Markovian Regime-Switching Models for South Australian Wholesale Electricity Prices

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Outline

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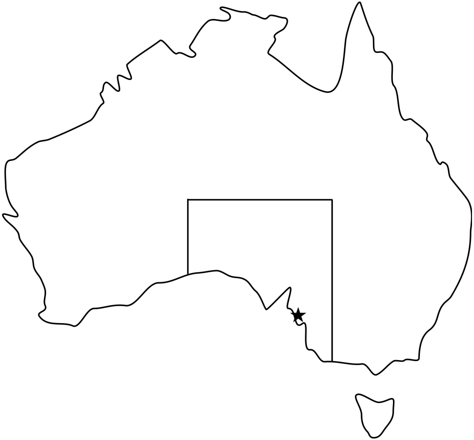
Australia and the Netherlands

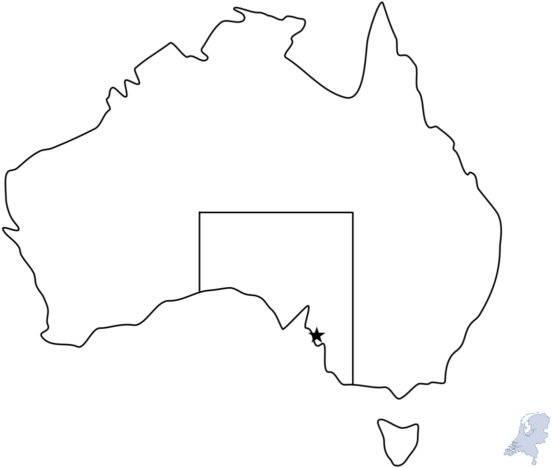


Population

- Australia: 24M
- Adelaide: 1.2M

- Netherlands: 17M
- Eindhoven: 220,000

Australia and the Netherlands



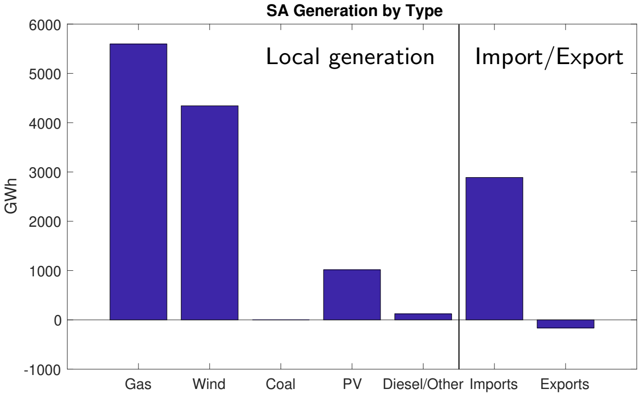
Population density

- Australia: 3.1ppl/km^2
- SA: 1.7ppl/km^2
- Netherlands: 488ppl/km^2

The National Electricity Market (NEM)

- SA has its own market
- Can trade via interconnectors

South Australia's Generation Resources

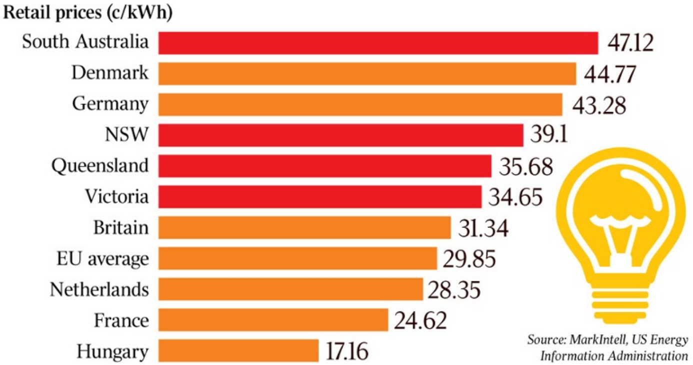


November 2017 - Generation (% of local generation only)

- 39.2% Wind
- 50.5% Gas
- 9.2% Rooftop solar
- No coal – 20.9% generation capacity withdrawn

