TAUTOLOGY :

In a compound statement of 2 or more propositions with the application of 1 or more logical and conditional connectives, if all the outcomes of a truth table comes true (T), then this compound proposition is called a tautology and if all outcomes will come False (F), then it is called a contradiction.

Algebric properties of propositional logic :

(i) $p \vee p \equiv p$, $p \wedge p \equiv p$ [Idempotent law]

(ii) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ [Associative law]

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(iii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ [Distributive Law]

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge q)$

(iv) $\sim(p \wedge q) = \sim p \vee \sim q$ [De-Morgan's Law]

$\sim(p \wedge q) = \sim p \wedge \sim q$

(v) $p \vee (p \wedge q) = p$ [Absorption Law]

$p \wedge (p \vee q) = p$

(vi) $p \vee$ Tautology $\equiv$ Tautology [Complement Law]

$p \wedge$ Tautology $\equiv p$

$p \vee$ Contradiction $\equiv p$

$p \wedge$ Contradiction $\equiv$ Contradiction

(vii) $p \vee \sim p$ = Tautology

$p \wedge \sim p$ = Contradiction

1. Prove that the expression is a tautology.

$$\{q \wedge (\sim(p \wedge q))\} \vee (p \wedge (\sim p \vee q))$$

$\equiv (q \wedge (\sim p \vee \sim q)) \vee ((p \wedge \sim p) \vee (p \wedge q))$ [de-Morgan's, distributive

$\equiv ((q \wedge \sim p) \vee (q \wedge \sim q)) \vee (\text{contradiction} \vee (p \wedge q)$

[Negative Law]

$\equiv ((q \wedge \sim p) \vee \text{contr.})) \vee (p \wedge q)$ [Complement]

$\equiv ((q \wedge \sim p)) \vee (p \wedge q)$

$\equiv (q \wedge \sim p) \vee (q \wedge p) \equiv q \wedge (\sim p \vee p) \equiv q \wedge \text{tautology} \equiv q$

$\therefore$ It is not tautology

(*) using Truth Table show that it is not a tautology.

2. Check that the proposition is contradiction or not.

$$(p \wedge q) \wedge \{\sim(p \vee q)\}$$

$\equiv (p \wedge q) \wedge \{(\sim p \wedge \sim q)\}$ [de-Morgan's Law]

$\equiv (p \wedge \sim p) \wedge (q \wedge \sim q)$ [Associative]

$\equiv$ Cont. $\wedge$ contri [Idempotent Law]

$\equiv$ Contradiction.

NOTE:

The conditional connective implication $p \rightarrow q \equiv \sim p \wedge q$

- If any compounded proposition P is respresented in the form of $P_1 \wedge P_2 \wedge \cdot \wedge P_R$, where all P_i's are either simple proposition or connected by only disjunction then the proposition, P will be called a conjunctive normal form.

$$P \equiv P_1 \wedge P_2 \wedge \dots \wedge P_R$$

- If any compounded proposition P is represented in the form $P \equiv P_1 \vee P_2 \vee \dots \vee P_R$, where all P_i's are either simple proposition or connected by only conjunction then the proposition, P will be <u>disjunctive normal form</u>

ARGUMENT AND VALID ARGUMENT:

If any proposition 'q' is deducted from a set of propositions $P_1, P_2, \dots, P_R$ (simple or compounded), then it is called an argument, when $P_1, P_2, \dots P_R$ are called premises and 'q' is called conclusion.

Moreover this argument will be called a valid argument when $P_1 \wedge P_2 \wedge \dots \wedge P_R$ is a tautology.

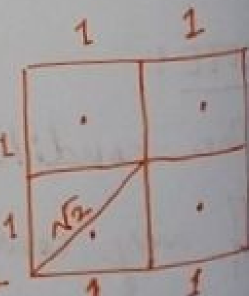
COUNTING PRINCIPLE:

Discreate anything can be countable.

PIGEON HOLE:

If $(n+1)$ no. of pigeons are placed in n no. of pigeon holes then in atleast one pigeon hole there would be 2 or more than 2 pigeons.

1. There are 5 points in a square of side length 2 inches. Prove that there exists 2 of them having a distance not more than $\sqrt{2}$ inches.



