

Problem 1

Suppose the time of the four stages of a pipeline are $\tau_1=60\text{ns}$, $\tau_2=70\text{ ns}$, $\tau_3=90\text{ns}$ and $\tau_4=80\text{ ns}$ respectively and the interface latch has a delay $\tau_d= 10\text{ns}$.

Then i) What would be the clock frequency of the above pipeline?

ii) What is the speed up of the pipeline over its equivalent non-pipeline Counterpart?

Solution:

The clock-period should at least $t = 90 + 10 = 100\text{ ns}$. So, the clock frequency, $f = 1/t = 1/100 = 10\text{ MHz}$.

In case of non-pipeline, the time delay = $t_1 + t_2 + t_3 + t_4 = 60 + 70 + 90 + 80 = 300\text{ ns}$.

So, the **speed-up** = $300/100 = 3$. This means that the pipeline processor is 3 times faster than its equivalent non-pipeline processor.

Problem 2

A program has a parallelizable fraction of 70%. If you have an infinite number of processors available, what is the maximum speedup that can be achieved?

Solution :

Understanding Amdahl's Law

Amdahl's Law describes the theoretical maximum speedup of a program when using multiple processors. It states that the speedup is limited by the sequential (non-parallelizable) portion of the program.

The formula for Amdahl's Law is:

$$\text{Speedup} = 1 / (S + (P / N))$$

Where:

- **S** is the sequential fraction of the program.
- **P** is the parallelizable fraction of the program.
- **N** is the number of processors.

Applying the Law to the Problem

1. Identify the given values:

- Parallelizable fraction (P) = 70% = 0.70
- Since the program is either parallelizable or sequential, the sequential fraction (S) is: $S = 1 - P = 1 - 0.70 = 0.30$
- Number of processors (N) = infinite (∞)

2. Apply the formula with infinite processors:

- When N approaches infinity, the term (P / N) approaches 0.
- Therefore, the formula simplifies to: $\text{Speedup} = 1 / S$

3. Calculate the maximum speedup:

- $\text{Speedup} = 1 / 0.30 = 3.33$ (approximately)

Conclusion

With an infinite number of processors, the maximum **speedup** that can be achieved is approximately **3.33** times.

Problem 3

Suppose a program has a **serial portion** that takes up 30% of the total execution time. If you parallelize this portion so that it now takes only 20% of the total execution time, calculate the speedup achieved.

Solution:

Initial State: The serial portion takes 30% of the total execution time.

Modified State: The serial portion now takes 20% of the total execution time.

We need to find the speedup, which is the ratio of the original execution time to the new execution time.

Calculations

1. **Original Execution Time:**

- Let's assume the original total execution time is 'T'.

- The serial portion's time is $0.30 * T$.
- Therefore, the parallel portion was $1 - 0.3 = 0.7$ or 70% of T .

2. **New Execution Time:**

- The new serial portion's time is $0.20 * T$.
- The parallel portion time has stayed the same. It is still $0.70 * T$.
- The new total execution time is $(0.20 * T) + (0.70 * T) = 0.90 * T$.

3. **Calculate Speedup:**

- $\text{Speedup} = (\text{Original Execution Time}) / (\text{New Execution Time})$
- $\text{Speedup} = T / (0.90 * T)$
- $\text{Speedup} = 1 / 0.90$
- $\text{Speedup} = 1.11$ (approximately)

Conclusion

The speedup achieved by reducing the serial portion from 30% to 20% of the total execution time is approximately **1.11** times.