Propositional Logic

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What is a Proposition?

A proposition (or statement) is a declarative sentence that is either **True (T)** or **False (F)**, but not both.

Logical Connectives

The main logical connectives are:

- **Conjunction (AND)**: $A \wedge B$
- **Disjunction (OR)**: $A \vee B$
- **Negation (NOT)**: $\neg A$
- **Implication (IF-THEN)**: $A \rightarrow B$
- **Biconditional (IF AND ONLY IF)**: $A \leftrightarrow B$

Truth Tables for Logical Connectives

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Implication, Converse, Contrapositive, Inverse

- **Implication**: If A then B, written as $A \to B$.
- **Converse**: $B \to A$
- **Contrapositive**: $\neg B \to \neg A$
- **Inverse**: $\neg A \to \neg B$

Example and Truth Table for Implication Variants

A	B	$A \to B$	$B \to A$	$\neg B \to \neg A$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Biconditional Statement

- A $\leftrightarrow$ B is true when both A and B have the same truth value.

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

Problems on Propositional Logic

Solve the following:

1. Prove that $(P \vee \neg P)$ is a tautology.
2. Show that $(P \wedge \neg P)$ is a contradiction.
3. Verify whether $(A \rightarrow B) \vee (B \rightarrow A)$ is always true.
4. Show that $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is always true.

Solutions to the Problems

1. Truth table for $(P \vee \neg P)$ shows it is always true.
2. Truth table for $(P \wedge \neg P)$ shows it is always false.
3. Constructing the truth table verifies $(A \to B) \vee (B \to A)$ is always true.
4. Truth table proves $((P \to Q) \wedge (Q \to R)) \to (P \to R)$ is always true.