The square is divided into 4 equal parts which are having length 1 inche. $\therefore$ Length of diagonal of each square is $\sqrt{2}$ inches. If there are 5 different points to be located inside this 4 squares, then by pigeon hole principle there ~~exists~~ ~~exi~~ exists one square which may contain atleast 2 points inside it. Hence distance between these 2 points is less than $\sqrt{2}$ ~~units~~ inches. $\therefore$ By pigeon hole principal

It is proved that there exists 2 points in the square whose distance is less than $\sqrt{2}$ inches.

Generalized pigeon hole principle:

If $(kn+1)$ no. of pigeons are placed in n no. of pigeon holes then atleast one ~~pigeon~~ hole will contain $(K+1)$ or more ~~pig~~ pigeons pigeons.

2. How many students each of whom comes from 1 of 29 states must be enrolled in a college to guarentee that there are atleast 35 who come from the same state.

Here no. of states $(n) = 29$

Atleast 35 students come from same state means $(K+1 = 35)$

or $k = 34$, by generalized pigeon hole principle.

$\therefore$ Hence total no. of students will be ~~29~~ $kn+1$

or, $29 \times 34 + 1 = 987$.

3. There are 3 coplaner lines. 4 Distinct points are on each of these lines. Find the maximum no. of triangles with vertices at these points.

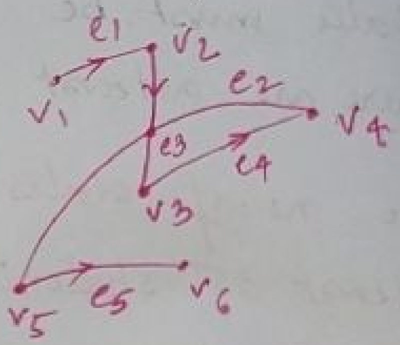
MODULE 04: GRAPH THEORY

A graph is a structure which consists of 2 sets V set of vertices & set of edges and is denoted in the form of $G(V,E)$

Parallel edge: If 2 edges will have same end vertices then these 2 edges will be called parallel edges.

Incidence: If any edge (e_1) has no vertex v_1 then we say e_1 is incident on v_1.

For undirected graph e_1 is incident on both v_1 & v_2 but for directed graph e_1 is only incident on v_2.



Self loop: Any edge whose starting and end vertices are same is called a self loop.

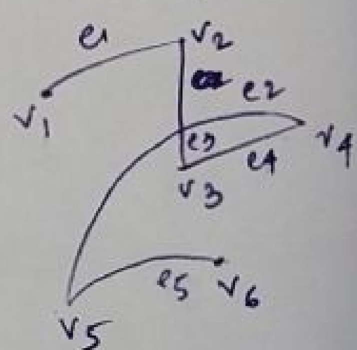
Degree of vertex: No. of edges incident on any particular vertex is the degree of that particular vertex.

Let G_1 is undirected graph.

$deg(v_1) = 1$ $\quad$ $deg(v_2) = 2$

$deg(v_3) = 2$ $\quad$ $deg(v_4) = 2$

$deg(v_5) = 2$ $\quad$ $deg(v_6) = 1$

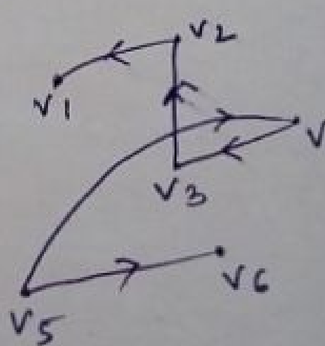


If the graph is improvised into a directed graph G_2, such that G_2 is formed like

$deg(v_1) = 1$ $\quad$ $deg(v_2) = 1$

$deg(v_3) = 1$ $\quad$ $deg(v_4) = 1$

$deg(v_5) = 0$ $\quad$ $deg(v_6) = 1$



If we insert a loop on any vertex then the degree of that vertex will be added by 2 for undirected Graph and for directed graph it will added by 1.

Simple Graph : If any graph will have no loop and no parallel edges then it is called a simple graph.

