Trend estimation for electricity prices

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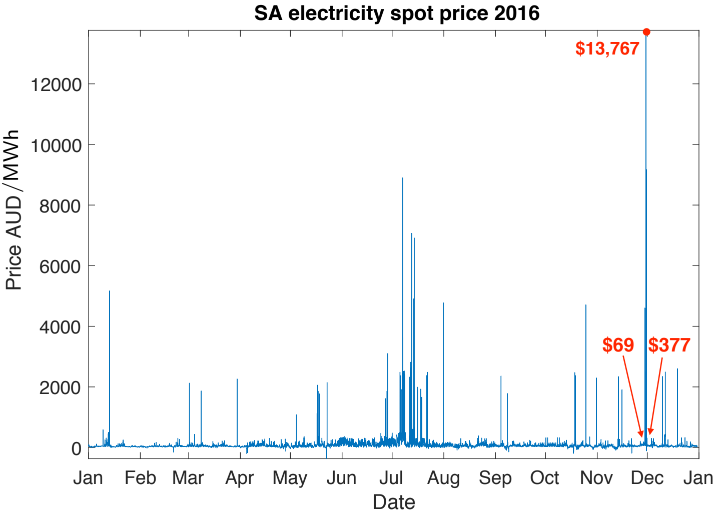
ANZIAM, February 2019, Nelson

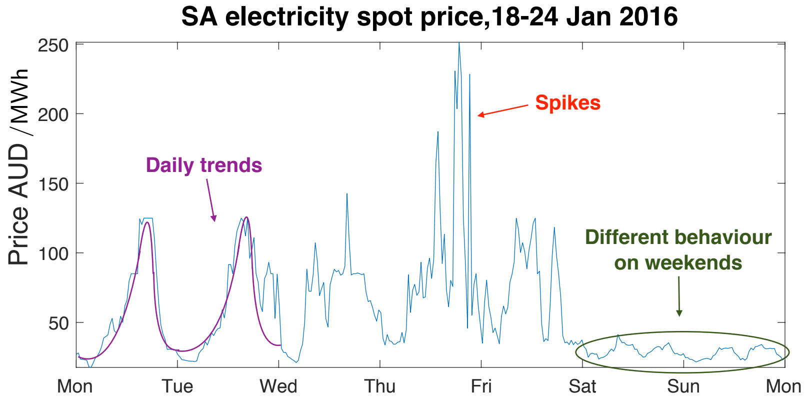
Motivation

- Forecasting
- Risk management
- Examine factors affecting prices

Electricity prices



Electricity prices



Price setting

- Every 5 minutes AEMO
  - Aggregates supply bids
  - Estimates demand
  - Matches supply and demand
    - Dispatches generators
    - Sets the *dispatch* price
    - Spot price = average over 30 minutes
- One-sided market
- Inelastic demand
- Cheap generators cannot vary supply easily – the most expensive generators can

Price Model

$$P_t = T_t + X_t$$

- T_t - Trend component
- X_t - Stochastic Component - MRS model

MRS models

$$X_t = \begin{cases} B_t & \text{when } R_t = 1, \\ S_t & \text{when } R_t = 2. \end{cases}$$

- R_t is a (latent) Markov chain: $p_{ij} = \mathbb{P}(R_t = j | R_{t-1} = i)$
- B_t
  - Base prices
  - AR(1) process – $B_t = \alpha + \phi B_{t-1} + \sigma \varepsilon_t$
  - $\varepsilon_t \sim$ i.i.d. $N(0, 1)$
- S_t
  - Spikes
  - i.i.d. – typically Gamma or Log-Normal

