Genetic Algorithm Based Optimization of Machining Parameters

Angshuman Pal
B. Mechanical E – IV
001411201048

Guide: Prof. Tapan K. Barman
Machine Design Specialization

Contents

1 Abstract

Optimization of a process output with reference to multiple input parameters is an important aspect of any manufacturing or machining process. This project examines and formulates a mechanism to optimize a given process output using the optimization technique of Genetic Algorithm. A new code for this purpose is formulated in MATLAB environment. The code is tested against some standardized functions and some case studies are performed to validate its performance against existing literature. The algorithm is then run over some real process performance data obtained from milling operation, and the optimum input parameters under given constraints required for achieving minimum surface roughness is proposed

Keywords: Turning operation, surface roughness, spindle speed, feed, depth of cut, genetic algorithm.

2 Introduction

Genetic algorithm is a metaheuristic optimization technique which borrows from the Darwinian theory of Natural Selection in the field of Evolution. It belongs to the larger class of evolutionary algorithms.

In a GA problem, there is a population of individuals in a generation who are all possible candidates for being the ideal candidate and who are also the potential parents for the next generation of individuals. These individuals are called “phenotypes”. Every phenotype is different from another by virtue of its “chromosome”, i.e. the properties that it carries. These properties are called “genotypes”. In our optimization problems, the genotype of a phenotype is essentially the coordinates of the location of the phenotype.

GA technique is based on the principle that out of all the phenotypes belonging to a single generation, only those ideally suited to the environment are successful of reproducing and passing on their own features to the next generation of offspring. Following this manner of propagation, only those ideally suited to the environment are successful of reproducing and passing on their own features to the next generation of offspring. Following this manner of propagation only the best features of successive generations are passed on and eventually the new generations are born with much advanced and “optimized” features for survival in that environment as compared to the previous generations.

Being a metaheuristic technique, GA comes with the obvious drawback that the solution it provides is not a fool proof one. By tuning the different parameters within the process, the ideal solution may be sought.

3 Methodology for Genetic Algorithm

3.1 Identification of number of phenotypes and genotype range

The first step to GA optimization is to identify the phenotype and genotype range.

The genotype is simply determined by the number of input variables that are a part of the problem. The number of independent coordinates of a phenotype is the same as the number of input parameters associated with the problem.

The genotype range has to be chosen judiciously. From observation of obtained data, an approximate range is to be identified where the function can be expected to be having a minimum. In addition, in most cases there exist limiting values for the extreme values which an input parameter can take, so the genotype can only remain between a particular range of values and try to achieve the optimum point. For example, the spindle speed has to remain within a range during turning operation, since if the value is too low the machining time will be too high and if the value is too high the tool wear will be sufficient and tool life will become low.

The number of phenotypes per generation depends on the convergence rate of the algorithm and the speed with which it can reach the minimum values.

3.2 Initialization of primary phenotypes

This step involves generation of a random number of phenotypes which will inhabit the first generation. Since this is the first generation there should be absolute randomness in the selection of the genotypes of these individuals. The range of the genotypes should be maintained within the range specified by the problem. This procedure is likened to “seeding” of a plant.

3.3 Identification of parents by Natural Selection

The objective function is the relationship between the input and output parameters of the problem. When the genotype of the function is provided as input to the function, it provides the value of the output function which in essence is the value we are trying to maximize or minimize.

Once the phenotypes of a generation are located, the values of the objective function for that generation are obtained. Out of these, a certain fraction of best suited phenotypes are selected to be the parents for the next generation of individuals, while the rest are rejected. This selection is done based on the objective function values. Commonly, half of the total number of phenotypes are selected as parents for the next generation.

3.4 Gene flow

Gene flow into the next generation occurs in three parts: Elite, Crossover, and Mutation. A predetermined and fixed ration is maintained for determining how many of the following generations of offspring will be generated by which method.

3.4.1 Elite

A fixed number of individuals from the parent generation are selected to be automatically transferred over to the next generation. These are called "elite" phenotypes. Elite phenotypes have the best values of the objective function in the parent generation.

The purpose of this is to ensure the propagation of the best-fit phenotypes of one generation into the next generation. This ensures the retention of the best features within a population.

3.4.2 Crossover

Crossover involves the combination of the genotype of two fit parent phenotypes. Two parent phenotypes are selected randomly from the parent set and their genotypes are combined or "interbred" in a fixed ratio. The two offspring produced are complements of each other with respect to the parent phenotypes. This ratio of recombination typically varies from 0.3-0.7 to 0.4-0.6.

The purpose of this is to identify the potential of the region in between two parents for optimization, and also to bring the algorithm to a convergence.

3.4.3 Mutation

Mutation involves standalone modification of the genotype of a parent phenotype. One parent is selected at random and its genotype is modified randomly, using a random parameter selected from a Gaussian distribution.

The purpose of this is to identify the potential of the region in the near vicinity of a parent for optimization by randomly testing their capability for optimization. It is optimistically expected that the mutation will shift the parent genotype towards a "more fit" offspring genotype.

3.5 Iteration

As a new generation of offspring are generated from the previous parent generation, the new generation now serves as the pool of phenotypes from which the parents of the subsequent generation will be selected through an identical process. This is called an iteration, i.e. the progress from one generation to the next. Through a number of subsequent iterations in the algorithm, the naturally selected and perfectly optimized values of the genotypes are obtained, which in turn is the required optimized output. With every passing iteration, the less fit genotypes are slowly eliminated and only the more fit individuals succeed in propagating their chromosomes to the next generation.

3.6 Termination

Termination of the iterations is done when the algorithm is expected to have reached the optimum value we are looking for. It basically halts further reproduction among the phenotypes and the final generation is considered to be the optimum generation for the function. The mean genotype properties, i.e. the mean coordinates of the final generation, are calculated, and this is the final result of the optimization problem.

Termination can be performed in a number of ways:

- A solution is obtained which satisfies the minimum criterion.
- Fixed number of generations is reached.
- Iterations no longer produce better results.
- Algorithm exceeds the stipulated time of running.
- A combination of more than one procedure.

4 Structure of MATLAB program for GA

4.1 Declaration of generation parameters

Number of input variables = *k*

Number of phenotypes per generation = *n*

Generation matrix = $[G]_{nxp}$, *where* $G_{ij} \in [-1,1]$

4.2 Objective function

$f(x_1, x_2, x_3 \ldots x_k)$ is defined as the objective function of [G] which gives the value of the output with a given input G_i.

$\{R\}_{nx1}$ *is* the matrix of values of objective function at G, where $R_{i,1} = f(G(i,:))$.

4.3 Selection of parents

$[P]_{mxk}$ is the set of ideally suited m phenotypes among as “naturally selected” and “genetically advanced” phenotypes for being the parents of the next generation. Here, half of the original generation is selected as parents. Assuming the scenario to be a minimization problem,

for i=1:n

if f(i) < f(median) (All phenotypes with objective function less than median value)

G(i) ϵ P

else

continue

end

end

4.4 New generation: Elite, Crossover and Mutation

$n = p + q + r$, where p q and r are the number of offspring produced by elite propagation, binary crossover and phenotype mutation respectively.

p number of elite offspring directly propagated from one generation to the next.

q (even) number of crossover offspring produced by mutual combination among two random parent phenotypes in a fixed ratio. $C_1 = \lambda P_i + (1-\lambda)P_j$; $C_2 = (1-\lambda)P_i + \lambda P_j$, *where* $\lambda \in [0,1]$.

r number of mutation offspring produced by slight random modification of their genes. $C_i = P_i + r.\Delta P_i$, where $r \in (-1,1)$.

5 Validation of GA code

In order to determine if the performance of the written algorithm is satisfactory or not, it is validated against some known results. Validation is performed on some commonly used optimization functions, as well as on some published results obtained through surveying the literature in this field.

5.1 Validation against standard optimization functions

A few tried and tested optimization functions are used for determining if the algorithm is functioning properly or not.

5.1.1 Sphere function

The sphere function is expressed as follows: $f(x,y) = x^2 + y^2$, within the domain $x, y \in [-1,1]$.

From simple observation, it can be identified that f(x,y) reaches a minima at (0,0) when f(x,y) = 0.