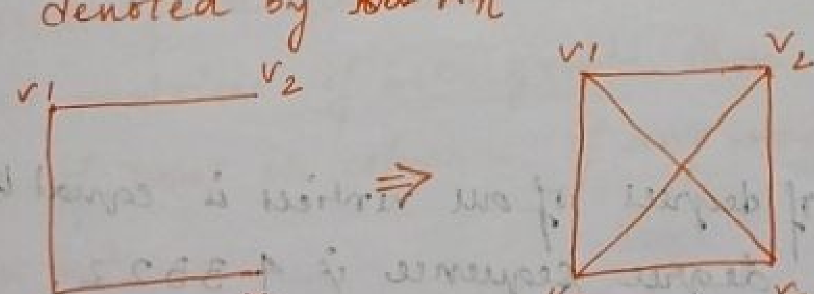
Regular Graph : If all the vertices of a Graph have same degree then it is called a regular Graph.

From the previous example, if ~~an~~ this graph has a connection between v_1 & v_6 then this graph will become a regular graph with degree 2

Isolated vertex and pendent vertex : If any vertex is not connected by any edge with rest of the vertices in the graph then this vertex is called an isolated vertex with degree 0.

If the degree of any vertex is 1, it is a pendent vertex.

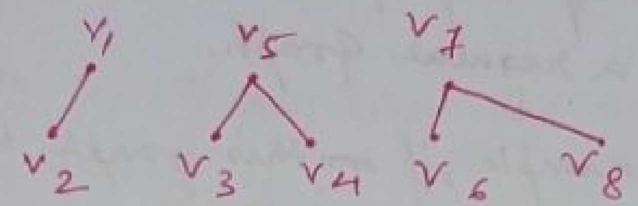
Complete Graph : Let us consider any simple graph. If each pair of vertices is connected by an edge, then the graph is called a complete graph. If there are n no. of vertices then the complete graph is denoted by ~~Kn~~ K_n



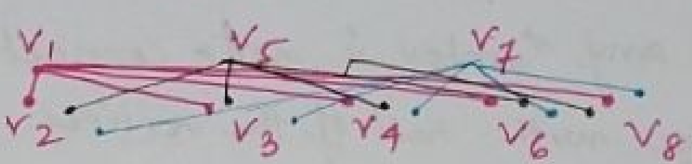
Every complete graph is a regular graph ~~but~~ but vice versa not possible.

If a complete graph has n vertices then degree of each vertex will be $n-1$.

Bi-partite Graph: Let us consider any graph $G(V, E)$, we divide the vertex set V into 2 subsets V_1 & V_2 in such a way that the vertices of V_1 will be connected with the vertices of V_2. But vertices of V_1 cannot be connected with each other and the similar thing happens for V_2, then the generated graph is called a bi-partite graph.



If the elements of V_1 are connected to with all elements of V_2 then the bi-partite graph becomes a complete bi-partite graph



If there are n ~~amd~~ and m no. of vertices in those 2 subsets then it is given in the form $K_{m,n}$.

If $m > n$ then the highest degree of any vertex in the set of m vertices is n. Similarly the highest degree of any vertex in the set of n vertices is m. Therefore degree of sequence of vertices will be $n, n-1, n-2, \ldots, m, m-1, \ldots 1$.

If this graph is a complete bipartite graph then the upper set vertices will have degree n and the lower set of vertices will be m

Q1 of Question Bank.

We know that sum of degrees of all vertices is equal to twice the no. of edges. Here the degree sequence is 4 3 3 2 2.

Let no. of edges be e, then total degree $= 2 \times e$. ~~$= 2\times 14 = 28$~~ 14

$\therefore e = 7$.

sum of the degree sequence

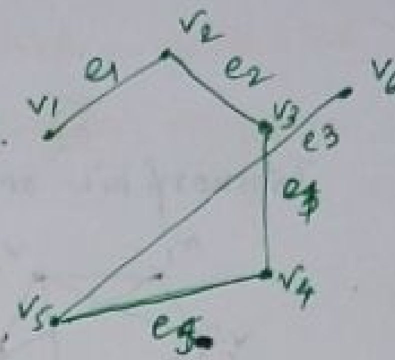
Connected and Disconnected Graph:

If in a graph there always exists a path between any 2 vertices. then the graph is called connected graph.

If there exists no path between ~~th~~ 2 vertices then the graph is called disconnected graph.

Walk, trail and path:

In a graph the alternating combination of vertices and edges are called walk.



A trail is a walk where no edges are repeated.

A path is a walk where no vertices will be repeated.

A circuit is a path where exceptionally the starting and the terminal vertices will be same.

