{2(1-\hbar)}h_{i,1}^2(\bar{x}_{i,1}(t)) \\
-\frac{1-\dot{\tau}_{i,1}(t)}{2(1-\hbar)}e^{-\Upsilon_{i,1}\tau_{i,1}}h_{i,1}^2(\bar{x}_{i,1}(t-\tau_{i,1})) + \tilde{\theta}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{\theta}}_{i,1} \\
-\Upsilon_{i,1}V_{h_{i,1}} + \tilde{w}_{i,1}^T\Lambda_{i,1}^{-1}\dot{\hat{w}}_{i,1} + \frac{1}{\gamma_{i,1}}\tilde{H}_{i,1}\dot{\hat{H}}_{i,1}\Big].
\end{aligned} \tag{18}$$

Substituting (15) into (18), using Young's inequality and Assumption 5 yields

$$\begin{aligned}
\dot{V}_{i,1} \leq S_{i,1}S_{i,2} + S_{i,1}X_{i,2} + S_{i,1}\alpha_{i,1} + S_{i,1}Q_{i,1}(\bar{S}_{i,1}) \\
+\psi_{i,1}^T\theta_{i,1}S_{i,1} + S_{i,1}d_{i,1} + \tilde{\theta}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{\theta}}_{i,1} + \frac{1}{\gamma_{i,1}}\tilde{H}_{i,1}\dot{\hat{H}}_{i,1} \\
-\Upsilon_{i,1}V_{h_{i,1}} - S_{i,1}\dot{y}_r + \tilde{w}_{i,1}^T\Lambda_{i,1}^{-1}\dot{\hat{w}}_{i,1},
\end{aligned} \tag{19}$$

where

$$Q_{i,1}(\bar{S}_{i,1}) = \frac{S_{i,1}(t)e^{\Upsilon_{i,1}\bar{\tau}_{i,1}}}{2} + \frac{1}{2(1-\hbar)S_{i,1}(t)}h_{i,1}^2(\bar{x}_{i,1}(t)). \tag{20}$$

and $\bar{S}_{i,1} = [S_{i,1}, \bar{x}_{i,1}]^{\mathrm{T}}$. By (19), the virtual control signal is designed as

$$\begin{aligned}
\alpha_{i,1}(\bar{x}_{i,1}(t)) = -c_{i,1}S_{i,1} - \psi_{i,1}^T\hat{\theta}_{i,1} + \xi_1\dot{y}_r \\
-\frac{S_{i,1}}{\sqrt{S_{i,1}^2 + \eta^2}}\hat{H}_{i,1} - Q_{i,1}(\bar{S}_{i,1}),
\end{aligned} \tag{21}$$

where $c_{i,1}$ is a positive constant, $\eta(t) > 0$, $\int_0^t \eta(\tau)\mathrm{d}\tau \leq \bar{\eta} < \infty$, $\forall t \geq 0$ and $\bar{\eta} > 0$. Note that the virtual control signal is designed in such a way that the time derivative of $V_{i,1}$ is upper bounded appropriately. It can be observed from (20) and (21) that $\alpha_{i,1}$ is not well-defined at $S_{i,1}(t) = 0$ because $Q_{i,1}$ becomes singular. To overcome this hurdle, the function $q_{i,m}(x)$ defined in Lemma 3 is used as follows:

$$\begin{aligned}
\alpha_{i,1}(\bar{x}_{i,1}(t)) = q_{i,1}(S_{i,1})\Big[-c_{i,1}S_{i,1} - \psi_{i,1}^T\hat{\theta}_{i,1} + \xi_1\dot{y}_r \\
-\frac{S_{i,1}}{\sqrt{S_{i,1}^2 + \eta^2}}\hat{H}_{i,1} - Q_{i,1}(\bar{S}_{i,1})\Big].
\end{aligned} \tag{22}$$

The virtual control signal (22) is not specified because $Q_{i,1}$ is unknown. It is straightforward to show that $Q_{i,1}$ is smooth for all

$\bar{x}_{i,1} \in R^N$ and $|S_{i,1}| \geq \lambda_{a_{i,1}}$, where $\lambda_{a_{i,1}}$ is a constant parameter in Lemma 3. Let $\bar{S}_{i,1} \in \Omega_{S_{i,1}} \times \Omega_{\bar{x}_{i,1}} \subset R^N \times R^N$ (where $\Omega_{S_{i,1}}$ and $\Omega_{\bar{x}_{i,1}}$ are compact subsets of R^N) and $\Omega'_{S_{i,1}} = \{S_{i,1} \big| |S_{i,1}| < \lambda_{a_{i,1}}\} \subset \Omega_{S_{i,1}}$, and define the compact set $\Omega^0_{S_{i,1}} = \Omega_{S_{i,1}} - \Omega'_{S_{i,1}}$. Therefore, $\Omega^0_{\bar{S}_{i,1}} = \Omega^0_{S_{i,1}} \times \Omega_{\bar{x}_{i,1}}$ is a compact subset of $R^N \times R^N$. Thus, $Q_{i,1}$ (that is the non-linear part of $\dot{V}_{i,1}$) can be approximated over $\Omega^0_{\bar{S}_{i,1}}$ by a RBFNN as

$$Q_{i,1}(\bar{S}_{i,1}) = w_{i,1}^T P_{i,1}(\bar{S}_{i,1}) + \varepsilon_{i,1}(\bar{S}_{i,1}), \tag{23}$$

where $P_{i,1}$ is a basis function vector and $\varepsilon_{i,1}$ is the bounded approximation error for $\bar{S}_{i,1} \in \Omega^0_{\bar{S}_{i,1}}$, ($\left|\varepsilon_{i,1}(\bar{S}_{i,1})\right| \leqslant \varepsilon^*_{\bar{S}_{i,1}}$, where $\varepsilon^*_{\bar{S}_{i,1}}$ is a positive real constant). Therefore, the virtual control law is reformulated as

$$\begin{aligned}\alpha_{i,1}(\bar{x}_{i,1}(t)) = q_{i,1}(S_{i,1}(t))\Big[&-c_{i,1}S_{i,1} - \psi_{i,1}^T\hat{\theta}_{i,1} + \xi_1\dot{y}_r \\ &- \frac{S_{i,1}}{\sqrt{S_{i,1}^2+\eta^2}}\hat{H}_{i,1} - \hat{w}_{i,1}^T P_{i,1}(\bar{S}_{i,1})\Big],\end{aligned} \tag{24}$$

and the parameter estimations or adaptive laws are given by

$$\dot{\hat{\theta}}_{i,1} = q_{i,1}(S_{i,1}(t))[-\eta_{i,1}\Gamma_{i,1}\hat{\theta}_{i,1} + \Gamma_{i,1}S_{i,1}\psi_{i,1}^T] \tag{25}$$

$$\dot{\hat{H}}_{i,1} = q_{i,1}(S_{i,1}(t))\left[\gamma_{i,1}\frac{S_{i,1}^2}{\sqrt{S_{i,1}^2+\eta^2}} - \gamma_{i,1}\hat{H}_{i,1}\right] \tag{26}$$

$$\dot{\hat{w}}_{i,1} = q_{i,1}(S_{i,1}(t))\left[\Lambda_{i,1}\left(S_{i,1}(t)P_{i,1}(\bar{S}_{i,1}) - \sigma_{i,1}\hat{w}_{i,1}\right)\right], \tag{27}$$

where $\eta_{i,1}, \Gamma_{i,1}, \gamma_{i,1}, \Lambda_{i,1}$, and $\sigma_{i,1}$ determine the convergence rate of $\hat{\theta}_{i,1}, \hat{H}_{i,1}$, and $\hat{w}_{i,1}$. Hence, using the function $q_{i,1}(S_{i,1})$, the control signal will be non-singular at all points. Now, using Lemma 3, the proof of stability is provided for three separate regions in the sequel.

Region 1. $\left\{S_{i,1} \in R \;\middle|\; |S_{i,1}| \geqslant \lambda_{a_{i,1}} + \lambda_{b_{i,1}}\right\}$. In this region, $q_{i,1}(S_{i,1}) = 1$. Thus, the parameter estimations become

$$\dot{\hat{\theta}}_{i,1} = -\eta_{i,1}\Gamma_{i,1}\hat{\theta}_{i,1} + \Gamma_{i,1}S_{i,1}\psi_{i,1}^T \tag{28}$$

$$\dot{\hat{H}}_{i,1} = \gamma_{i,1}\frac{S_{i,1}^2}{\sqrt{S_{i,1}^2+\eta_{i,1}^2}} - \gamma_{i,1}\hat{H}_{i,1} \tag{29}$$

$$\dot{\hat{w}}_{i,1} = \Lambda_{i,1}\left(S_{i,1}(t)P_{i,1}(\bar{S}_{i,1}) - \sigma_{i,1}\hat{w}_{i,1}\right), \tag{30}$$

and the virtual control signal turns out to be

$$\begin{aligned}\alpha_{i,1}(\bar{x}_{i,1}(t)) = &-c_{i,1}S_{i,1} - \psi_{i,1}^T\hat{\theta}_{i,1} + \xi_1\dot{y}_r \\ &- \frac{S_{i,1}}{\sqrt{S_{i,1}^2+\eta^2}}\hat{H}_{i,1} - \hat{w}_{i,1}^T P_{i,1}(\bar{S}_{i,1}).\end{aligned} \tag{31}$$

Substituting (28)–(31) into (19), and using Young's inequalities

$$\begin{aligned}&-\eta_{i,1}\tilde{\theta}_{i,1}^T\hat{\theta}_{i,1} \leq \frac{-1}{2}\eta_{i,1}\|\tilde{\theta}_{i,1}\|^2 + \frac{1}{2}\eta_{i,1}\|\theta_{i,1}\|^2, \\ &-\tilde{H}_{i,1}\hat{H}_{i,1} \leq \frac{-1}{2}\left|\tilde{H}_{i,1}\right|^2 + \frac{1}{2}|H_{i,1}|^2, \\ &-\sigma_{i,1}\tilde{w}_{i,1}^T\hat{w}_{i,1} \leq \frac{-1}{2}\sigma_{i,1}\|\tilde{w}_{i,1}\|^2 + \frac{1}{2}\sigma_{i,1}\|w_{i,1}\|^2,\end{aligned}$$