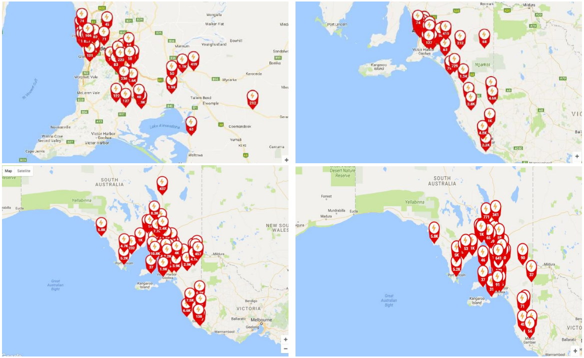
Other Problems in SA

- Highest *retail* electricity price in the world! – August 2016



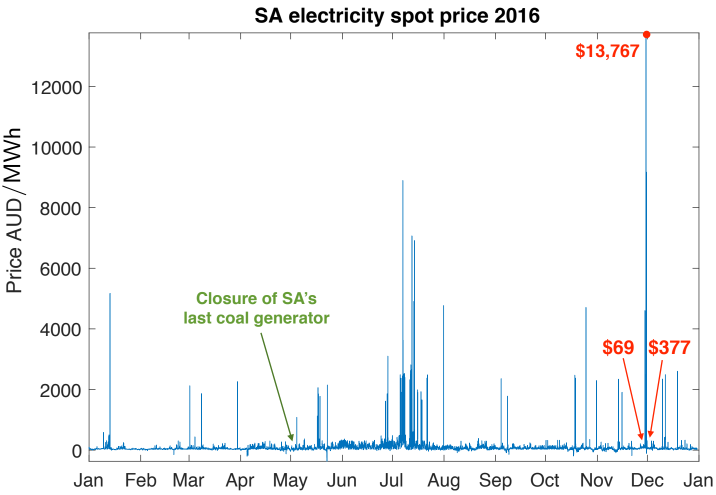
Other Problems in SA

- Other frequent blackouts – Every year!
  - It gets hot - 13 days $> 40^{\circ}$c in 2014
  - *'Another day, another blackout for angry South Australians'* – The Australian
  - *'Median cost of the blackout on SA businesses was $5,000'* – ABC News



Outline

Wholesale spot prices



- AUD$13,767 = EUR€8,785

About the market

Every 5 minutes AEMO

- Aggregates supply bids
- Estimates demand
- Matches supply and demand
  - Dispatches generators
  - Sets the *dispatch* price
  - Spot price = average over 30 minutes

Spikes occur due to

- Unprecedented demand/Incorrect supply forecast
- Low marginal cost generators cannot vary supply quickly

Our aims

What I am doing

- Model South Australian wholesale electricity prices

Why I am doing it

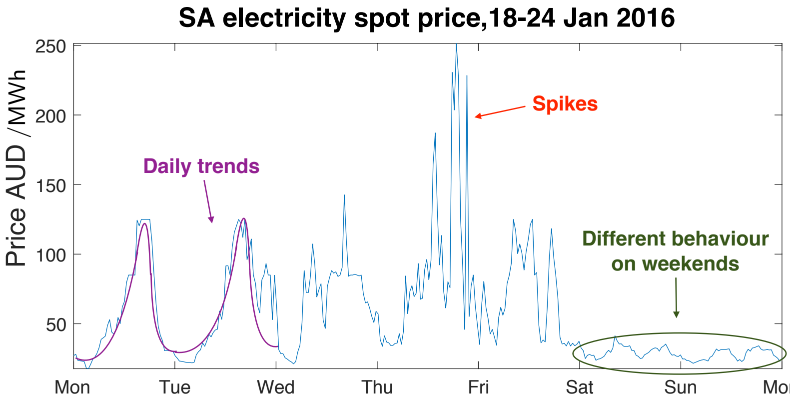
- Risk management
- Value contracts & investments
- Examine affects of exogenous factors on prices

How I am doing it

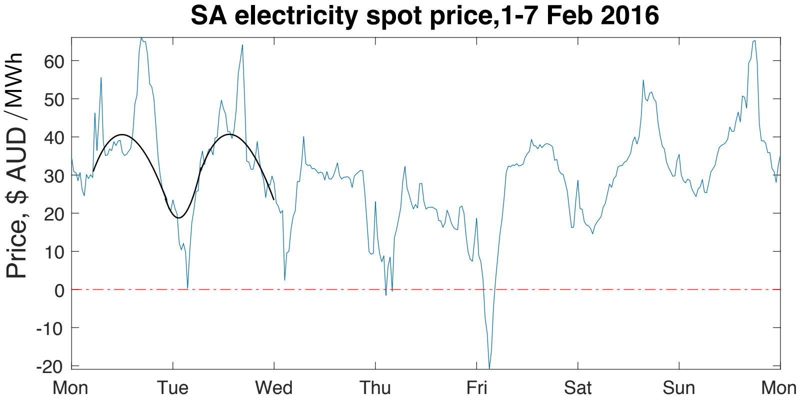
- Use regime-switching models with independent regimes
- 'Exact' inference using data augmented MCMC

