Module-1

Part-A

Question Number	Question	CO	Marks
1	Let A = {1,3,5,7} and B={2,3,4,5}, find $A \setminus B$.	CO1	2
2	Let A = {1,3,5,7} and B={2,3,4,5}, find $A \cup B$.	CO1	2
3	Let A = {1,3,5,7} and B={4,5}, find $A \cap B$	CO1	2
4	Let A = {1,3,5,7} and B={2,3,4,5} find A X B.	CO1	2
5	Let A and B be two finite sets such that n(A) = 20, n(B) = 28 and n(A$\cap$B) = 36, find n(A$\cup$ B).	CO1	2
6	Let A = {x: x is a natural number and a factor of 18} and B = {x: x is a natural number and less than 6}. Find A $\cap$B.	CO1	2
7	If A = {2, 3, 4, 5, 6, 7} and B = {3, 5, 7, 9, 11, 13}, then find (i) A \ B and (ii) B \ A.	CO1	2
8	Draw the Venn diagram of (A $\cup$B)$\cap$C.	CO1	2
9	How many total relations can be defined on a set with n elements?	CO1	2
10	If A is a set having n elements then what is the cardinality of the power set of A?	CO1	2
11	If A is a set having n elements and B is a set having m elements then what is the cardinality of A X B?	CO1	2
12	Draw the Venn diagram of (A\ B) \ C.	CO1	2
13	Draw the Venn diagram of (A $\cup$B) $\cap$C	CO1	2
14	What is the power set of the set {0, 1, 2}?	CO1	2
15	State Division algorithm with an example.	CO1	2
16	How many different factors does 48 have?	CO1	2
17	Define GCD of two integers.	CO1	2
18	Define relatively prime integers with an example.	CO1	2
19	State “Fundamental theorem of arithmetic”.	CO1	2
20	Define “Well ordering principle” .	CO1	2
21	Find the G.C.D of -34 and 48 .	CO1	2

22	When a number is divided by 36, it leaves a remainder of 19. What will be the remainder when the number is divided by 12?	CO1	2
23	Perform Prime factorization of 7007.	CO1	2
24	Define Mersenne prime.	CO1	2
25	Find the remainder of 263^{25} when it is divided by 7.	CO1	2
26	Find the digit at the unit place of 457^{243}.	CO1	2
27	Find whether the solution of the following congruence equation exists or not $4x \equiv 3 \pmod 6$	CO1	2
28	Find the value of x where $5^{241} \equiv x \pmod 7$.	CO1	2
29	"x^2 gives a remainder 2 when it is divided by 3". Verify whether this statement is true or false.	CO1	2
30	If 12x47 is divisible by 9, then what is the value of x?	CO1	2

Part-B

Question Number	Question	CO	Marks
1	Prove that if n is a composite integer, than n has a prime divisor less than or equal to $\sqrt{n}$.	CO1	5
2	For integers a,b,c show that if a|b and a|c, then a|bx+cy for arbitrary integers x and y. 5 marks.	CO1	5
3	If a|c and b|c with gcd(a,b)=1,then prove that ab|c,for some integers a,b,c (with at least one in between a and b should be non zero).	CO1	5
4	Prove that there are infinitely many primes	CO1	5
5	Prove that $\sqrt{2}$ is not rational	CO1	5
6	If m and n are two positive integers and m=qn+r where $0 \le r < n$ then prove that gcd(m,n)=gcd(n,r).	CO1	5
7	Let A = {1,2,3,4}. Find the equivalence relation generated by the partition {(1,4),(2,3)}.	CO1	5
8	Show that (with the help of Venn diagram and proper explanation) the following argument(S) is valid based on the	CO1	5

	assumptions S_1, S_2, S_3 S_1 :Babies are illogical. S_2 :Nobody is despised who can manage a crocodile. S_3 :Illogical people are despised. S :Babies cannot manage crocodile.		
9	Define a POSET. Let R be a relation on the set of integers such that a R b if and only a|b . Then verify whether R is a partially order relation or not.	CO1	5
10	What do you mean by an equivalence class? Let R be a relation on the set of two dimensional straight lines such that a line is related to another if and only if they are parallel. Then verify whether R is an equivalence relation or not.	CO1	5
11	Define an equivalence relation. Let R be a relation on the set of integers such that a R b iff a-b is divisible by 3 where a and b are arbitrary integers. Then verify whether R is an equivalence relation or not.	CO1	5
12	Let A=$\{1,2,3,4\}$ and R=$\{(1,1),(1,2), (2,3),\{3,2),(4,3),(3,4)\}$. Find the transitive closure of R.	CO1	55
13	Prove that a 4 digit number $a_1a_2a_3a_4$ is divisible by 4 if and only if a_3a_4 is divisible by 4.	CO1	5
14	Prove that $2^k - 1$ is prime if k is prime	CO1	5
15	Find general solution of each of the following congruence equation: (i) $2x \equiv 1 \pmod 3$ (ii) $3x \equiv 2 \pmod 7$	CO1	5
16	Verify whether f(x)=2x+3 is bijective or not and if bijective find its inverse.	CO1	5
17	Consider the following relation R on the set A=$\{1,2,3,4\}:\{(1,1),(1,3),(2,4),(3,1),(3,3),(4,3)\}$. Find the reflexive closure of R.	CO1	5
18	Consider the following relation R on the set A=$\{1,2,3\}:\{(1,2),(2,3),(3,3)\}$. Find the transitive closure of R.	CO1	5
19	Prove that the number of injective functions from a set having n elements to another set having m elements is $^m p_n$.	CO1	5
20	Let A=$\{1,2,3,4\}$. Define an equivalence relation on A.	CO1	5