Equation (23), Lemma 1, and the relation $H_{i,1} = D_{i,1} + (1-\xi_i)H_{r,1}$, result in

$$S_{i,1}(D_{i,1} + (1-\xi_1)\dot{y}_r) - \frac{S_{i,1}^2}{\sqrt{S_{i,1}^2+\eta^2}}H_{i,1} \leq H_{i,1}\eta, \tag{32}$$

the time derivative of $V_{i,1}$ can be written as

$$\begin{aligned}\dot{V}_{i,1} \leq &\frac{S_{i,2}^2}{2} + \frac{X_{i,2}^2}{2} - \Upsilon_{i,1}V_{h_{i,1}} - c_{i,1}^*S_{i,1}^2 + \mu_{i,1} \\ &-\frac{1}{2}\left|\tilde{H}_{i,1}\right|^2 - \frac{1}{2}\eta_{i,1}\|\tilde{\theta}_{i,1}\|^2 - \frac{1}{2}\sigma_{i,1}\|\tilde{w}_{i,1}\|^2,\end{aligned} \tag{33}$$

where

$$\begin{aligned}&c_{i,1}^* = c_{i,1} - \frac{3}{2}, \\ &\mu_{i,1} = \frac{1}{2}\eta_{i,1}\|\theta_{i,1}\|^2 + \frac{1}{2}\varepsilon_{i,1}^2 + \frac{1}{2}\sigma_{i,1}\|w_{i,1}\|^2 + \frac{1}{2}|H_{i,1}|^2 \\ &\qquad + H_{i,1}\eta.\end{aligned} \tag{34}$$

As can be seen in (33), the boundedness of $V_{i,1}(t)$ is dependent on $S_{i,2}$ and $X_{i,2}$, which will be investigated in the next step.

Region 2. $\left\{S_{i,1} \in R \;\middle|\; \lambda_{a_{i,1}} < |S_{i,1}| < \lambda_{a_{i,1}} + \lambda_{b_{i,1}}\right\}$. In this region, $|S_{i,1}|$ is bounded from both sides, and consequently, V_{S_1} and V_{h_1} are bounded as well. To prove the boundedness of $V_1(t)$, some inequalities on $\dot{V}_{\theta_{i,1}}$, $\dot{V}_{H_{i,1}}$, and $\dot{V}_{w_{i,1}}$ are derived in the following.

From (17) and (25), the time derivative of $V_{\theta_{i,1}}$ is given by

$$\dot{V}_{\theta_{i,1}} = \tilde{\theta}_{i,1}^T\Gamma_{i,1}^{-1}\dot{\hat{\theta}}_{i,1} = q_{i,1}(S_{i,1})\tilde{\theta}_{i,1}^T(-\eta_{i,1}\hat{\theta}_{i,1} + S_{i,1}\psi_{i,1}^T). \tag{35}$$

From Young's inequality

$$\begin{aligned}q_{i,1}(S_{i,1})\tilde{\theta}_{i,1}^T S_{i,1}\psi_{i,1}^T \leq &\frac{1}{2c'_{i,1}}q_{i,1}(S_{i,1})\|\tilde{\theta}_{i,1}\|^2 \\ &+\frac{c'_{i,1}}{2}q_{i,1}(S_{i,1})\psi_{i,1}^T\psi_{i,1}S_{i,1}^2,\end{aligned} \tag{36}$$

where $c'_{i,1}$ is a positive real constant. On the other hand, from the inequality $-\eta_{i,1}\tilde{\theta}_{i,1}^T\hat{\theta}_{i,1} \leq \frac{-1}{2}\eta_{i,1}\|\tilde{\theta}_{i,1}\|^2 + \frac{1}{2}\eta_{i,1}\|\theta_{i,1}\|^2$, and hence

$$\begin{aligned}\dot{V}_{\theta_{i,1}} \leq &\frac{-1}{2}q_{i,1}(S_{i,1})\left(\eta_{i,1} - \frac{1}{c'_{i,1}}\right)\|\tilde{\theta}_{i,1}\|^2 \\ &+\frac{1}{2}q_{i,1}(S_{i,1})(\eta_{i,1}\|\theta_{i,1}\|^2 + c'_{i,1}\psi_{i,1}^T\psi_{i,1}S_{i,1}^2).\end{aligned} \tag{37}$$

In this region, $\psi_{i,1}$ is smooth and bounded, and $q_{i,1}(S_{i,1}) \in (0,1)$. Thus, by choosing $c'_{i,1}$ such that $\eta_{i,1}^* = \eta_{i,1} - \frac{1}{c'_{i,1}} > 0$ (the parameter $\eta_{i,1}$ is designed in (28)), the following inequality is obtained:

$$\dot{V}_{\theta_{i,1}} \leq -C^*_{\theta_{i,1}}V_{\theta_{i,1}}(t) + \lambda_{\theta_{i,1}}, \tag{38}$$

where

$$\begin{aligned}&C^*_{\theta_{i,1}} = q_{i,1}(S_{i,1})\frac{\eta_{i,1}^*}{\lambda_{\max}(\Gamma_{i,1}^{-1})}, \\ &\lambda_{\theta_{i,1}} = \frac{1}{2}q_{i,1}(S_{i,1})(\eta_{i,1}\|\theta_{i,1}\|^2 + c'_{i,1}\psi_{i,1}^T\psi_{i,1}S_{i,1}^2).\end{aligned} \tag{39}$$

Since $\lambda_{a_{i,1}} < |S_{i,1}| < \lambda_{a_{i,1}} + \lambda_{b_{i,1}}$, hence $V_{\theta_{i,1}}(t)$ in the following inequality:

$$0 \leq V_{\theta_{i,1}}(t) \leq \left[V_{\theta_{i,1}}(0) - \rho_{\theta_{i,1}}\right]e^{-C^*_{\theta_{i,1}}t} + \rho_{\theta_{i,1}} \tag{40}$$

is bounded, where $\rho_{\theta_{i,1}} = \frac{\lambda_{\theta_{i,1}}}{C^*_{\theta_{i,1}}}$. Similar to (37), the inequality

$$\begin{aligned}\dot{V}_{H_{i,1}} \leq &\frac{-1}{2}q_{i,1}(S_{i,1})\left|\tilde{H}_{i,1}\right|^2 + \frac{1}{2}q_{i,1}(S_{i,1})\Big(|H_{i,1}|^2 \\ &+\left|\tilde{H}_{i,1}\right|\frac{S_{i,1}^2}{\sqrt{S_{i,1}^2+\eta^2}}\Big)\end{aligned} \tag{41}$$

is derived to prove the boundedness of $V_{H_{i,1}}(t)$. In this region, $q_{i,1}(S_{i,1}) \in (0,1)$, and $\frac{1}{2}q_{i,1}(S_{i,1})\left(\left|H_{i,1}\right|^2 + \left|\tilde{H}\right|\frac{S_{i,1}^2}{\sqrt{S_{i,1}^2+\eta^2}}\right)$ is positive and bounded. Consequently,

$$
\begin{aligned}
\dot{V}_{H_{i,1}} &\le -C^*_{H_{i,1}}V_{H_{i,1}}(t) + \lambda_{H_{i,1}}, \\
C^*_{H_{i,1}} &= q_{i,1}(S_{i,1})\frac{1}{\lambda_{\max}(\gamma_{i,1}^{-1})}, \\
\lambda_{H_{i,1}} &= \frac{1}{2}q_{i,1}(S_{i,1})\left(\left|H_{i,1}\right|^2 + \left|\tilde{H}\right|\frac{S_{i,1}^2}{\sqrt{S_{i,1}^2+\eta_{i,1}^2}}\right).
\end{aligned} \tag{42}
$$

Since $\lambda_{a_{i,1}} < |S_{i,1}| < \lambda_{a_{i,1}} + \lambda_{b_{i,1}}$, thus $V_{H_{i,1}}(t)$ in the following inequality:

$$
\begin{aligned}
0 \le V_{H_{i,1}}(t) &\le \left[V_{H_{i,1}}(0) - \rho_{H_{i,1}}\right]e^{-C^*_{H_{i,1}}t} + \rho_{H_{i,1}}, \\
\rho_{H_{i,1}} &= \frac{\lambda_{H_{i,1}}}{C^*_{H_{i,1}}}
\end{aligned} \tag{43}
$$

is bounded for all $t \in \mathfrak{R}^+$. Similar to the previous approaches for the boundedness of $V_{\theta_{i,1}}(t)$ and $V_{H_{i,1}}(t)$, the following inequality is obtained to prove the boundedness of $V_{w_{i,1}}(t)$:

$$
\begin{aligned}
\dot{V}_{w_{i,1}} &\le -C^*_{w_{i,1}}V_{w_{i,1}}(t) + \lambda_{w_{i,1}}, \\
C^*_{w_{i,1}} &= q_{i,1}(S_{i,1})\frac{\sigma^*_{i,1}}{\lambda_{\max}(\Lambda_{i,1}^{-1})}, \\
\lambda_{w_{i,1}} &= \frac{1}{2}q_{i,1}(S_{i,1})(\sigma_{i,1}\| w_{i,1}\|^2 + c''_{i,1}S_{i,1}^2P_{i,1}^2),
\end{aligned} \tag{44}
$$

where $\sigma^*_{i,1} = \sigma_{i,1} - \frac{1}{c''_{i,1}} > 0$, and $c''_{i,1}$ is a positive constant. Therefore, it results from (16) that $V_1(t)$ is bounded in this region.

Region 3. $\left\{S_{i,1} \in R \;\middle|\; |S_{i,1}| \le \lambda_{a_{i,1}}\right\}$. In this region, $S_{i,1}$ is clearly bounded, and from (9) and Lemma 2, $x_{i,1}$ is bounded as well. Consequently, V_{S_1} and $V_{h_{i,1}}$ are also bounded. In this region, $q_{i,1}(S_{i,1}) = 0$; hence $\dot{\hat{\theta}}_{i,1} = 0$, $\alpha_{i,1} = 0$, and $\hat{\theta}_{i,1}$ is bounded. Similarly, $\hat{w}_{i,1}$ and $\hat{H}_{i,1}$ are bounded. Thus, $V_{\theta_{i,1}}$, $V_{w_{i,1}}$, and $V_{H_{i,1}}$ are bounded. Accordingly, $V_1(t)$ is bounded.

