asumiendo aproximación perfecta

$$\frac{2n+1}{2}\int_{-1}^{1} P_n(x)\,f(x)\,dx = \int_{-1}^{1}\sum_{m=0}^{\infty} P_m C_m \cdot P_n\,dx \cdot \frac{2n+1}{2} = C_n$$

$$\frac{2n+1}{2}\int_{-1}^{1} C_m \underbrace{\sum_{m=0}^{\infty} P_m\cdot P_n}_{\delta}\,dx$$

por ser conjunto ortonormal $\delta\begin{cases} 0, & m\neq n \\ \dfrac{2}{2n+1}, & m=n \end{cases}$ $\therefore$

$$\frac{2n+1}{2}\int_{-1}^{1} C_m \frac{2}{2n+1}\,\delta(m=n)\,dx$$

$$\int_{-1}^{1} \underbrace{C_m\,\cancel{2}\,\delta(m=n)}_{\delta}\,dx = \underline{\underline{C_n}} = \underline{\underline{C_n}}$$

La integral será 0 a menos que $\underline{\underline{m=n}}$