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A Model for South Australian Electricity Spot Prices

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Introduction

Electricity is a unique commodity because it cannot be stored effectively. This causes strange behaviour not seen in other financial markets. The South Australian electricity market is a particularly interesting example: we see huge price spikes rarely seen in other electricity markets due to our relative isolation, weather and generation mix. (see Figure 1)

- Aim: To model randomness in electricity prices and explicitly incorporate key factors that cause extreme price events (spikes and drops).
- Impact: Modelling prices helps understand price spikes and their causes.
- Impact: Price models are necessary for pricing derivative contracts which are key to managing risk in financial markets.

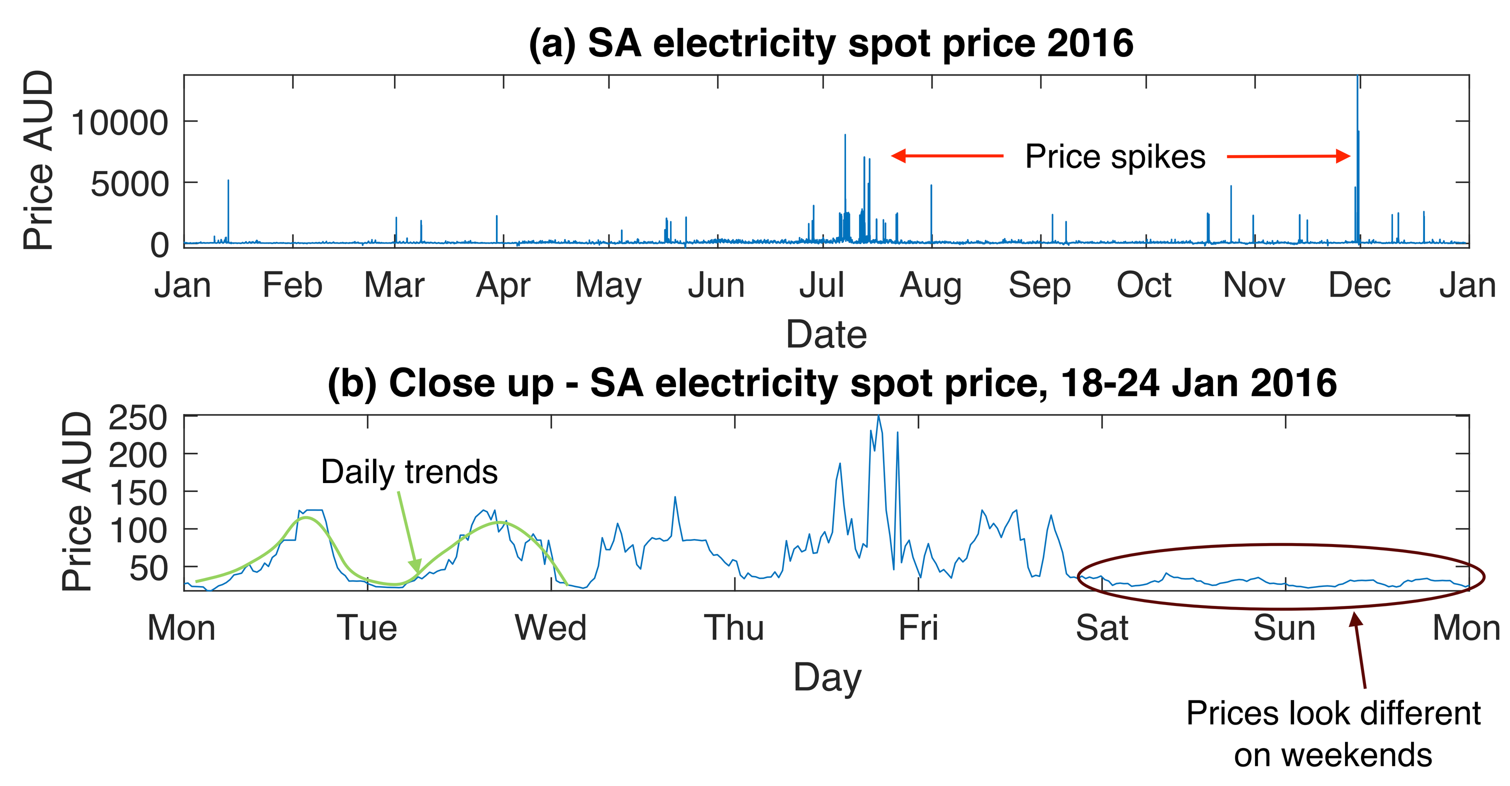

Figure 1: (a) SA electricity market prices for 2016. (b) SA electricity market price data for the week 18th-24th of January.

The Model

Price process $P(t)$ = seasonal $y(t)$ + stochastic $X(t)$

The seasonal component $y(t)$ is made up of two parts

1. A long term component to capture price changes over years and months.
2. A short term component to capture trends over days and weeks.

The stochastic component $X(t)$ is modelled as a *regime-switching time series.*

- At least one regime is as an *autoregressive process of order one*:

$$B(t) = a + bB(t-1) + \sigma\epsilon(t)$$

  where a, b and σ are parameters and $\epsilon(t)$ are Normal$(0,1)$. This regime models 'normal' prices.
- Other regimes model different behaviour such as spikes spikes: $S(t)$ are i.i.d. lognormal random variables with parameters μ_Y and σ_Y.
- The regime switching process $R(t)$ is modelled as a *Markov chain* with transition probabilities $p_{ij} = p(R(t+1) = j | R(t) = i)$. Note that $R(t)$ cannot be observed, only inferred.