Part-C

Question Number	Question	CO	Marks
1	Find d=gcd(453,1650). Express d=453x+1650y where x, y are integers.	CO 1	3+7=10
2	Find d=gcd(363,1500). Express d=363x+1500y where x, y are integers.	CO 1	3+7=10
3	Using Chinese remainder theorem to fine the general solution from the following congruence equations: $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$, $x \equiv 4 \pmod{7}$	CO 1	10
4	Convert the following congruence relations into a single congruence relation: $x \equiv 2 \pmod{5}$, $x \equiv 3 \pmod{7}$, $x \equiv 4 \pmod{11}$	CO 1	10
5	Find the condition for which the number $a_1 a_2 a_3 a_{.4} \ldots\ldots\ldots\ldots\ldots\ldots a_{n-1} a_n$ (i) is divisible by 9 (ii) is divisible by 11.	CO 1	5+5=10
6	$x \equiv 0 \pmod{2}$, $x \equiv 5 \pmod{7}$, $x \equiv 1 \pmod{11}$, *then find a such that* $x \equiv a \pmod{154}$	CO 1	10
7	(i)Show that the set of rational numbers is countable. (ii) Show that the set of integers is countable.	CO 1	5+5=10
8.	Find d=gcd(432,1176). Express d=432x+1176y where x, y are integers.	CO 1	3+7=10
9.	148 elderly people live in an old age home. Here is the information regarding different medicines they take for different illness: 62 people take medicines of diabetes, 51 take medicines for high blood pressure, 25 take medicine for anxiety, 22, 13, 11 people take medicine for both diabetes and high blood pressure, diabetes and anxiety, high blood pressure and anxiety respectively.6 people takes all three medicines. Then	CO 1	2+2+2+2+2=10

	(i)How many people take either of the medicines? (ii)How many people take none of the medicines? (iii) How many people take only medicine for high blood pressure? (iv) How many people take only medicine for anxiety (v) How many people take only medicine for diabetes.		
10.	176 people live in a housing society. Here is the information regarding the newspaper they take regularly: 78 people study The Hindu, 61 study The Telegraph, 35 study The Statesman.35, 18, 15 are the numbers of people who study The Hindu and The Telegraph, The Hindu and The Statesman, The Telegraph and The Statesman respectively.10 people study all. Then (i)How many people study either of the newspaper? (ii)How many people like none of the newspaper? (iii) How many people study The Telegraph and The Statesman but not The Hindu? (iv)How many people study The Telegraph and The Hindu but not The Statesman? (v) How many People study The Statesman and The Hindu but not The Telegraph?	CO 1	2+2+2+2+2=10
11.	Sumit surveyed 220 people to see which sports they like. Here is the information that Sumit has got: 85 people like football, 75 like cricket, 65 like tennis.30,15,12 are the numbers of people who like cricket and football, football and tennis, cricket and tennis respectively.10 people like all three sports.	CO 1	2+2+2+2+2=10

	Then (i)How many people like either of the sports? (ii)How many people like none of the sports? (iii) How many people like cricket and football but not tennis? (iv) How many people like only cricket but not tennis and football? (v) How many people like only tennis but not cricket and football?		
12.	Let A={1,2,3,4} , determine whether the following relations on A are reflexive, symmetric, transitive, symmetric or anti-symmetric (a) {(1,1),(2,2),(2,3),(3,2)} (b) {(3,2), (1,1),(1,3),(3,3),(2,3),(3,1)} (c) {(1,1), (2,2,),(3,3),(2,3) } (d) {(1,2), (3,2,),(3,4) } (e){(1,3),(3,4),(3,1),(1,4)}	CO 1	2+2+2+2+2=10
13.	Prove or disprove the following statement (i) $a \equiv b \pmod{n}, c \equiv d \pmod{n}, then \;\; ac \equiv bd \pmod{n}$ (ii) $a = b \pmod{n}$ $then$ $a^k \equiv b^k \pmod{n}$, $where$ k is a $natural$ $number$.	CO 1	5+5=10
14.	What can you say about the sets A , B and C if we know that a) $A \cup B = A$? b) $A \cap B = A$? c) $A \setminus B = A$? d) $A \cup B = A \cup C$ and $A \cap B = A \cap C$ e) $A \setminus B = B \setminus A$?	CO 1	2+2+2+2+2=10
15.	Find the largest common factor of 540 and 168. Represent that number as a linear combination of 540 and 168.	CO 1	3+7=10

Module 2 Questions

Sl. No.	Question	Bloom's Taxonomy
2 Marks		
1	What are the contrapositive, the converse and the inverse of the conditional statement "If I study then I will pass in Exam"?	BL3
2	Construct the truth table of $[(p \wedge q) \vee (\neg r)]$	BL2
3	Write the symbolic representation and give its contra positive statement of "If it rains today, then I buy an umbrella".	BL3
4	Find the truth table for the statement $p \to \neg q$.	BL3
5	Write the negation of the statement $(\exists x)(\forall y)P(x, y)$.	BL3
6	When do you say that two compound propositions are equivalent?	BL4
7	Is $\neg p \wedge (p \vee q) \to q$ a tautology?	BL4
8	Check whether the following is a tautology or not. $(p \vee q) \to (q \wedge p)$	BL4
9	State an equivalent statement of $(P \vee Q)$.	BL2
10	Show that $(p \vee q) \wedge (\neg p \wedge \neg q)$ is contradiction.	BL4
11	Is the following a tautology? Justify $(p \vee q) \to (q \wedge r)$.	BL4
12	Is the following a contradiction? Justify $(p \to q) \to (q \to r)$.	BL4
13	Write the symbolic form and negate the following statements. i) Everyone should help his neighbors, or his neighbors will not help him. ii) Everyone agrees with someone, and someone agrees with everyone.	BL3