Q7. $\sum \text{degree} = 2 \times e$

$\therefore 4V = 2 \times 10$

$\Rightarrow V = 5$

Isomorphism of 2 graphs:

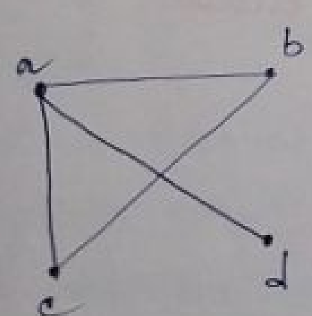
2 simple graphs will be isomorphic if the following conditions will be satisfied.

(i) Both the graphs will have same no. of edges and vertices.

(ii) Both of them will have same pattern of degree of vertices.

This can be shown from incidence matrix. In 1st step we will find the incident matrices of both graph then we need to check that permutation of rows or columns of any matrix generates the 2nd matrix or not.

Q18.

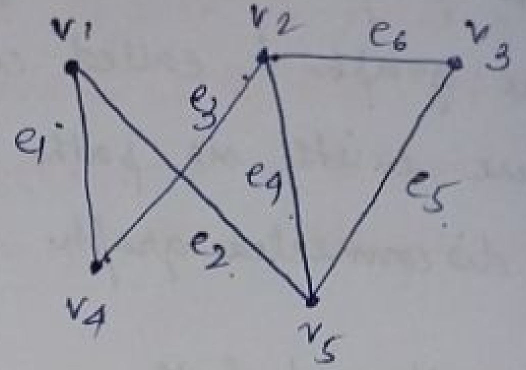


$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 1 & 0 & 0 \\ d & 1 & 0 & 0 & 0 \end{array}$$

Adjacent matrix

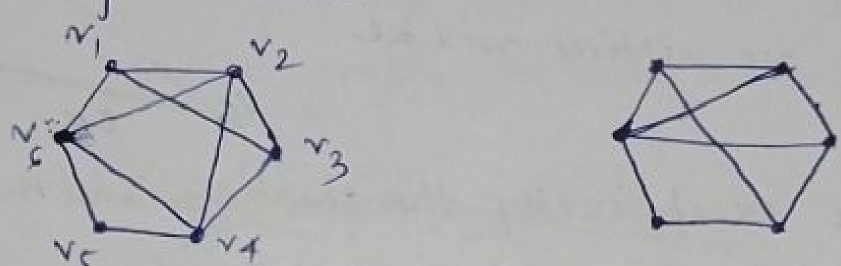
19. In incident matrix the rows are represented by vertices and columns are represented by edges.

$$
\begin{array}{c|cccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\
\hline
v_1 & 1 & 1 & 0 & 0 & 0 & 0 \\
v_2 & 0 & 0 & 1 & 1 & 0 & 1 \\
v_3 & 0 & 0 & 0 & 0 & 1 & 1 \\
v_4 & 1 & 0 & 1 & 0 & 0 & 0 \\
v_5 & 0 & 1 & 0 & 1 & 1 & 0
\end{array}
$$



20.

Isomorphic or not.



Both g1 and g2 have 6 vertices.

	degree
v_1	3
v_2	4
v_3	3
v_4	4
v_5	2
v_6	4

	Degree
u_1	3
u_2	4
u_3	3
u_4	4
u_5	2
u_6	4

Since there are 2 no. of 3, 3 no. of 4 and 1 no. of 2 in both the graphs it is

<u>Eulerian</u> <u>Circuit</u>: If in a graph a walk is generated which visits all the edges only at once then that walk is called an Eulerian walk. If the walk is ~~an~~ circuit then it becomes an Eulerian circuit.

If in that walk all the vertices are visited only at once then it becomes a Hamilton path. If this path has same starting and ending vertex then it is called a Hamilton circuit.

