Upper bounding the commuting operator value of a nonlocal game

Angus Lowe

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Overview

- Nonlocal games, classical strategies, linear programming

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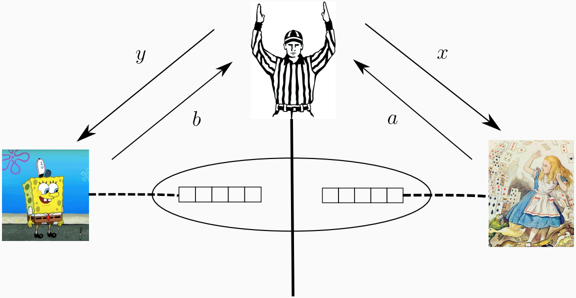
- Convergence to the commuting operator value
- Conclusion

Nonlocal games



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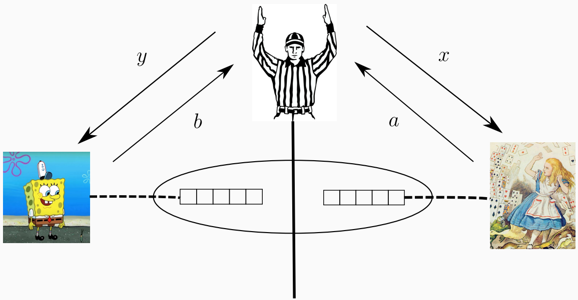

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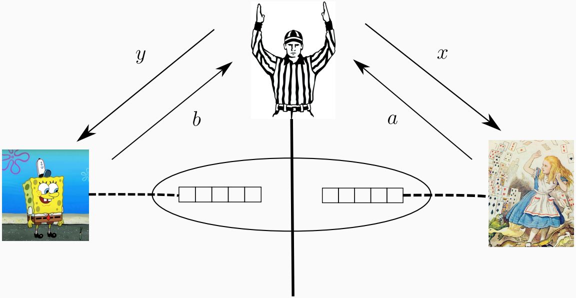

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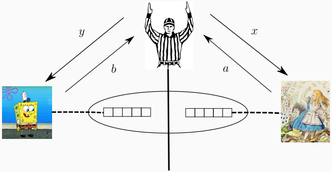
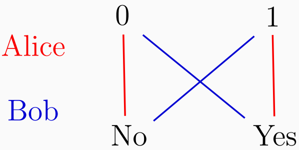
$$\text{win prob.} = \sum_{x,y} \pi(x, y) \sum_{a,b} p_{abxy} V(a, b, x, y) \tag{1}$$

Example: CHSH game





- $a, b, x, y \in \{0, 1\}$. x, y given uniformly at random.

$$V(a, b, x, y) = \begin{cases} 1 & \text{if } a \oplus b = x \wedge y \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

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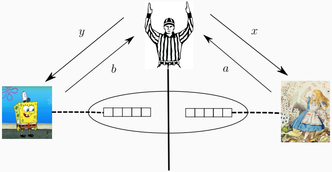

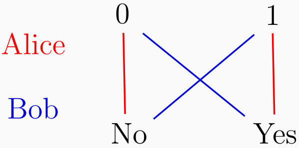

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- The maximum winning probability is $3/4$

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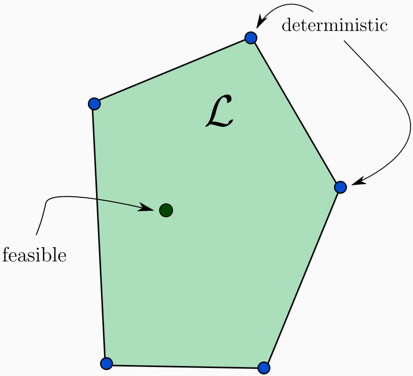

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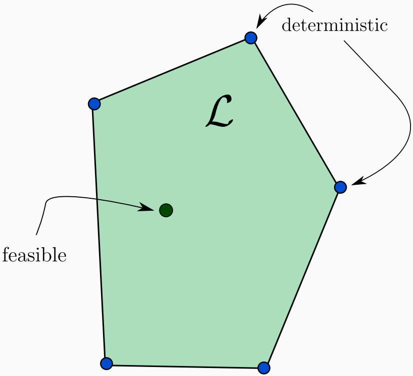