14	Negate each of the following statements: (i) If the teacher is absent, then some students do not complete their homework (ii) All the students completed their homework and the teacher is present.	BL3
15	Let p denote "He is rich" and let q denote "He is happy". Write each statement in symbolic form using p and q. (i) If he is rich, then he is unhappy. (ii) He is neither rich nor happy.	BL3
16	Construct the truth table for the compound proposition $(P \rightarrow Q) \leftrightarrow (\neg P \rightarrow \neg Q)$.	BL3
17	Formulate the contrapositive, the converse and the inverse of the conditional statement "If you work hard then you will be rewarded".	BL3
18	Write the symbolic form and negate the following statements. i) Everyone who is healthy can do all kinds of work. ii) Some people are not admired by everyone.	BL3
19	Construct the truth table of $(p \vee \sim q) \rightarrow (p \wedge q)$.	BL3
20	Construct the truth table for the compound proposition $(p \rightarrow q) \rightarrow (q \rightarrow p)$.	BL3
21	State the converse, contrapositive, and inverse of the conditional statement - "If it snows tonight, then I will stay at home".	BL3
22	Find r if $5 \times 4_{P_r} = 6 \times 5_{P_{r-1}}$.	BL3
23	Explain Principle of Inclusion–Exclusion.	BL2
24	Explain The Pigeonhole Principle.	BL2
25	Determine which of these are linear homogeneous recurrence	BL2

	relations with constant coefficients: 1) $a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$ 2) $a_n = a_{n-1} + 2$	
26	State the absorption law in Boolean algebra.	BL1
27	Define **Boolean Algebra**.	BL2
28	Prove x+y.z=(x+y).(x+z) using Boolean laws.	BL3
29	Write the **dual** of the expression: $A + (B \cdot C)$.	BL3
30	State **De Morgan's Theorems** for **Boolean algebra**.	BL1

5 Marks

1	Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.	BL3
2	Show that the following conditional statement is tautology by using truth table. $$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$	BL3
3	Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded "imply the conclusion "It rained."	BL4
4	For the following sets of premises, what relevant conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."	BL4
5	Show, without using truth table, that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.	BL4
6	Use resolution to show that the hypotheses "Jasmine is skiing or it is not snowing" and "It is snowing or Bart is playing hockey" imply that "Jasmine is skiing or Bart is playing hockey".	BL4
7	Show that $\sim\left(p \vee (\sim p \wedge q)\right)$ and $(\sim p \wedge \sim q)$ are logically equivalent by developing a series of logical equivalences (and not using truth table).	BL4
8	Let Q(x, y) denote "$x + y = 0$." What are the truth values of the quantifications $\exists y \forall x Q(x, y), \forall x \exists y Q(x, y)$, where the domain for all variables consists of all real numbers?	BL4
9	In how many ways one can select 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour?	BL5
10	A man has 3 friends. Evaluate in how many ways he can invite one friend every day for dinner on 6 successive nights so that no friend is invited more than 3 times.	BL5
11	Find the number of 7 lettered words each consisting of 3 vowels and 4consonant which can be forms using the letters of the word "DIFFERENTIATION".	BL5
12	For each positive integer k, let S_k denotes the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example S_3 is the sequence 1, 4, 7, 10,... . Find the number of values of k for which S_k contains the term 361.	BL5

13	There are n triangles of positive area that have one vertex A(0, 0) and the other two vertices whose coordinates are drawn independently with replacement from the set {0, 1, 2, 3, 4} e.g. (1, 2), (0, 1) (2, 2) etc. What is the value of n?	BL5
14	There are 5 points in a square of side length 2. Prove that there exist 2 of them having a distance not more than $\sqrt{2}$.	BL5
15	Prove that in a set containing n positive integers there must be a subset such that the sum of all numbers in it is divisible by n.	BL5

16	There is a sequence of 100 integers. Prove that there is a sequence of consecutive terms such that the sum of these terms is divisible by 99.	BL5
17	Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.	BL5
18	How many numbers must be selected from the set {1, 2,3,4,5,6} to guarantee that at least one pair of these numbers add up to 7?	BL5
19	In a discrete mathematics class every student is a major in computer science or mathematics or both. The number of students having computer science as a major (possibly along with mathematics) is 25; the number of students having mathematics as a major (possibly along with computer science) is 13; and the number of students majoring in both computer science and mathematics is 8. How many students are in the class?	BL5
20	A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken a course in at least one of Spanish French and Russian, how many students have taken a course in all 3 languages.	BL5
21	What is the solution of the recurrence relation $a_n = 6\,a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 6$?	BL5
22	Solve the following recurrence relation together with the initial conditions given. $a_{n+2} = -4a_{n+1} + 5a_n$ for $n \geq 0, a_0 = 2, a_1 = 8$	

22	Solve the following recurrence relation together with the initial conditions given. $a_{n+2} = -4a_{n+1} + 5a_n$ for $n \geq 0, a_0 = 2, a_1 = 8$	

23	Show that $x\bar{y} + y\bar{z} + \bar{x}z = \bar{x}y + \bar{y}z + x\bar{z}$.	BL5
24	Show that De Morgan's laws hold in a Boolean algebra.	BL5
25	Show that in a Boolean algebra, every element x has a unique complement $\bar{x}$ such that $x \vee \bar{x} = 1$ and $x \wedge \bar{x} = 0$.	BL5
26	Use a table to express the values of the following Boolean function: $f(x,y,z) = \bar{y}\,(xz + \bar{x}\,\bar{z})$	BL5
27	Show that $x \oplus y = y \oplus x$.	BL5
28	Find the duals of these Boolean expressions. 1) $xyz + \bar{x}\,\bar{y}\,\bar{z}$ 2) $x + y$	BL5

29	Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 3^n; a_1 = 5$.	BL5
30	Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7, a_1 = -4$, and $a_2 = 8$.	BL5

