Pre Placement Training and Aptitude

Surds:-

Problem:-

1) If $a+\sqrt{b} = c+\sqrt{d}$

$a=c$ & $b=d$

2) If $ax^2+bx+c = px^2+qx+r$

$a=p$, $b=q$, $c=r$

3) If $x = \sqrt{3}+\sqrt{2}$

$\frac{1}{x} = \sqrt{3}-\sqrt{2}$

Q) $\sqrt{50}-\sqrt{45}$ ④, $\sqrt{75}-\sqrt{70}$ ⑥, $\sqrt{35}-\sqrt{30}$ ②, $\sqrt{15}-\sqrt{10}$ ①, $\sqrt{40}-\sqrt{35}$ ③, $\sqrt{60}-\sqrt{55}$ ⑤

A) $\sqrt{3}-\sqrt{2}$, $\sqrt{2}-\sqrt{1}$

$1.732 - 1.414 = 0.318$

$1.414 - 1.000 = 0.414$

$\therefore 0.414 > 0.318$

[~~no ascending to~~ arranged in ascending order]

If $x = \sqrt{3}+\sqrt{2}$

Q) Find a) $x+\frac{1}{x}$ b) $x-\frac{1}{x}$ c) $x^2+\frac{1}{x^2}$ d) $x^3+\frac{1}{x^3}$

e) ~~$x^3 \frac{1}{x^3}$~~ $x^3-\frac{1}{x^3}$

Q) If $x = 5 + 2\sqrt{6}$ ~~(X) Cont'd next page~~

Find i) $\sqrt{x} + \frac{1}{\sqrt{x}}$ ~~ii) $x = 2 + \sqrt{3}$~~

Q) If $x = 2 + \sqrt{3}$ Find i) $x^2 + \frac{1}{x^2}$ iii) $x^3 - \frac{1}{x^3} = ?$

ii) $x^3 + \frac{1}{x^3} = ?$

$\Rightarrow \frac{1}{x} = 2 - \sqrt{3}$

$x + \frac{1}{x} = 4 \Rightarrow$ (A i) $\left(x^2 + \frac{1}{x^2}\right) = 16 - 2$

$= 14$ (Ans)

(A ii) $x^3 - \frac{1}{x^3}$

$= k^3 + 3k$

$64 + 12 = 76$ (Ans)

(A iii) $x^3 + \frac{1}{x^3} = k^3 - 3k$

$= 64 - 12 = 52$

(Ans)

Q3) $x = 5 + 2\sqrt{6}$

$= (\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3}\sqrt{2} \Rightarrow x = (\sqrt{3} + \sqrt{2})^2$

$\sqrt{x} = \sqrt{3} + \sqrt{2} \quad | \quad \frac{1}{\sqrt{x}} = \sqrt{3} - \sqrt{2}$

$\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$

Q) $\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = a - b\sqrt{2}$ then $a - b$?

$\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = a - b\sqrt{2}$

$2 - 2\sqrt{2} + 1 = a - b\sqrt{2}$

$3 - 2\sqrt{2} = a - b\sqrt{2}$

$\Rightarrow a = 3$ and $b = 2$

Q) $\sqrt{7-4\sqrt{3}} = a\sqrt{3} - b$

$\therefore a - b = ?$

Q) $\sqrt{21-8\sqrt{5}} = \sqrt{5}a - b$

$a + b = ?$

Q) $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}} = \sqrt{p} + \sqrt{q} + \sqrt{r}$

A) We can write $\sqrt{7-4\sqrt{3}}$ as

$\sqrt{(2)^2 + (\sqrt{3})^2 - 2 \times 2\sqrt{3}} = \sqrt{(2-\sqrt{3})^2}$

$= \sqrt{3}a - b$

$\therefore a = -1$ and $b = -2$

A) $\sqrt{4^2 - 2 \cdot 4 \cdot \sqrt{5} + (\sqrt{5})^2} \Rightarrow \sqrt{(4-\sqrt{5})^2} = \sqrt{5}a - b$

$\Rightarrow a = -1 \quad b = -4$

$\therefore a + b = -5$

$a - b = -3$ (Ans)

A) If $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}}$ then lets assume,