Remark 1: Unlike undirected graphs, the Laplacian matrix of a directed graph is not symmetric. As a result, the matrix $(L+\xi)$ is not symmetric positive definite [39]. Thus, V_{S_1} in the Lyapunov function in (17) will be $V_{S_1} = \sum_{i=1}^{N} V_{S_{i,1}} = \sum_{i=1}^{N}\frac{S_{i,1}^2}{2}$, and the derivative of $S_{i,1}$ is obtained as

$$
\begin{aligned}
\dot{S}_{i,1} &= \sum_{j=1}^{N} a_{ij}(\dot{x}_{i,1} - \dot{x}_{j,1}) + \xi_i(\dot{x}_{i,1} - \dot{y}_r) \\
&= b_i\dot{x}_{i,1} - \sum_{j=1}^{N} a_{ij}\dot{x}_{j,1} - \xi_i\dot{y}_r,
\end{aligned} \tag{45}
$$

where $b_i = \xi_i + \sum_{j=1}^{N} a_{ij}$, $i \in \{1,2,\ldots,N\}$. Moreover, $\dot{x}_{j,1}$ in (45) introduces a new term in the Lyapunov function as

$$
\begin{aligned}
V_{h_{N_i,1}} &= \frac{1}{2(1-\hbar)}\sum_{j=1}^{N} a_{i,j}V_{h_{j,1}} \\
&= \frac{1}{2(1-\hbar)}\sum_{j=1}^{N} a_{i,j}\int_{t-\tau_{j,1}(t)}^{t} e^{\Upsilon_{j,1}(\varrho - t)}h_{j,1}^2(\bar{x}_{j,1}(\varrho))\mathrm{d}\varrho\,.
\end{aligned}
$$

This change affects step 1, but steps 2 and 3 (will be explained in the next steps) remain the same as those in the undirected graph.

Step 2. Similar to the first step, here by considering (7), (11), and (13) for $m = 2,\ldots,n-1$, the time derivative of $S_{i,m}$ is computed as

$$
\begin{aligned}
\dot{S}_{i,m} = {} & S_{i,m+1} + X_{i,m+1} + \alpha_{i,m} + h_{i,m}(\bar{x}(t-\tau_{i,m})) \\
& + \psi_{i,m}^T\theta_{i,m} + d_{i,m} - \dot{z}_{i,m}\,.
\end{aligned} \tag{46}
$$

The Lyapunov function candidate for this subsystem is defined as

$$
V_m = \sum_{i=1}^{N} V_{i,m}, \tag{47}
$$

where

$$
\begin{aligned}
V_{i,m} = {} & V_{i,m-1} + V_{S_{i,m}} + V_{\theta_{i,m}} + V_{H_{i,m}} + V_{h_{i,m}} + V_{w_{i,m}} \\
& + V_{X_{i,m}}, \\
V_{S_{i,m}} = {} & \frac{S_{i,m}^2}{2},\ V_{\theta_{i,m}} = \frac{\tilde{\theta}_{i,m}^T\Gamma_{i,m}^{-1}\tilde{\theta}_{i,m}}{2},\ V_{H_{i,m}} = \frac{\tilde{H}_{i,m}^2}{2\gamma_{i,m}}, \\
V_{h_{i,m}} = {} & \frac{1}{2(1-\hbar)}\int_{t-\tau_{i,m}(t)}^{t} e^{\Upsilon_{i,m}(\varrho - t)}h_{i,m}^2(\bar{x}_{i,m}(\varrho))\mathrm{d}\varrho, \\
V_{X_{i,m}} = {} & \frac{X_{i,m}^2}{2}, V_{w_{i,m}} = \frac{1}{2}\tilde{w}_{i,m}^T\Lambda_{i,m}^{-1}\tilde{w}_{i,m},
\end{aligned} \tag{48}
$$

note that the term $V_{X_{i,m}}$ was not part of the Lyapunov function of the previous step. From (48), the time derivative of $V_{i,m}$ is obtained as follows:

$$
\begin{aligned}
\dot{V}_{i,m} = {} & \dot{V}_{i,m-1} + S_{i,m}(t)\dot{S}_{i,m}(t) + X_{i,m}(t)\dot{X}_{i,m}(t) \\
& + \frac{1}{\gamma_{i,m}}\tilde{H}_{i,m}\dot{\hat{H}}_{i,m} + \frac{1}{2(1-\hbar)}h_{i,m}^2(\bar{x}_{i,m}(t)) \\
& - \frac{1-\dot{\tau}_{i,m}(t)}{2(1-\hbar)}e^{-\Upsilon_{i,m}\tau_{i,m}}h_{i,m}^2(\bar{x}_{i,m}(t-\tau_{i,m})) \\
& - \Upsilon_{i,m}V_{h_{i,m}} + \tilde{w}_{i,m}^T\Lambda_{i,m}^{-1}\dot{\hat{w}}_{i,m} + \tilde{\theta}_{i,m}^T\Gamma_{i,m}^{-1}\dot{\hat{\theta}}_{i,m}\,.
\end{aligned} \tag{49}
$$

Let

$$
\alpha_{i,m-1} = \zeta_{i,m}\dot{z}_{i,m} + z_{i,m}, \qquad \alpha_{i,m-1}(0) = z_{i,m}(0), \tag{50}
$$

and use (13) to obtain

$$
\begin{aligned}
\dot{X}_{i,m} = \dot{z}_{i,m} - \dot{\alpha}_{i,m-1} = {} & -\frac{X_{i,m}}{\zeta_{i,m}} + E_{i,m}(S_{i,1},\ldots,S_{i,m},X_{i,2} \\
& ,\ldots,X_{i,m},\hat{w}_{i,1},\ldots,\hat{w}_{i,m},\hat{H}_{i,1},\ldots,\hat{H}_{i,m},\hat{\theta}_{i,1},\ldots,\hat{\theta}_{i,m},\eta,\dot{\eta}),
\end{aligned} \tag{51}
$$

where

$$
E_{i,m}(\,.\,) = -\dot{\alpha}_{i,m-1}(t)\,. \tag{52}
$$

From (46), (49), (51) and Assumption 5, the time derivative of $V_{i,m}$ is obtained as

$$
\begin{aligned}
\dot{V}_{i,m} \le {} & \dot{V}_{i,m-1} + S_{i,m}(t)S_{i,m+1}(t) + S_{i,m}(t)\alpha_{i,m}(t) \\
& + S_{i,m}(t)X_{i,m+1}(t) + S_{i,m}(t)Q_{i,m}(\bar{S}_{i,m}) \\
& + S_{i,m}\psi_{i,m}^T\theta_{i,m} + S_{i,m}(t)d_{i,m} + \tilde{\theta}_{i,m}^T\Gamma_{i,m}^{-1}\dot{\hat{\theta}}_{i,m} \\
& + X_{i,m}\left(-\frac{X_{i,m}}{\zeta_{i,m}} + E_{i,m}(\,.\,)\right) + \frac{1}{\gamma_{i,m}}\tilde{H}_{i,m}\dot{\hat{H}}_{i,m} \\
& + \tilde{w}_{i,m}^T\Lambda_{i,m}^{-1}\dot{\hat{w}}_{i,m} - S_{i,m}\dot{z}_{i,m} - \Upsilon_{i,m}V_{h_{i,m}}\,.
\end{aligned} \tag{53}
$$

In this step, $Q_{i,m}$ is defined as follows:

$$
\begin{aligned}
Q_{i,m}(\bar{S}_{i,m}) = {} & \frac{S_{i,m}(t)e^{\Upsilon_{i,m}\bar{\tau}_{i,m}}}{2} \\
& + \frac{1}{2(1-\hbar)S_{i,m}(t)}h_{i,m}^2(\bar{x}_{i,m}(t)),
\end{aligned} \tag{54}
$$

where $\bar{S}_{i,m} = [S_{i,m}, \bar{x}_{i,m}]^{\mathrm{T}}$. Similar to the previous step, $Q_{i,m}$ is not well-defined at $S_{i,m}(t) = 0$ and the resultant virtual controller is singular at this point. To overcome this problem, $q_{i,m}(S_{i,m})$ defined in Lemma 3 is used. Thus, the virtual controller is given by

$$\begin{aligned}\alpha_{i,m}(\bar{x}_{i,m}(t)) = q_{i,m}(S_{i,m}(t))\Big[- c_{i,m}S_{i,m} - \psi_{i,m}^T\hat{\theta}_{i,m} \\ - \frac{S_{i,m}}{\sqrt{S_{i,m}^2 + \eta^2}}\hat{H}_{i,m} - Q_{i,m} + \dot{z}_{i,m}\Big],\end{aligned} \tag{55}$$

where $c_{i,m}$ is a positive constant. The virtual controller in (55) is not specified, because $Q_{i,m}$ is unknown, although it is smooth for all $\bar{x}_{i,m} \in R^N$ and $|S_{i,m}| \geq \lambda_{a_{i,m}}$, where $\lambda_{a_{i,m}}$ is one of the constant parameters in Lemma 3. Let $\bar{S}_{i,m} \in \Omega_{S_{i,m}} \times \Omega_{\bar{x}_{i,m}} \subset R^N \times R^N$ and $\Omega'_{S_{i,m}} = \{S_{i,m} \big| |S_{i,m}| < \lambda_{a_{i,m}}\} \subset \Omega_{S_{i,m}}$ (note that $\Omega_{S_{i,m}}$ and $\Omega_{\bar{x}_{i,m}}$ are compact subsets of R^N). Then $\Omega^0_{S_{i,m}} = \Omega_{S_{i,m}} - \Omega'_{S_{i,m}}$ is also a compact set. Therefore, $\Omega^0_{\bar{S}_{i,m}} = \Omega^0_{S_{i,m}} \times \Omega_{\bar{x}_{i,m}}$ is a compact subset of $R^N \times R^N$. Thus, $Q_{i,m}$ (i.e. the non-linear part of $\dot{V}_{i,m}$) can be approximated over $\Omega^0_{\bar{S}_{i,m}}$ by a RBFNN as

$$Q_{i,m}(\bar{S}_{i,m}) = w_{i,m}^T P_{i,m}(\bar{S}_{i,m}) + \varepsilon_{i,m}(\bar{S}_{i,m}), \tag{56}$$

where $w_{i,m}$ is an unknown weight vector, as noted before, $P_{i,m}$ is a basis function vector, and $\varepsilon_{i,m}$ is the bounded approximation error for $\bar{S}_{i,m} \in \Omega^0_{\bar{S}_{i,m}}$ (i.e. $\left|\varepsilon_{i,m}(\bar{S}_{i,m})\right| \leqslant \varepsilon^*_{\bar{S}_{i,m}}$, where $\varepsilon^*_{\bar{S}_{i,m}}$ is a positive real constant). The parameter estimations are then given by

$$\dot{\hat{\theta}}_{i,m} = q_{i,m}(S_{im}(t))[- \eta_{i,m}\Gamma_{i,m}\hat{\theta}_{i,m} + \Gamma_{i,m}S_{i,m}\psi_{i,m}^T], \tag{57}$$

$$\dot{\hat{H}}_{i,m} = q_{i,m}(S_{i,m}(t))\left[\gamma_{i,m}\frac{S_{i,m}^2}{\sqrt{S_{i,m}^2 + \eta^2}} - \gamma_{i,m}\hat{H}_{i,m}\right], \tag{58}$$

$$\dot{\hat{w}}_{i,m} = q_{i,m}(S_{i,m}(t))\left[\Lambda_{i,m}\big(S_{i,m}(t)P_{i,m}(\bar{S}_{i,m}) - \sigma_{i,m}\hat{w}_{i,m}\big)\right], \tag{59}$$

where $\eta_{i,m}, \Gamma_{i,m}, \gamma_{i,m}, \Lambda_{i,m}$, and $\sigma_{i,m}$ determine the convergence rate of the above parameters.