Outline

Price data characteristics



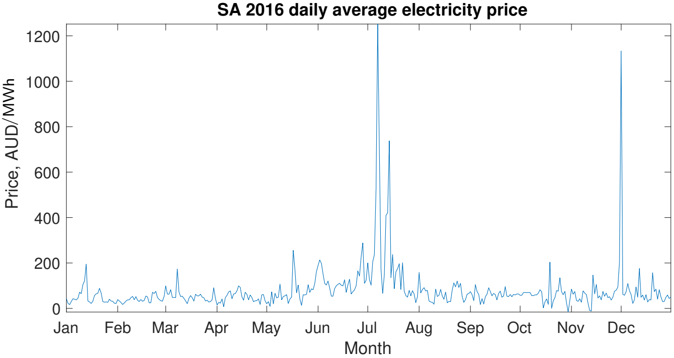
Price data characteristics



- Mean reversion - to a trend line
- Negative prices!

Simplifying the problem

- Model average daily price
  - Justification: Some contracts are valued on daily average prices - e.g. EEX futures
  - Drawback: Not all contracts are valued in this way
- Displays mean reversion, spikes, drops and trends



Outline

The model

$$\overbrace{P_t}^{\text{price process}} = \overbrace{y_t}^{\text{trend component}} + \overbrace{X_t}^{\text{stochastic component}}$$

Trend, y_t

- Capture different behaviour on weekends/weekdays
- Seasonal fluctuations

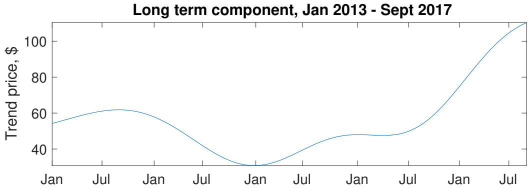
Regime-switching model, X_t

- Mean reversion
- Spikes
- Drops

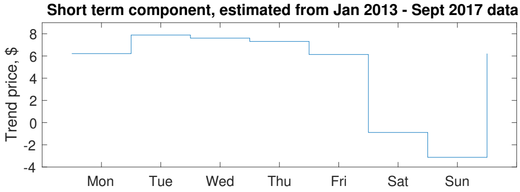
Trend

$$y_t = \text{weekly trend} + \text{long-term trend}$$

- Long-term trend
  - Estimated using wavelet filtering



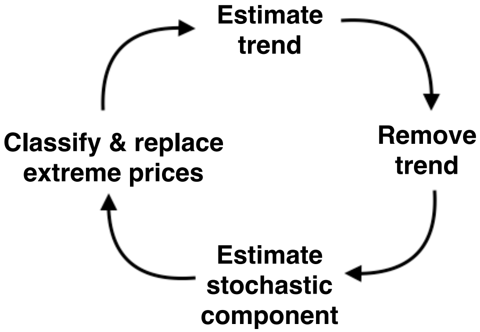
- Weekly trend



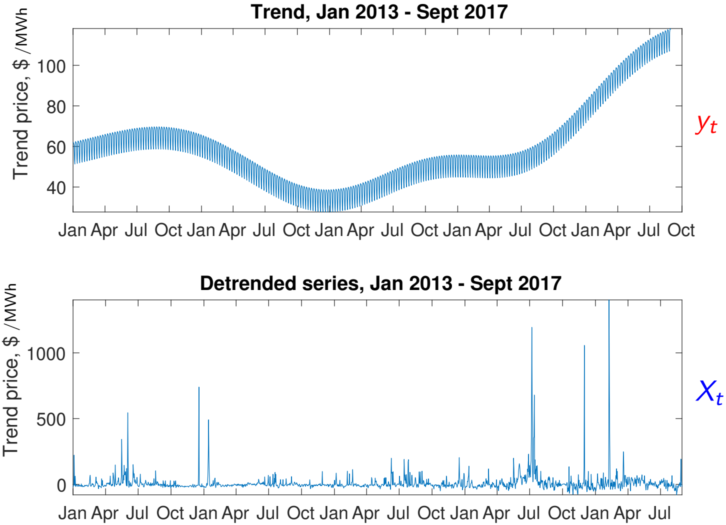
Trend estimation

Extreme prices bias our estimate of the trend.