Trend Model, T_t

- Deterministic
- Must capture long/short term movements

Common models

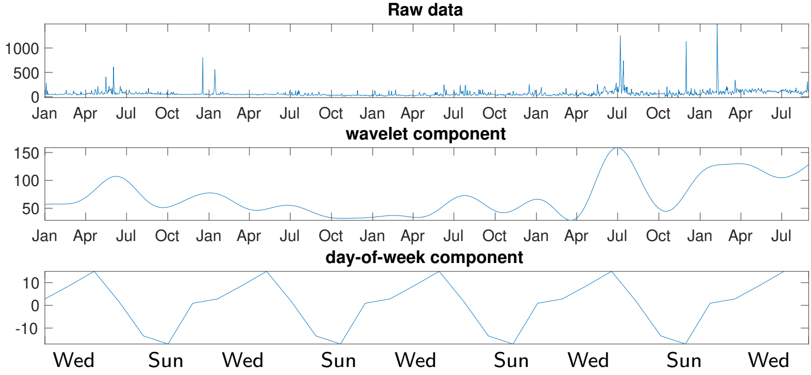
- Wavelets
  - Non-parametric
  - Flexible, non-periodic – localised in frequency and time
  - Can perform well for parameter estimation
- Fourier/other sinusoidal models - Periodic
- Indicator functions

We use wavelets + day-of-week indicator functions

Simple trend estimation

Output:

- trend – wavelet component + day-of-week component
- residual noise



Extreme prices may have a large effect

Filtering extreme prices

- How do we identify extreme prices?

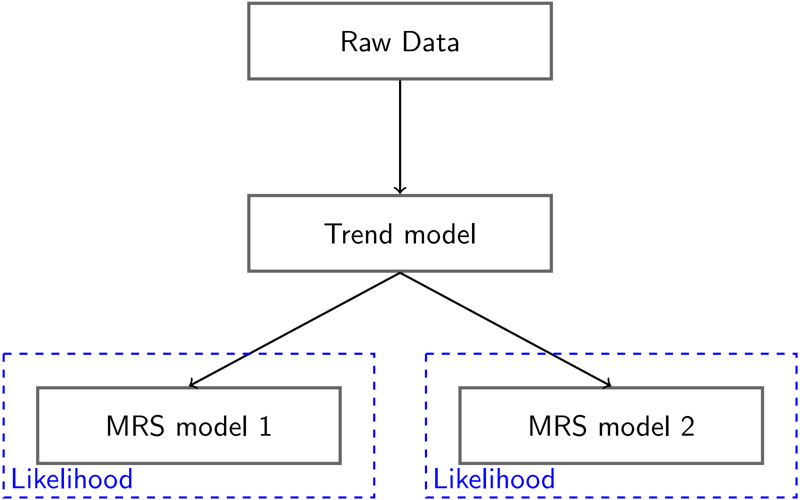
How do we identify extreme prices?[1]

Stochastic model independent

- Fixed price threshold: $> 3\sigma$ from mean
- Variable price threshold: $> 3\sigma$ from moving average
- Threshold on price changes: top 10% of price changes
  - What threshold to use? Somewhat arbitrary

[1] Janczura *et at.* 2013, *Identifying spikes and seasonal components in electricity spot price data: A guide to robust modeling*, Energy Economics

Model-independent trends



How do we identify extreme prices?[2]

Stochastic model-based

- Fit an MRS model and use the output to classify prices

[2] Janczura *et at.* 2013, *Identifying spikes and seasonal components in electricity spot price data: A guide to robust modeling*, Energy Economics

Model estimation

Stochastic model

- Bayesian inference using Data-Augmented MCMC
- Maximum likelihood

Both output parameter estimates and

$$P(R_t = i | x_0, ..., x_T)$$

Use these to classify prices into regime k if

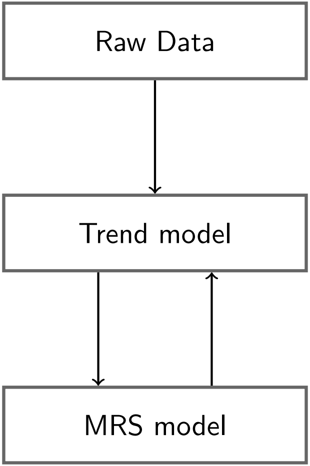
$$k = \text{argmax}_i P(R_t = i | x_0, ..., x_T).$$

Def: A price is classified as extreme if k does not belong to a base regime.