10 Marks

1	a) What are the negations of the statements "There is an honest politician" and "All Americans eat cheeseburgers"? Explain with proper justification. b) Show that $\neg\forall x\big(P(x) \rightarrow Q(x)\big)$ and $\exists x\big(P(x) \wedge \neg Q(x)\big)$ are logically equivalent.	BL4
2	For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. (a) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive. (b) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephants running down the road." (c) There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre.	BL5
3	(a) Determine whether $\big(\neg q \wedge (p \rightarrow q)\big) \rightarrow \neg p$ is a tautology. (b) Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.	BL3
4	Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."	BL5
5	a) State the converse, contrapositive, and inverse of the conditional statement. – "When I stay up late, it is necessary that I sleep until noon." b) Show, without using truth table, that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.	BL4

6	Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."	BL5
7	In an election for the managing committee of a reputed club, the number of candidates contesting elections exceeds the number of members to be elected by r (r > 0). If a voter can vote in 967 different ways to elect managing committee by voting at least 1 of them & can vote in 55 different ways to elect (r – 1) candidates by voting in the same manner. What is the number of candidates contesting the election & the number of candidates losing the elections?	BL6
8	A shop sells 6 different flavors of ice-creams. In how many ways can a customer can choose 4 ice-cream cones if (i) they are all of different flavors (ii) they are not necessarily of different flavors (iii) they contain only three different flavors (iv) they contain only two or three different flavors	BL6
9	There are 3 different cars available to transport 3 girls and 5 boys on a field trip. Each car can hold up to 3 children. (i) Find the numbers of ways in which they can be accommodated. (ii) Find the number of ways in which they can be accommodated if 2 or 3 girls are assigned to one of the cars. In both the cases the internal arrangement of the children inside the car is considered to be immaterial.	BL6
10	A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them. a) How many balls must she select to be sure of having at least three balls of the same color? b) How many balls must she select to be sure of having at least three blue balls?	BL5
11	Solve the following recurrence relation: $f_n = f_{n-1} + f_{n-2}; n \geq 2$ and $f_0 = 0, f_1 = 1$.	BL5
12	Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2, a_1 = 5$, and $a_2 = 15$.	BL5
13	Find the solution to the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with initial conditions $a_0 = 1, a_1 = -2$, and $a_2 = -1$.	BL5
14	Find all solutions of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 7n$.	BL5
15	Find the sum-of-products expansion for the function $F(x, y, z) = (x + y)\bar{z}$.	BL5

Module-3

Part-A

Question Number	Question	CO	Marks
1	Define Group.	CO3	2
2	Prove that in a group, identity element is unique.	CO3	2
3	Prove that in a group inverse of every element is unique	CO3	2
4	Set of all 2×2 matrices, i.e., $M_2(R)$ forms a group under multiplication. Justify.	CO3	2
5	Set of all natural numbers form a field under addition and multiplication. Justify	CO3	2
6	Let $(G, *)$ be any abelian group with 4 elements, i.e., $G = \{e, a, b, c\}$. Let $a * b = c, b * c = a$. Then $(b * a) * b = ?$	CO3	2
7	Set of all odd integers is closed under addition. Verify the statement.	CO3	2
8	Set off all irrational numbers is closed under multiplication. Justify the statement.	CO3	2
9	Find the identity element with respect to the binary operation '$*$' on Z, where '$*$' is defined as $a * b = a + b - 2, a, b \in Z$.	CO3	2
10	In the composition table of $(Z_6, +)$, what is the inverse of 2?	CO3	2
11	What is the multiplicative inverse of 7 in $(U_8, \times)$.	CO3	2
12	What do you mean by subgroup?	CO3	2
13	In a group $(G, *)$, what is the smallest subgroup?	CO3	2
14	Give an example of a non-commutative group of order 6.	CO3	2
15	The number of elements in $(S_5, \circ)$ is 24 where S_5 denotes the set of all bijections from $\{1,2,3,4,5\}$ to itself. Justify.	CO3	2
16	Cyclic groups are commutative. Verify.	CO3	2
17	Set of all rational numbers form a group under multiplication. With proper justification, prove that the	CO3	2

	given statement is false.		
18	Give examples of two non-zero elementsa, b in $(Z_6, +, \times)$, where $a \times b = 0$.	CO3	2
19	A group of order 6 consist an element of order 4. Justify.	CO3	2
20	Give an example of an infinite abelian group.	CO3	2

Part-B

Question Number	Question	CO	Marks
1	Prove that $(Z, +)$ forms a group.	CO3	5
2	Show that $(Q^*, \times)$ forms a group, where $Q^* = Q - \{0\}$.	CO3	5
3	Write the composition table of $(Z_8, +)$.	CO3	5
4	Prove that intersection of two subgroups is also a subgroup.	CO3	5
5	Let $(G, *)$ be any group and let $(H, *)$, $(K, *)$ be two subgroups of $(G, *)$. Prove that $(H \cup K, *)$ is a subgroup if and only if H is a subset of K or K is a subset of H.	CO3	5
6	Prove that the set of all non-zero real numbers form a group under multiplication.	CO3	5
7	Prove that the set of all rational numbers form a group under addition.	CO3	5
8	Prove that $(R, +)$ forms an abelian group.	CO3	5
9	Prove that if $(R_1, +, .)$ and $(R_2, +, .)$ are two subrings of a ring $(R, +, .)$, then $(R_1 \cap R_2, +, .)$ is also a subring.	CO3	5
10	If a group is commutative then it is normal. Verify.	CO3	5
11	Find the orders of $(1,2)$ and $(1,2,3)$ in S_3.	CO3	5
12	Let $M = \{set\ of\ all\ 2 \times 2\ matrices\ with\ determinant\ 1\}$. Prove that M forms a group under matrix multiplication.	CO3	5
13	Let $a, b \in Z$ and let ' $*$ ' denotes a binary relation on Z. Let $a * b = a + b - 2$. Check whether the binary operation is (i) commutative nor not (ii) associative or not.	CO3	5
14	Let $a, b \in Z$ and let ' $*$ ' denotes a binary relation on Z. Let $a * b = a + b + 3$. Find the identity element and also find	CO3	5

	the inverse of 7.		
15	Prove that if $(Z_n, +\,,.)$ is a field then n is prime.	CO3	5

