

Pre Placement Training and Aptitude

$$1) (n+1)^2 = n^2 + 2n + 1$$

$$2) (n-1)^2 = n^2 - 2n + 1$$

$$3) (n+1)^3 = n^3 + 3n^2 + 3n + 1$$

$$4) (n-1)^3 = n^3 - 3n^2 + 3n - 1$$

$$5) (n+y+z)^3 = n^3 + y^3 + z^3 + 3(n+y)(y+z)(z+n)$$

$$6) (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$7) (a+b)^2 - (a-b)^2 = 4ab$$

$$8) a^3 + b^3 + c^3 - 3abc$$

$$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= \frac{1}{2} (a+b+c) \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \}$$

$$10) \text{ If } a+b+c=0 \text{ then } a^3 + b^3 + c^3 = 3abc$$

$$8) a^2 + b^2 + c^2 - ab - bc - ca$$

$$\frac{1}{2} \{ (a-b)^2 + (b-c)^2 + (c-a)^2 \}$$

 X

$$n + \frac{1}{n} = k$$

$$n^2 + \frac{1}{n^2} = k^2 - 2$$

$$(n + \frac{1}{n})^2 - 2 \times \frac{1}{n} \Rightarrow k^2 - 2$$

$$n - \frac{1}{n} = k$$

$$n^2 + \frac{1}{n^2} = k^2 + 2$$

$$\text{If } n + \frac{1}{n} = k$$

$$n^3 + \frac{1}{n^3} = k^3 - 3k$$

$$\text{If } n - \frac{1}{n} = k$$

$$n^3 - \frac{1}{n^3} = k^3 + 3k$$

$$\text{If } n + \frac{1}{n} = 1$$

then ~~$n^3 = -1$~~ $n^3 = -1$

TO
REMEMBER

$$\text{If } n + \frac{1}{n} = -1$$

then ~~$n^3 = +1$~~ $n^3 = +1$

ie $n + \frac{1}{n} = 1$

$$n^2 + 1 = n$$

$$n^2 - n + 1 = 0$$

$$(n+1)(n^2 - n + 1) = 0$$

$$n^2 + 1 = 0$$

$$n^2 = -1$$

$$n^3 = ?$$

$$n + \frac{1}{n} = 2$$

$$n^2 + 1 = n$$

$$n^3 + n = n^2$$

$$\begin{array}{r} n^3 + 1 = 0 \\ \hline n^3 = -1 \end{array}$$

① If $n + \frac{1}{n} = 2$ then $n = 1$

② If $n + \frac{1}{n} = -2$ then $n = -1$

③ If $n + \frac{1}{n} = \sqrt{3}$

or

$$\left(n + \frac{1}{n}\right)^2 = 3$$

$$n^3 + \left(\frac{1}{n}\right)^3 = \left(n + \frac{1}{n}\right)^3 - 3n \cdot \frac{1}{n} \left(n + \frac{1}{n}\right)$$