- Solution: remove and replace extreme values



Stochastic component



Markovian Regime-switching model

3-regimes with shifted log-normal spikes and drops

$$X_t = \begin{cases} B_t & \text{when } R_t = 1, \\ S_t & \text{when } R_t = 2, \\ D_t & \text{when } R_t = 3. \end{cases}$$

Markovian Regime-switching model

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- $B_t = \alpha + \phi B_{t-1} + \sigma \varepsilon_t$ - AR(1) base regime
  - to capture mean reversion

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- $\log(S_t - q_3) \sim \mathcal{N}(\mu_S, \sigma_S^2)$ - Shifted log-normal spikes
  - Support $[q_3, \infty)$

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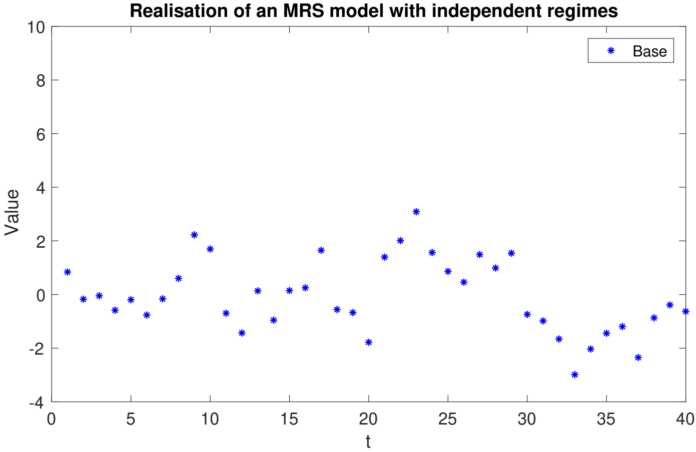
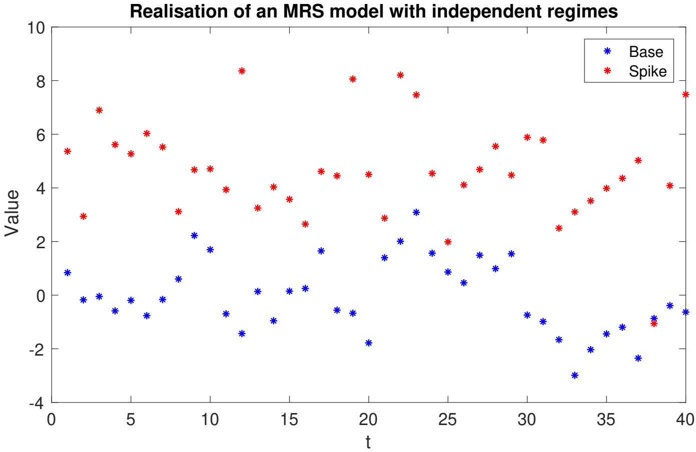
- $\log(S_t - q_3) \sim \mathcal{N}(\mu_S, \sigma_S^2)$ - Shifted log-normal spikes
  - Support $[q_3, \infty)$
- $\log(q_1 - D_t) \sim \mathcal{N}(\mu_D, \sigma_D^2)$ - Shifted log-normal drops
  - Support $(-\infty, q_1]$
- Note the independent regimes.