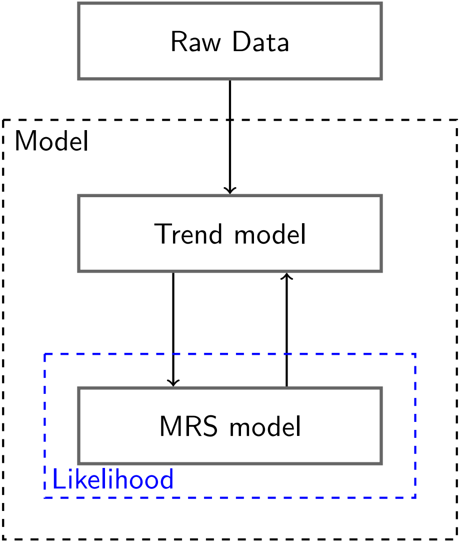
MRS classification

- Iterative scheme
  0. Estimate T_t from raw data
  1. Estimate MRS model on residuals
  2. Classify and replace extreme prices in raw data
  3. Estimate T_t from filtered prices
  4. Repeat 1.-3.
- Replace extreme prices with the current value of the trend estimate
- Simulations suggest this is good for recovering parameters
- Note: trend model depends on MRS model!

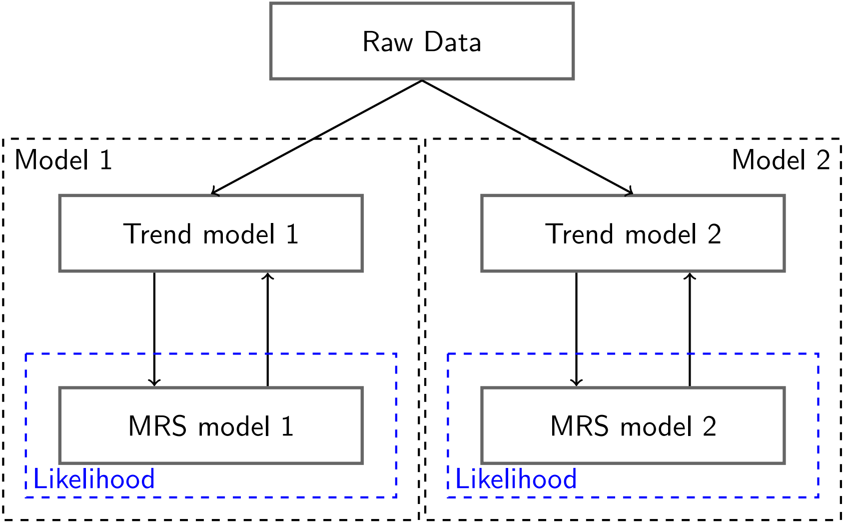
MRS model-dependent trends



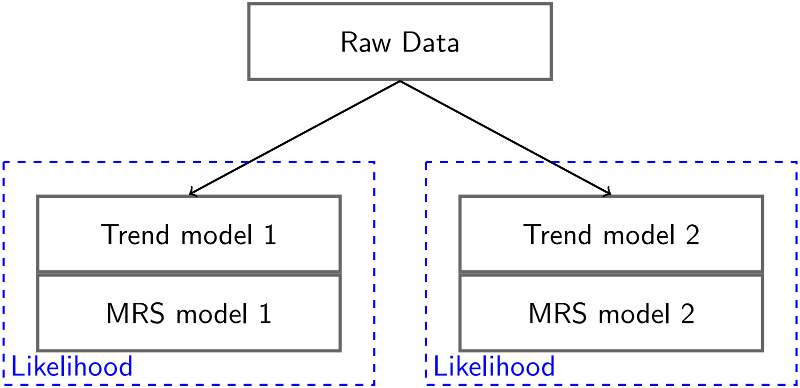
MRS model-dependent trends



MRS model-dependent trends – Model comparison



Model-dependent trend estimation



A proposed approach

- Include trend in MRS model and estimate with ML or Bayesian inference.

$$X_t = \begin{cases} T_t + B_t & R_t = 1, \\ S_t & R_t = 2 \end{cases}$$

- Complexity of T_t is limited
  - identifiability
  - it's just not practical
  - can we still use wavelets?

