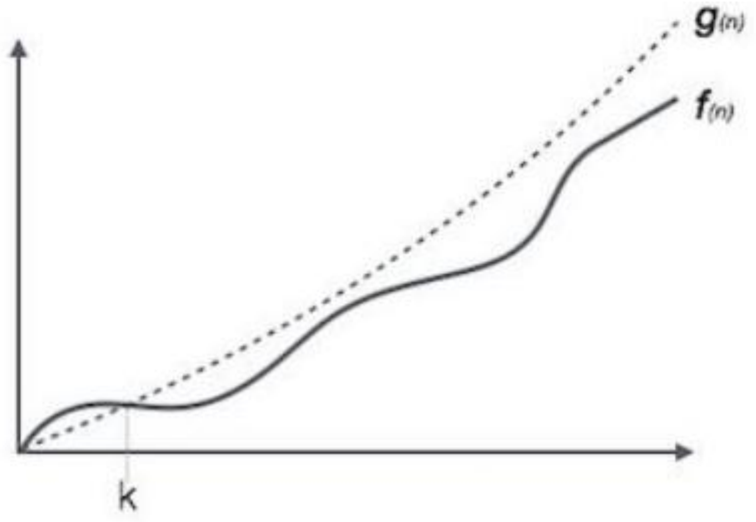
ASYMPTOTIC NOTATION

- Asymptotic analysis refers to computing the running time of any operation in mathematical units of computation. Example: $T_A(N)=O(n^2)$
- Types
- Big oh or upper bound
- Big theta or tight bound
- Big omega or lower bound

BIG OH OR UPPER BOUND

- The function $f(n)=O(g(n))$ iff $\exists$ positive constant c and n_0 such that $f(n)<=cg(n)$ for $\forall$ $n> n_0$.



- growth
- n_0 input

BIG OH OR UPPER BOUND

- The function $f(n)=O(g(n))$ iff $\exists$ positive constant c and n_0 such that $f(n) <= cg(n)$ for $\forall$ $n > n_0$.

  Example- $f(n)=6n+7$

  $6n+7 <= 6n+7n$

  $c=13$ $g(n)=n$ so complexity= $O(n)$, $n_0 = 1$

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BIG OH OR UPPER BOUND

- The function $f(n)=O(g(n))$ iff $\exists$ positive constant c and n_0 such that $f(n)<=cg(n)$ for $\forall$ $n> n_0$.

Example- $f(n)=6n+7$

$6n+7 <= 6n^2 +7n^2$

$c=13$ $g(n)=n^2$ so complexity= $O(n^2)$, $n_0 =1$

BIG OH OR UPPER BOUND

- The function $f(n)=O(g(n))$ iff $\exists$ positive constant c and n_0 such that $f(n)<=cg(n)$ for $\forall$ $n> n_0$

Example- $f(n)=6n+7$

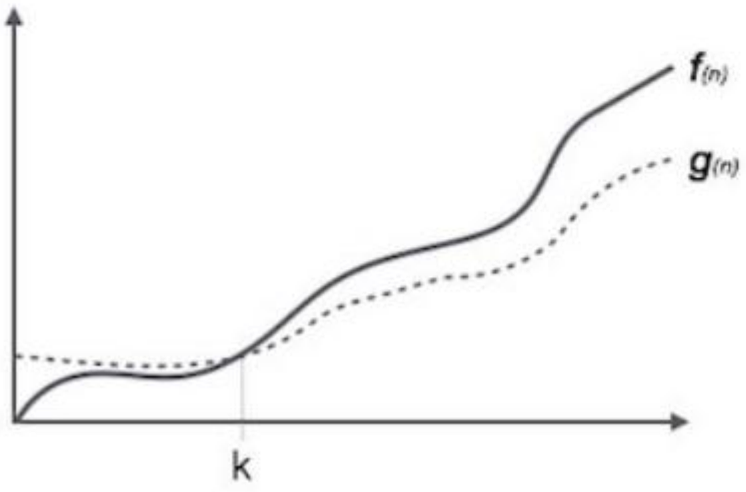
so complexity= $O(n^2)$ or $O(n)$, $n_0 = 1$

$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \ldots\ldots\ldots\ldots\ldots\ldots < 2^n < 3^n < n^n$

- N.B- choose Closest one

BIG OMEGA OR LOWER BOUND

- The function $f(n)= \Omega(g(n))$ iff $\exists$ positive constant c and n_0 such that $f(n)>=cg(n)$ for $\forall$ $n>n_0$.



