Discrete Mathematics

Prime numbers

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Problems on division algorithm

- ✓ Prove that product of any m consecutive integers is divisible by m.

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- ✓ By division algorithm, show that square of an odd integer is of the form 8k+1, where k is an integer.

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- ✓ By division algorithm, show that square of an odd integer is of the form $8k+1$, where k is an integer.

- ✓ Show that $\gcd(a, a+2) = 1$ or 2 for all integers a.

- ✓ If k is a positive integer, then $\gcd(ka, kb) = k \gcd(a,b)$

Relatively prime numbers

Two integers a and b are said to be relatively prime when gcd(a,b) =1.

Theorem: Let a and b are two nonzero integers. Then a and b are said to be relatively prime if and only if there exist another two integers u and v such that au+bv=1.

Proof: Let a and b are relatively prime and hence, gcd(a,b) =1. Then, by Bezouts equation, there exist two integers u and v such that au+bv=1 (If condition satisfied).

Conversely, we consider 1=au+bv. Let d is the gcd of a and b. Then, by division algorithm, for any x and y, d|(ax+by). Therefore, d|1 which implies d=1. Thus, gcd(a,b)=1 and hence, a and b are relatively prime. (Only if condition is satisfied).

Relatively prime numbers

✓ If $a|bc$ and gcd(a,b)=1, then $a|c$.

Soln: Since gcd(a,b)=1, therefore there exist two integers u and v such that au+bv=1. thus, c=acu+bcv.

Again, $a|ac$ and $a|bc$ implies $a|(acu+bcv)$ for some integers u and v

Which implies $a|c$. (Proved)

❖ If a is prime to b and a is prime to c, then a is prime to bc.

✓ If a is prime to b show that a+b is prime to ab.

Since a is prime to b, there exist two integers u and v for which au+bv=1$\Longrightarrow$ $(a+b)u+(v-u)b$. Since u and v-u are integers, therefore a+b is prime to b. Similarly, a(u-v)+(a+b)v=1 implies that a and a+b are relatively prime. Therefore a+b is prime to ab.

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Relatively prime numbers

✓ Prove that product of any three consecutive integers is divisible by 6.

Soln: By division algorithm, if any number is divided by 3 then there will be three remainders 0,1,2. Thus, the number n may be of the form 3k, 3k+1, or 3k+2.
When $n=3k$, then $3|n$,
When $n=3k+1$, then $n+2=3k+3$, and $3|n+2$,
When $n=3k+2$, then $n+1=3k+3$, and $3|n+1$. Hence whatever be the value of n, 3 divides any one of n, n+1, or n+2. Hence, $3|[n(n+1)(n+2)]$.
Again, product of any two consecutive integers is divided by 2. Thus $2|\, n(n+1)(n+2)$. Again $\gcd(2,3)=1$. Therefore $6|n(n+1)(n+2)$. (Proved)

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When n=3k+2, then n+1=3k+3, and 3|n+1. Hence whatever be the value of n, 3 divides any one of n, n+1, or n+2. Hence, 3|[n(n+1)(n+2)].
Again, product of any two consecutive integers is divided by 2. Thus 2| n(n+1)(n+2). Again gcd(2,3)=1. Therefore 6|n(n+1)(n+2). (Proved)

Prime number

An integer p greater than 1 is called *prime* if the only positive factors of p are 1 and p. A positive integer that is greater than 1 and is not prime is called *composite.*

The integer 7 is prime because its only positive factors are 1 and 7, whereas the integer 9 is composite because it is divisible by 3.

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