Modelling Electricity Prices Using Regime Switching Models

Angus Lewis
Supervised by
Prof. Nigel Bean & Dr. Giang Nguyen

AUSTRALIAN RESEARCH COUNCIL CENTRE OF EXCELLENCE FOR MATHEMATICAL AND STATISTICAL FRONTIERS
THE UNIVERSITY of ADELAIDE

Electricity Spot Markets - Market participants

- Generators

Wind

Gas Turbine

Gas-fired

- Buyers

Steelworks

Retailers

AGL Energy in action®

origin

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Electricity Spot Markets - Market Operator

Electricity Spot Markets - Generators

Base load generation

- Short-term supply: fixed
- Low cost $\sim$ \$30 to \$45/MWh

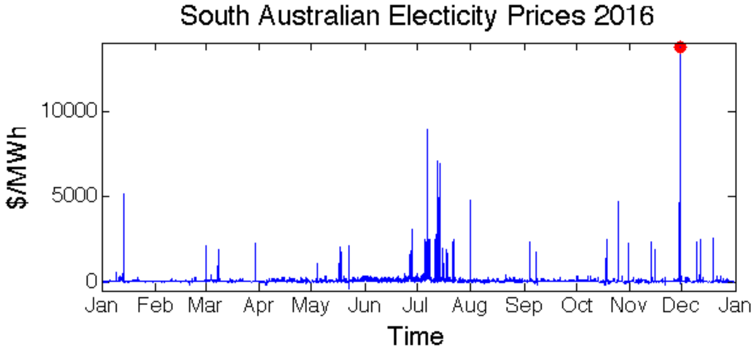
Gas-fired

Peak demand generation

- Short-term supply: variable
- High cost $\sim$ \$120 to \$185/MWh

Gas Turbine

Electricity Spot Markets



- December 1st, 2am - $13,766.58/MWh - Interconnector failure

Why we model prices

- Market participants face significant market risk
  - Cannot pass on to residential consumers
- Derivative contracts used to hedge risk
  - Valuation requires a model of the spot price

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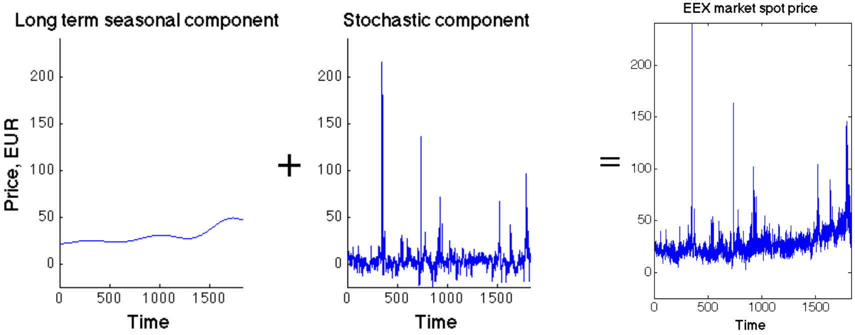
Modelling

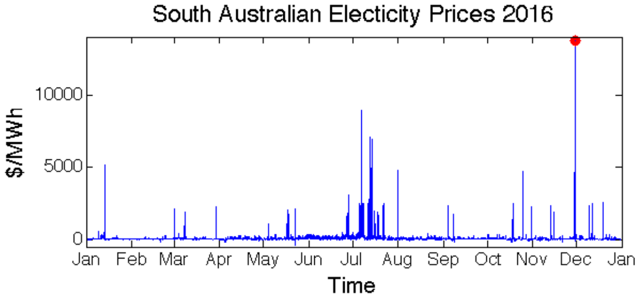
Model prices as the sum of two pieces

$$p_t = f_t + X_t$$

- f_t: Long term seasonal component (wavelet decomposition)
- X_t: Stochastic component



Stochastic component



Regime switching models

- base regime
- spike regime

R_t: regime process at time t,
$R_t, t \in \mathbb{Z}_+$: Markov chain

$$P = \begin{pmatrix} p_{b,b} & p_{b,s} \\ p_{s,b} & p_{s,s} \end{pmatrix}$$

Stochastic component

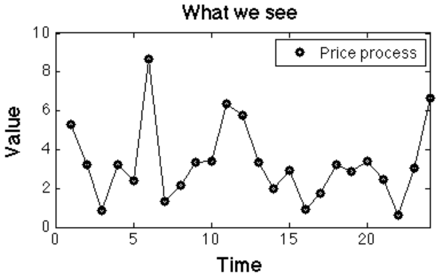
X_t: stochastic component

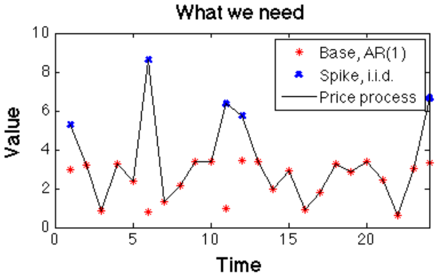
$$X_t = \begin{cases} B_t & \text{in the base regime, } R_t = \text{base} \\ S_t & \text{in the spike regime, } R_t = \text{spike} \end{cases}$$