The mathematical expression for the stochastic model is

$$X(t) = \begin{cases} B(t) & \text{when } R(t) = 1, \text{ the base regime,} \\ S(t) & \text{when, } R(t) = 2, \text{ the spike regime.} \end{cases}$$

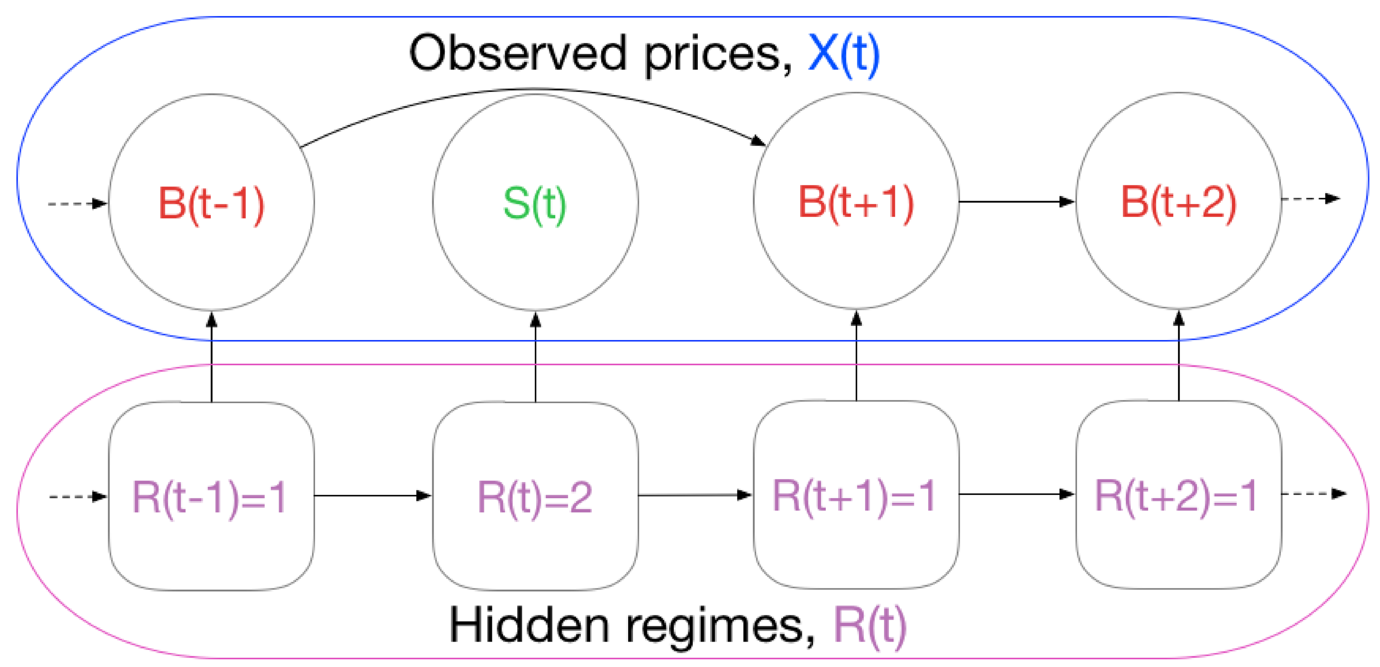

Figure 2: An example of how an regime switching model evolves. The arrows represent dependence between variables.

Estimation

- Estimate the seasonal component $y(t)$ by a technique called *wavelet filtering* for the long term trend, and by averaging for the short term trend.
- Estimate the stochastic component $X(t)$, by using *Bayesian inference*. This methodology has not been used in the electricity pricing literature for this regime switching model.

Bayesian inference

- We assume uniform prior distributions, $p(\theta)$; θ are parameters.
- The likelihood is used to construct the prior but is not easy to compute.
- Instead, we use *data augmentation* which allows us to work with the augmented likelihood instead – we infer the joint posterior of the hidden regimes and the parameters.

$$\overbrace{p(\theta, \boldsymbol{R}|\boldsymbol{x})}^{\text{joint posterior}} = \frac{\overbrace{p(\boldsymbol{x}|\boldsymbol{R}, \theta)p(\boldsymbol{R}|\theta)}^{\textit{augmented likelihood}}\, p(\theta)}{\underbrace{p(\boldsymbol{x})}_{\textit{constant}}}.$$

- The algorithm is called *adaptive-data-augmented-block-Metropolis-Hastings.*
- The *adaptive* and *block* steps allow us to optimise our algorithm and make it easier to implement.

Our Best Model

$$X(t) = \begin{cases} B^{(1)}(t) & \text{when } R(t) = 1, \text{ a base regime,} \\ B^{(2)}(t) & \text{when } R(t) = 2, \text{ another base regime,} \\ S^{(3)}(t) & \text{when, } R(t) = 3, \text{ a spike regime for small spikes,} \\ S^{(4)}(t) & \text{when, } R(t) = 4, \text{ a spike regime for extreme spikes.} \end{cases}$$

		Posterior Mean			Posterior Mean
Base 1	α_1	-3.93	Spike 1	q_3	3.59
	ϕ_1	0.59		μ_3	3.94
	σ_1^2	55.94		σ_3^2	0.51
Base 2	α_2	-2.59	Spike 2	q_4	159.61
	ϕ_2	0.55		μ_4	4.77
	σ_2^2	482.53		σ_4^2	3.10

$$\hat{\mathbb{P}} = \begin{array}{r} \text{base 1} \\ \text{base 2} \\ \text{spike 1} \\ \text{spike 2} \end{array} \begin{bmatrix} 0.9175 & 0.0197 & 0.0613 & 0.0015 \\ 0.0127 & 0.9112 & 0.0698 & 0.0062 \\ 0.4367 & 0.1954 & 0.2924 & 0.0755 \\ 0.0797 & 0.1206 & 0.3785 & 0.4212 \end{bmatrix}$$