$\sqrt{(\sqrt{2}+\sqrt{3}+\sqrt{5})^2} = \sqrt{p} + \sqrt{q} + \sqrt{r}$

$\Rightarrow p = 2, q = 3, r = 5$

$\therefore p + q + r = 10$ (Ans)

Infinite series under Surds

1 a) $\sqrt{56+\sqrt{56+\sqrt{56+\sqrt{56+\cdots}}}}\ \infty$

b) $\sqrt{42-\sqrt{42-\sqrt{42-\sqrt{42-\cdots}}}}\ \infty$

c) $\sqrt{15\sqrt{15\sqrt{15\cdots}}}\ \infty$

d) $\sqrt{21+\sqrt{13+\sqrt{2+\sqrt{49}}}}$

e) $\sqrt{110+\sqrt{110+\sqrt{110+\sqrt{110}}}}\cdots\ \infty$

f) $\sqrt{90-\sqrt{90-\sqrt{90-\cdots}}}\ \infty$

g) $\sqrt{40\cdot\sqrt{40\sqrt{40\sqrt{\ }}}}\ \infty$

i) $\sqrt{31+\sqrt{21+\sqrt{8+\sqrt{64}}}}$

A1 a) $x=\sqrt{56+x}$

then

$x^2=56+x$

$x^2-x-56=0$

$x=-7$ or 8 then $x=8$ (Ans)

OR

Steps -> Divide the number into the product of 2 numbers having a difference of 1.

↳ For the higher number is answer

↳ For the lower number is the answer

↳ For the number present in the question is the answer itself

A 1) a) $56 = 7 \times (8)$ → Ans

b) $42 = 6 \times 7$

↳ 6 is the answer

c) 15 (Ans)

e) $110 \rightarrow 10 \times (11)$

11 is the answer

f) $90 \rightarrow (9) \times 10$

↳ 9 is the answer

g) 40 Answer

d) $\sqrt{21+\sqrt{13+\sqrt{2+7}}}$

$= \sqrt{21+\sqrt{13+3}} = \sqrt{21+4} = 5$ (Ans)

i) $\sqrt{31+\sqrt{21+\sqrt{8+8}}}$

$= \sqrt{31+\sqrt{21+4}}$

$= \sqrt{31+5} = 6$ (Ans)

2 i) $\sqrt{7+\sqrt{7+\sqrt{7+}}}$ ---- ∞

ii) $\sqrt{11+\sqrt{11+\sqrt{11+}}}$ ---- ∞

iii) $\sqrt{5-\sqrt{5-\sqrt{5-}}}$ ---- ∞

iv) $\sqrt{13-\sqrt{13-\sqrt{13}}}$ ---- ∞

Note → $\sqrt{n+\sqrt{n+}\cdots} \Rightarrow \frac{\sqrt{4n+1}+1}{2}$

↳ $\sqrt{n-\sqrt{n}\cdots} \Rightarrow \frac{\sqrt{4n+1}-1}{2}$

This is the direct formula for odd numbers

~~Q3 i) $\frac{\sqrt{4\times7+1}+1}{2}$~~

A2 i) $\frac{\sqrt{4\times7+1}+1}{2} = \frac{\sqrt{29}+1}{2}$

ii) $\frac{\sqrt{4\times11+1}+1}{2} = \frac{\sqrt{45}+1}{2}$

iii) $\frac{\sqrt{21}-1}{2}$

iv) $\frac{\sqrt{53}-1}{2}$

Q3a) $\sqrt{7+\sqrt{7-\sqrt{7+}}}\cdots$

b) $\sqrt{11+\sqrt{11-\sqrt{11+\sqrt{11-}}}}\cdots$

c) $\sqrt{7-\sqrt{7+\sqrt{7}}}\cdots\infty$

d) $\sqrt{11-\sqrt{11+\sqrt{11-\sqrt{11+}}}}\cdots\infty$

Formula $\sqrt{a+\sqrt{a-\sqrt{a+}}}\cdots \Rightarrow \frac{\sqrt{4a-3}+1}{2}$

$\sqrt{a-\sqrt{a+\sqrt{a}}}\cdots \Rightarrow \frac{(\sqrt{(4a-3)}-1)}{2}$