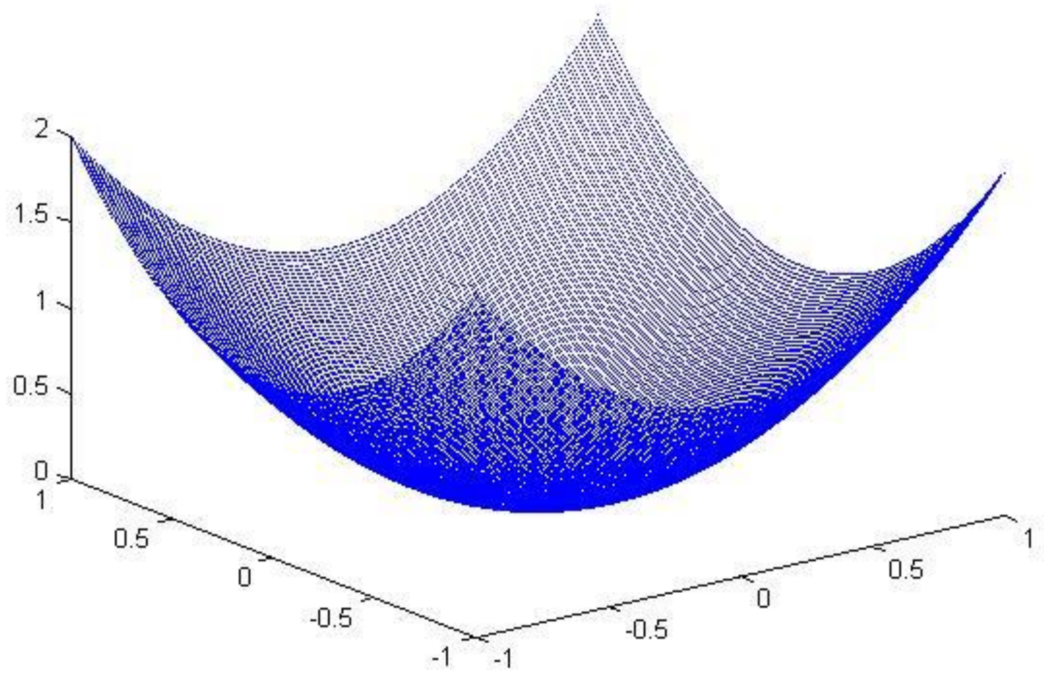

Fig 1. Contour plot of sphere function

The output minimum for this function is given by the code at the point (-0.005, 0.0016), and the minimum value of the objective function is given as 3.00×10^{-6}. The result, therefore, is quite accurate and near enough to the absolute minimum of f(0,0) = 0.

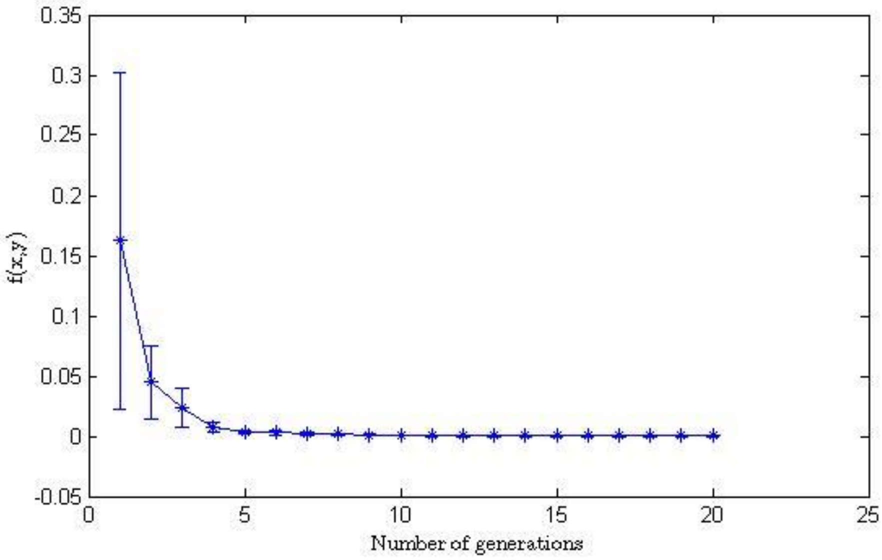

Fig 2. Convergence plot for sphere function minimization

The objective function can be observed to be converging to zero by the time the program reaches the 6th or 7th generation.

The scatter plot for the final generation and the plot showing the reduction of objective function value with passing of the generation are shown below:

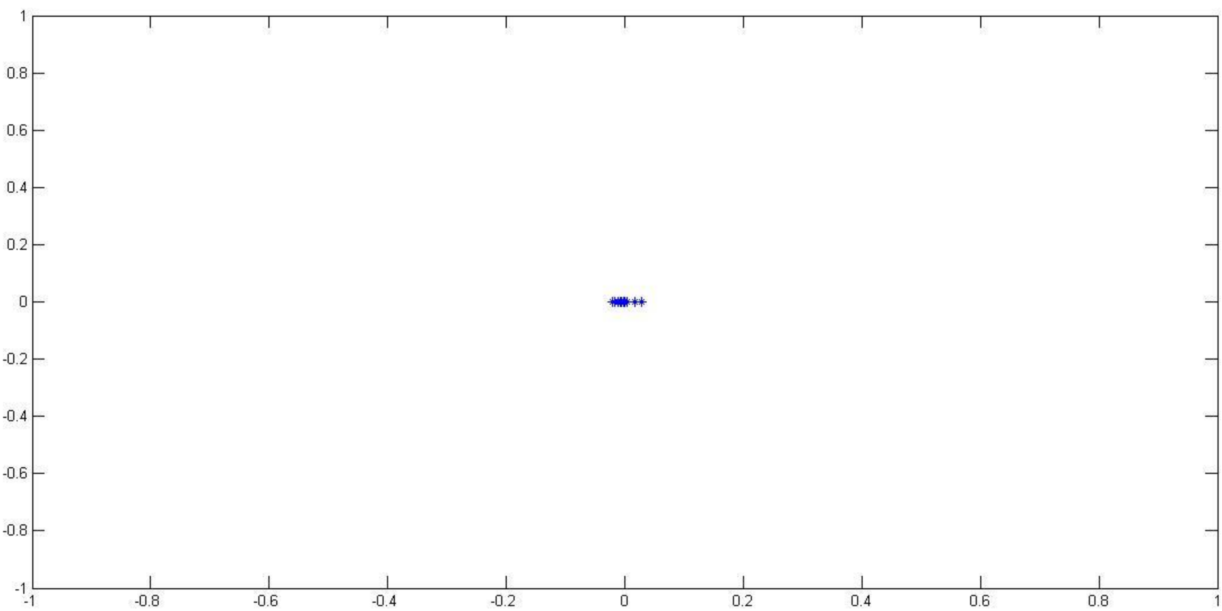

Fig 3. Scatter plot for sphere function optimization

5.1.2 Booth function

The Booth function is expressed as follows: $f(x,y) = (x + 2y - 7)^2 + (2x + y - 5)^2$, within the domain $x, y \in [-10,10]$.

From solving a linear simultaneous equation, it can be observed that f(x,y) reaches a minimum at (1,3) when f(x,y) = 0.

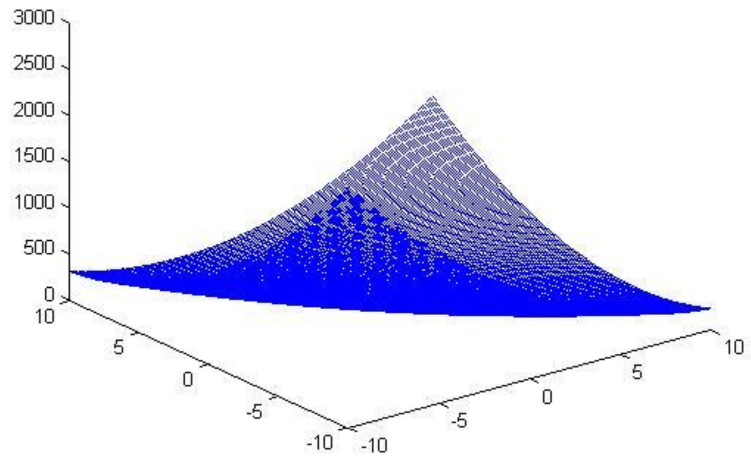

Fig 4. Contour plot for Booth function

The output minimum for this function is given by the code at the point (0.9995, 3.0005), and the minimum value of the objective function is given as 1.41×10^{-6}. The result, therefore, is quite accurate and near enough to the absolute minimum of f(1,3) = 0.