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Summary of methods

MRS model independent (i.e. threshold methods)

- somewhat arbitrary
- no likelihood $\Longrightarrow$ no likelihood-based model comparisons for **trends**
- can use likelihood-based model comparisons for **stochastic components**

MRS model-based

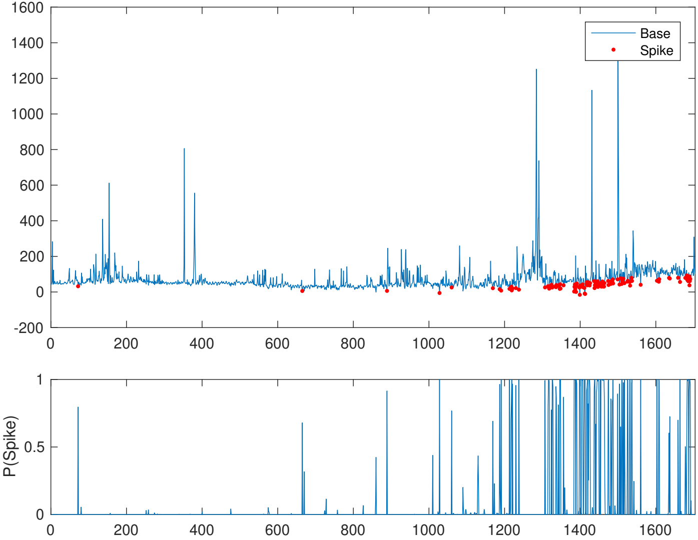
- natural classification of extreme prices
- trend depends on MRS specification
- cannot use likelihood-based methods to compare models – trend or stochastic

Model selection

Start with a simple model from literature

$$X_t = \begin{cases} B_t = \alpha + \phi B_{t-1} + \sigma \varepsilon_t & R_t = 1 \\ S_t - q_3 \text{ i.i.d. } LN(\mu_2, \sigma_2^2) & R_t = 2 \\ q_1 - D_t \text{ i.i.d. } LN(\mu_3, \sigma_2^2) & R_t = 3. \end{cases}$$

Model Selection



Model selection

Remove the drops

$$X_t = \begin{cases} B_t = \alpha + \phi B_{t-1} + \sigma\varepsilon_t & R_t = 1 \\ S_t - q_3 \text{ i.i.d. } LN(\mu_2, \sigma_2^2) & R_t = 2 \end{cases}$$

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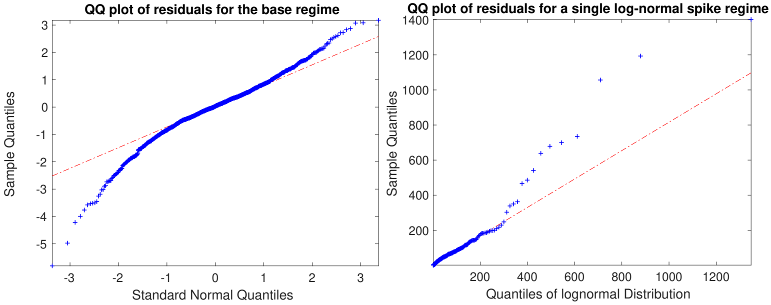

Model selection

Add base regime

$$X_t = \begin{cases} B_t^{(1)} = \alpha_1 + \phi_1 B_{t-1}^{(1)} + \sigma_1 \varepsilon_t^{(1)} & R_t = 1 \\ B_t^{(2)} = \alpha_2 + \phi_2 B_{t-1}^{(2)} + \sigma_2 \varepsilon_t^{(2)} & R_t = 2 \\ S_t - q_3 \text{ i.i.d. } LN(\mu_3, \sigma_3^2) & R_t = 3 \end{cases}$$