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BIG OMEGA OR LOWER BOUND

- The function $f(n)= {}_{\Omega}(g(n))$ iff ∃ positive constant c and n_0 such that $f(n)>=cg(n)$ for ∀ $n>n_0$.

Example- $f(n)=6n+7$

$6n+7 >= n$, $n0=0$

$c=1$ $g(n)=n$ so complexity= $\Omega(n)$, $n_0 =0$

BIG OMEGA OR LOWER BOUND

- The function $f(n) = \Omega(g(n))$ iff $\exists$ positive constant c and n_0 such that $f(n) >= cg(n)$ for $\forall$ $n > n_0$.

Example- $f(n)=6n+7$

$6n+7 >= \log n$

$c=1$ $g(n)=\log n$ so complexity= $\Omega(\log n)$, $n_0 = 1$

BIG OMEGA OR LOWER BOUND

- The function $f(n)= \Omega(g(n))$ iff $\exists$ positive constant c and n_0 such that $f(n)>=cg(n)$ for $\forall$ $n>n_0$

Example- $f(n)=6n+7$

so complexity= $O(\log n)$ or $O(n)$, $n_0 = 1$

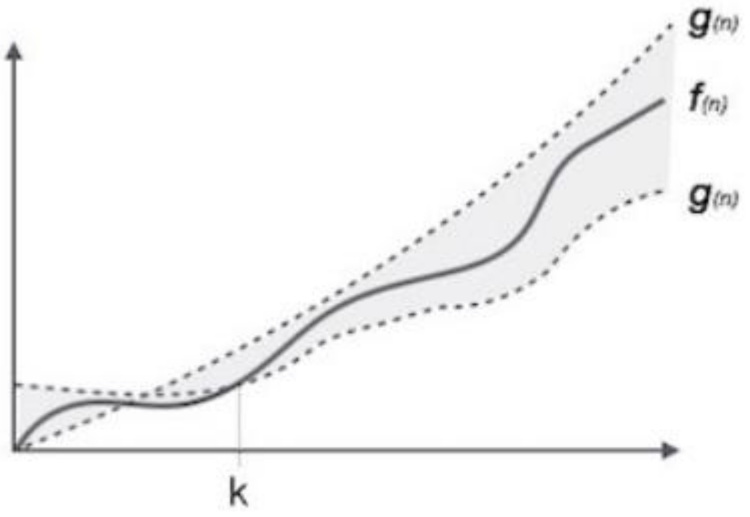
$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \ldots\ldots\ldots\ldots < 2^n < 3^n < n^n$$

- N.B- choose Closest one

BIG THETA OR TIGHT BOUND

- The function $f(n)=\Theta(g(n))$ iff $\exists$ positive constant c1,c2 and n_0 such that

  $c2g(n) =< f(n)<=c1g(n)$ for $\forall$ $n>n_{0.}$



- n_0

BIG THETA OR TIGHT BOUND

- The function $f(n)=\Theta(g(n))$ iff $\exists$ positive constant c1,c2 and n_0 such that

  $c2g(n) =< f(n)<=c1g(n)$ for $\forall$ $n>n_0$.

Example- $f(n)=6n+7$

$n <= 6n+7 <=13n$

c1=1, g(n)=n f(n) c2=13, g(n)=n $f(n)=\Theta(n)$

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BIG THETA OR TIGHT BOUND

- The function $f(n)=\Theta(g(n))$ iff $\exists$ positive constant c1,c2 and n_0 such that

  $c2g(n) =< f(n) <= c1g(n)$ for $\forall$ $n>n_0$.