Using Lemma 3, the following regions are considered and the proof of stability is provided for each region, accordingly.

Region 1. In this region, $q_{i,m}(S_{i,m}) = 1$. Hence, the virtual controller becomes

$$\begin{aligned}\alpha_{i,m}(\bar{x}_{i,m}(t)) = -c_{i,m}S_{i,m} - \psi_{i,m}^T\hat{\theta}_{i,m} - \frac{S_{i,m}}{\sqrt{S_{i,m}^2 + \eta^2}}\hat{H}_{i,m} \\ - \hat{w}_{i,m}^T P_{i,m}(\bar{S}_{i,m}) + \dot{z}_{i,m},\end{aligned} \tag{60}$$

and the parameter estimations

$$\dot{\hat{\theta}}_{i,m} = - \eta_{i,m}\Gamma_{i,m}\hat{\theta}_{i,m} + \Gamma_{i,m}S_{i,m}\psi_{i,m}^T, \tag{61}$$

$$\dot{\hat{H}}_{i,m} = \gamma_{i,m}\frac{S_{i,m}^2}{\sqrt{S_{i,m}^2 + \eta^2}} - \gamma_{i,m}\hat{H}_{i,m}, \tag{62}$$

$$\dot{\hat{w}}_{i,m} = \Lambda_{i,m}\big(S_{i,m}(t)P_{i,m}(\bar{S}_{i,m}) - \sigma_{i,m}\hat{w}_{i,m}\big). \tag{63}$$

Consequently, the time derivative of $V_{i,m}(t)$ satisfies the following inequality:

$$\begin{aligned}\dot{V}_{i,m} \leq \dot{V}_{i,m-1} + \frac{1}{2}S_{i,m+1}^2 + \frac{1}{2}X_{i,m+1}^2 - \Upsilon_{i,m}V_{h_{i,m}} \\ -c^*_{i,m}S_{i,m}^2 - \frac{1}{2}\sigma_{i,m}\|\ \tilde{w}_{i,m}\ \|^2 - \frac{1}{2}\left|\tilde{H}_{i,m}\right|^2 \\ -\frac{1}{2}\eta_{i,m}\|\ \tilde{\theta}_{i,m}\ \|^2 + \mu_{i,m} - \frac{X_{i,m}^2}{\zeta_{i,m}} + X_{i,m}E_{i,m}(\,.\,),\end{aligned} \tag{64}$$

where

$$\begin{aligned}c^*_{i,m} &= c_{i,m} - \frac{3}{2}, \\ \mu_{i,m} &= \frac{1}{2}\varepsilon_{i,m}^2 + \frac{1}{2}\sigma_{i,m}\|\ w_{i,m}\ \|^2 + \frac{1}{2}|H_{i,m}|^2 \\ &\quad + \frac{1}{2}\eta_{i,m}\|\ \theta_{i,m}\ \|^2 + H_{i,m}\eta\,.\end{aligned} \tag{65}$$

It can be observed from (64) that the boundedness of $V_{i,m}$ is dependent on that of $S_{i,m+1}$, $X_{i,m+1}$, which can be determined in the (m+1)th state equation investigation.

Region 2. In this region, $\lambda_{a_{i,m}} < |S_{i,m}| < \lambda_{a_{i,m}} + \lambda_{b_{i,m}}$, which means that $S_{i,m}$ is bounded. Thus, $V_{S_{i,m}}$ is bounded and so is $V_{h_{i,m}}$. Similarly, from (11), (13), and (60), $X_{i,m}$ is bounded and so is $V_{X_{i,m}}$. Therefore, the boundedness of $V_{\theta_{i,m}}$, $V_{w_{i,m}}$ and $V_{H_{i,m}}$ is concluded in a way similar to the previous step. Consequently, $V_{i,m}(t)$ is bounded in this region.

Region 3. In this region, $|S_{i,m}| < \lambda_{a_{i,m}}$ and $q_{i,m}(S_{i,m}) = 0$, which means that $S_{i,m}$ is bounded and $\dot{\hat{\theta}}_{i,m} = 0$. As a result, $\hat{\theta}_{i,m}$ is bounded and so is $V_{\theta_{i,m}}$. Similarly, one can conclude that $V_{w_{i,m}}$ and $V_{H_{i,m}}$ are also bounded. Furthermore, $V_{X_{i,m}}$, $V_{S_{i,m}}$, and $V_{h_{i,m}}$ are bounded because $S_{i,m}$ is bounded, which implies that $V_{i,m}(t)$ is bounded in this region.

Step 3. In the last step, using (7) and (11), $S_{i,n}$ and the time derivative of $S_{i,n}$ are computed as

$$S_{i,n} = x_{i,n} - z_{i,n}, \tag{66}$$

$$\dot{S}_{i,n} = \beta_i u_i(t) + \psi_{i,n}^T\theta_{i,n} + d_{i,n} + h_{i,n}(\bar{x}(t - \tau_{i,n})) - \dot{z}_{i,n}\,. \tag{67}$$

The Lyapunov function is considered as follows:

$$\begin{aligned}V &= \sum_{i=1}^N \sum_{k=1}^n V_{i,k} = V_{i,n-1} + V_{i,n} = V_{i,n-1} + V_{S_{i,n}} \\ &\quad + V_{\theta_{i,n}} + V_{H_{i,n}} + V_{h_{i,n}} + V_{w_{i,n}} + V_{X_{i,n}}, \\ V_{S_{i,n}} &= \frac{S_{i,n}^2}{2}\ ,\ V_{\theta_{i,n}} = \frac{\tilde{\theta}_{i,n}^T\Gamma_i^{-1}\tilde{\theta}_{i,n}}{2}, \\ V_{H_{i,n}} &= \frac{\tilde{H}_{i,n}^2}{2\gamma_{i,n}}\ ,\ V_{w_{i,n}} = \frac{1}{2}\tilde{w}_{i,n}^T\Lambda_{i,n}^{-1}\tilde{w}_{i,n},\ \ V_{X_{i,n}} = \frac{X_{i,n}^2}{2}, \\ V_{h_{i,n}} &= \frac{1}{2(1 - \hbar)}\int_{t - \tau_{i,n}(t)}^t e^{\Upsilon_{i,n}(\varrho - t)}h_{i,n}^2(\bar{x}_{i,n}(\varrho))\mathrm{d}\varrho\,.\end{aligned} \tag{68}$$

From (67), (68), and by using a first-order filter similar to (50) and (51), the time derivative of $V_{i,n}$ is obtained as

$$\begin{aligned}\dot{V} &\leq \dot{V}_{i,n-1} + S_{i,n}\beta_i u_i(t) + S_{i,n}\psi_{i,n}^T\theta_{i,n} + S_{i,n}d_{i,n} \\ &\quad + S_{i,n}(t)Q_{i,n}(\bar{S}_{i,n}) - S_{i,n}\dot{z}_{i,n} + X_{i,n}(t)\bigg(- \frac{X_{i,n}}{\zeta_{i,n}} \\ &\quad + E_{i,n}(\,.\,)\bigg) + \tilde{\theta}_{i,n}^T\Gamma_{i,n}^{-1}\dot{\hat{\theta}}_{i,n} + \frac{1}{\gamma_{i,n}}\tilde{H}_{i,n}\dot{\hat{H}}_{i,n} - \Upsilon_{i,n}V_{h_{i,n}} \\ &\quad + \tilde{w}_{i,n}^T\Lambda_{i,n}^{-1}\dot{\hat{w}}_{i,n},\end{aligned} \tag{69}$$

where

$$Q_{i,n}(\bar{S}_{i,n}) = \frac{S_{i,n}(t)e^{\Upsilon_{i,n}\bar{\tau}_{i,n}}}{2} + \frac{1}{2(1 - \hbar)S_{i,n}(t)}h_{i,n}^2(\bar{x}_{i,n}(t)), \tag{70}$$

and $\bar{S}_{i,n} = [S_{i,n}, \bar{x}_{i,n}]^{\mathrm{T}}$. The following control law is proposed for agent i, $\forall i \in \{1, \ldots, N\}$

$$u_i(t) = \frac{1}{\beta_i(x_i)}\Big[-\psi_{i,n}^T\hat{\theta}_{i,n} - c_{i,n}S_{i,n} - Q_{i,n} - \frac{S_{i,n}}{\sqrt{S_{i,n}^2+\eta^2}}\hat{H}_{i,n} + \dot{z}_{i,n}\Big], \tag{71}$$