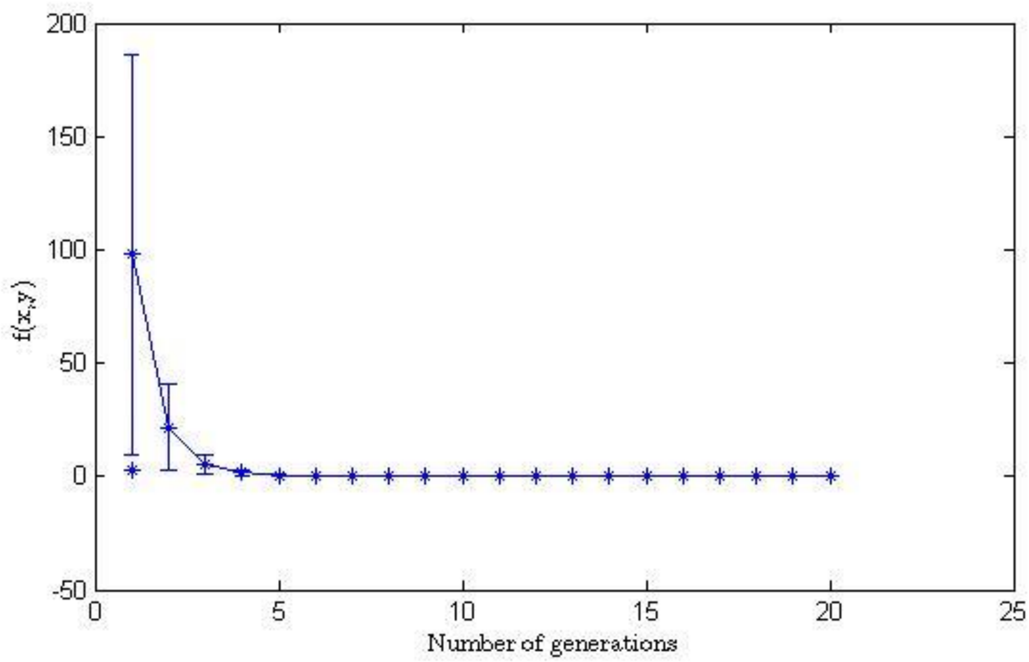

Fig 5. Convergence plot for Booth function

The objective function can be observed to be converging to zero by the time the program reaches the 6^{th} or 7^{th} generation.

The scatter plot for the final generation and the plot showing the reduction of objective function value with passing of the generation are shown below:

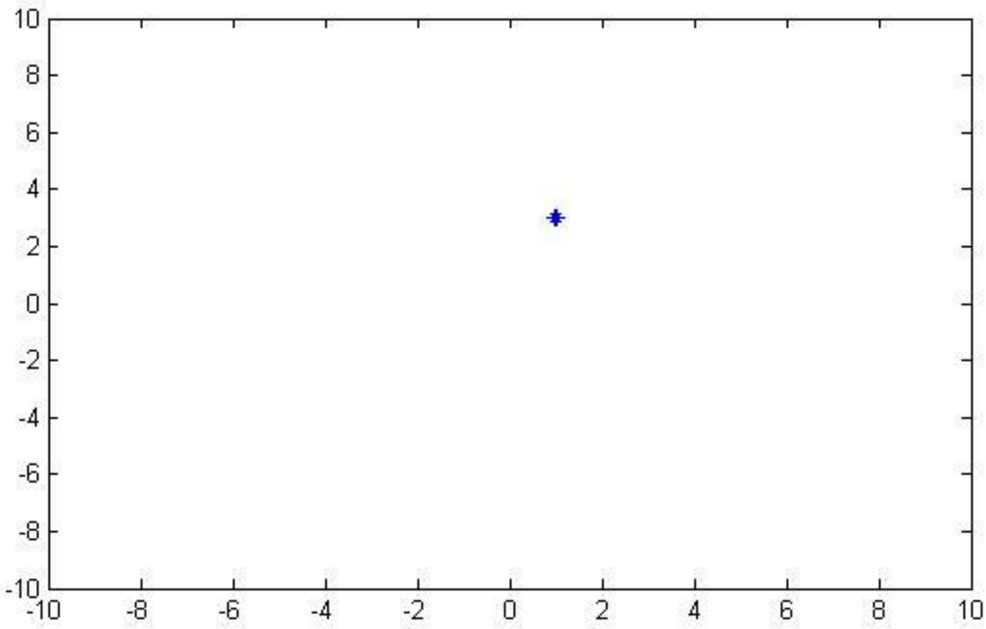

Fig 6. Scatter plot for Booth function

5.2 Validation against existing literature

A few case studies are presented here which have been drawn from the existing body of research already performed on this topic after an extensive literature survey. The results obtained using the formulated algorithm are used to validate the existing published results.

5.2.1 Case Study I

Title of Paper: *Modelling and optimisation of machining parameters for composite pipes using artificial neural network and genetic algorithm*

Authors*: Kumar, K. Vijay, and A. Naveen Sait*

Journal: *International Journal on Interactive Design and Manufacturing (IJIDeM)* 11.2 (2017): 435-443.

In their work on machining of composite pipes, Kumar and Sait have studied the variation of cutting force during machining of composite pipes with variation of cutting speed, feed and depth of cut. The authors have modelled the machining data using ANN and have predicted the optimum combination of input parameters using GA.

The parametric equation as modelled using ANN is given as follows:

$F_c = F(v,f,d) = \quad 763.43 + 0.04361v - 2312.05f - 22.8375d$

The R^2 value of the fitness of the parametric equation is given as 95.56%.

The ranges of the three parameters are as follows:

- Cutting speed (v) (m/min) : 75 – 120
- Feed (f) (mm/revolution): 0.05 – 0.20
- Depth of cut (d) (mm): 0.50 – 1.25

Using the developed code, optimum machining parameters are obtained as follows:

v = 78.6903 m/min f = 0.2006 mm/revolution d = 0.5362 mm

Minimum cutting force as given by the algorithm = F(v,f,d) = 315.46 N

Experimental Cutting Force (N)	**Optimum Cutting Force from GA**		**Error**	
	From existing research (N)	From developed code (N)	From existing research (N)	From developed code (N)
315.72	315.70	315.46	0.02	0.26

We can conclude that the results shown by the algorithm is satisfactorily near to the actual value with an error percent of 0.082%, but the error is slightly more from the existing literature.

