$I = \int_a^b f(x)\,dx$

$f(x) \approx p_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

$I \simeq \int_a^b p_1(x)\,dx = \int_a^b \frac{x-b}{a-b} f(a)\,dx + \int_a^b \frac{x-a}{b-a} f(b)\,dx$

$\frac{f(a)}{a-b}\int_a^b x\,dx - \frac{b f(a)}{a-b}\int_a^b 1\,dx + \frac{-f(b)}{a-b}\int_a^b x\,dx + \frac{a f(b)}{a-b}\int_a^b 1\,dx =$

$\checkmark \qquad \sim \qquad \checkmark \qquad \sim$

$\frac{(f(a)-f(b))}{a-b}\int_a^b x\,dx + \frac{(a f(b) - b f(a))}{a-b}\int_a^b 1\,dx =$

$\frac{f(a)-f(b)}{a-b}\left(\frac{1}{2}b^2 - \frac{1}{2}a^2\right) + \frac{a f(b) - b f(a)}{a-b}(b-a) =$

$\frac{b^2 f(a)}{2(a-b)} + \frac{a^2 f(b)}{2(a-b)} + \frac{ab\, f(b)}{a-b} + \frac{ab f(a)}{a-b} - \frac{b^2 f(b)}{2(a-b)} - \frac{a^2 f(a)}{2(a-b)} - \frac{b^2 f(a)}{(a-b)} - \frac{a^2 f(b)}{a-b}$

$f(a)\left(\frac{b^2}{2(a-b)} + \frac{ab}{a-b} - \frac{a^2}{2(a-b)} - \frac{b^2}{a-b}\right) + f(b)\left(\frac{a^2}{2(a-b)} + \frac{ab}{a-b} - \frac{b^2}{2(a-b)} - \frac{a^2}{a-b}\right) =$

$f(a)\left(-\frac{a^2}{2(a-b)} - \frac{b^2}{2(a-b)} + \frac{ab}{a-b}\right) + f(b)\left(-\frac{a^2}{2(a-b)} - \frac{b^2}{2(a-b)} + \frac{ab}{a-b}\right) =$

$-\frac{(f(a)+f(b))}{2(a-b)}\left(\frac{a^2 - 2ab + b^2}{1}\right) = \frac{-[f(a)+f(b)]}{2}\,\frac{(a-b)^2}{a-b} = (f(a)+f(b))\,\frac{b-a}{2}$