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$$\begin{aligned} \max_{p} \quad & \text{win prob.} \\ \text{s.t.} \quad & p \in \mathcal{L} \end{aligned}$$

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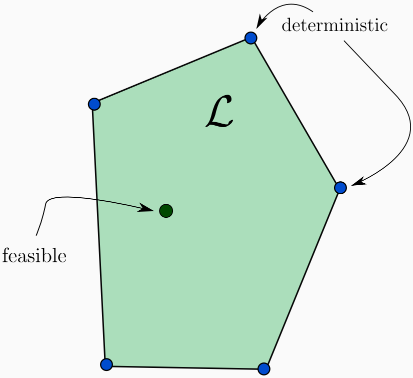

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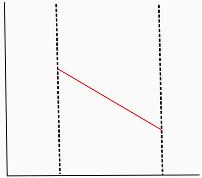
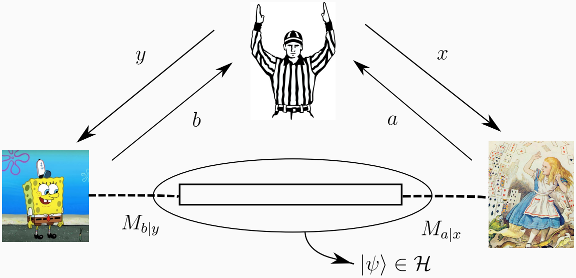


Quantum strategies (commuting operator)



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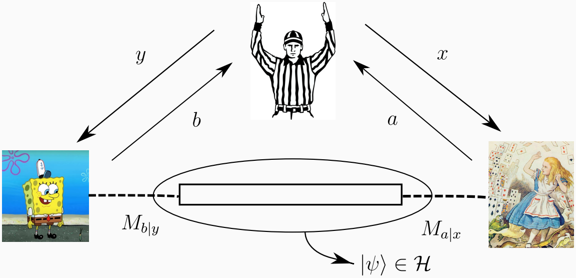

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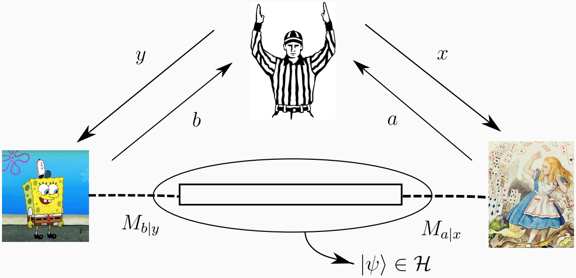

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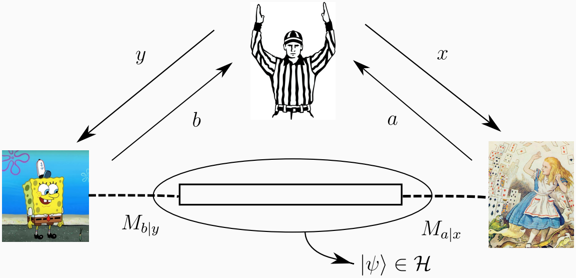

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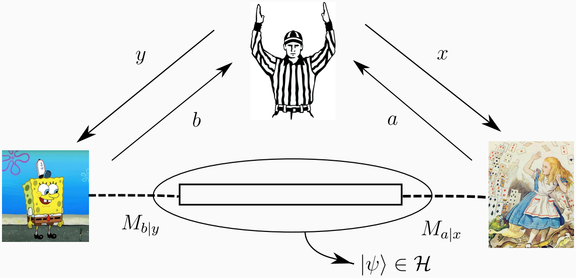

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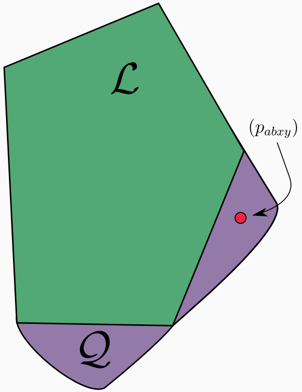
From now on, we refer to these as the 3 CO requirements.