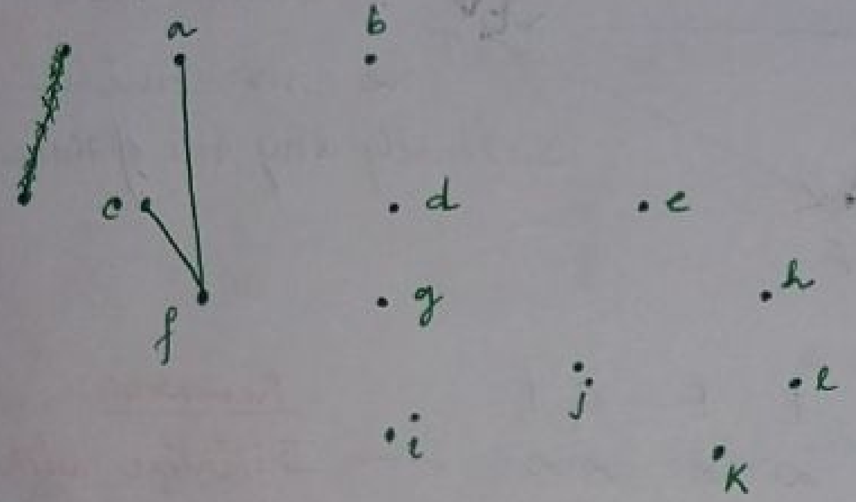
4) ~~xop~~ A and B are 2 person approaching to a point perpendicular to each other
~~xop~~: when A is at 0 . After 9 min A and B are in
after 7 min A and B are in again

TREE :

A connected graph without any cycle is called a Tree

Spanning Tree : It is formed from a graph which is connected, all vertices are visited. but there won't be any circuit or cycle.

Q6.



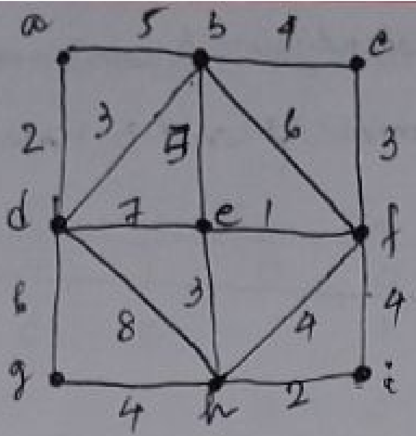
Minimum spanning Tree: It is a connected graph where the spanning from the graph is done on the basis of minimum cost or minimum weight of the edge. To determine MST from a graph there are 2 algorithms.

(i) Prim's algorithm
(ii) Kruskal's algorithm.

— In prim's algorithm we set arbitrary nodes as the initial vertex. when we use prim's algorithm, then we start from a single vertex and compare the next minimum cost to the adjacent vertex. and the process goes no with no repetation of vertices and all vertices will be visited

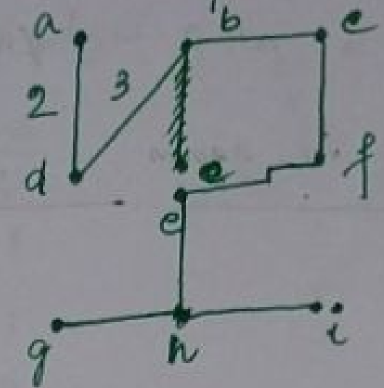
— But in Kruskal, initially we consider all the vertices and we arrange the weights between the connected vertices. in ascending order and we must consider those edges which are having minimum weight. Maybe we ~~note it~~ don't need to check all the edges if all vertices are visited already.

Q8.



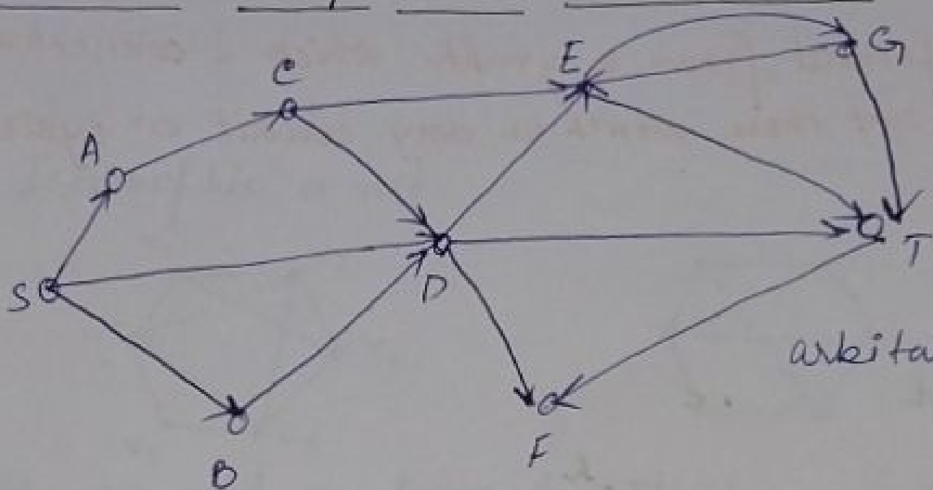
We start from d,

Step 1:



DJIKSTRKS' ALGO FOR SHORTEST PATH:

Q14.



