Markovian Regime-Switching Models for South Australian Wholesale Electricity Prices

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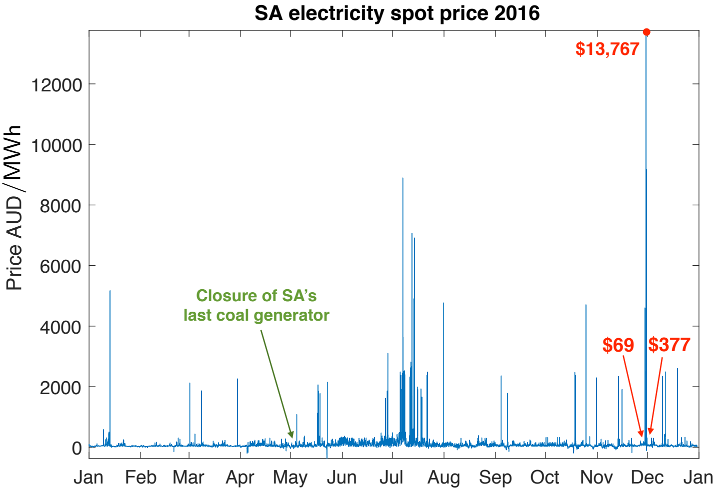
Outline

Wholesale spot prices



About the market

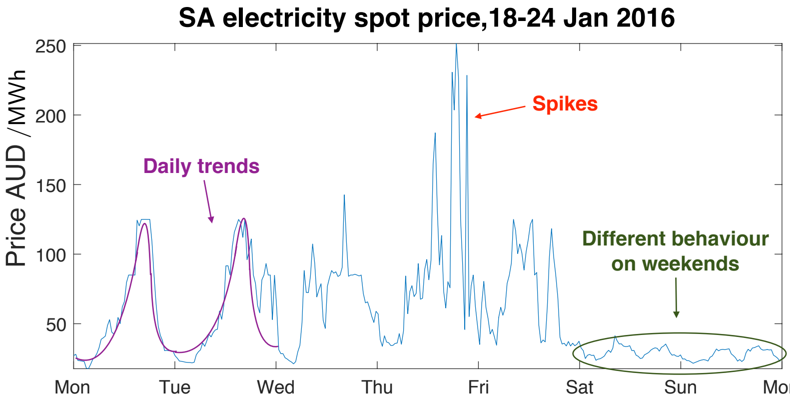
Every 5 minutes AEMO

- Aggregates supply bids
- Estimates demand
- Matches supply and demand
  - Dispatches generators
  - Sets the *dispatch* price
  - Spot price = average over 30 minutes

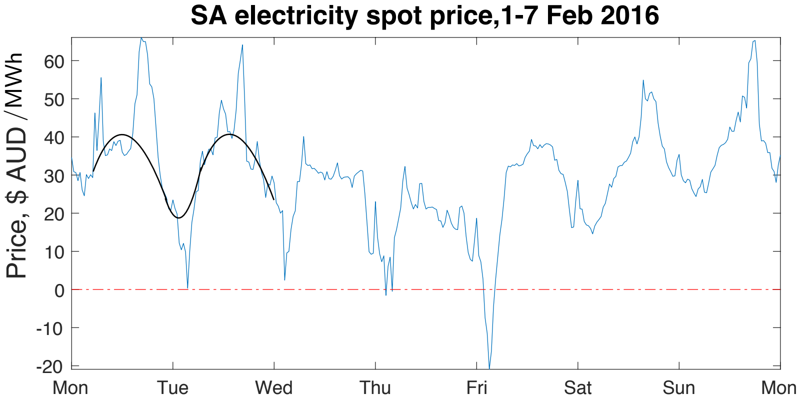
Spikes occur due to

- Unprecedented demand/Incorrect supply forecast
- Low marginal cost generators cannot vary supply quickly