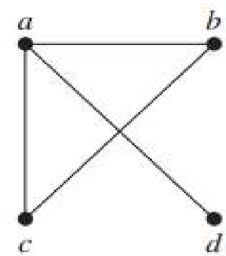
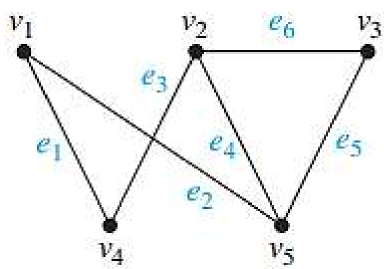
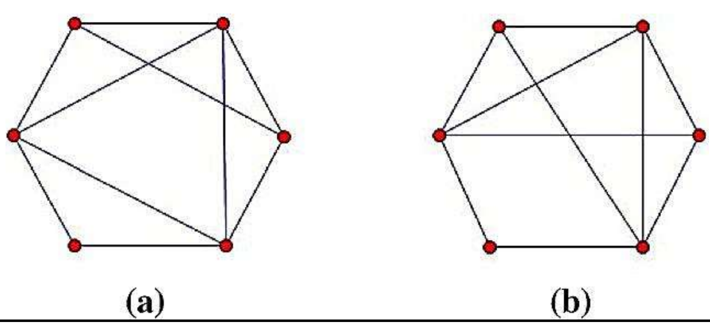
Part-C

Question Number	Question	CO	Marks
1	Prove that $(Z, +\,,.)$ forms a ring.	CO3	10
2	Prove that $(M_2(R), +\,,.)$ is a non-commutative ring, where $M_2(R)$ denotes the set of all 2×2 real matrices.	CO3	10
3	Prove that $(R[x], +\,,.)$ is a commutative ring, where $R[x] = \{a_0 + a_1x + a_2x^2 + \dots + a_nx^n: a_0, a_1, a_2, \dots, a_n \in R \text{ and } n \in N\}$.	CO3	10
4	Prove that the set of all rational numbers form a field under addition and multiplication.	CO3	10
5	Prove that the set of all real numbers form a field under addition and multiplication.	CO3	10
6	Prove that $(Z_6, +\,,.)$ forms a ring.	CO3	10
7	Let $M = \{\begin{pmatrix} a & a \\ a & a \end{pmatrix}: a \neq 0, a \in R\}$ Prove that M forms a commutative group under matrix multiplication.	CO3	10
8.	Prove that $(Z_7, +\,,.)$ forms a field.	CO3	10
9.	Find the composition table of $(S_3, \circ)$ where S_3 denotes the set of all bijections from $\{1,2,3\}$ to itself. Is it commutative? Find the inverse of every element.	CO3	10
10.	Prove that the set of all continuous functions defined on $[0,1]$ form a ring under function addition and function multiplication.	CO3	10

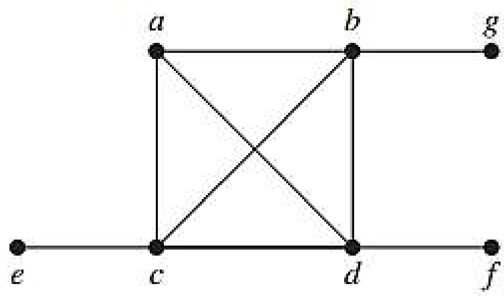
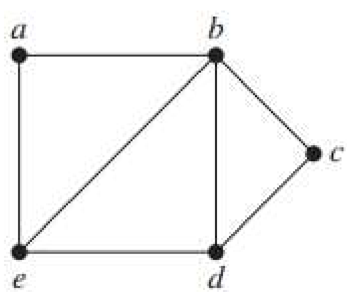
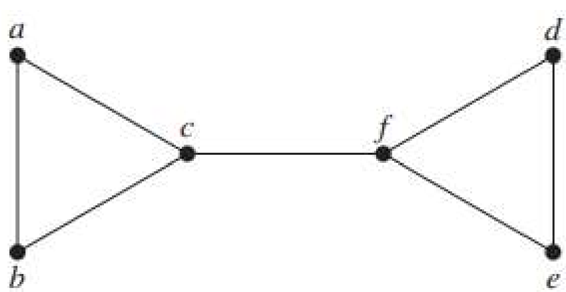
Module 4

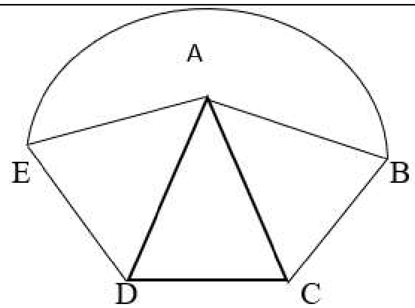
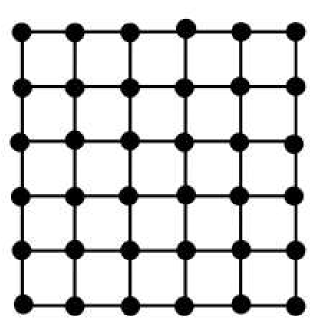
Part A