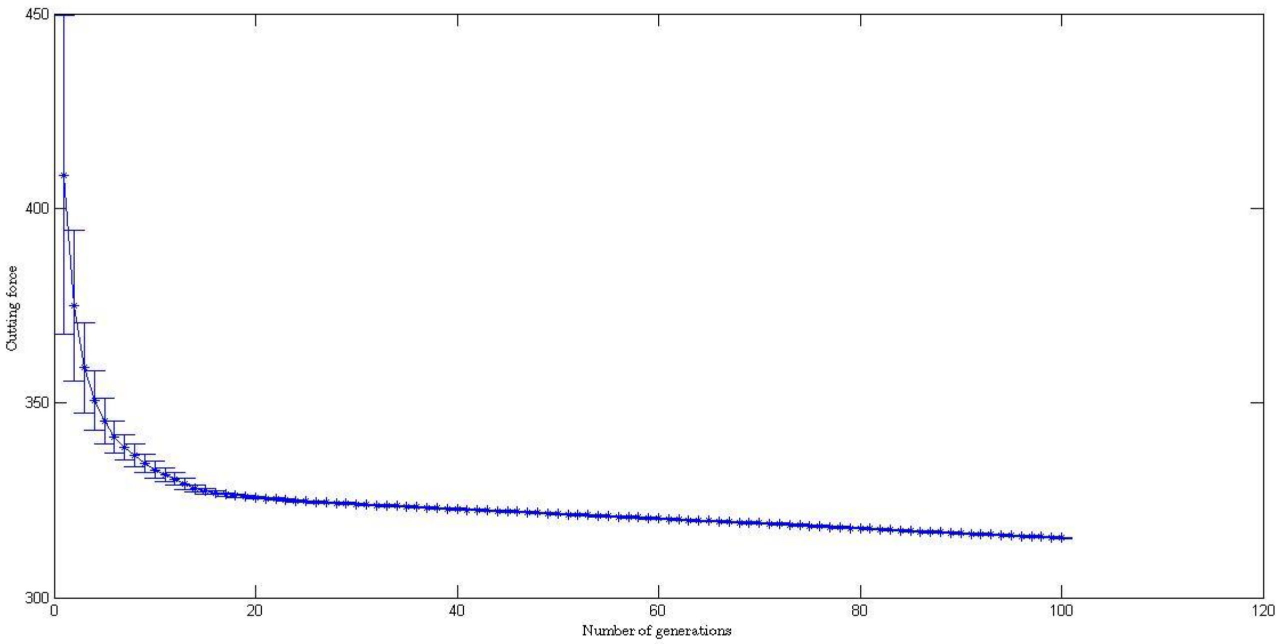

Fig 7. Convergence plot for Case Study I

Figure 1

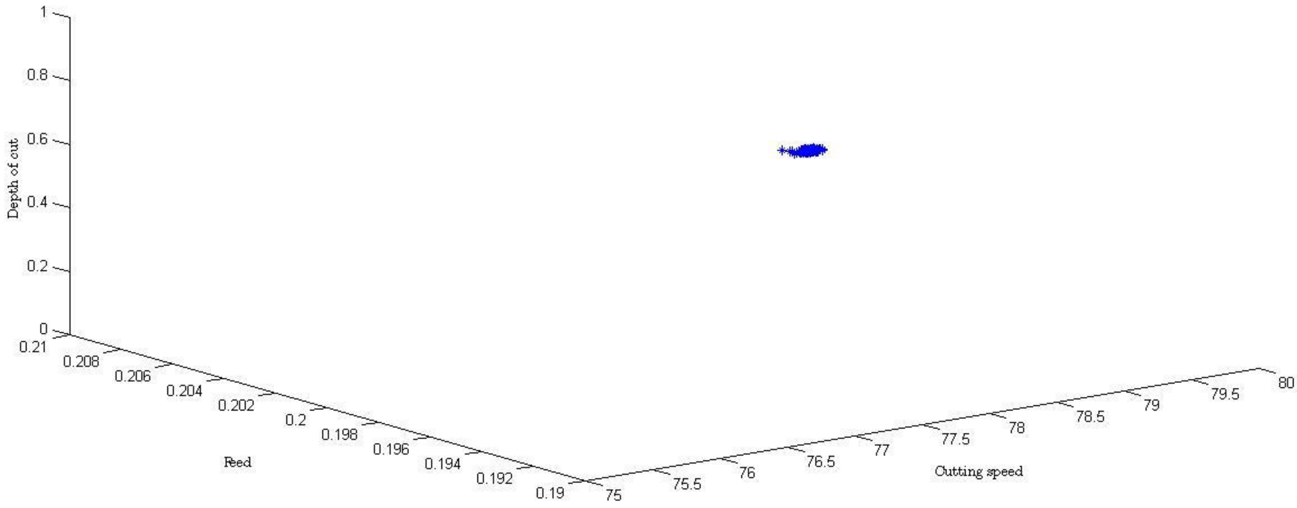

Fig 8. Scatter plot for Case Study I

5.2.2 Case Study II

Title of Paper: *Evaluation of lipase production by genetic algorithm and particle swarm optimization and their comparative study.*

Authors*: Garlapati, Vijay Kumar, Pandu Ranga Vundavilli, and Rintu Banerjee*

Journal: *Applied biochemistry and biotechnology* 162.5 (2010): 1350-1361.

In their work on evaluation of Lipase production, Garlapati *et al* have used Particle Swarm Optimization to maximise the lipase activity with variation of temperature, liquid-to-solid ratio, pH, and fermentation time.

The parametric equation modelled after the collected data is quadratic in nature and has four input variables, which makes the optimization process even more challenging.

The parametric equation is given as follows:

$$L_a = F(T,r,p,d) = -842.551 + 31.6692T + 103.673r + 75.2450p + 41.1417d$$
$$- 0.381250Tr + 1.10875Tp - 0.156125Td + 0.687500rp + 2.93375rd + 4.61875pd$$
$$- 0.508562T^2 - 35.9412r^2 - 13.0713\, p^2 - 6.66031d^2$$

The R^2 value of the fitness of the parametric equation is given as 95.0%.

The ranges of the three parameters are as follows:

- Temperature (T) (°C) : 35-37
- Liquid-to-solid ratio (r): 1-2

- pH (p): 5-7
- Time (d) (days) 4-6

Using the developed code, optimum fermentation parameters are obtained as follows:

T = 35.73 °C r = 1.50 p = 5.29 d = 4.84 days

Maximum lipase activity as given by the algorithm = F(T,r,p,d) = 96.89 U/gds.

Experimental lipase activity (U/gds)	**Optimum lipase activity from GA**		**Error**	
	From existing research (U/gds)	From developed code (U/gds)	From existing research (U/gds)	From developed code (U/gds)
95.34	96.89	96.89	1.55	1.55

We can conclude that the results shown by the algorithm is are highly satisfactory and show absolute correspondence to the optimum values obtained by Garlapati *et al* in their investigations.

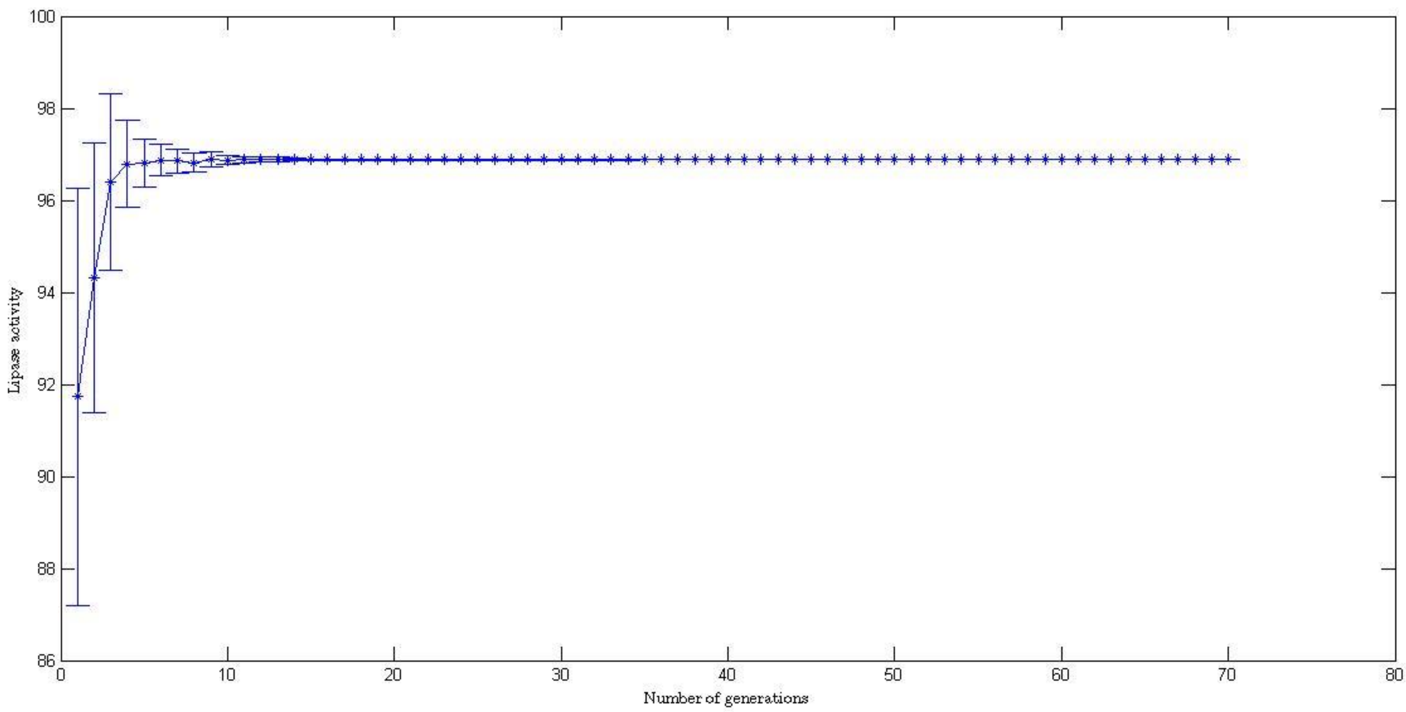

Fig 9. Convergence plot for Case Study II

In addition, this particular optimization problem is one of maximization. Therefore it is clear that the generated algorithm is watertight enough for application to both sorts of optimization problems. The code has a high degree of flexibility and requires only a small degree of tuning to adjust it to the requirements of the user.