Price data characteristics



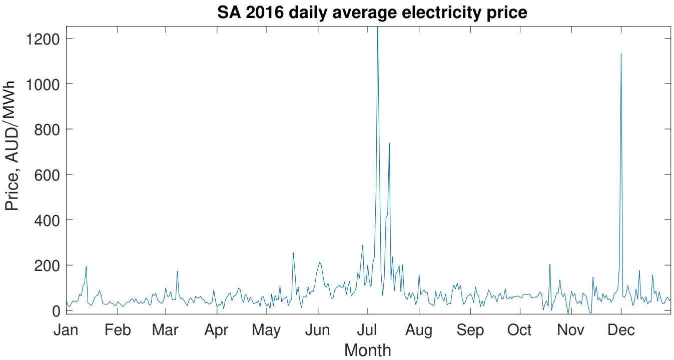
Price data characteristics



- Mean reversion - to a trend line
- Negative prices!

Simplifying the problem

- Model average daily price
  - Justification: Some contracts are valued on daily average prices - e.g. EEX futures
  - Drawback: Not all contracts are valued in this way
- Displays mean reversion, spikes, drops and trends



The model

$$\overbrace{P_t}^{\text{price process}} = \overbrace{y_t}^{\text{trend component}} + \overbrace{X_t}^{\text{stochastic component}}$$

Trend, y_t

- Capture different behaviour on weekends/weekdays
- Seasonal fluctuations

Regime-switching model, X_t

- Mean reversion
- Spikes
- Drops

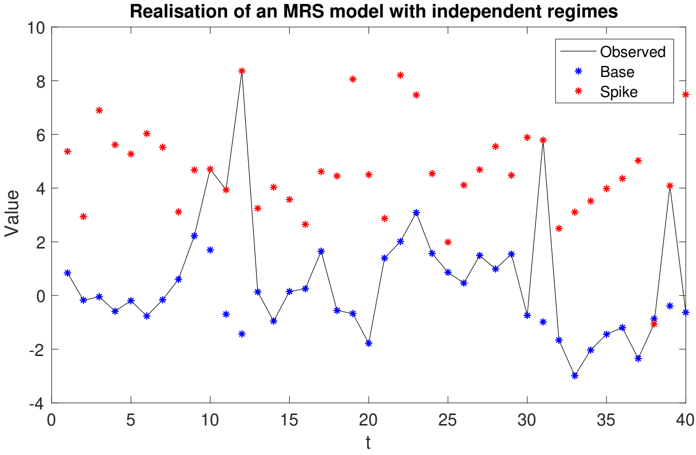
Markovian Regime-switching model

3-regimes with shifted log-normal spikes and drops

$$X_t = \begin{cases} B_t & \text{when } R_t = 1, \\ S_t & \text{when } R_t = 2, \\ D_t & \text{when } R_t = 3. \end{cases}$$

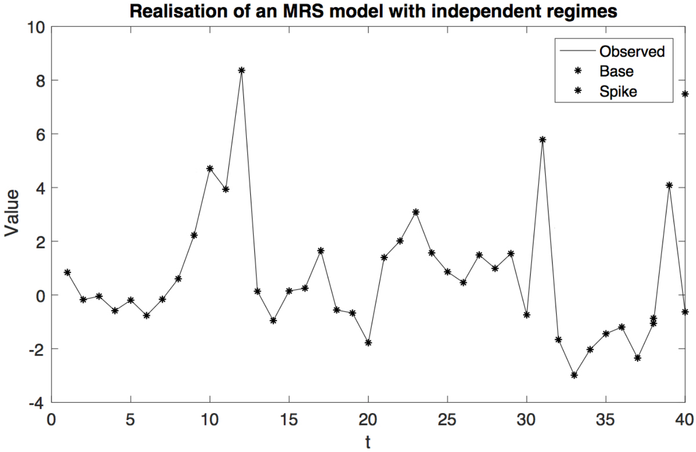
- R_t evolves with probabilities $p_{ij} = \mathbb{P}(R_t = j | R_{t-1} = i)$
- $B_t = \alpha + \phi B_{t-1} + \sigma \varepsilon_t$ - AR(1) base regime
  - to capture mean reversion
- $\log(S_t - q_3) \sim \mathcal{N}(\mu_S, \sigma_S^2)$ - Shifted log-normal spikes
  - Support $[q_3, \infty)$
- $\log(q_1 - D_t) \sim \mathcal{N}(\mu_D, \sigma_D^2)$ - Shifted log-normal drops
  - Support $(-\infty, q_1]$
- Note the independent regimes.