- B_t: AR(1).
$$B_t = c + \phi B_{t-1} + \sigma_B \varepsilon_t,$$
  $\varepsilon_t \sim N(0,1)$
- S_t : i.i.d. LogNormal(μ, σ_s).

Parameter Estimation

- Don't know the function $\log L(\boldsymbol{\theta}|\mathbf{x})$,
  $\mathbf{x}$: Observed price data
- Regime process, R_t, is unobserved
- Some prices are unobserved





Expectation-Maximisation for regime switching models

- $\mathbf{x}$, observed prices.
- Regimes, $\mathbf{R}$.
- Missing prices, $\mathbf{Y}$.

1. Initialise a guess of $\hat{\boldsymbol{\theta}}$, $\boldsymbol{\theta}_0$, set $n = 0$
2. Given the current value of the sequence, $\boldsymbol{\theta}_n$, calculate

$$\begin{aligned} Q(\boldsymbol{\theta};\boldsymbol{\theta}_n) &= \mathbb{E}_{\mathbf{Y},\mathbf{R}}[\ell(\mathbf{x},\mathbf{Y},\mathbf{R}|\boldsymbol{\theta})|\boldsymbol{\theta}_n,\mathbf{x}] \\ &= \int_{\mathcal{Y}} \sum_{\mathbf{R}} \ell(\mathbf{x},\mathbf{Y},\mathbf{R}|\boldsymbol{\theta}) p(\mathbf{Y},\mathbf{R}|\boldsymbol{\theta}_n,\mathbf{x}) d\mathbf{Y}. \end{aligned}$$

3. Set $n = n + 1$ and

$$\boldsymbol{\theta}_{n+1} = \max_{\boldsymbol{\theta}\in\boldsymbol{\theta}} Q(\boldsymbol{\theta};\boldsymbol{\theta}_n),$$

return to Step 2.

Approximations

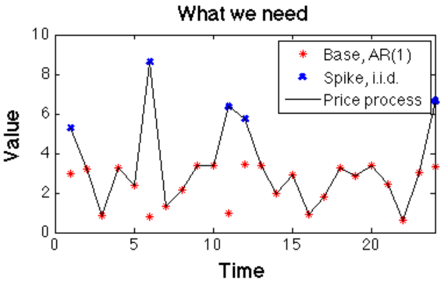
- Janczura and Weron [3] infer missing data.
  - Replace $\mathbf{Y}$ with $\bar{\mathbf{y}} = \mathbb{E}_{\mathbf{R}}[\mathbf{Y}|\boldsymbol{\theta}_n]$.

$$Q(\boldsymbol{\theta}; \boldsymbol{\theta}_n) = \sum_{\mathbf{R}} \ell(\mathbf{x}, \bar{\mathbf{y}}, \mathbf{R}|\boldsymbol{\theta}) p(\mathbf{R}|\mathbf{x}, \boldsymbol{\theta}_n)$$

- This method works well numerically, but no theoretical results.
- Further approximations are still needed to evaluate $p(\mathbf{R}|\mathbf{x}, \boldsymbol{\theta}_n)$.

Expectation-Maximisation algorithm output

- $\hat{\boldsymbol{\theta}} = (\hat{p}_{b,b}, \hat{p}_{s,s}, \hat{c}, \hat{\phi}, \hat{\sigma}_B, \hat{\mu}, \hat{\sigma}_S)$: Maximum likelihood estimate of the parameters
- Approximation of $P(R_t|\mathbf{x})$ - *'Smoothed inferences'*
  - The probability with which the observation x_t belongs to a regime
    (*soft* classification of states)



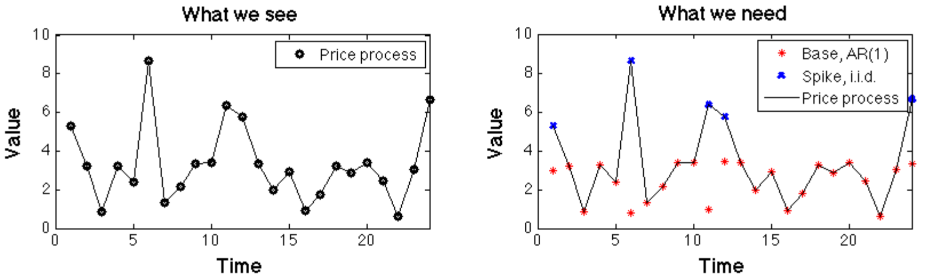
Parameter Estimation - Bayesian

Problem: Find the posterior $P(\boldsymbol{\theta}|\mathbf{x}) \propto L(\boldsymbol{\theta}|\mathbf{x})P(\boldsymbol{\theta})$.