Since the scatter plot has four independent dimensions, it is not possible to show them on the three dimensional coordinate system.

6 GA based Optimization of Machining Parameters for Milling Operation

6.1 Design of Experiment

The three input variables are designated as follows:

Spindle Speed = N (rpm); Feed = f (mm/min); Depth of cut = f (mm)

Experiments are conducted based on full factorial design of experiments, with three parameters and five levels. A total of 125 sets of input and output data are obtained with every possible combination accounted for.

6.2 Data in coded form

The obtained data are converted to coded form for ease of optimization. The five levels and their respective coded values of the three variables are as follows:

Coded value	Speed (rpm)	Feed	Depth of cut
-1.0	2500	300	0.150
-0.5	2750	350	0.175
0.0	3000	400	0.200
0.5	3250	450	0.225
1.0	3500	500	0.250

6.3 Objective Function

6.3.1 Parametric Equation

Using design expert, the experimental data are fit in a quadratic parametric equation in the form of

$F(x_1, x_2, x_3) = a_0 + a_1d + a_2N + a_3f + a_{11}d^2 + a_{22}N^2 + a_{33}f^2 + a_{12}dN + a_{23}Nf + a_{13}df.$

The values of the constants in the equation are obtained using the software by inputting the experimental data points. The data set is attached in the Appendix.

The parametric equation comes out to be

$$
\begin{aligned}
F(d, N, f) = \; & 0.5937 \\
& + 0.1427d + 0.004818N + 0.02f \\
& + 0.004902d^2 + 0.06602N^2 + 0.01293f^2 \\
& - 0.02901dN - 0.002912df - 0.03872Nf
\end{aligned}
$$

6.3.2 Goodness of Fit

The fit of the parametric equation is checked by performing Chi-square test on the equation.

$$\chi^2 = \sum_{i=1}^{125} \frac{(\mathrm{Fo} - \mathrm{Fe})^2}{\mathrm{Fe}}$$

The value of χ^2 comes to be 1.6018.

At 95% confidence level with d.f. = 124, $\chi^2_{0.05,124}$ = 150.98.

So since $\chi^2_e << \chi^2_{0.05,124}$, the goodness of fit of the curve is satisfactory.

6.3.3 Two-parameter variation

Since the parametric equation contains three input and one output variables, it is not possible to plot the points of the function in the three-dimensional coordinate system. In order to gauge the kind of variation of the input parameters, two-parameter variation plots are created.

The coded value of one variable at a time was set at zero, while surface roughness (R_a) value was plotted along the Z axis for variation of the remaining two coded parameters on the X and Y axes. The plots obtained are as follows:

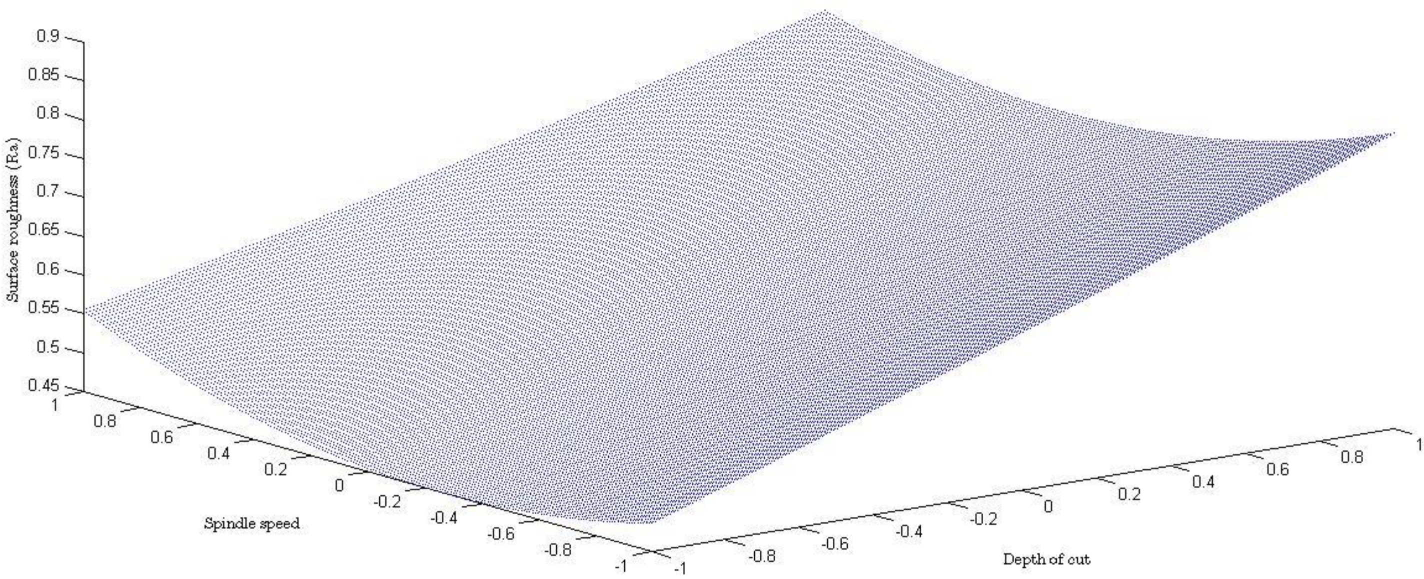

Fig 10. Surface plot of Surface roughness with constant feed

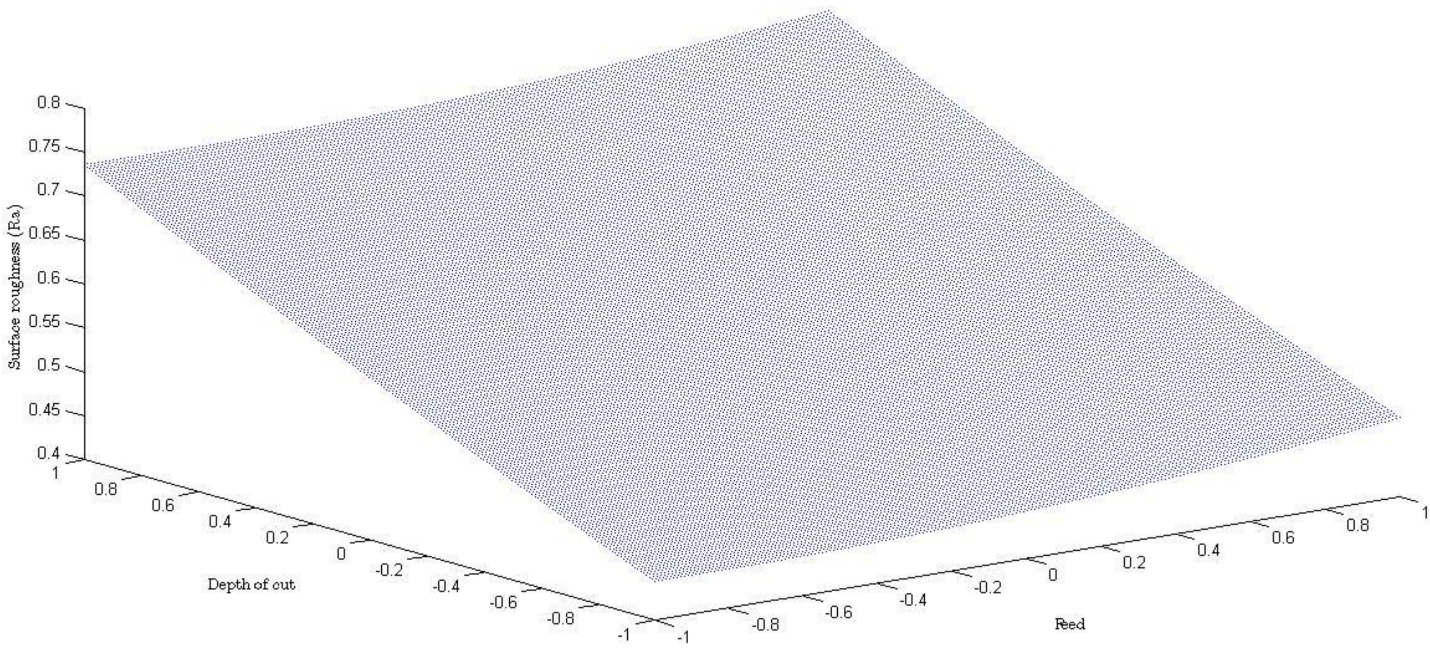

Fig 11. Surface plot of Surface roughness with constant spindle speed

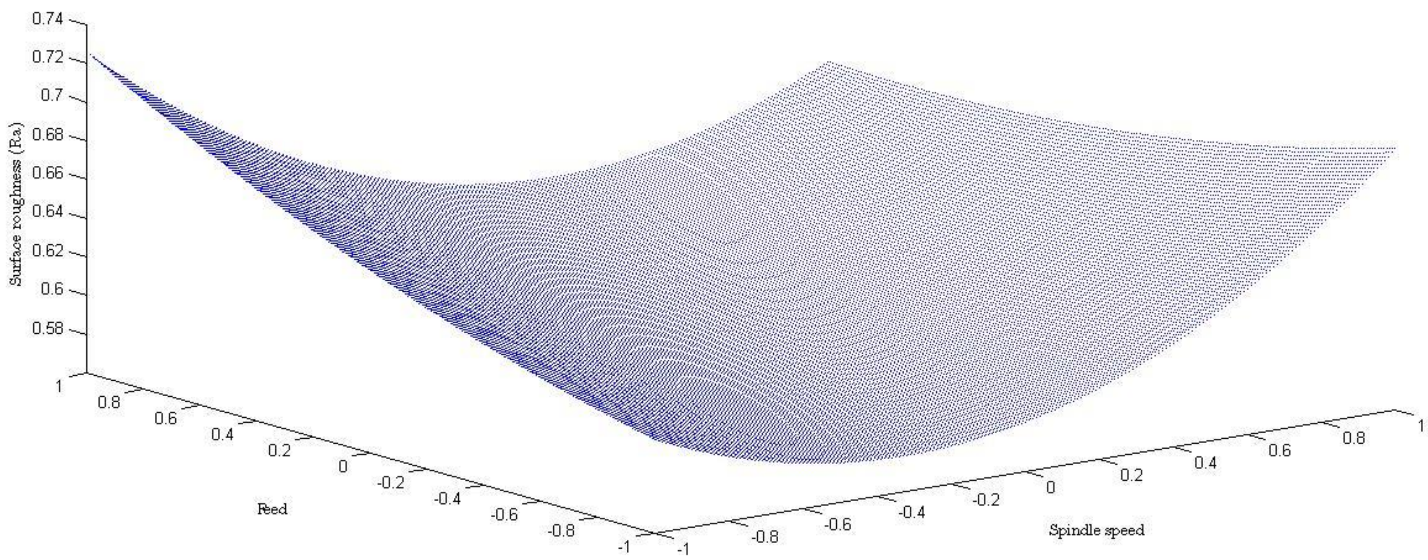

Fig 12. Surface plot of Surface roughness with constant depth of cut

6.4 Genetic Algorithm parameters

The crucial parameters for satisfactory optimization are the ratios of elite offspring, crossover offspring and mutation offspring in every new generation of phenotypes. Moreover, the function being optimized is quadratic in nature and is likely to have a large number of local minima, as is apparent from the two-parameter variation shown in the Fig. 9 10 and 11.

The number of elite offspring was fixed at 1 for all runs. This is because considering the highly non-linear form of the equation, if higher number of elite offspring is used there is higher tendency of stagnation of the algorithm at a local minimum.

For similar reasons, the ratio of crossover and mutation offspring is also kept at 1:1. For every pair of crossover offspring born from a pair of parent phenotypes, two individual mutation offspring are born from two individual parents. Fraction of mutation offspring is kept high in order to track the non-linearity in the function.

6.5 Genetic Algorithm based optimized results

6.5.1 Variation in number of phenotypes

For satisfactory determination of minimized value of objective function, it is important to have a threshold number of phenotypes in each generation. If this condition is not met then the lesser number of phenotypes would be unsuccessful in spanning the entire range of parameters and identify the global minimum from among several possible instances of local minima.

To investigate this parameter, several runs are conducted for different values of phenotypes per generation, starting from n = 41 and proceeding at intervals of 20 till n = 141. The number of generations is kept fixed at 70, which is a standard value of number of generations when termination is done after a fixed number of generations. Five runs were conducted for each value of number of phenotypes. The mean of the five runs and their standard deviation is plotted in the form of a bar chart.

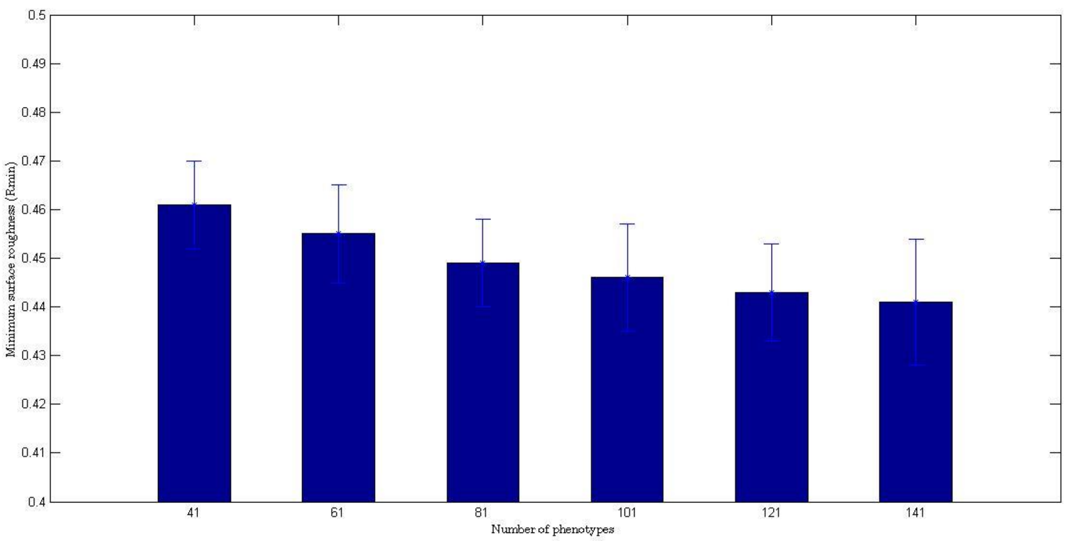

Fig 13. Variation of minima with increase in phenotypes per generation

It is observed that with increase in number of phenotypes successfully reduces the minimum value obtained by the algorithm. The reduction is sharp in the initial stages, but gradually reduces to become steady. The nature of the curve is exponential.

The investigation is not carried on at lesser than 41 phenotypes because at such low values it was evident upon just observation that the obtained minimum value was not sufficiently low enough.

The trend is not continued after reaching 141 phenotypes, since beyond that range the optimized genotype was increasingly tending to go beyond the permissible range of input parameter values.

6.5.2 Determination of optimized function

Through a process of trial-and-error, it was found that with further increase in the number of iterations (generations), the genotypes tend to leave the specified range of inputs hence rendering the optimization redundant. From the previous result, it can be observed that at around 150 phenotypes, maximum reduction in minimized value of the equation can be obtained while staying within the permitted range. As a result, repeated runs were performed for 70 iterations and 141 phenotypes per generation. This eventually gave the optimized minimum value within the range of input parameters.