Example- $f(n)=6n+7$

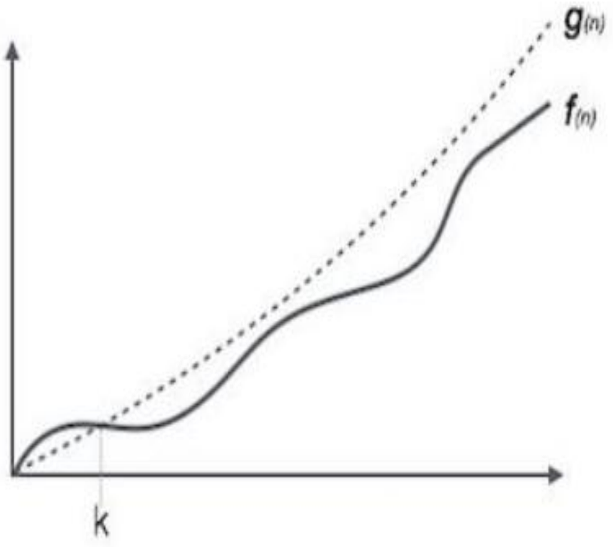
$n <= 6n+7 <=13n$

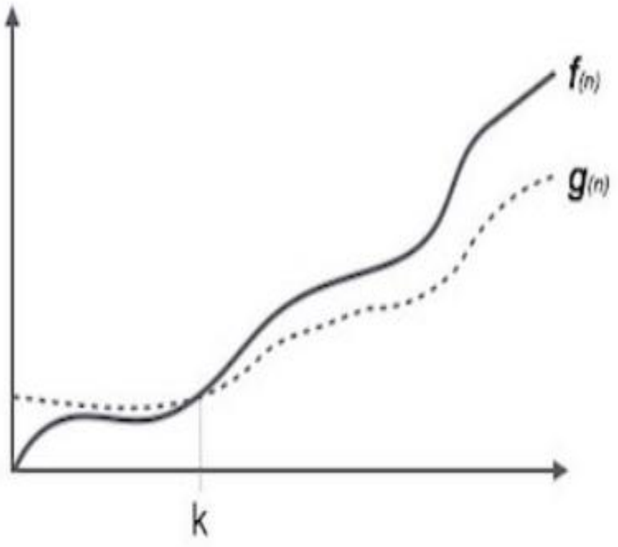
c1=1, g(n)=n f(n) c2=13, g(n)=n $f(n)=\Theta(n)$

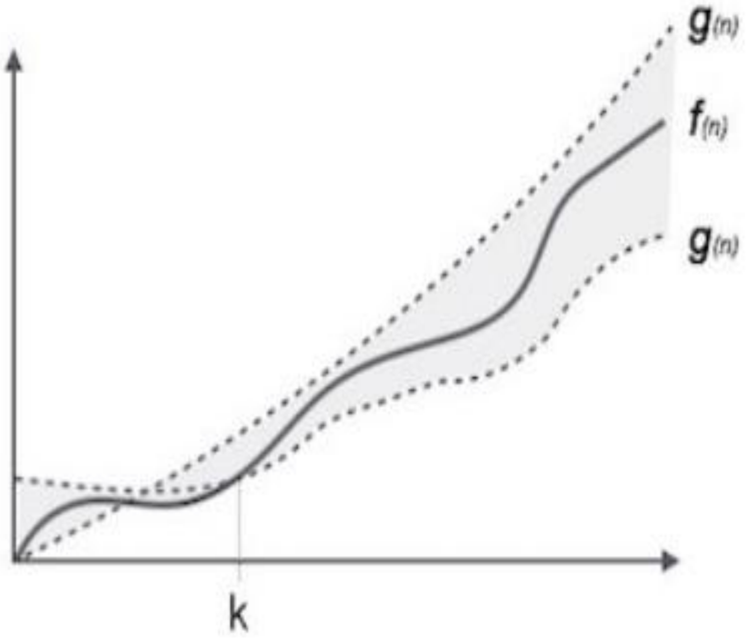
- $1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \ldots\ldots\ldots\ldots < 2^n < 3^n < n^n$