where $c_{i,n}$ is a positive constant. Similar to the previous step, $Q_{i,n}$ is not well-defined at $S_{i,n}(t)=0$, which means that the above control law is singular at this point. To address this problem, the function $q_{i,n}(S_{i,n})$, defined in Lemma 3, is used here. To this end, note that $Q_{i,n}$ is smooth for all $\bar{x}_{i,n}\in R^N$, and $|S_{i,n}|\ge\lambda_{a_{i,n}}$, where $\lambda_{a_{i,n}}$ is a constant parameter introduced in Lemma 3. Similar to the previous step, let $\bar{S}_{i,n}\in\Omega_{S_{i,n}}\times\Omega_{\bar{x}_{i,n}}\subset R^N\times R^N$ (that $\Omega_{S_{i,n}}$ and $\Omega_{\bar{x}_{i,n}}$ are compact subsets of R^N) and $\Omega'_{S_{i,n}}=\{S_{i,n}\,\big|\,|S_{i,n}|<\lambda_{a_{i,n}}\}\subset\Omega_{S_{i,n}}$. This means that $\Omega^0_{S_{i,n}}=\Omega_{S_{i,n}}-\Omega'_{S_{i,n}}$ is a compact set. Therefore, $\Omega^0_{\bar{S}_{i,n}}=\Omega^0_{S_{i,n}}\times\Omega_{\bar{x}_{i,n}}$ is a compact subset of $R^N\times R^N$. Thus, $Q_{i,n}$ can be approximated over $\Omega^0_{\bar{S}_{i,n}}$ by a RBFNN as follows:

$$Q_{i,n}(\bar{S}_{i,n}) = w_{i,n}^T P_{i,n}(\bar{S}_{i,n}) + \varepsilon_{i,n}(\bar{S}_{i,n}), \tag{72}$$

where like the previous step, $w_{i,n}$ is an unknown weight vector, $P_{i,n}$ is a basis function vector, and $\varepsilon_{i,n}$ is the bounded approximation error for $\bar{S}_{i,n}\in\Omega^0_{\bar{S}_{i,n}}$. Thus, the resultant control rule is

$$u_{i,n}(t) = \frac{1}{\beta_i(x_i)}q_{i,n}(S_{i,n})\Big[-\psi_{i,n}^T\hat{\theta}_{i,n} - c_{i,n}S_{i,n} - \hat{w}_{i,n}^T P_{i,n} - \frac{S_{i,n}}{\sqrt{S_{i,n}^2+\eta^2}}\hat{H}_{i,n} + \dot{z}_{i,n}\Big]. \tag{73}$$

Similar to (57)–(59), the following update laws are proposed

$$\dot{\hat{\theta}}_{i,n} = q_{i,n}(S_{i,n}(t))[-\eta_{i,n}\Gamma_{i,n}\hat{\theta}_{i,n} + \Gamma_{i,n}S_{i,n}\psi_{i,n}^T] \tag{74}$$

$$\dot{\hat{H}}_{i,n} = q_{i,n}(S_{i,n}(t))\Big[\gamma_{i,n}\frac{S_{i,n}^2}{\sqrt{S_{i,n}^2+\eta^2}} - \gamma_{i,n}\hat{H}_{i,n}\Big] \tag{75}$$

$$\dot{\hat{w}}_{i,n} = q_{i,n}(S_{i,n}(t))\big[\Lambda_{i,n}\big(S_{i,n}(t)P_{i,n}(\bar{S}_{i,n}) - \sigma_{i,n}\hat{w}_{i,n}\big)\big], \tag{76}$$

where the convergence rate of the estimated parameters $\hat{\theta}_{i,n},\hat{H}_{i,n}$, and $\hat{w}_{i,n}$ are dependent on the choice of $\eta_{i,n},\Gamma_{i,n},\gamma_{i,n},\Lambda_{i,n}$, and $\sigma_{i,n}$. Using Lemma 3, the following regions are considered for the proof of stability.

Region 1. In this region, $q_{i,n}(S_{i,n})=1$. Let the following update laws be used

$$\dot{\hat{\theta}}_{i,n} = -\eta_{i,n}\Gamma_{i,n}\hat{\theta}_{i,n} + \Gamma_{i,n}S_{i,n}\psi_{i,n}^T, \tag{77}$$

$$\dot{\hat{H}}_{i,n} = \gamma_{i,n}\frac{S_{i,n}^2}{\sqrt{S_{i,n}^2+\eta^2}} - \gamma_{i,n}\hat{H}_{i,n}, \tag{78}$$

$$\dot{\hat{w}}_{i,n} = \Lambda_{i,n}\big(S_{i,n}(t)P_{i,n}(\bar{S}_{i,n}) - \sigma_{i,n}\hat{w}_{i,n}\big). \tag{79}$$

The control law is then given by

$$u_{i,n}(t) = \frac{1}{\beta_i(x_i)}\Big[-\psi_{i,n}^T\hat{\theta}_{i,n} - c_{i,n}S_{i,n} - \hat{w}_{i,n}^T P_{i,n} - \frac{S_{i,n}}{\sqrt{S_{i,n}^2+\eta^2}}\hat{H}_{i,n} + \dot{z}_{i,n}\Big], \tag{80}$$

and the time derivative of $V(t)$ is calculated as

$$\dot{V} \le \dot{V}_{i,n-1} - c^*_{i,n}S_{i,n}^2 - \frac{1}{2}\eta_{i,n}\|\tilde{\theta}_{i,n}\|^2 - \frac{1}{2}\big|\tilde{H}_{i,n}\big|^2 - \frac{1}{2}\sigma_{i,n}\|\tilde{w}_{i,n}\|^2 + X_{i,n}(t)\Big(-\frac{X_{i,n}}{\zeta_{i,n}} + E_{i,n}(\,.\,)\Big) - \Upsilon_{i,n}V_{h_{i,n}} + \mu_{i,n}, \tag{81}$$

where

$$c^*_{i,n} = c_{i,n} - \frac{3}{2},\quad \mu_{i,n} = \frac{1}{2}\eta_{i,n}\|\theta_{i,n}\|^2 + \frac{1}{2}|H_{i,n}|^2 + H_{i,n}\eta + \frac{1}{2}\sigma_{i,n}\|w_{i,n}\|^2 + \frac{1}{2}\varepsilon_{i,n}^2. \tag{82}$$

Considering the compact sets $\prod$ (in Assumption 2) and

$$\widehat{\prod} := \sum_{i=1}^{N}\sum_{k=1}^{n}\Big\{\Big(\frac{S_{i,k}^2}{2} + \frac{1}{2}\tilde{\theta}_{i,k}^T\Gamma_{i,k}^{-1}\tilde{\theta}_{i,k} + \frac{1}{2\gamma_{i,k}}\tilde{H}_{i,k}^2 + \frac{1}{2}\tilde{w}_{i,k}^T\Lambda_{i,k}^{-1}\tilde{w}_{i,k} + \frac{1}{2(1-\hbar)}\int_{t-\tau_{i,k}(t)}^{t}e^{\Upsilon_{i,k}(\partial - t)}h_{i,k}^2(\bar{x}_{i,k}(\partial))\Big)\Big\} + \frac{1}{2}\sum_{i=1}^{N}\sum_{k=2}^{n}X_{i,k}^2 \le p, \tag{83}$$

where $p>0$ is a design parameter, and knowing that for any $k\in\{2,3,\dots,n\}$, $E_{i,k}$ in (52) is a continuous function, there is a positive finite constant $M_{i,k}$ such that $|E_{i,k}|\leqslant M_{i,k}$. Moreover, by applying Young's inequality (from Lemma 5 and considering $a=X_{i,k}E_{i,k}(\,.\,)$, $b=1$, $p=q=2$ and $\epsilon=(\lambda_{i,k}/2)$) the following inequality:

$$|X_{i,k}E_{i,k}(\,.\,)| \le \frac{X_{i,k}^2E_{i,k}^2}{\lambda_{i,k}} + \frac{\lambda_{i,k}}{4} \leqslant \frac{X_{i,k}^2M_{i,k}^2}{\lambda_{i,k}} + \frac{\lambda_{i,k}}{4}, \tag{84}$$

is obtained, where $\lambda_{i,k}$ is an arbitrary positive constant, and on noting that

$$2\tilde{\theta}_{i,k}^T\hat{\theta}_{i,k} \ge \|\tilde{\theta}_{i,k}\|^2 - \|\theta_{i,k}\|^2, \tag{85}$$

$\dot{V}(t)$ is simplified as follows:

$$\dot{V} \le \sum_{i=1}^{N}\Big(\sum_{k=2}^{n}\Big\{\mu_{i,k} - c^*{}_{i,k}S_{i,k}^2 - X_{i,k}^2\Big(\frac{1}{\zeta_{i,k}} - \frac{M_{i,k}^2}{\lambda_{i,k}} - 1\Big) + \frac{\lambda_{i,k}}{4} - \frac{1}{2}\sigma_{i,k}\|\tilde{w}_{i,k}\|^2 - \frac{1}{2}\eta_{i,k}\|\tilde{\theta}_{i,k}\|^2 - \frac{1}{2}\big|\tilde{H}_{i,k}\big|^2 - \Upsilon_{i,k}V_{h_{i,k}}\Big\}\Big) + \mu_{i,1} - c^*{}_{i,1}S_{i,1}^2 - \frac{1}{2}\sigma_{i,1}\|\tilde{w}_{i,1}\|^2 - \frac{1}{2}\eta_{i,1}\|\tilde{\theta}_{i,1}\|^2 - \frac{1}{2}\big|\tilde{H}_{i,1}\big|^2 - \Upsilon_{i,1}V_{h_{i,1}}. \tag{86}$$

By choosing $\zeta_{i,k}$ and $\lambda_{i,k}$ such that $\frac{1}{\zeta_{i,k}} - \frac{M_{i,k}^2}{\lambda_{i,k}} - 1 > 0$, the time derivative of $V(t)$ satisfies the following inequality:

$$\dot{V}(t) \le -CV(t) + \mu, \tag{87}$$

where

$$C = \min\Big\{2c^*{}_{i,1},\dots,2c^*{}_{i,n},\frac{\sigma_{i,1}}{\lambda_{\max}(\Lambda_{i,1}^{-1})},\dots,\frac{\sigma_{i,n}}{\lambda_{\max}(\Lambda_{i,n}^{-1})}, \frac{\eta_{i,1}}{\lambda_{\max}(\Gamma_{i,1}^{-1})},\dots,\frac{\eta_{i,n}}{\lambda_{\max}(\Gamma_{i,n}^{-1})},\gamma_{i,1},\dots,\gamma_{i,n}, 2\Big(\frac{1}{\zeta_{i,2}} - \frac{M_{i,2}^2}{\lambda_{i,2}} - 1\Big),\dots,2\Big(\frac{1}{\zeta_{i,n}} - \frac{M_{i,n}^2}{\lambda_{i,n}} - 1\Big)\Big\}, \tag{88}$$

and

$$\mu = \sum_{i=1}^{N}\sum_{m=1}^{n}\mu_{i,m} = \sum_{i=1}^{N}\Big(\sum_{m=1}^{n}\Big\{\frac{1}{2}\varepsilon_{i,m}^2 + \frac{1}{2}\sigma_{i,m}\| w_{i,m} \|^2 + \frac{1}{2}|H_{i,m}|^2 + \frac{1}{2}\eta_{i,m}\| \theta_{i,m} \|^2 + H_{i,m}\eta\Big\} + \sum_{m=2}^{n}\frac{\lambda_{i,m}}{4}\Big). \quad (89)$$

If $C > \mu/p$ and $V(t) = p$, then $\dot{V}(t) < 0$ (according to (87)). Thus, $V(t) \le p$ is an invariant set (if $V(0) \le p$, then $V(t) \le p$ for all $t \ge 0$). Furthermore, it can be easily verified that C and μ are finite positive constants, which implies that $V(t)$ is bounded in this region.