Stochastic model – Evolution & Dependence Structure



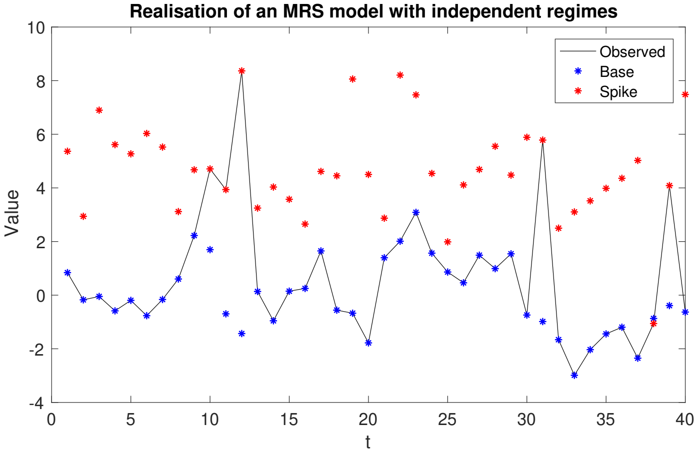
- AR(1) Base process

Stochastic model – Evolution & Dependence Structure



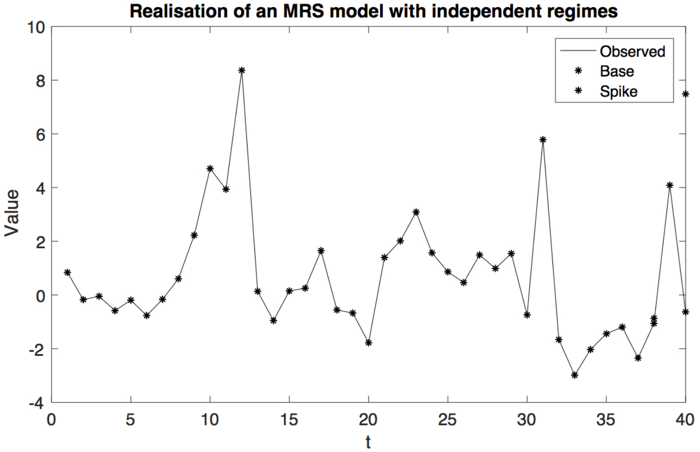
- Spike process – Independent of Base process

Stochastic model – Evolution & Dependence Structure



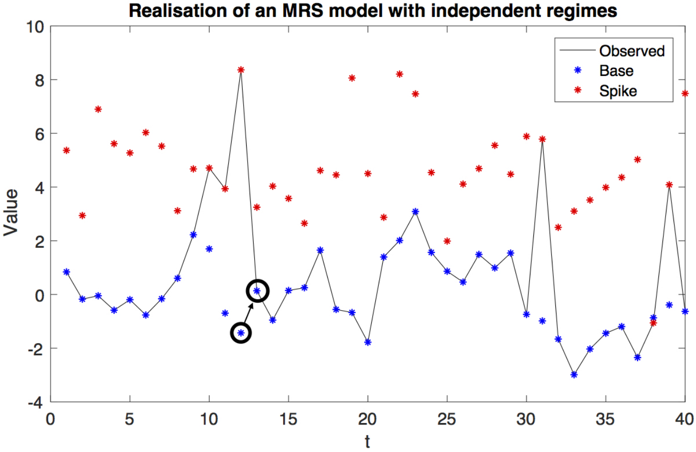
- The regime sequence determines which points we observe

Stochastic model – Evolution & Dependence Structure



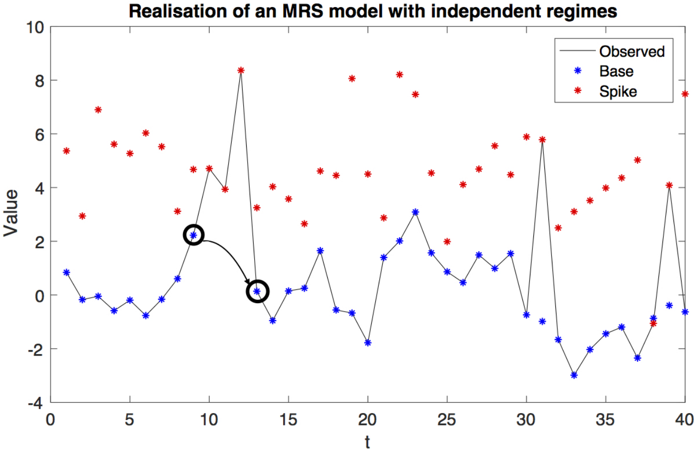
- But this is all we actually observe

Stochastic model – Evolution & Dependence Structure



- B_t depends on B_{t-1}, but it might be unobserved

Stochastic model – Evolution & Dependence Structure



- Can integrate unobserved prices away
- B_t depends on a random lagged observation

Outline

Inference

The likelihood

$$L(\theta; \boldsymbol{x}) = f(\boldsymbol{x}|\theta) = \sum_{\boldsymbol{R}} f(\boldsymbol{x}, \boldsymbol{R}|\theta) = \sum_{\boldsymbol{R}} f(\boldsymbol{x}|\boldsymbol{R}, \theta)\mathbb{P}(\boldsymbol{R}|\theta)$$

- Sum over all sequences of length $T = 1704 =$ the number of observations
- $3^{1704} = 1.034 \times 10^{813}$ such sequences!