Stochastic model – Evolution & Dependence Structure



- The regime sequence determines which points we observe

Stochastic model – Evolution & Dependence Structure



- But this is all we actually observe

Inference

We use Bayesian Inference and specify uniform (improper) prior distributions and MCMC.

A three-regime model

$$X_t = \begin{cases} B_t^{(1)} & R_t = 1 \\ Y_t^{(3)} & R_t = 3 \\ Y_t^{(5)} & R_t = 5 \end{cases}$$

$$B_t^{(1)} = \alpha_1 + \phi_1 B_{t-1}^{(1)} + \sigma_1 \varepsilon_t,$$

$$Y_t^{(3)} - q_3 = LN(\mu_3, \sigma_3)$$

$$q_5 - Y_t^{(5)} = LN(\mu_5, \sigma_5)$$

- Common in the literature
- Our inference allocated very little mass to the drop regime

A two-regime model

$$X_t = \begin{cases} B_t^{(1)} & R_t = 1 \\ Y_t^{(3)} & R_t = 3 \end{cases}$$

$$B_t^{(1)} = \alpha_1 + \phi_1 B_{t-1}^{(1)} + \sigma_1 \varepsilon_t,$$

$$Y_t^{(3)} - q_3 = LN(\mu_3, \sigma_3)$$

- We cannot use typical model comparisons
  - e.g. AIC, BIC, likelihood ratio
- We check the model with Posterior Predictive Checks

Posterior Predictive Checks

Constructing PPCs

- Sample $\boldsymbol{\theta}^*$ and $\boldsymbol{R}^*$ from $p(\boldsymbol{\theta}, \boldsymbol{R}|\boldsymbol{x})$
- Produce statistics using $\boldsymbol{\theta}^*, \boldsymbol{R}^*$ and $\boldsymbol{x}$.
- Compare statistics to what we expect under the model
- Repeat for many samples and assess overall

Pros & Cons

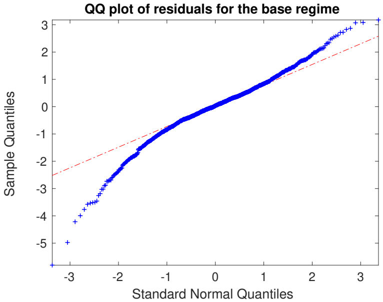
\+ Very flexible

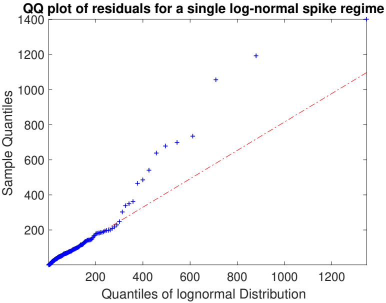
\+ Can tells us where a model fails

− Tend to make models look better than they are

A two-regime model – Posterior Predictive Checks

- Use the sample $\boldsymbol{R}^*$ to classify observations in to each regime
- Calculate residuals
- Create QQ plots





A two-regime model – Posterior Predictive Checks

- You can use PPCs to check almost any assumption you can think of!
- You can also calculate '*Bayesian p-values*'
  - Beware, they may be misleading!