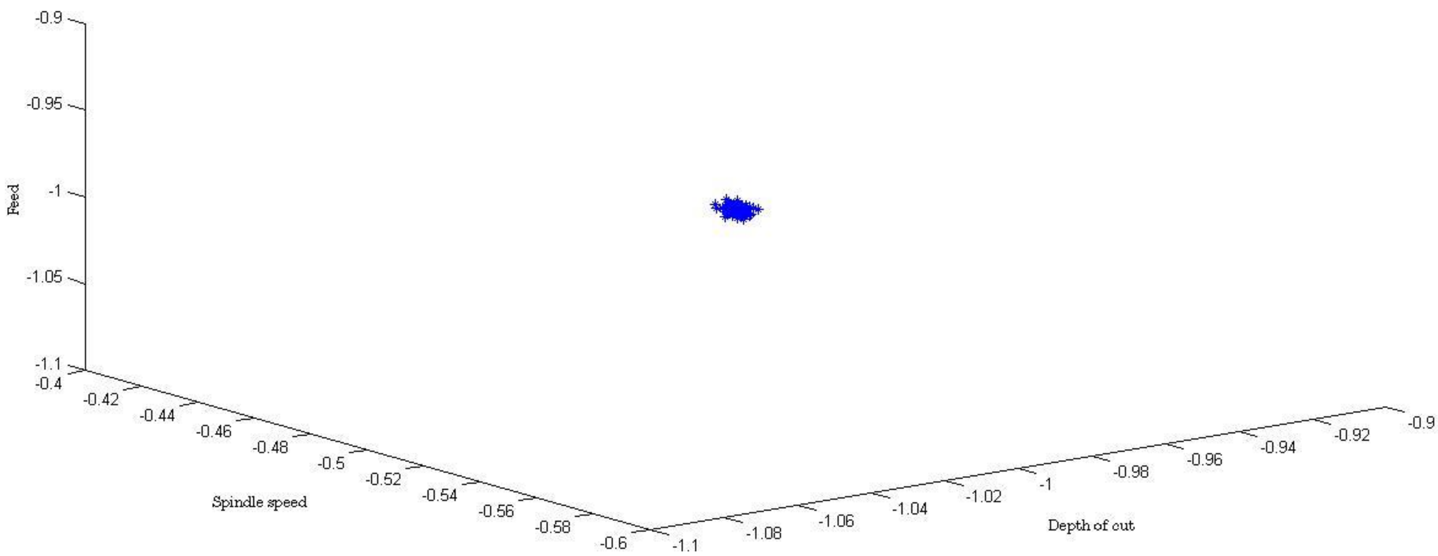

Fig 14. Scatter plot for optimized function

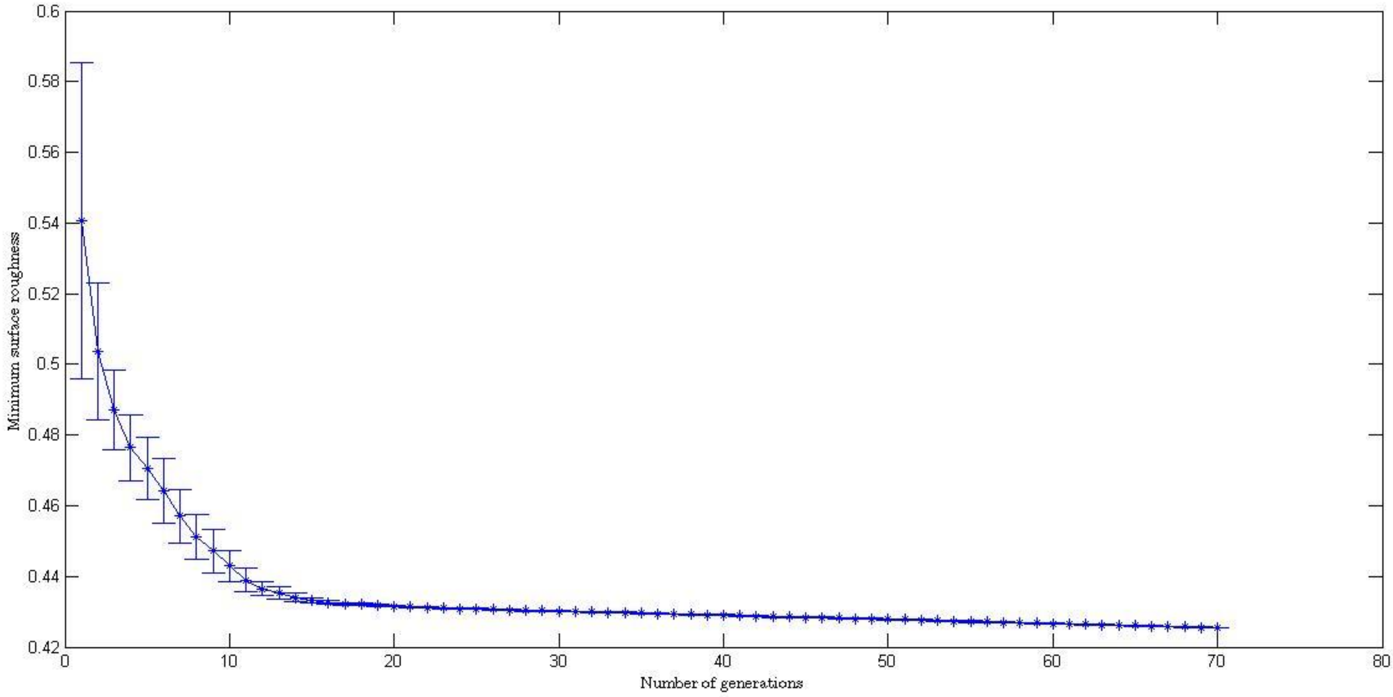

Fig 15. Generation plot for optimized function

The optimum value in coded form is as follows: (-0.9963, -0.4966, -0.9990). In coded form, it is nearest to the data point (-1.0, -0.5, -1.0).

Converting the coded forms to the real values:

Depth of cut = 0.1501 Spindle speed = 2751 rpm Feed = 300.1

The surface roughness value at the optimum condition is:

$R_a = 0.4255$

The obtained R_a value is lesser than the theoretical R_a's calculated at each of the 125 data points in the set.

7 Conclusion

In this study, a complete algorithm was devised in MATLAB environment which inherently has a great deal of flexibility which allows the user to modify and tune the GA parameters as per his or her requirements. The code was verified by well established optimization functions.

To validate the efficacy of the code with real data, to accepted investigations and results pertaining to different optimization processes were validated by the algorithm written for this project. The work of previous authors coincided with the results given by this study.

It was also shown that the optimization function works equally well for both minimization as well as maximization problems.

In the final parts of the study, the optimization technique was applied on a new set of data related to milling operation of a workpiece. Some conclusive statements could be made regarding the effect of spindle speed, feed and depth of cut upon the average roughness value of the workpiece. Within the stipulated range given with the data, the optimum value of the parameters was determined. The minimum attainable value of surface roughness was accordingly predicted with a high degree of confidence.

8 References

- Kumar, K. Vijay, and A. Naveen Sait. "Modelling and optimisation of machining parameters for composite pipes using artificial neural network and genetic algorithm." *International Journal on Interactive Design and Manufacturing (IJIDeM)* 11.2 (2017): 435-443.
- Garlapati, Vijay Kumar, Pandu Ranga Vundavilli, and Rintu Banerjee. "Evaluation of lipase production by genetic algorithm and particle swarm optimization and their comparative study." *Applied biochemistry and biotechnology* 162.5 (2010): 1350-1361.
- Yu-long, Chen, et al. "Optimization of WEDM Process Parameters for Monocrystalline Silicon by Response Surface Method and Genetic Algorithm." *Proceedings of the 2017 International Conference on Artificial Intelligence, Automation and Control Technologies*. ACM, 2017.

9 Acknowledgement

I would like to acknowledge the following individuals and institutions for giving me the scope for undertaking this Advanced Design Project:

- Prof. Tapan K Barman, my guide for this Project, for providing me with the idea for the project and guiding me during its execution.
- Prof. Dipankar Sanyal, Head of the Mechanical Engineering Department, Jadavpur University, for the giving me the chance to undertake this Project.
- The Mechanical Engineering Department, Jadavpur University, for all assistance throughout the duration of the Project.