$$\left(\sqrt{3}\right)^3 = 3\sqrt{3}$$

$$3\sqrt{3} - 3\sqrt{3} = 0$$

$$n^3 + \frac{1}{n^3} = 0 \Rightarrow n^6 + 1 = 0$$

$$n^6 = -1$$

X

$$n + \frac{1}{n} = 1$$

$$n^3 = -1$$

$$n^3 + 1 = 0$$

$$n + \frac{1}{n} = \sqrt{3}$$

$$n^3 + \frac{1}{n^3} = 0$$

$$n^6 = -1$$

$$n^6 + 1 = 0$$

$$\text{If } n^{79} + \frac{1}{n^{79}} = 3$$

$$n^{79} - \frac{1}{n^{79}} = ?$$

$$\text{If } n^{113} - \frac{1}{n^{113}} = 4$$

$$n^{113} + \frac{1}{n^{113}} = ?$$

$$\sqrt{4^2 + 4}$$

$$\text{If } n^n + \frac{1}{n^n} = k$$

$$n^n - \frac{1}{n^n} = \sqrt{k^2 - 4}$$

$$\text{If } n^n - \frac{1}{n^n} = k$$

$$n^n + \frac{1}{n^n} = \sqrt{k^2 + 4}$$

1) If $x^y + y^x = 50$
 $9^1 + 1^{49} = 50$

2) $x^y + y^x = 100$
 $9^1 + 1^{99} = 100$

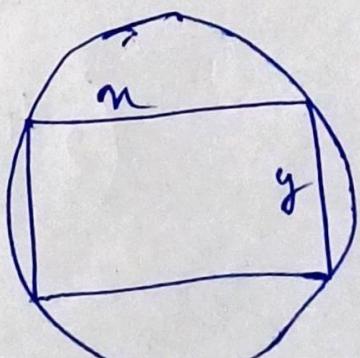
3) If $x+y=4$ $x^3+y^3+xy=?$
 then find maximum value of ~~x^3+y^3+xy~~

Ex. $2^3 + 2^3 + 2 \times 2$

4) If $x+y=6$ the find max value
 of x^3+y^3+xy

$$x+y = \frac{6}{2} = 3$$

$$3^3 + 3^3 + 3 \times 3$$



Max
Area

$$n + \frac{1}{n} = 1$$

$$n^{36} + \frac{1}{n^{36}} = ?$$

$$(n^3)^{12} + \left(\frac{1}{n^3}\right)^{12}$$

$$n + \frac{1}{n} = -1 \Rightarrow (-1)^{12} + \left(\frac{1}{-1}\right)^{12} \Rightarrow 1 + 1 = 2$$

$$n^3 = 1$$

$$n^{35} + \frac{1}{n^{35}} = (n^3)^{11} \times n^2 + \frac{1}{(n^3)^{11} n^2}$$

$$n^{60} + \frac{1}{n^{60}} = ?$$

$$(n^3)^{20} + \left(\frac{1}{n^3}\right)^{20} = 1 + 1 = 2$$

$$16^{41} + 2^{7925} + 16^n \text{ is a}$$

$$(2^4)^{41} + 2^1 \cdot 2^{82} \cdot 2^{2n} + (2^{2n})^2$$

$$2^{83+2n} = 2^{7925}$$

If $n + \frac{1}{n} = 4$

then find the value of i) $n^5 + \frac{1}{n^5} = ?$

ii) $n^7 + \frac{1}{n^7} = ?$

$$\left(n^3 + \frac{1}{n^3} \right) \left(n^2 + \frac{1}{n^2} \right) - \left(n + \frac{1}{n} \right)$$
$$= (4^3 - 3 \times 4) (4^2 - 2) - 4$$

$$\left(n^7 + \frac{1}{n^7} \right) \left(n^3 + \frac{1}{n^3} \right) - \left(n + \frac{1}{n} \right)$$
$$= n^5 + n + \frac{1}{n} + \frac{1}{n^3} - n - \frac{1}{n}$$

$$= n^5 + \frac{1}{n^5}$$

Aptitude

Level 0
chap: 1 Variation

2 Indices

3 Surds

4 Logarithm Imps (4 qs?)

5 Binomial

6 Theory of Equations

Informations

$$\textcircled{1} \quad (x+1)^2 = x^2 + 2x + 1$$

$$\textcircled{2} \quad (x-1)^2 = x^2 - 2x + 1$$

$$\textcircled{3} \quad (x+1)^3 = x^3 + 3x^2 + 3x + 1$$

$$\textcircled{4} \quad (x-1)^3 = x^3 - 3x^2 + 3x - 1$$

$$\textcircled{5} \quad (a+y+z)^3 = a^3 + y^3 + z^3 + 3(a+y)(y+z)(z+a)$$

$$\textcircled{6} \quad (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$\textcircled{7} \quad (a+b)^2 - (a-b)^2 = 4ab$$