ASYMPTOTIC NOTATION

- $f(n)=O(g(n))$ $f(n)=\Omega(g(n))$ $f(n)=\Theta(g(n))$







COMPLEXITY CALCULATION USING ASYMPTOTIC NOTATION

- Find the all possible complexities of $f(n)=4n^3+3n+5$
- $<=f(n)$ $,n_0=$ $f(n)<=$ $, n_0=$
- $<=f(n)<=$ $, n_0=$

COMPLEXITY CALCULATION USING ASYMPTOTIC NOTATION

- Find the all possible complexities of $f(n)=4n^3\log n+2n+5$
- ?<=f(n) ,n_0= ? f(n)<= ? , n_0=?
- ? <=f(n)<= ? , n_0= ?

COMPLEXITY CALCULATION USING ASYMPTOTIC NOTATION

- Find the all possible complexities of $f(n)=4n^3\log n+2n+5$
- $n^3\log n <= f(n)$, $n_0 = 1$ || $f(n) <= 11n^3\log n$, $n_0=1$
- $n^3\log n <= f(n) <= 11n^3\log n$, $n_0 = 1$
- $\Omega(n^3\log n)$ $\Theta(n^3\log n)$ $O(n^3\log n)$

COMPLEXITY CALCULATION USING ASYMPTOTIC NOTATION

- Find the all possible complexities of $f(n)=n!$
- $f(n)=n.(n-1).(n-2)\ldots\ldots\ldots 3.2.1$
- $1.1.1\ldots.1 <= f(n)<=n.n.n\ldots.. n \quad , n_0=1$
- $\Omega(\ ?)$ $\Theta(?)$ $O(n^n)$

COMPLEXITY CALCULATION USING ASYMPTOTIC NOTATION

- Find the all possible complexities of $f(n)=\log(n!)$
- $f(n)=\log(n.(n-1).(n-2)\ldots\ldots\ldots 3.2.1)$
- $\log() <= f(n) <= \log()$, $n_0=$
- $\Omega(?)$ $\Theta(?)$ $O(?)$

PROPERTIES OF ASYMPTOTIC NOTATION

- If f(n) is O(g(n)) Then af(n)=O(g(n))

  eg. if f(n) =n   f(n)=O(n)

  :. 15f(n)   f(n)=O(n)

PROPERTIES OF ASYMPTOTIC NOTATION

- Reflexive -

  If $f(n)$ is given Then $f(n)=O(f(n))$

  Example- $f(n)=n$ $f(n)=O(n)$

PROPERTIES OF ASYMPTOTIC NOTATION

- Transitive-

  If f(n) is O(g(n)) and g(n)=O(h(n))   Then f(n)=O(h(n))

  Example- f(n)=5n   g(n)=n^2   h(n)=n^3

- O(n^3)

PROPERTIES OF ASYMPTOTIC NOTATION

- Symmetric- (only true for Ө notation)

  If f(n) is Ө(g(n)) then g(n)= Ө(f(n)).

  Example- f(n)=5n^2 g(n)=n^2

PROPERTIES OF ASYMPTOTIC NOTATION

- Transpose Symmetric- (only true for O & Ω notation)

  If $f(n)$ is $O(g(n))$ then $g(n)= \Omega(f(n))$.

  Example- $f(n)=5n$ $g(n)=n^2$ Then

  $n=O(n^2)$ and $n^2= \Omega(n)$

PROPERTIES OF ASYMPTOTIC NOTATION

If $f(n) = O(g(n))$ and $d(n)=O(e(n))$ then

$$f(n)+d(n)=O(max(g(n),e(n))$$

Example- $f(n)=5n$ $d(n)=n^2$ Then

$$f(n)+d(n)=O(n) +O(n^2)= O(n^2)$$

PROPERTIES OF ASYMPTOTIC NOTATION

If $f(n) = O(g(n))$ and $d(n)=O(e(n))$ then

$$f(n)*d(n)=O(g(n)*d(n))$$

Example- $f(n)=5n$ $d(n)=n^2$ Then

$$f(n)*d(n)=O(n^3)$$