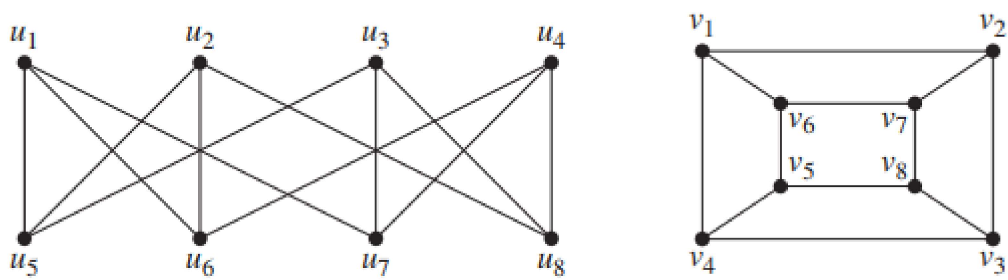
SL No.	Question	Marks
1	How many edges does a graph have if its degree sequence is $4, 3, 3, 2, 2$? Draw such a graph.	2
2	What is the degree sequence of K_n, where n is a positive integer? Explain your answer.	2
3	What is the degree sequence of the bipartite graph $K_{m,n}$, where m and n are positive integers? Explain your answer.	2
4	What is the chromatic number of K_n?	2
5	Define Euler Circuit with an example.	2
6	Define Hamiltonian Circuit with an example.	2
7	How many vertices does a regular graph of degree four with 10 edges have?	2
8	What does it mean for two simple graphs to be isomorphic? Explain with an example	2
9	State Hall's marriage theorem.	2
10	Explain planar graph with an example. Write down one application of planar graphs in real life.	2
11	What is the chromatic number a complete graph with 15 vertices?	2
12	Is the following polynomial a chromatic polynomial? $$x^2 2 + 2x - 3$$ Justify your answer.	2
13	Define maximal matching and maximum matching with an example.	2

14	Define a perfect matching. Is every maximal matching perfect matching? explain with an example.	2
15	Define binary tree with an example.	2
16	What will be the chromatic number star graph S_n and a cyclic graph C_{2n}?	2
17	Which of the graphs K_n, C_n, and W_n are bipartite? Answer with explanation	2
18	Use an adjacency matrix to represent the graph 	2
19	Use an incidence matrix to represent the graph 	2
20	Check whether the following two graphs are isomorphic or not. 	2

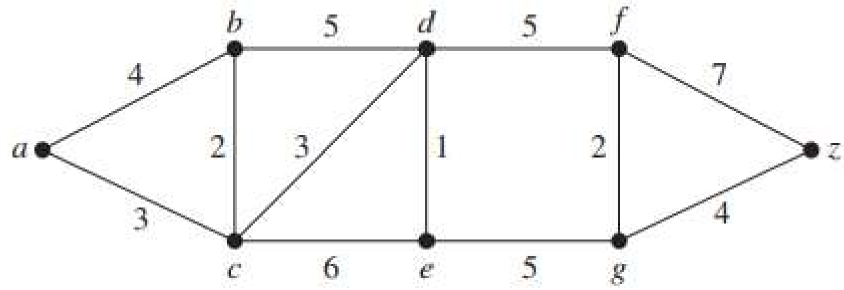
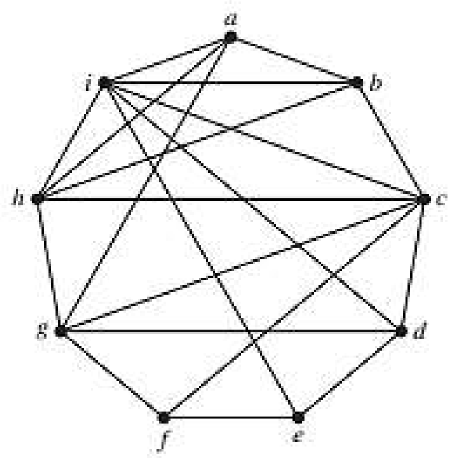
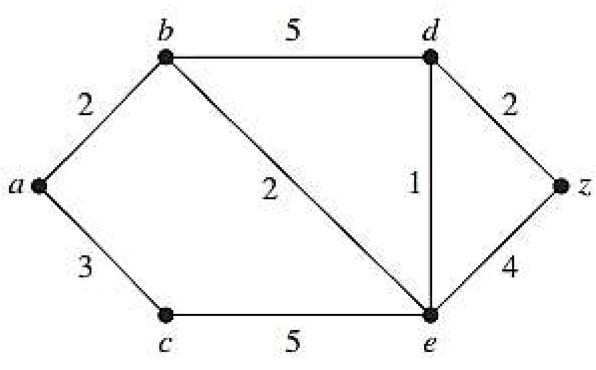
Part B

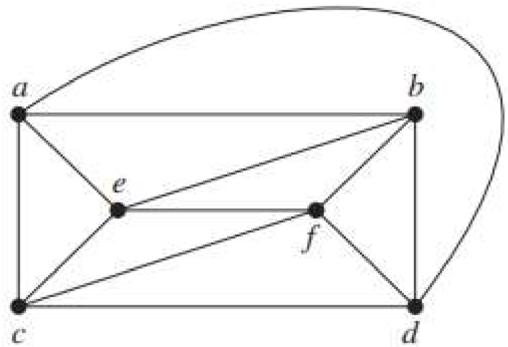
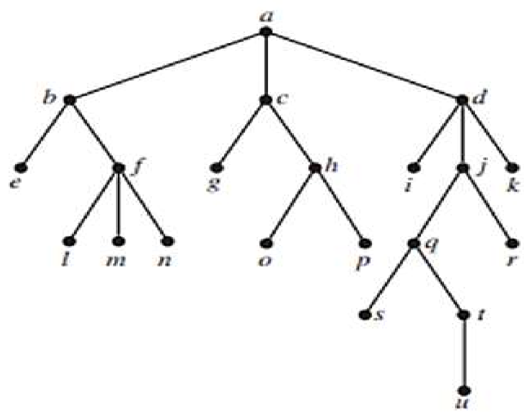
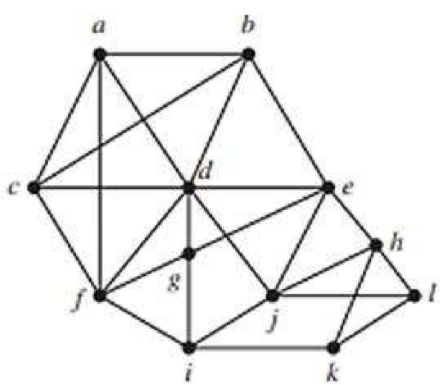
SL	Question	Marks
1	Does the graph given below have a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists. 	5
2	With the help of at least two different examples, explain how graph colouring can be used in modelling.	5
3	Check if the graph given below has a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists. 	5
4	Check if the graph given below has a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists. 	5
5	Find Chromatic Polynomial of the following graph and hence find its chromatic number:	5