$$\textcircled{8} \quad a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} \left\{ (a-b)^2 + (b-c)^2 + (c-a)^2 \right\}$$

$$\textcircled{9} \quad a^3 + b^3 + c^3 - 3abc = (a+b+c) \left(a^2 + b^2 + c^2 - ab - bc - ca \right)$$

$$= \frac{1}{2} (a+b+c) \left((a-b)^2 + (b-c)^2 + (c-a)^2 \right)$$

$$\textcircled{10} \quad \text{If } a+b+c=0$$

$$a^3 + b^3 + c^3 = 3abc$$

\textcircled{11}

$$\frac{x+1}{x} = k$$

$$\frac{x^2 + 1}{x^2} = k^2 - 2$$

$$\left(\frac{x^2 + 1}{x} \right)^2 - 2 \frac{x^2}{x} = k^2 - 2$$

$$(12) \quad x - \frac{1}{x} = k$$

$$x^2 + \frac{1}{x^2} = k^2 + 2$$

$$(13) \quad \text{If } x + \frac{1}{x} = k$$

$$x^3 + \frac{1}{x^3} = k^3 - 3k$$

$$(14) \quad \text{If } x - \frac{1}{x} = k$$

$$x^3 - \frac{1}{x^3} = k^3 + 3k$$

$$(15) \quad \text{If } x + \frac{1}{x} = -1$$

$$x^3 = ? - 1$$

$$\text{If } x + \frac{1}{x} = -1$$

$$x^3 = ?$$

~~$$(16) \quad \text{if method}$$~~

$$x + \frac{1}{x} = 1$$

$$x^2 + 1 = x$$

$$x^2 - x + 1 = 0$$

this is ~~zero~~.

$$(x+1)(x^2 - x + 1) = 0$$

$$x^3 = -1$$

direct method \rightarrow

$$x \cancel{x} \rightarrow x + \frac{1}{x} = 1$$

$$x \cancel{x} \rightarrow x^2 + 1 = x$$

$$(+) \rightarrow x^3 + x = x^2$$

$$\rightarrow \underline{\underline{x^3 + 1 = 0}}$$

$$\rightarrow x^3 = -1$$

1. (10) If $x + \frac{1}{x} = 2$

$$\boxed{x=1}$$

ii. If $x + \frac{1}{x} = -2$

$$\boxed{x=-1}$$

iii. If $x + \frac{1}{x} = \sqrt{3}$

or

$$\boxed{(x + \frac{1}{x})^2 = 3}$$

then $x^3 + \left(\frac{1}{x}\right)^3 = \left(x + \frac{1}{x}\right)^3 - 3x \cdot \frac{1}{x} \cdot (x + \frac{1}{x})$

$$= (\sqrt{3})^3 - 3\sqrt{3}$$

≈ 0

i) $x^3 + \frac{1}{x^3} = 0$

$$x^6 + 1 = 0$$

ii) $x^6 = -1$

(17) $x + \frac{1}{x} = 1$

$$x^3 = -1$$

$$x^3 + 1 = 0$$

(power diff = 3)
combo = 0

If $x + \frac{1}{x} = \sqrt{3}$

$$x^3 + \frac{1}{x^3} = 0$$

$$x^6 = -1$$

$$x^6 + 1 = 0$$

Tricks

If $x^n + \frac{1}{x^n} = k$

$$x^n - \frac{1}{x^n} = \sqrt{k^2 - 4}$$

If $x^n - \frac{1}{x^n} = k$

$$x^n + \frac{1}{x^n} = \sqrt{k^2 + 4}$$

(18)