10 Appendix

Sl. No.	Depth of cut	Spindle speed	Feed	R_a experimental	R_a theoretical	Error fraction	Chi squared
1	-1	-1	-1	0.5160	0.4394	0.148	0.013357
2	-1	-1	-0.5	0.4830	0.4605	0.047	0.001098
3	-1	-1	0	0.5070	0.4881	0.037	0.000732
4	-1	-1	0.5	0.4540	0.5221	-0.150	0.008893
5	-1	-1	1	0.4530	0.5627	-0.242	0.021371
6	-1	-0.5	-1	0.3590	0.4262	-0.187	0.010581
7	-1	-0.5	-0.5	0.4760	0.4376	0.081	0.003372
8	-1	-0.5	0	0.4350	0.4555	-0.047	0.000922
9	-1	-0.5	0.5	0.5610	0.4799	0.145	0.013719
10	-1	-0.5	1	0.4740	0.5107	-0.077	0.002637
11	-1	0	-1	0.3760	0.4459	-0.186	0.010963
12	-1	0	-0.5	0.4300	0.4477	-0.041	0.000698
13	-1	0	0	0.5630	0.4559	0.190	0.025159
14	-1	0	0.5	0.4340	0.4706	-0.084	0.002845
15	-1	0	1	0.4890	0.4917	-0.006	0.000015
16	-1	0.5	-1	0.4970	0.4987	-0.003	0.000006
17	-1	0.5	-0.5	0.4880	0.4908	-0.006	0.000016
18	-1	0.5	0	0.5680	0.4893	0.139	0.012651
19	-1	0.5	0.5	0.5350	0.4943	0.076	0.003346
20	-1	0.5	1	0.5010	0.5058	-0.010	0.000046
21	-1	1	-1	0.5830	0.5845	-0.003	0.000004
22	-1	1	-0.5	0.5800	0.5669	0.023	0.000303
23	-1	1	0	0.5420	0.5558	-0.025	0.000340
24	-1	1	0.5	0.5670	0.5511	0.028	0.000460
25	-1	1	1	0.4880	0.5529	-0.133	0.007612
26	-0.5	-1	-1	0.5320	0.5230	0.017	0.000154
27	-0.5	-1	-0.5	0.6350	0.5434	0.144	0.015435
28	-0.5	-1	0	0.4250	0.5703	-0.342	0.037007
29	-0.5	-1	0.5	0.6400	0.6036	0.057	0.002196
30	-0.5	-1	1	0.5190	0.6434	-0.240	0.024045
31	-0.5	-0.5	-1	0.4990	0.5025	-0.007	0.000025
32	-0.5	-0.5	-0.5	0.5930	0.5132	0.134	0.012394
33	-0.5	-0.5	0	0.6360	0.5304	0.166	0.021016
34	-0.5	-0.5	0.5	0.6580	0.5541	0.158	0.019499
35	-0.5	-0.5	1	0.6330	0.5842	0.077	0.004083
36	-0.5	0	-1	0.4530	0.5150	-0.137	0.007475
37	-0.5	0	-0.5	0.5430	0.5161	0.050	0.001404
38	-0.5	0	0	0.5090	0.5236	-0.029	0.000406
39	-0.5	0	0.5	0.6580	0.5375	0.183	0.026996
40	-0.5	0	1	0.7030	0.5580	0.206	0.037702
41	-0.5	0.5	-1	0.5850	0.5606	0.042	0.001064

42	-0.5	0.5	-0.5	0.6020	0.5519	0.083	0.004543
43	-0.5	0.5	0	0.5180	0.5497	-0.061	0.001833
44	-0.5	0.5	0.5	0.4580	0.5540	-0.210	0.016643
45	-0.5	0.5	1	0.7150	0.5648	0.210	0.039963
46	-0.5	1	-1	0.6620	0.6391	0.035	0.000820
47	-0.5	1	-0.5	0.5000	0.6208	-0.242	0.023500
48	-0.5	1	0	0.4940	0.6089	-0.233	0.021688
49	-0.5	1	0.5	0.5990	0.6035	-0.008	0.000034
50	-0.5	1	1	0.5920	0.6046	-0.021	0.000262
51	0	-1	-1	0.5170	0.6091	-0.178	0.013929
52	0	-1	-0.5	0.5530	0.6288	-0.137	0.009132
53	0	-1	0	0.6090	0.6549	-0.075	0.003217
54	0	-1	0.5	0.6670	0.6875	-0.031	0.000611
55	0	-1	1	0.6520	0.7266	-0.114	0.007650
56	0	-0.5	-1	0.6130	0.5814	0.052	0.001721
57	0	-0.5	-0.5	0.6240	0.5913	0.052	0.001803
58	0	-0.5	0	0.5260	0.6078	-0.156	0.011008
59	0	-0.5	0.5	0.6430	0.6307	0.019	0.000240
60	0	-0.5	1	0.5530	0.6601	-0.194	0.017373
61	0	0	-1	0.5230	0.5866	-0.122	0.006902
62	0	0	-0.5	0.5310	0.5869	-0.105	0.005330
63	0	0	0	0.6810	0.5937	0.128	0.012837
64	0	0	0.5	0.6600	0.6069	0.080	0.004640
65	0	0	1	0.4750	0.6266	-0.319	0.036691
66	0	0.5	-1	0.6590	0.6249	0.052	0.001860
67	0	0.5	-0.5	0.4800	0.6155	-0.282	0.029840
68	0	0.5	0	0.4840	0.6126	-0.266	0.027002
69	0	0.5	0.5	0.5890	0.6162	-0.046	0.001198
70	0	0.5	1	0.4870	0.6262	-0.286	0.030937
71	0	1	-1	0.8090	0.6962	0.139	0.018280
72	0	1	-0.5	0.6990	0.6771	0.031	0.000706
73	0	1	0	0.6540	0.6645	-0.016	0.000167
74	0	1	0.5	0.7340	0.6584	0.103	0.008678
75	0	1	1	0.7170	0.6587	0.081	0.005151
76	0.5	-1	-1	0.5600	0.6976	-0.246	0.027159
77	0.5	-1	-0.5	0.8270	0.7166	0.134	0.017014
78	0.5	-1	0	0.6970	0.7420	-0.065	0.002727
79	0.5	-1	0.5	0.8900	0.7738	0.131	0.017434
80	0.5	-1	1	0.7880	0.8122	-0.031	0.000720
81	0.5	-0.5	-1	0.8900	0.6627	0.255	0.078002
82	0.5	-0.5	-0.5	0.7220	0.6719	0.069	0.003735
83	0.5	-0.5	0	0.5720	0.6876	-0.202	0.019442
84	0.5	-0.5	0.5	0.9400	0.7098	0.245	0.074651
85	0.5	-0.5	1	1.1400	0.7385	0.352	0.218341

86	0.5	0	-1	0.5780	0.6607	-0.143	0.010343
87	0.5	0	-0.5	0.6150	0.6602	-0.074	0.003099
88	0.5	0	0	0.6550	0.6663	-0.017	0.000191
89	0.5	0	0.5	0.7350	0.6788	0.076	0.004656
90	0.5	0	1	0.8170	0.6977	0.146	0.020381
91	0.5	0.5	-1	0.5450	0.6917	-0.269	0.031107
92	0.5	0.5	-0.5	0.7480	0.6816	0.089	0.006473
93	0.5	0.5	0	0.5660	0.6779	-0.198	0.018482
94	0.5	0.5	0.5	0.5640	0.6808	-0.207	0.020026
95	0.5	0.5	1	0.6810	0.6901	-0.013	0.000119
96	0.5	1	-1	0.9210	0.7557	0.179	0.036150
97	0.5	1	-0.5	0.6520	0.7359	-0.129	0.009572
98	0.5	1	0	0.7680	0.7226	0.059	0.002851
99	0.5	1	0.5	0.7290	0.7158	0.018	0.000245
100	0.5	1	1	0.6840	0.7154	-0.046	0.001375
101	1	-1	-1	0.8740	0.7886	0.098	0.009240
102	1	-1	-0.5	0.9650	0.8068	0.164	0.031002
103	1	-1	0	0.7410	0.8315	-0.122	0.009853
104	1	-1	0.5	0.8340	0.8627	-0.034	0.000952
105	1	-1	1	0.8330	0.9003	-0.081	0.005024
106	1	-0.5	-1	0.6640	0.7464	-0.124	0.009094
107	1	-0.5	-0.5	0.8250	0.7549	0.085	0.006507
108	1	-0.5	0	0.7390	0.7699	-0.042	0.001240
109	1	-0.5	0.5	0.7400	0.7914	-0.069	0.003333
110	1	-0.5	1	0.7930	0.8193	-0.033	0.000843
111	1	0	-1	0.5690	0.7371	-0.296	0.038354
112	1	0	-0.5	0.8130	0.7360	0.095	0.008058
113	1	0	0	0.6520	0.7413	-0.137	0.010758
114	1	0	0.5	0.6670	0.7531	-0.129	0.009839
115	1	0	1	0.8280	0.7713	0.068	0.004165
116	1	0.5	-1	0.7750	0.7609	0.018	0.000261
117	1	0.5	-0.5	0.7750	0.7501	0.032	0.000828
118	1	0.5	0	0.6010	0.7457	-0.241	0.028082
119	1	0.5	0.5	0.6770	0.7478	-0.105	0.006705
120	1	0.5	1	0.6990	0.7564	-0.082	0.004351
121	1	1	-1	0.8070	0.8177	-0.013	0.000140
122	1	1	-0.5	1.0250	0.7972	0.222	0.065108
123	1	1	0	0.8460	0.7831	0.074	0.005047
124	1	1	0.5	0.7950	0.7755	0.024	0.000488
125	1	1	1	0.7800	0.7744	0.007	0.000040