Inference

The likelihood

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- Sum over all sequences of length $T = 1704 =$ the number of observations
- $3^{1704} = 1.034 \times 10^{813}$ such sequences!

Consequences

- A MLE approach is computationally intractable
- EM algorithm is computationally intractable
- Existing literature uses an approximation to EM
- Instead we use data-augmented MCMC

Inference

Data-augmented block-wise MCMC

- Recall:

$$p(\theta|\boldsymbol{x}) = \frac{\overbrace{p(\boldsymbol{x}|\boldsymbol{\theta})}^{\text{likelihood}}\, p(\boldsymbol{\theta})}{p(\boldsymbol{x})}$$

- Likelihood:

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \sum_{\boldsymbol{R}} p(\boldsymbol{x}|\boldsymbol{R}, \boldsymbol{\theta}) p(\boldsymbol{R}|\boldsymbol{\theta})$$

where $\boldsymbol{R} = (R_0, R_1, ..., R_T)$ is a sequence of hidden regimes

- Solution:

$$p(\boldsymbol{R}, \boldsymbol{\theta}|\boldsymbol{x}) = \frac{\overbrace{p(\boldsymbol{x}|\boldsymbol{R}, \boldsymbol{\theta}) p(\boldsymbol{R}|\boldsymbol{\theta})}^{\text{augmented likelihood}}\, p(\boldsymbol{\theta})}{p(\boldsymbol{x})}$$

Inference

Data-augmented block-wise MCMC

- A block-wise structure (aka. Metropolis-within-Gibbs) makes our MCMC more efficient

Outline

A three-regime model

$$X_t = \begin{cases} B_t^{(1)} & R_t = 1 \\ Y_t^{(3)} & R_t = 3 \\ Y_t^{(5)} & R_t = 5 \end{cases}$$

$$B_t^{(1)} = \alpha_1 + \phi_1 B_{t-1}^{(1)} + \sigma_1 \varepsilon_t,$$

$$Y_t^{(3)} - q_3 = LN(\mu_3, \sigma_3)$$

$$q_5 - Y_t^{(5)} = LN(\mu_5, \sigma_5)$$

- Common in the literature
- Our inference allocated very little mass to the drop regime

A two-regime model

$$X_t = \begin{cases} B_t^{(1)} & R_t = 1 \\ Y_t^{(3)} & R_t = 3 \end{cases}$$

$$B_t^{(1)} = \alpha_1 + \phi_1 B_{t-1}^{(1)} + \sigma_1 \varepsilon_t,$$

$$Y_t^{(3)} - q_3 = LN(\mu_3, \sigma_3)$$

- We cannot use typical model comparisons
  - e.g. AIC, BIC, likelihood ratio
- We check the model with Posterior Predictive Checks

Posterior Predictive Checks

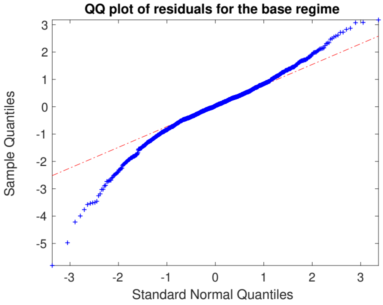
Constructing PPCs

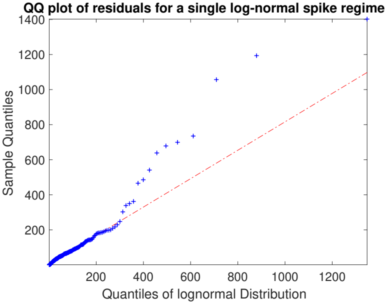
- Sample $\boldsymbol{\theta}^*$ and $\boldsymbol{R}^*$ from $p(\boldsymbol{\theta}, \boldsymbol{R}|\boldsymbol{x})$
- Produce statistics using $\boldsymbol{\theta}^*, \boldsymbol{R}^*$ and $\boldsymbol{x}$.
- Compare statistics to what we expect under the model
- Repeat for many samples and assess overall