Region 2. In this region, $\lambda_{a_{i,n}} < |S_{i,n}| < \lambda_{a_{i,n}} + \lambda_{b_{i,n}}$, which means that $S_{i,n}$ is bounded. Similar to the previous step, $V_{S_{i,n}}$, $V_{X_{i,n}}$, $V_{h_{i,n}}$, $V_{\theta_{i,n}}$, $V_{w_{i,n}}$, and $V_{H_{i,n}}$ are bounded, and hence so is $V_{i,n}$. The boundness of $V_{i,n-1}$ was proved in the previous steps; therefore, V is also bounded.

Region 3. In this region, $|S_{i,n}| < \lambda_{a_{i,n}}$ and $q_{i,n}(S_{i,n}) = 0$, which means that $S_{i,n}$ is bounded and consequently $V_{S_{i,n}}$ and $V_{h_{i,n}}$ are bounded too. Thus from (13) and (66), $X_{i,n}$ is bounded, and so is $V_{X_{i,n}}$. On the other hand, from (74), $\dot{\hat{\theta}}_{i,n} = 0$, meaning that $\hat{\theta}_{i,n}$ is bounded, which in turn implies the boundedness of $V_{\theta_{i,n}}$. Using a similar approach, it can be concluded that $V_{w_{i,n}}$ and $V_{H_{i,n}}$ are bounded as well. Therefore, $V_{i,n}$ is bounded in this region. As a result, V is bounded, on noting that the boundness of $V_{i,n-1}$ was proved in the previous steps.

Theorem 1: Consider the non-linear MAS described by (7), and let Assumptions 1–4 hold. Consider also the controller law (73) and the adaptive parameter estimators (57)–(59). By choosing appropriate control parameters, semi-global uniform boundedness of all the signals in the proposed closed-loop control system and the boundedness of $Z_{i,m} = [S_{i,m}, X_{i,m}]^{\mathrm{T}}$ in the compact set

$$\Omega_{i,m} = \{Z_{i,m}, \tilde{w}_{i,m}, \tilde{\theta}_{i,m}, \tilde{H}_{i,m} \Big| \ \| Z_{i,m} \| \ \le 2\sqrt{\Big[V(0) + \frac{\mu}{C}\Big]}, \ \| \tilde{w}_{i,m} \| \le \sqrt{\frac{2\big[V(0) + \frac{\mu}{C}\big]}{\lambda_{\min}(\Lambda_{i,m}^{-1})}}, \ \| \tilde{\theta}_{i,m} \| \le \sqrt{\frac{2\big[V(0) + \frac{\mu}{C}\big]}{\lambda_{\min}(\Gamma_{i,m}^{-1})}}, \ \big|\tilde{H}_{i,m}\big| \le \sqrt{2\gamma_{i,m}\Big[V(0) + \frac{\mu}{C}\Big]}\} \quad (90)$$

are guaranteed, and hence the distributed consensus tracking errors converge to a bounded neighbourhood of the origin given by

$$\bar{\Lambda}_\delta = \left\{\delta \middle| \ \| \delta \| \le \ \max\left\{\sqrt{\frac{2\mu}{C\underline{\sigma}(L+\xi)}}, \frac{\sqrt{N}(\lambda_{a_{i,m}} + \lambda_{b_{i,m}})}{\underline{\sigma}(L+\xi)}\right\}\right\}. \quad (91)$$

Proof: The proof is carried out by considering three different regions for $|S_{i,m}|$.

Region 1. In this region, $|S_{i,m}| \ge \lambda_{a_{i,m}} + \lambda_{b_{i,m}}$. By taking the integral of both sides of (87) on $[0, t]$, one obtains

$$V(t) \le V(0)e^{-Ct} + (1 - e^{-Ct})\frac{\mu}{C}. \quad (92)$$

Therefore, $V(t)$ is bounded as follows:

$$V(t) \le V(0) + \frac{\mu}{C}. \quad (93)$$

Also, according to (68) and (93)

$$\sum_{i=1}^{N}\sum_{m=1}^{n} S_{i,m}^2 \le 2V(t) \le 2\Big(V(0) + \frac{\mu}{C}\Big)$$
$$\sum_{i=1}^{N}\sum_{m=1}^{n} X_{i,m}^2 \le 2V(t) \le 2\Big(V(0) + \frac{\mu}{C}\Big)$$
$$\sum_{i=1}^{N}\sum_{m=1}^{n} \| \tilde{w}_{i,m} \|^2 \le \frac{2V(t)}{\lambda_{\min}(\Lambda_{i,m}^{-1})} \le \frac{2\big[V(0) + \frac{\mu}{C}\big]}{\lambda_{\min}(\Lambda_{i,m}^{-1})} \quad (94)$$
$$\sum_{i=1}^{N}\sum_{m=1}^{n} \| \tilde{\theta}_{i,m} \|^2 \le \frac{2V(t)}{\lambda_{\min}(\Gamma_{i,m}^{-1})} \le \frac{2\big[V(0) + \frac{\mu}{C}\big]}{\lambda_{\min}(\Gamma_{i,m}^{-1})}$$
$$\sum_{i=1}^{N}\sum_{m=1}^{n} \tilde{H}_{i,m}^2 \le 2\gamma_{i,m}V(t) \le 2\gamma_{i,m}\Big(V(0) + \frac{\mu}{C}\Big).$$

From (17), (93) and Lemma 4

$$\lim_{t\to\infty} \| \delta \| \le \sqrt{\frac{2\mu}{C\underline{\sigma}(L+\xi)}}, \quad (95)$$

which means that in this region $\| \delta \|$ is bounded.

Region 2. In this region, $S_{i,m}$ is bounded. Thus, $V_{s_{i,m}}$ and $V_{h_{i,m}}$ are bounded as well. Note that in Region 2 of Step 2, $V_{\theta_{i,m}}$, $V_{X_{i,m}}$, $V_{w_{i,m}}$, and $V_{H_{i,m}}$ are bounded, and consequently, $V(t)$ is bounded. In this region, $\lambda_{a_{i,m}} < |S_{i,m}| < \lambda_{a_{i,m}} + \lambda_{b_{i,m}}$, which yields

$$\| S_1 \| \le \sqrt{N}(\lambda_{a_{i,m}} + \lambda_{b_{i,m}}). \quad (96)$$

Now, according to (5) and (96)

$$\lim_{t\to\infty} \| \delta \| \le \frac{\sqrt{N}(\lambda_{a_{i,m}} + \lambda_{b_{i,m}})}{\underline{\sigma}(L+\xi)}. \quad (97)$$

In other words, in this region also $\| \delta \|$ is bounded.

Region 3. In this region, $S_{i,m}$ is bounded and $q_{i,m}(S_{i,m}) = 0$; therefore, $\dot{\hat{\theta}}_{i,m} = 0$. As a result, $\hat{\theta}_{i,m}$ is bounded, and so is $V_{\theta_{i,m}}$. Similarly, it is concluded that $V_{w_{i,m}}$ and $V_{H_{i,m}}$ are bounded. Furthermore, the boundedness of $V_{X_{i,m}}$, $V_{S_{i,m}}$, and $V_{h_{i,m}}$ results from that of $S_{i,m}$, which means that $V(t)$ is bounded as well. Since in this region $|S_{i,m}| < \lambda_{a_{i,m}}$, hence

$$\| S_1 \| \le \sqrt{N}(\lambda_{a_{i,m}}), \quad (98)$$

and according to (5) and (98)

$$\lim_{t\to\infty} \| \delta \| \le \frac{\sqrt{N}(\lambda_{a_{i,m}})}{\underline{\sigma}(L+\xi)}. \quad (99)$$

This concludes the proof. □

5 Simulations

To demonstrate the effectiveness of the proposed design scheme, two examples are presented. First, a dynamic system consisting of three one-link manipulators [22, 45] is considered, and then a numerical example is given.