Q. If $x^{29} + \frac{1}{x^{29}} = 3$

$$x^{29} - \frac{1}{x^{29}} = ?$$

$$x^{29} = \sqrt{3^2 - 1} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

Q. If $x^{113} - \frac{1}{x^{113}} = 4$

$$x^{113} + \frac{1}{x^{113}} = ?$$

$$\sqrt{4^2 + 4} = \sqrt{20}$$

Application

1. $x^y + y^x = 50$

$$x=?$$

$$y=?$$

~~$x+y$~~ $50 = 49 + 1$

$$x = 49 / 1$$

$$y = 1 / 49$$

2. $x^y + y^x = 100$

$$x=?$$

$$y=?$$

~~$x+y$~~ $100 = 99 + 1$

$$x = 99 / 1$$

$$y = 1 / 99$$

3. $x+y = 4$

Max value of $\frac{1}{2}$

$$x^3 + y^3 + xy = ?$$



Max value of area
of a rect. in a circle

$$\Rightarrow x=y$$

$$x+y=4 \leftarrow \text{circle.}$$

$$x=y$$

$$\text{rad} = 20$$

4. If $x+y = 6$

Max value

$$\text{if } x^3y^3 + xy = ?$$

↓

738.

$$x=y=3$$

$$= 738$$

$$27$$

$$27$$

$$189$$

$$54$$

$$729$$

Level 1.

Q. 1. $x + \frac{1}{x} = 1$

$$x^3 = 1$$

$$\frac{x^{36} + 1}{x^{36}} = ?$$

$$\begin{aligned} (x^3)^{12} + \frac{1}{(x^3)^{12}} &= ? \\ (-1)^{12} + \frac{1}{(-1)^{12}} &= ? \\ &= 2 \end{aligned}$$

2. $x + \frac{1}{x} = -1$ $\Rightarrow x^3 = -1$

$$\therefore x^{35} + \frac{1}{x^{35}} = ?$$

$$\begin{aligned} (x^3)^{11} x^2 + \frac{1}{(x^3)^{11} x^2} &= ? \\ \Rightarrow x^2 + \frac{1}{x^2} &= (-1)^2 - 1 \\ &= -1 \end{aligned}$$

III. $x^{60} + \frac{1}{x^{60}} = ?$

$$\begin{aligned} (x^3)^{20} + \frac{1}{(x^3)^{20}} &= 1 + 1 \\ (x^3)^{20} &= 2 \end{aligned}$$

Q. $16^{41} + 2^{7925} + 16^n$

[TCS - 2012]

is a perfect square.

then, what is the value of n

A: 82

16⁴¹

Step 1 → bring to base 2.

$$(2)^{164} + 2^{7925} + (2)^{8n}$$

$$3(2^{82})^2 + 2^2 \cdot 2^{7n} (2^n)^2$$

~~$$2^{7925} \cdot 2^{164} + 2^{8n}$$~~

~~$$2^{164} (1 + 2^{7925})$$~~

$$2^{7925} = 2^1 \cdot 2^{82} \cdot 2^{7n}$$

$$7925 = 83 + 2n$$

$$7842 = 2n$$

$$\therefore n = 3921$$

practise
at home

If $x + \frac{1}{x} = y$

Find the value of $x^5 + \frac{1}{x^5}$, $x^7 + \frac{1}{x^7}$

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$$\cdot x^7 + x^2 \frac{1}{x^7} =$$

$$\left(x^3 + \frac{1}{x^3} \right) \left(x^2 + \frac{1}{x^2} \right) - \left(x + \frac{1}{x} \right)$$

()

$$\left[x^5 + \frac{1}{x^5} + \left(x + \frac{1}{x} \right) - \left(x + \frac{1}{x} \right) \right]$$

~~$$(4^3 - 3 \cdot 4) (4^2 - 1) - 4 = 724$$~~

$$(4^3 - 3 \cdot 4) (4^2 - 1) - 4 = 724$$

$$x^7 + \frac{1}{x^7} =$$

$$= \underline{\textcircled{1}}$$

$$\left(x^5 + \frac{1}{x^5} \right) \left(x^2 + \frac{1}{x^2} \right) - \left(x^3 + \frac{1}{x^3} \right)$$