Q. Find quotient and reminder when

(i) $777 \div 21$

(ii) $-2002 \div 87$

$87 \overline{) -2002 }$

① $777 = 7 \times 111$ $\Rightarrow ac \equiv bd \pmod m$

$111 \equiv 6 \pmod{21}$

$7 \equiv 7 \pmod{21}$

$777 \equiv 42 \pmod{21}$

$\equiv 0 \pmod{21}$

(ii) $-2002 = -11 \times 182$

$182 \equiv 8 \pmod{87}$

$-11 \equiv 76 \pmod{87}$

$\therefore 182 \times -11 \equiv 8 \times 76 \pmod{87}$

$-2002 \equiv 608 \pmod{87}$

$-2002 \equiv 86 \pmod{87}$

reminder

Prime no:

Any positive integer which is having +ve divisors as one and itself, is callد prime no.

Greatest common divisor

let, non zero int a, b such that a and b has multiple divisors. Among these divisions d is common division of a and b also highest among all divisons. then we write

$d = GCD(a, b)$ d is +ve.

* two int a, b are said to be relativel prime when $GCD(a,b) = 1$

* BEZOUT's equation

If there exist 2 ints a, b then to determine their gcd, Bezout's equation states that, $\exists$ two integers s and t for which GCD of a, b = (sa + tb) for two relatively prime numbers a and b.

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Ques

Q. prove that product of any m consecutive int is divisible by m

⇒ product of m consecutive no is:

$$P = n(n+1)(n+2)\cdots(n+m-1)$$

since, n is an arbitary no. and is divided by m, then there are possible no. in the form,

$$n = mk$$
$$n = mk+1$$
$$n = mk+2$$
$$\vdots$$
$$n = mk+(m-1)$$

if $n = mk$, P is divisible by m.

If $n = mk+1$, then $n+m-1 = mk+1+m-1$
$= m(k+1)$
is divisible by m.

If $n = mk+(m-1)$, then $(n+1) = mk+m$
$= m(k+1)$.

In the product P any one of the consecutive terms are always divisible by m, and hence the product P i.e. product of m consuctive int are divisible by m.

Q. By division algorithm, show square of an odd int is of the form $8k+1$ where k is an int.

$\Rightarrow$ Any odd integer may be represented in the form $x = 4n \pm 1$ $\quad 2 \in \mathbb{Z}$

$$x^2 = (4n \pm 1)^2$$
$$= 16n^2 \pm 8n + 1$$
$$= 8(2n^2 \pm n) + 1$$
$$= 8k + 1$$

Since $2n^2 \pm n = k$ is also a integer.

Q. Show that $\gcd(a, a+2) = 1$ or 2 for all integers a.

Let us consider any no. a is even.

So that $a = 2m$, $m \in \mathbb{Z}$

$a + 2 = 2(m+1)$; $m \in \mathbb{Z}$

$$\gcd(a, a+2)$$
$$= \gcd(2m, 2(m+1))$$
$$= 2(\gcd(m, m+1)) = 2$$

Let a be an odd int. $\therefore a = 2m+1$

$a + 2 = 2m + 3$

$m \in \mathbb{Z}$

$\therefore \gcd(a, a+2)$

$= \gcd(2m+1, 2m+3)$

$= 1$

$\therefore \gcd(a, a+2) = 1$ or 2.

Relatively prime no. $\rightarrow$ proof.

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any equation $ax+by=c$ where $a,b,c \in \mathbb{Z}$ is called linear Diophantine Equation.

This equation is solvable only if $\gcd(a,b)$ divides c.

$$258x + 147y = 369 \quad \text{--- (I)}$$

$\gcd(258, 147) = 3$

$258 = 147 \times 1 + 111$

$147 = 111 \times 1 + 36$

$111 = 36 \times 3 + 3$

$36 = 3 \times 12 + 0$

$3 = 111 - 36 \times 3$

$= 111 - (147 - 111) \times 3$

$= 111 \times 4 - 147 \times 3$

$= (258 - 147) \times 4 - 147 \times 3$

$= 258 \times 4 - 147 \times 7$

$$258 \times (492) - 147 \times (861) = 369 \quad \text{--- (II)}$$

$\Rightarrow 258x + 147y = 258(492) - 147(861)$

$\Rightarrow 258(x - 492) = -147(y + 861)$

$\Rightarrow \dfrac{x - 492}{-147} = \dfrac{y + 861}{258} = t$

t is any int

$x = 492 - 147t$

$y = -861 + 258t$

$t = 0, \quad x = 492, \quad y = -861$

Linear Congruence

Any congruent form written as, $ax \equiv b \pmod{m}$ where a, b & m are int and x is a variable

$$ax \equiv b \pmod{m}$$

$$\frac{ax - b}{m} = y$$

$\rightarrow ax - b = my$

$$ax - b = my$$

any linear congruence can be solvable when $\gcd(a, m)$ will divide by b.