Our best model

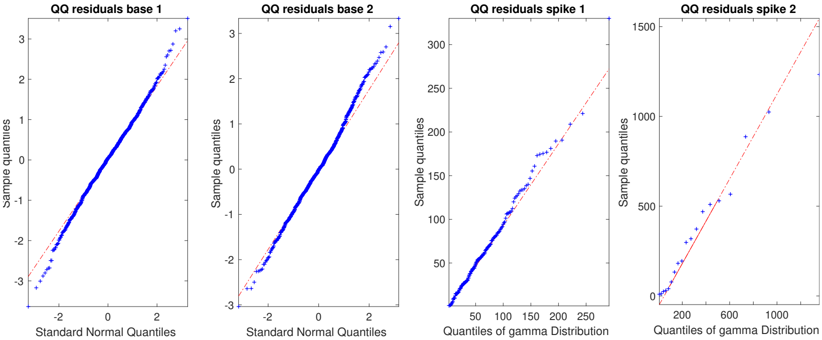
$$X_t = \begin{cases} B_t^{(1)} & R_t = 1 \\ B_t^{(2)} & R_t = 2 \\ Y_t^{(3)} & R_t = 3 \\ Y_t^{(4)} & R_t = 4 \end{cases}$$

$$B_t^{(i)} = \alpha_i + \phi_i B_{t-1}^{(i)} + \sigma_i \varepsilon_t, \qquad \sigma_1 < \sigma_2$$

$$Y_t^{(i)} - q_i = gamma(\mu_i, \sigma_i), \qquad q_3 < q_4$$

- Two AR(1) base regimes
- Two spike regimes

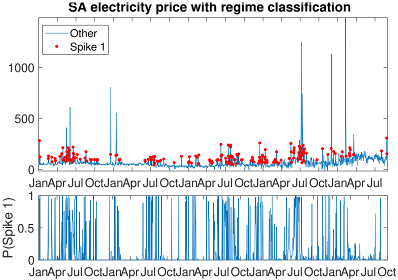
Our best model



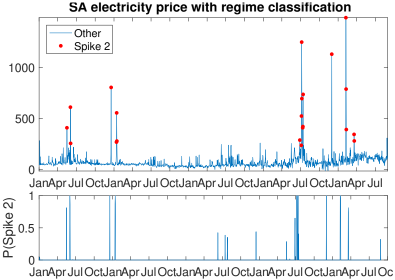
Better...

Our best model

We can use our inference to classify points into regimes.



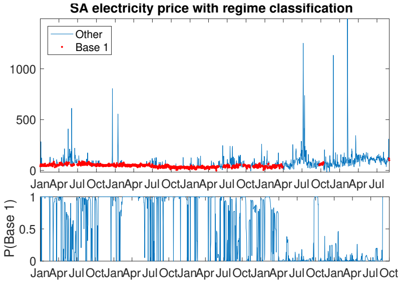
Spike regime 1

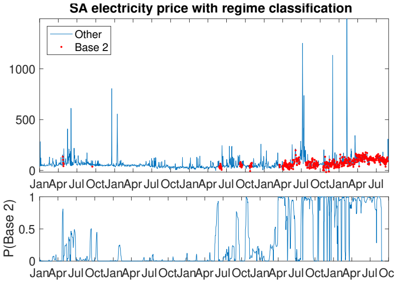


Spike regime 2 – Extreme spikes

Our best model

We can use our inference to classify points into regimes.





Base regime 1 – low volatility
$\sigma_1^2 = 53.5$

Base regime 2 – high volatility
$\sigma_2^2 = 535.9$

Final words

- For the SA market we found a 4-regime model is best
  - 2 base regimes, 2 spike regimes
  - The model automatically uncovers a structural change in volatility
  - Elon Musk to the rescue!
    - The battery should smooth generation & reduce market volatility
- Posterior predictive checks are good!
- Future work
  - Extend our model to actual spot prices
  - Extend our model to incorporate exogenous factors

THANKS!