Pros & Cons

+ Very flexible
+ Can tells us where a model fails
− Tend to make models look better than they are

A two-regime model





Our best model

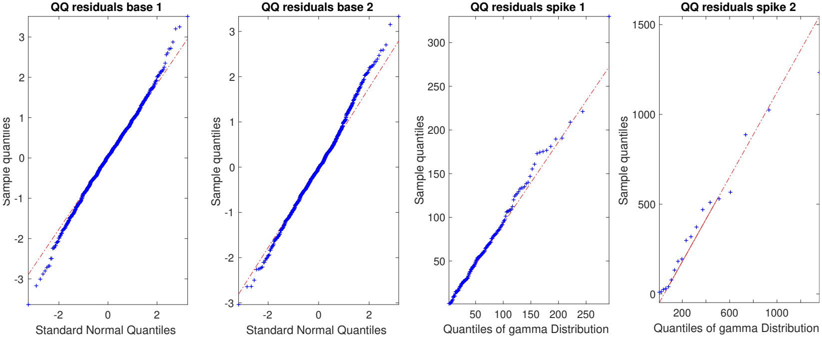
$$X_t = \begin{cases} B_t^{(1)} & R_t = 1 \\ B_t^{(2)} & R_t = 2 \\ Y_t^{(3)} & R_t = 3 \\ Y_t^{(4)} & R_t = 4 \end{cases}$$

$$B_t^{(i)} = \alpha_i + \phi_i B_{t-1}^{(i)} + \sigma_i \varepsilon_t, \qquad \sigma_1 < \sigma_2$$

$$Y_t^{(i)} - q_i = gamma(\mu_i, \sigma_i), \qquad q_3 < q_4$$

- Two AR(1) base regimes
- Two spike regimes

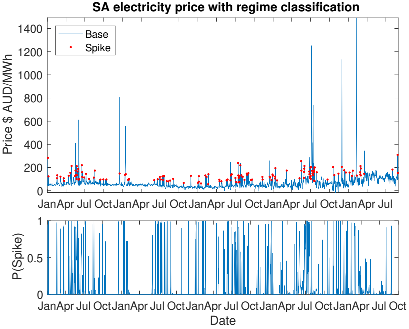
Our best model



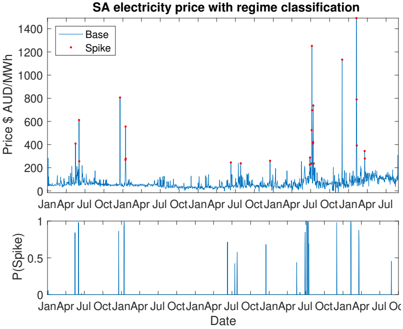
Better...

Our best model

We can use our inference to classify points into regimes.



Spike regime 1



Spike regime 2 – Extreme spikes

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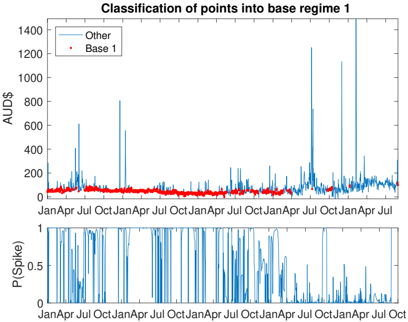

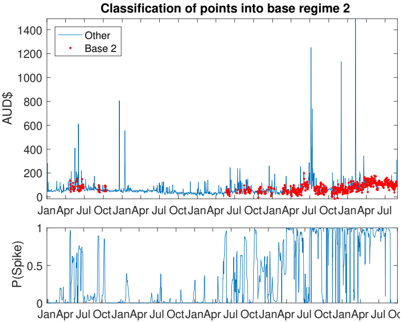

Base regime 1 – low volatility
$$\sigma_1^2 = 55.94$$

Base regime 2 – high volatility
$$\sigma_2^2 = 482.53$$

Final words

- For the SA market we found a 4-regime model is best
  - 2 base regimes, 2 spike regimes
  - The model automatically uncovers a structural change in volatility
  - Elon Musk to the rescue!
    - The battery should smooth generation & reduce market volatility
- Future work
  - Extend our model to actual spot prices
  - Extend our model to incorporate exogenous factors

THANKS!