15/1/25

Function has three categories -

(1) One to one / into / injective.

The definition states for any function f, A to B will be 1 to 1 function or injective function when $f(a) = f(b)$ $\Rightarrow a = b$ where $a, b \in A$

(2) Onto / surjective function.

A function $f: A \to B$ will be surjective if $f(A) = B$ [all elements are mapped to all elements of B]

In surjective function co-domain and range coinsides. If any function follows both surjective & injective property then the function will be called bijective

Q. Determine whether the function $f(x) = x + 1$, where $f: \mathbb{R} \to \mathbb{R}$ is injective or not.

$\Rightarrow$ let, $x_1, x_2 \in \mathbb{R}$ such that $f(x_1) = f(x_2)$

⇒ $x_1 + 1 = x_2 + 1$

⇒ $x_1 + x_2$

∴ The function is injective from $\mathbb{R} \to \mathbb{R}$

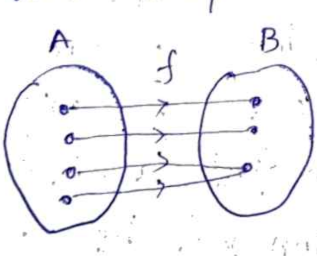
Q. If $f(x) = x^2$ from the set of integers to set of integer ($\mathbb{Z} \to \mathbb{Z}$) is onto or not.

⇒ since f is a square function, therefore, It maps any integer set into any +ve integer set. hence, the −ve integers ~~are~~ in range are not mapped with other integers through this function.

∴ It is not onto.

Inverse of a function:

let us consider $f: A \to B$ such that for every element of A there will be an unique image in B such that $f(A) = B$ then the inverse function will map $f^{-1}(B) = A$



one to one ✗
onto ✓

∴ f^{-1} doesn't exist

Q. Let $f: \mathbb{Z} \to \mathbb{Z}$ such that $f(x) = x+1$ find the inverse of the function, if there exist any

⇒ The function here defined from

set of integers such that, in the range set $f(x) = y = x + 1$

The pre image of the element $y-1$ of the range, is x in domain.

$\therefore$ From the theory of inverse function,

$f^{-1}(y) = y - 1$