Instead, find the posterior

$$P(\boldsymbol{\theta}, \mathbf{R}|\mathbf{x}) \propto L(\boldsymbol{\theta}, \mathbf{R}|\mathbf{x})P(\boldsymbol{\theta}, \mathbf{R}) = P(\mathbf{x}|\boldsymbol{\theta}, \mathbf{R})P(\boldsymbol{\theta}, \mathbf{R}).$$

- Note we no longer need the unobserved prices, $\mathbf{Y}$
- Expectation-Maximisation gives *soft* classification data
- The Bayesian approach proposes a *hard* classification of data



Markov Chain Monte Carlo

- Construct Markov Chain with stationary distribution

$$\boldsymbol{\pi} = P(\boldsymbol{\theta}, \mathbf{R}|\mathbf{x}).$$

- Problem: Find $P(p_{b,b}, p_{s,s}, c, \phi, \sigma_B, \mu, \sigma_S, \mathbf{R}|\mathbf{x})$.
  i.e. explore $\Theta \times \{0,1\}^T$.
  - Θ is the parameter space.
  - T is the number of data points.

Hybrid Metropolis-Hastings/Gibbs Sampler

1. Initialise and set $n = 0$.
2. Gibbs sampler: $p_{b,b}, p_{s,s}, \mathbf{R}$.
   - Conditional proposal can be derived.
3. MH algorithm: $c, \phi, \sigma_B, \mu, \sigma_S$.
4. Set $n = n + 1$, go to 2.

Parameter Estimation

$$X_t = \begin{cases} B_t & \text{when } R_t = \text{base}, \\ S_t & \text{when } R_t = \text{spike}, \end{cases}$$

$$B_t\text{: AR(1)}, \; B_t = c + \phi B_{t-1} + \sigma_B \varepsilon_t,$$

$$S_t \sim \text{i.i.d Normal}(\mu, \sigma_s).$$

Table: Simulated data

	True parameters	MCMC mean	J&W (EM-like)
$P_{b,b}$	0.95	0.962	0.963
$P_{s,s}$	0.9	0.890	0.894
c	10	10.20	9.758
ϕ	0.2	0.183	0.219
σ_B	1	1.067	1.053
μ	16	15.98	15.97
σ_s	1	1.210	1.192

Parameter Estimation

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$$B_t\text{: AR(1), } B_t = c + \phi B_{t-1} + \sigma_B \varepsilon_t,$$

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Table: European Energy Exchange

	MCMC mean	J&W (EM-like)
$P_{b,b}$	0.9663	0.9773
$P_{s,s}$	0.5618	0.7878
c	0.5645	0.6036
ϕ	0.7189	0.7080
σ_B	14.63	15.15
μ	22.07	19.72
σ_s	1049	880.6

Summary

- Introduced the spot market
- Introduced regime switching models
- Expectation-Maximisation
  - Cannot be applied directly
  - Approximations used, EM-*like* algorithm
- Bayesian Inference
  - Does not suffer the same problems
  - MCMC methods: approximate posterior distributions

Future Work

- Model Comparison
- Spike frequency dependent on exogenous variables
  - Temperature
  - Season
  - Generation methods available (e.g. wind, gas, coal)
- Janczura & Weron [2]: Regime switching is time dependent. Becker *et al.* [1]: Base regime is not auto regressive.
  - Combine these models

References

Ralf Becker, Stan Hurn, and Vlad Pavlov.
Modelling spikes in electricity prices.
Economic Record, 83(263):371–382, 2007.

Joanna Janczura and Rafał Weron.
An empirical comparison of alternate regime-switching models for electricity spot prices.
Energy Economics, 32(5):1059–1073, 2010.

Joanna Janczura and Rafał Weron.
Efficient estimation of Markov regime-switching models: An application to electricity spot prices.
AStA Advances in Statistical Analysis, 96(3):385–407, 2012.

Chang-Jin Kim.
Dynamic linear models with Markov-switching.
Journal of Econometrics, 60(1-2):1–22, January-February 1994.