Example 1: Let the dynamic model of each manipulator in the three one-link manipulator system noted above be given by the following non-linear equations [22]:

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2},\\ \dot{x}_{i,2} &= x_{i,3} - [\sin(x_{i,1}), x_{i,2}]\theta_{i,1} + \frac{I_{d,i}}{D_i},\\ \dot{x}_{i,3} &= b_i u_i - [x_{i,2}, x_{i,3}]\theta_{i,2} + \sin(x_{i,3}(t-\tau(t))),\end{aligned} \quad (100)$$

where $\tau(t) = (0.05 + 0.001\sin(0.1t))$,

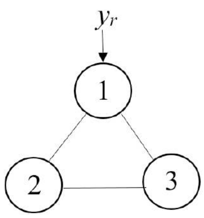

Fig. 1 *Communication topology of the MAS in Example 1*

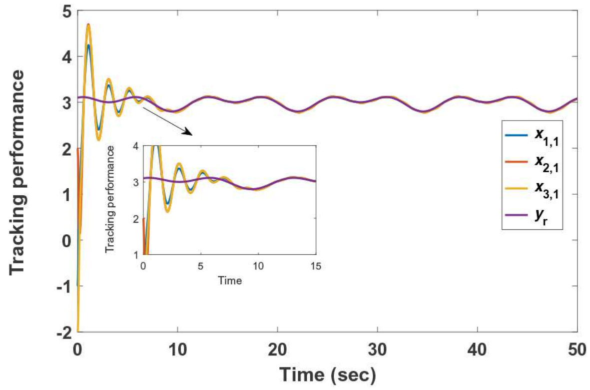

Fig. 2 *Tracking performance of the agents' outputs in Example 1*

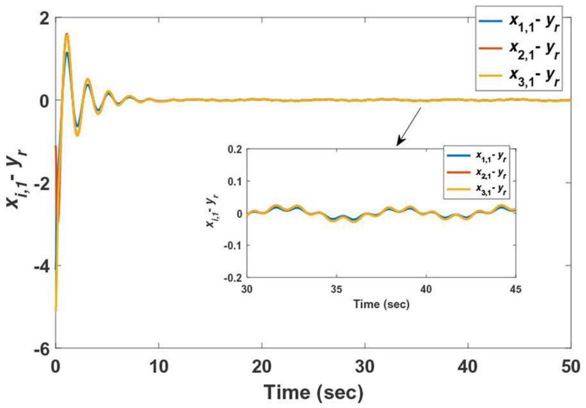

Fig. 3 *Tracking errors in Example 1*

$$\theta_{i,1} = \left[\frac{N_i}{D_i}, \frac{B_i}{D_i}\right]^{\mathrm{T}}, \; b_i = \frac{1}{M_i D_i}, \; \theta_{i,2} = \left[\frac{K_{m,i}}{M_i D_i}, \frac{H_i}{M_i}\right]^{\mathrm{T}}.$$

The system parameters are $D_i = 1$, $N_i = 2$, $B_i = 1$, $K_{m,i} = 10$, $M_i = 10$, and $H_i = 0.5$. Let the external disturbance be $I_{d,i} = 0.1\cos(5t)$ for $i = 1, 2, 3$. The control parameters are chosen as $c_{i,1} = 3$, $c_{i,2} = c_{i,3} = 2$, $\Lambda_{i,1} = \Lambda_{i,2} = \Lambda_{i,3} = 10I$, $\Gamma_{i,1} = \Gamma_{i,2} = \Gamma_{i,3} = 0.1I$, $\gamma_{i,1} = \gamma_{i,2} = \gamma_{i,3} = 0.1$, $\sigma_{i,1} = \sigma_{i,2} = \sigma_{i,3} = 0.1$, $\lambda_{a_{i,m}} = \lambda_{b_{i,m}} = 0.001$, and $\eta(t) = e^{-0.01t}$ for $i = 1, 2, 3$ and $m = 1, 2, 3$. The initial states are assumed to be $x_{1,1}(0) = -1$, $x_{2,1}(0) = 2$, $x_{3,1}(0) = -2$, $x_{2,3}(0) = 0.001$, $x_{3,2}(0) = -0.5$,
and $x_{3,3}(0) = -2$; all other initial states are set to zero. The leader trajectory is given by $y_r(t) = 3 + 0.1\sin(0.5t) + 0.1\cos(t)$. The RBFNN contains 25 nodes with centres are distributed evenly in the variation range of $[-12, \ldots, 12]$ and $\sigma_k = 2\sqrt{2}$, $k = 1, \ldots, 25$ is the width of the Gaussian function.

Fig. 1 shows the communication topology of this MAS. Fig. 2, on the other hand, provides a comparison of the tracking performance of the followers' output with respect to the desired trajectory, and the tracking errors are demonstrated in Fig. 3, which confirm the convergence of the tracking errors to a neighbourhood of the origin. The parameter estimates, i.e. $\hat{\theta}_{i,2}$, $\hat{\theta}_{i,3}$, $\hat{H}_{i,1}$, $\hat{H}_{i,2}$, $\hat{H}_{i,3}$, are depicted in Figs. 4–6. Also, the control signals are shown in Fig. 7. The reason for the peaks of the control signal in Fig. 7 is that there is a trade-off between the size of the output tracking errors and control cost. Example 1 is simulated again with new design parameters ($c_{i,1} = c_{i,2} = c_{i,3} = 1.7$, $\Lambda_{i,1} = \Lambda_{i,2} = \Lambda_{i,3} = I$, $\sigma_{i,1} = \sigma_{i,2} = \sigma_{i,3} = 0.1$, $\Gamma_{i,1} = \Gamma_{i,2} = \Gamma_{i,3} = 0.1I$). It is observed that the error signals are increased (by $\sim 6\%$) in Fig. 8, and the control signals, in Fig. 9, are decreased compared to the ones in Fig. 7. Semi-global uniform boundedness of all signals in the resultant closed-loop system can be observed from these figures.

Remark 2: In [22], leader tracking is investigated in the absence of communication delay. Then, in the presence of time-delay in three one-link manipulators example in [22], it is shown that the system can become unstable, and hence, the tracking performance of the followers cannot be ensured. Fig. 10 demonstrates the tracking method's performance in [22], applied to the MAS with non-linear dynamics, in the absence of communication delay. Fig. 11, on the other hand, shows the tracking performance, this time in the presence of communication delay. It can be observed that unlike Fig. 10, the tracking objective is not achieved in Fig. 11 as the system becomes unstable. Since the semi-global uniform boundedness of the resultant closed-loop signals in the presence of the unknown time-delays has been shown in our work, the robustness of the strategy w.r.t. delays is guaranteed.

Example 2: Consider a MAS with one leader and with four followers. The communication topology of the network is shown in Fig. 12. The dynamic model of each follower is given by the following non-linear equations:

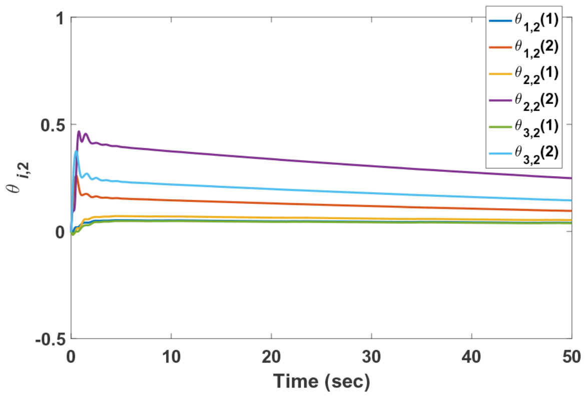

Fig. 4 *Parameter estimates* $\hat{\theta}_{i,2}$ *for the three agents in Example 1*

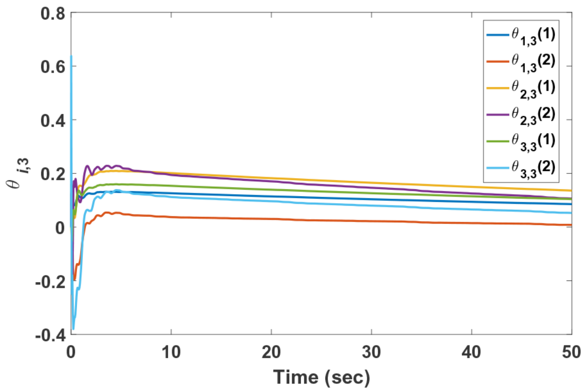

Fig. 5 *Parameter estimates* $\hat{\theta}_{i,3}$ *for the three agents in Example 1*

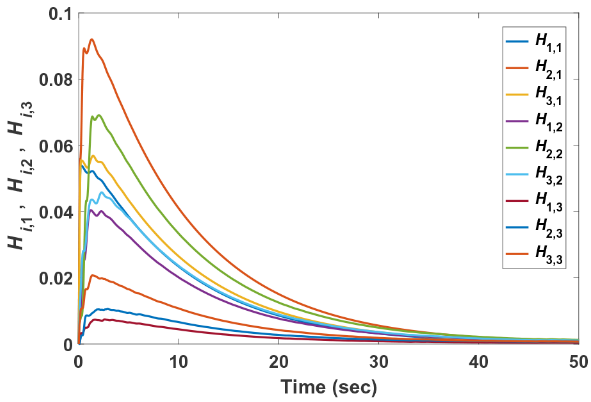

Fig. 6 *Parameter estimates* $\hat{H}_{i,1}$, $\hat{H}_{i,2}$, $\hat{H}_{i,3}$ *for the three agents in Example 1*

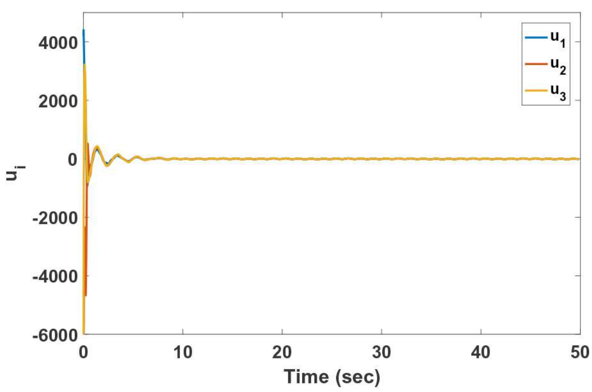

Fig. 7 *Control signals for the three agents in Example 1*

$$
\begin{aligned}
\dot{x}_{i,1} &= x_{i,2} + 0.1x_{i,1}^2 + 0.1\cos(x_{i,1}(t-\tau_{i,1}(t))) + I_{d,1},\\
\dot{x}_{i,2} &= \sin(x_{i,1}) - (x_{i,2}^2+1)u_i + 0.2x_{i,2}(t-\tau_{i,2}(t)),\\
y_i &= x_{i,1},\\
\tau_{i,1}(t) &= 0.01, \quad \tau_{i,2}(t) = 0.9\sin(t) + 2,
\end{aligned}
$$

and the leader trajectory is $y_r = 0.5\sin(0.05t)$. The external disturbance is $I_{d,1} = 0.1\cos(0.01t)$. The control parameters are chosen as $c_{i,1} = c_{i,2} = c_{i,3} = 10$, $\Lambda_{i,1} = \Lambda_{i,2} = \Lambda_{i,3} = I$, $\Gamma_{i,1} = \Gamma_{i,2} = \Gamma_{i,3} = I$, $\gamma_{i,1} = \gamma_{i,2} = \gamma_{i,3} = 0.01$, $\sigma_{i,1} = \sigma_{i,2} = \sigma_{i,3} = 0.01$, $\lambda_{a_{i,m}} = \lambda_{b_{i,m}} = 0.001$, and $\eta(t) = e^{-0.01t}$ for $i = 1, 2, 3, 4$, and $m = 1, 2$. By increasing the values of the design