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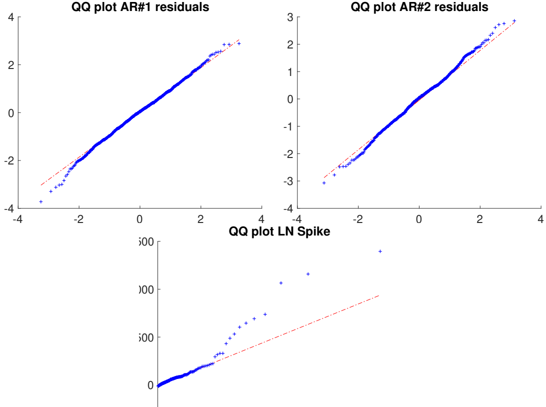

Model selection

Add spike regime

$$X_t = \begin{cases} B_t^{(1)} = \alpha_1 + \phi_1 B_{t-1}^{(1)} + \sigma_1 \varepsilon_t^{(1)} & R_t = 1 \\ B_t^{(2)} = \alpha_2 + \phi_2 B_{t-1}^{(2)} + \sigma_2 \varepsilon_t^{(2)} & R_t = 2 \\ S_t^{(1)} - q_3 \text{ i.i.d. } Gamma(\mu_3, \sigma_3^2) & R_t = 3 \\ S_t^{(2)} - q_{98} \text{ i.i.d. } Gamma(\mu_4, \sigma_4^2) & R_t = 4 \end{cases}$$

Model selection

Add spike regime

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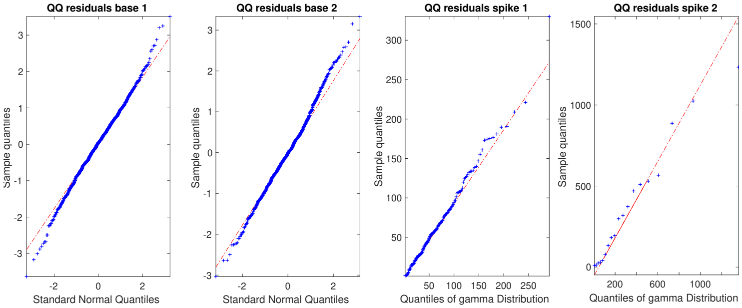
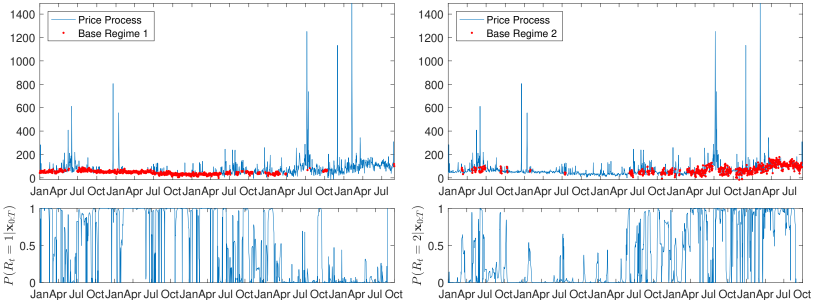


Model selection



Model selected!

Parameter		Parameter	
α_1	-0.0658	q_3	11.9
ϕ_1	0.532	μ_3	2.50
σ_1^2	50.6	σ_3^2	21.9
α_2	0.280	q_4	168
ϕ_2	0.415	μ_4	2.50
σ_2^2	382	σ_4^2	104.6

$$\text{Transition Matrix} \begin{pmatrix} 0.929 & 0.008 & 0.062 & 0.000 \\ 0.000 & 0.906 & 0.092 & 0.002 \\ 0.313 & 0.260 & 0.377 & 0.050 \\ 0.062 & 0.048 & 0.456 & 0.433 \end{pmatrix}$$