If there is a tie between the values of 2 nodes then we can consider arbitarlly any one of them

S	A	B	C	D	E	F	G	T	Remarks
∞	$\boxed{0}$	∞	∞	∞	∞	∞	∞	∞	Initialize with vertex A and permananthy labelled
∞	$\boxed{0}$	∞	1	∞	∞	∞	∞	∞	C is having temporary labeled
∞	$\boxed{0}$	∞	$\boxed{1}$	∞	∞	∞	∞	∞	C is permanently labeled.
∞	$\boxed{0}$	∞	$\boxed{1}$	4	2	∞	∞	∞	E and D are temporarily labeled.
∞	$\boxed{0}$	∞	$\boxed{1}$	4	$\boxed{2}$	∞	4	∞	E is permanantely labeled.
∞	$\boxed{0}$	∞	$\boxed{1}$	4	$\boxed{2}$	∞	4	6	G and T are temporarily labeled.
∞	$\boxed{0}$	∞	$\boxed{1}$	$\boxed{4}$	$\boxed{2}$	∞	4	6	D is permanently labeled.
∞	$\boxed{0}$	∞	$\boxed{1}$	$\boxed{4}$	$\boxed{2}$	9	4	6	F is temporarily labeled
∞	$\boxed{0}$	∞	$\boxed{1}$	$\boxed{4}$	$\boxed{2}$	9	$\boxed{4}$	6	G is permanently labeled.
∞	$\boxed{0}$	∞	$\boxed{1}$	$\boxed{4}$	$\boxed{2}$	9	$\boxed{4}$	$\boxed{6}$	

Now to find the shortest path we need to backtrack, check where last row (7) changed from ∞, here E is permanantly labelled from C than A.

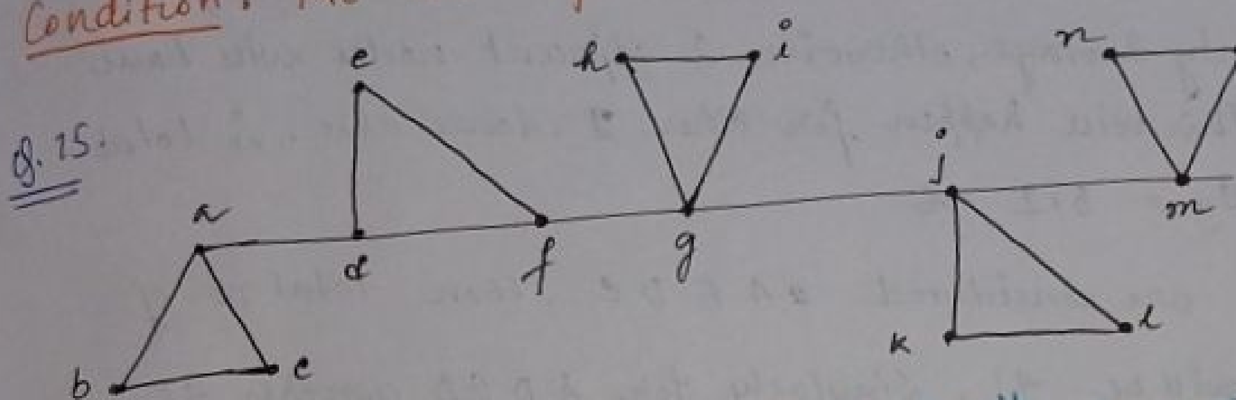
Graph Colouring:

i) Chromatic Number

If any Graph is coloured with different colours then, chromatic no. represents minimum no. of colours required for colouring all the vertices.

Condition: No two adjacent vertices will have same colour.

Q. 15.



We are taking A B C as the initial circuit from the Graph which can be coloured by 3 colours say R, G, B. In forward d, e they have either G or B and correspondingly the circuit g e d f may be coloured by R G B. The similar process will continue with rest of the circuits GHI, jkl and mno. Thus using minimum 3 colours this graph can be coloured so that the chromatic no. of this graph is 3.