		
6	Find chromatic polynomial of a connected graph on three vertices.	5
7	Draw the graph whose incidence matrix is $$\begin{bmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$	5
8	Draw the graph represented by the given adjacency matrix: $$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$	
9	Show that $K_{3,3}$ is not planar.	5
10	Write down the Euler's Formula for planar graphs and verify the result for the following graph 	5
11	Show that if $\chi(G) = 2$, then G is bipartite.	5

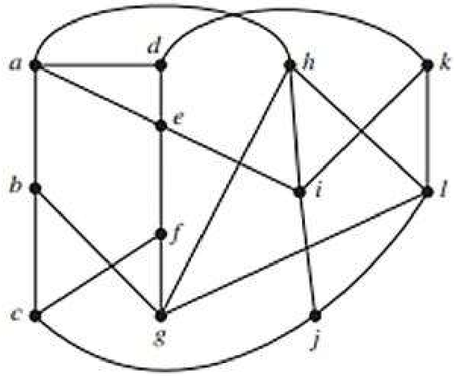
12	How many vertices and how many edges do these graphs have? a) K_n b) C_n c) W_n d) $K_{m,n}$ e) Q_n	5
13	Prove that the cyclic graph C_{2n+1} is not Bipartite.	5
14	Determine whether two given graphs are isomorphic.  	5
15	Show that a connected graph is an Euler graph if and only if every vertex of the graph has an even degree.	

Part C

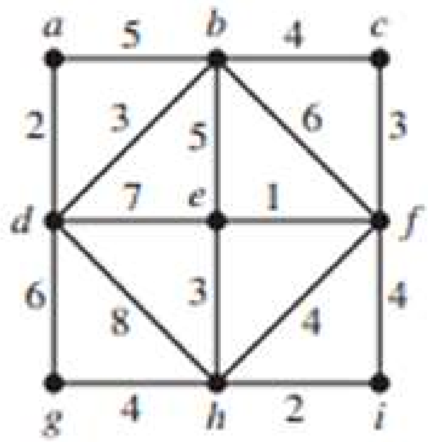
SL	Question	Marks
1	In the graph given below, find the length of a shortest path between a and z in the given weighted graph. 	10
2	Find the chromatic number of the given graph: 	10
3	In the graph given below, find the length of a shortest path between a and z in the given weighted graph. 	10

4	Find the chromatic number of the given graph:  	10
5	Answer the following about the rooted tree illustrated.  a) Which vertex is the root? b) Which vertices are internal? c) Which vertices are leaves? d) Which vertices are children of j? e) Which vertex is the parent of h? f) Which vertices are siblings of o? g) Which vertices are ancestors of m? h) Which vertices are descendants of b?	10
6	Find a spanning tree for the graph shown by removing edges in simple circuits  	10

<table>
<tr><td>7</td><td>Find a spanning tree for the graph shown by removing edges in simple circuits

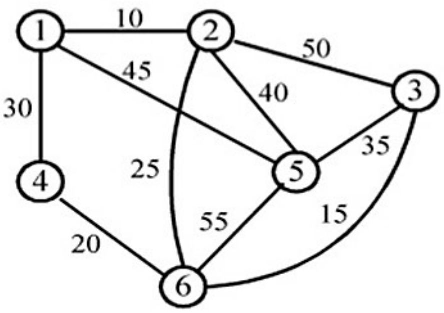
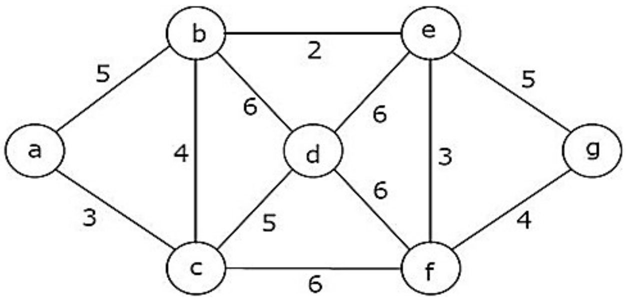
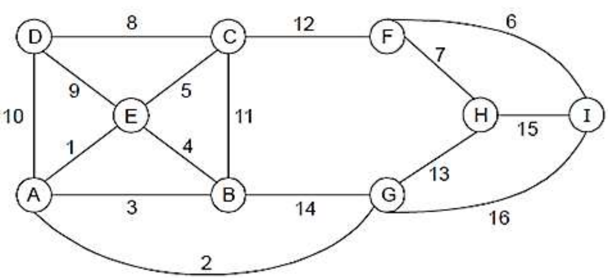
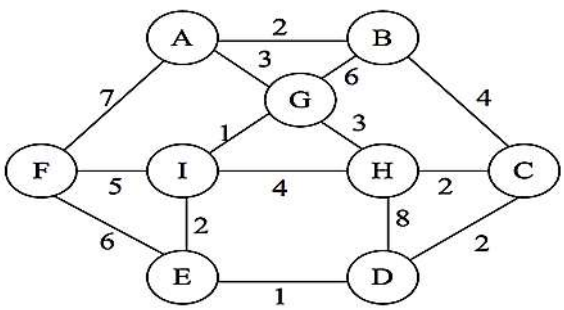