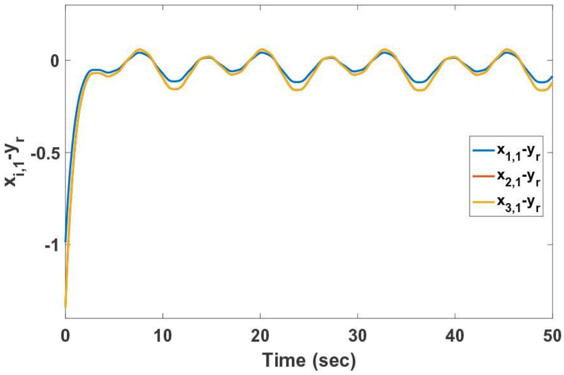

Fig. 8 *Tracking errors in Example 1 with new design parameters*

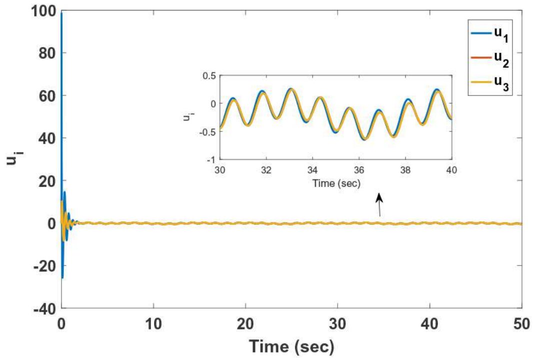

Fig. 9 *Control signals for the three agents of Example 1 with new design parameters*

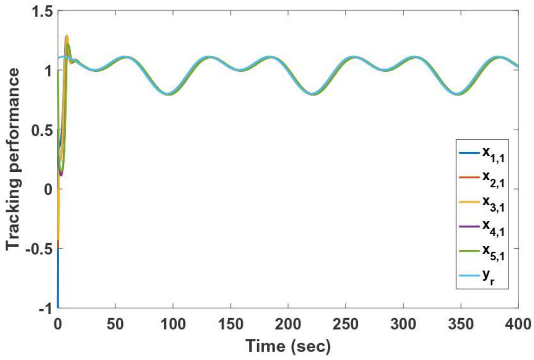

Fig. 10 *Tracking performance of the agents' outputs in the absence of delay [22]*

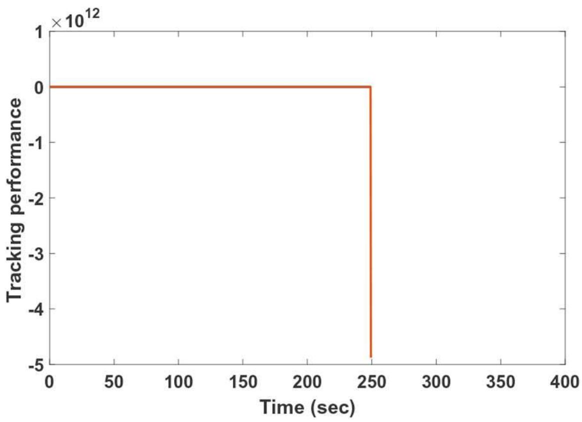

Fig. 11 *Tracking performance of the agents' outputs in the presence of delay with the used protocol in [22]*

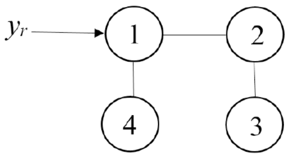

Fig. 12 *Communication topology of the MAS in Example 2*

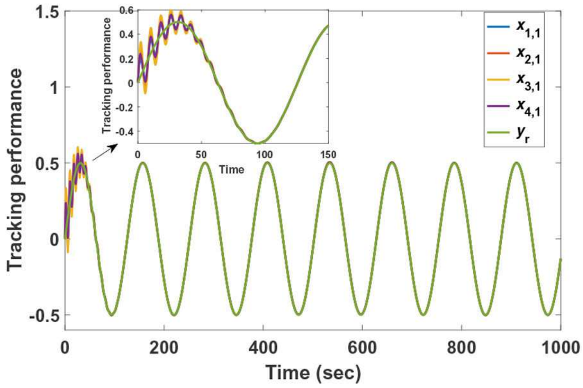

Fig. 13 *Tracking performance of the agents' outputs in Example 2*

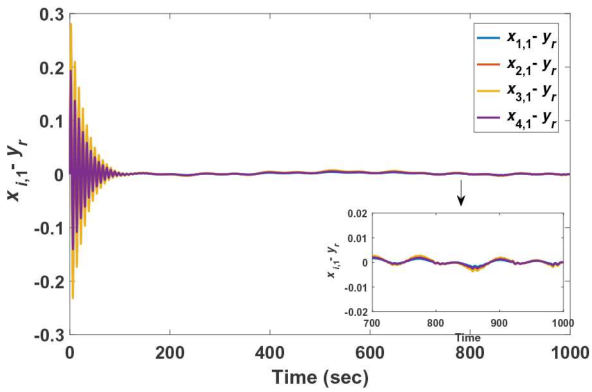

Fig. 14 *Tracking errors in Example 2*

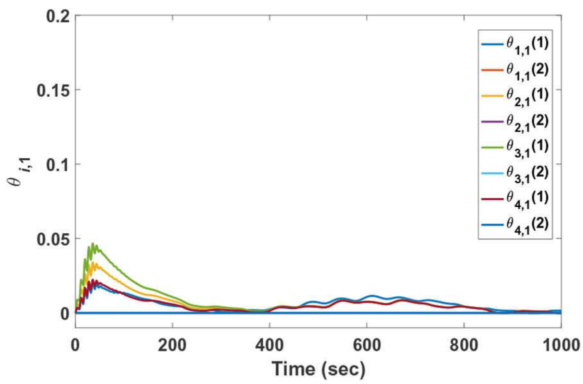

Fig. 15 *Parameter estimates* $\hat{\theta}_{i,1}$ *for the four agents in Example 2*

parameters such as $\Gamma_{i,m}$, $\gamma_{i,m}$ and $\sigma_{i,m}$, the convergence rate of tracking performance can be improved, but this leads to larger control signals. Fig. 13 shows the comparison of the tracking performance of the followers' output with the desired trajectory. Fig. 14 shows the convergence of the tracking errors to a neighbourhood of the origin. The estimations of $\hat{\theta}_{i,1}$, $\hat{\theta}_{i,2}$, $\hat{H}_{i,1}$, and $\hat{H}_{i,2}$ are shown in Figs. 15–17, respectively. Also, the control signals are shown in Fig. 18. The semi-global uniform boundedness of all the closed-loop signals and the convergence of the tracking errors to a neighbourhood of the origin can be observed.

6 Conclusions

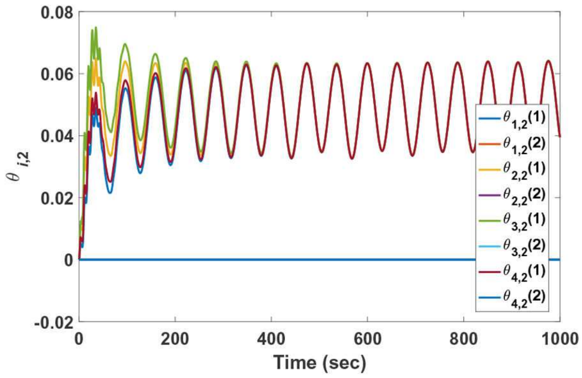

Fig. 16 *Parameter estimates* $\hat{\theta}_{i,2}$ *for the four agents in Example 2*

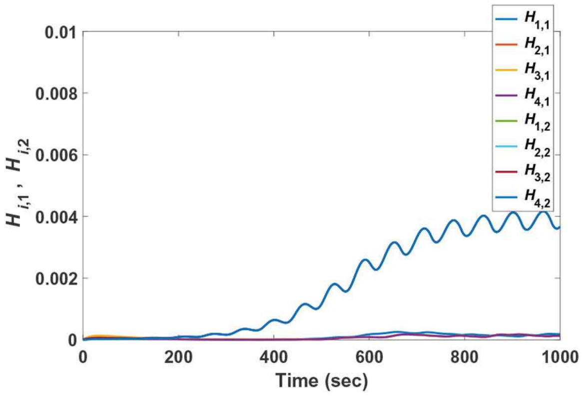

Fig. 17 *Parameter estimates* $\hat{H}_{i,1}$, $\hat{H}_{i,2}$ *for the four agents in Example 2*

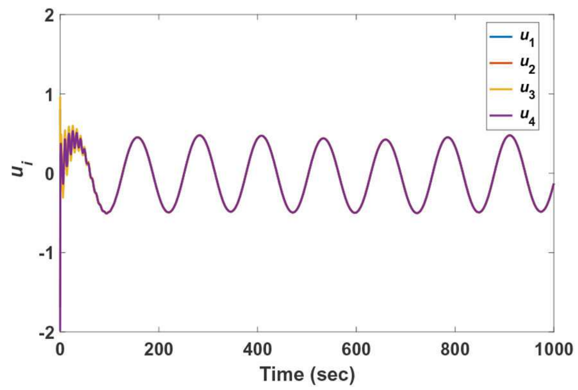

Fig. 18 *Control signals for the four agents in Example 2*

In this study, a distributed tracking adaptive controller for high-order non-linear MASs with unknown non-linear functions, uncertain disturbances and unknown time-varying delays is presented. The DSC technique is employed to design the new adaptive laws, with a NN to model non-linear uncertainties. Also, the Lyapunov–Krasovskii functional is used for the stability analysis of the MAS in the presence of state delay. The proposed technique is computationally efficient and does not require any assumption on time-delay terms of the non-linear dynamic system. By choosing the appropriate design parameters, the semi-global uniform boundedness of all the closed-loop signals and the convergence of the tracking errors to a neighbourhood of the origin are proved. Simulations demonstrate the effectiveness of the proposed adaptive control method. In future work, the consensus problem of unknown non-linear MAS with time-varying delays under switching topologies and input restrictions such as input saturation will be studied.

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