(ii) Chromatic polynomial:

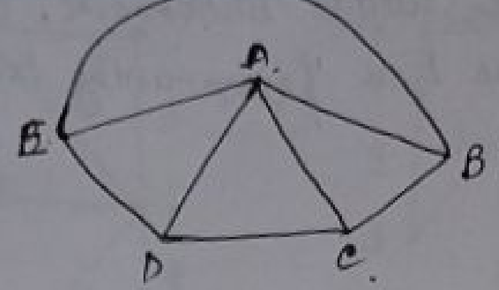
In a graph G there are n vertices and x no. of colours are present for colouring the graph, than the chromatic polynomial shows the possible no. of ways of colouring the graph with x no. of colours, and it is given in the form:

$$c_1 \, {}^{x}C_1 + c_2 \, {}^{x}C_2 + \dots \; c_n \, {}^{x}C_n$$

are the arrangement of colouring the graph with exactly i no. of colours.

5 marks

5.



With one colour it is impossible to colour the whole graph and $\therefore$ $c_1 = 0$

$c_2 = 0$

If 3 colours are taken R, G, B, we fix a particular node with a colour say R then other 4 nodes can be arranged / coloured in only 2 ways, otherwise 2 adjacent nodes will have same colour. This will happen for other 2 colours also. $\therefore$ total arrangement $c_3 = 3 \times 2 = 6$.

If 4 vertices are considered A E D C, then total no. of arrangement will be 4!. Similarly for A D C B another 4! combination is required. (ie) $c_4 = 2 \times 4! = 48$.

If 5 colours are given for 5 vertices then total no. of arrangements will be $5! = 120$. $\therefore$ The chromatic polynomial

$$f(G,x) = c_1 \, {}^xC_1 + c_2 \, {}^xC_2 + c_3 \, {}^xC_3 + c_4 \, {}^xC_4 + c_5 \, {}^xC_5$$

$$= 0 \times {}^xC_1 + 0 \times {}^xC_2 + 6 \times \frac{x(x-1)(x-2)}{6} + \frac{48 \times x(x-1)(x-2)(x-3)}{48}$$

$$+ 120 \times \frac{x(x-1)(x-2)(x-3)(x-4)}{120}$$

$$= x(x-1)(x-2)\left[1 + 2(x-3) + (x-3)(x-4)\right]$$

$f(G,1) = 0$

$f(G,2) = 0$

$f(G,3) = 6 \neq 0$.

Since $f(G,3) = 6 \neq 0$, $\therefore$ the chromatic no. of the graph is 3.

PLANAR GRAPH :

A planar graph is drawn over a plane where 2 edges of the graph will not intercept. If 2 graphs G_1 and G_2 are isomorphic such that G_1 is planar then G_2 will also be a planar graph.

Euler's formula for a planar's graph.

If a graph G is connected planar graph such that it has n no. of vertices and e no. of edges so that it is divided into f no. of regions, then by Euler's formula we may write

$f = e - n + 2$

I) ABJC II) DJCB (curved area) III) ACHG

IV) IKGI V) AIKG which covers up the whole graph

∴ It does not follow the Euler's theorem and not a planar graph.

Q.15

To find any hamiltonian circuit we can start from any arbitary vertex. The intial vertex is a.

$a \to b \to d \to c \to e \to a$

$a \to c \to b \to d \to e \to a$

$a \to e \to d \to c \to b \to a$

$a \to d \to e \to c \to b \to a$

GROUP

A group.. Let us consider a non-empty set G with an operation '$*$' such that –

(i) for all $a, b \in G$, $a * b \in G$ [Closure property]

(ii) for all $a, b, c \in G$, $a * (b * c) \in G$ $= (a * b) * c \in G$ [Associative property]

(iii) for all $a \in G$, ~~such that~~ there exists one unique element e_g, for which $a * e = e * a \in G$ (Identity elements)

(iv) for all $a \in G$, there exists ($\exists$) an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

The group will be called commutative or Abelian, $a * b = b * a$ for all $a, b \in G$.

$a, b \in Z$

then $a + b \in Z$ (Closure property satisfied)

for $a, b \in Z$

$a + (b + c) = (a + b) + c$

Let $a + e = a$, then by left cancellation property

$-a + a + e = -a + a$

$e = 0 \in Z$ [Identity element exists]

We consider, $b \in G$ such that

$a + b = 0$ [Inverse exists]

$\therefore b = -a \in Z$

$\therefore$ All 4 properties are satisfied and $Z+$ forms a group.