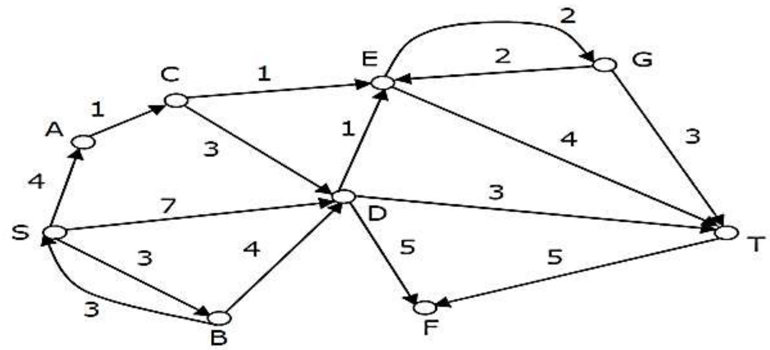
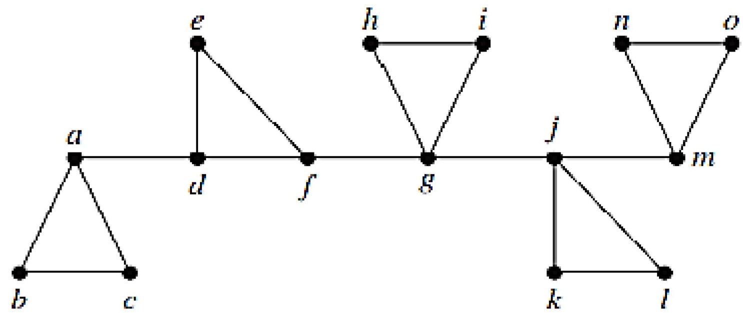
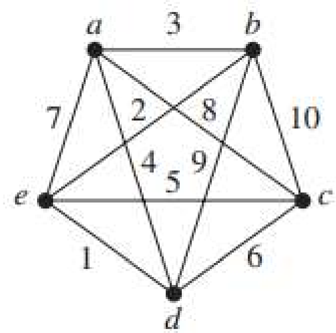
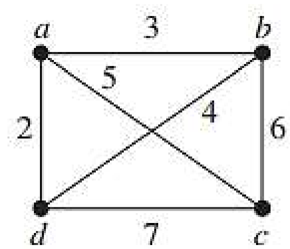
</td><td>10</td></tr>
<tr><td>8</td><td>Use Prim’s algorithm to find a minimum spanning tree for the given weighted graph:

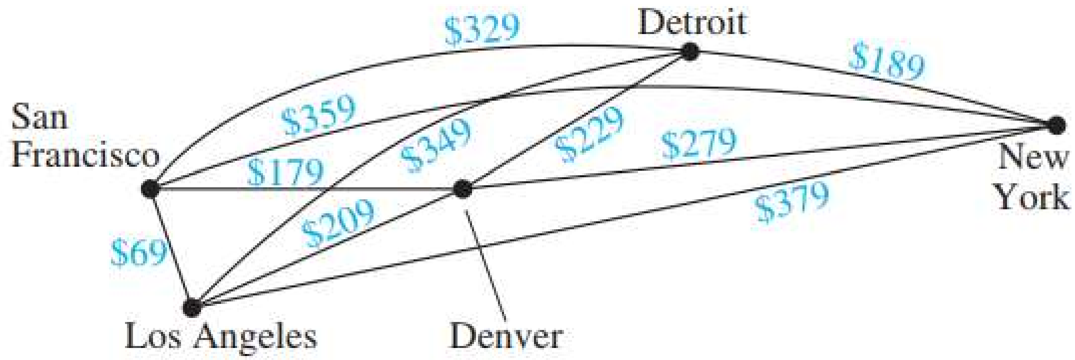
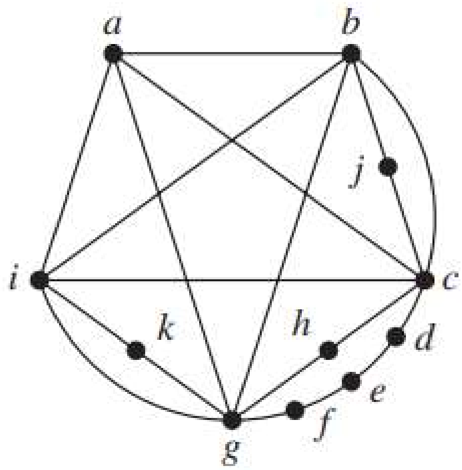


</td><td>10</td></tr>
<tr><td>9</td><td>Draw the graph whose incidence matrix is

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$
</td><td>10</td></tr>
<tr><td>10</td><td>Find the minimal spanning tree (MST) using Kruskal’s algorithm for the following graph</td><td>10</td></tr>
</table>

		
11	Find the minimal spanning tree (MST) using Kruskal's algorithm for the following graph 	10
12	Find the minimal spanning tree (MST) using Kruskal's algorithm for the following graph 	10
13	Find the minimal spanning tree (MST) using Prim's algorithm for the following graph 	10

14	Find the shortest path from A vertex to T vertex using Dijkstra's algorithm  	10
15	Find the chromatic number of the given graph:  	10
16	Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.  	10
17	Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.  	10
18	Find a route with the least total airfare that visits each of the cities in this graph, where the weight on an edge is the least price available for a flight between the two cities.	10

<table>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>19</td><td>Determine whether the following graph G is planar or not.


G</td><td>10</td></tr>
<tr><td>20</td><td>Suppose that a connected planar simple graph with e edges and v vertices contains no simple circuits of length 4 or less. Show that $e \le \frac{5v}{3} - \frac{10}{3}$, $v \ge 4$ if.</td><td>10</td></tr>
</table>