Preview: the (commuting operator) quantum set



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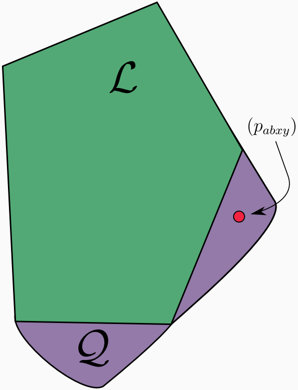

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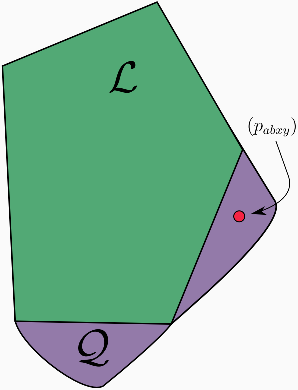

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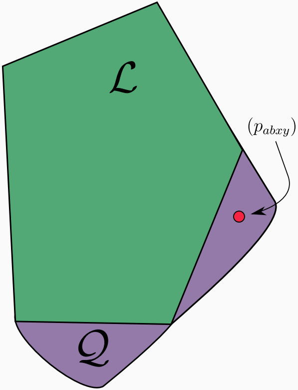

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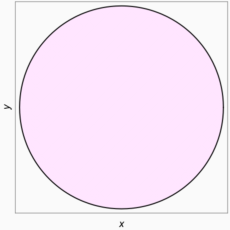

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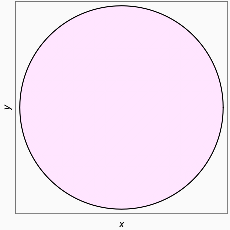

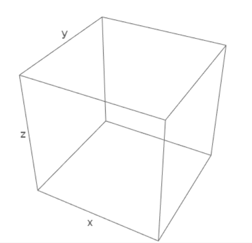

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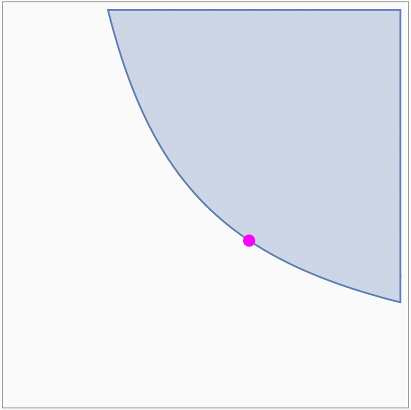

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- Then we can define the $|\mathcal{S}_n| \times |\mathcal{S}_n|$ moment matrix Γ^n with entries $\Gamma^n_{S,T} = \langle\psi|S^\dagger T|\psi\rangle$ for all $S, T \in \mathcal{S}_n$. [BCP$^+$14]

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1. $\Gamma^n \succeq 0$. Why? $\sum_{S,T} v_S^* \langle \psi | S^\dagger T | \psi \rangle v_T = \langle \psi | A^\dagger A | \psi \rangle \geq 0$.

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3. There is a submatrix of Γ^n containing all values p_{abxy}.

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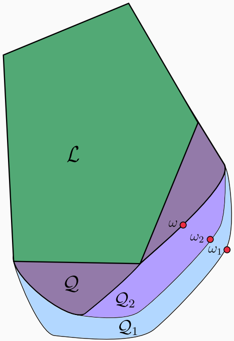

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CHSH example revisited

- Tsirelson's bound [Cir80] tells us the quantum value of the CHSH game is ≈ 0.85.
- For this game, it happens that $\mathcal{Q}^1 = \mathcal{Q}$!

```
In[28]:= res =
       -SemidefiniteOptimization[g[ps], cons[Γ, ps],
         {Γ ∈ Matrices[{5, 5}, Reals], ps ∈ Vectors[16, Reals]}, "PrimalMinimumValue"]

Out[28]= 0.853553
```

Convergence to the quantum value

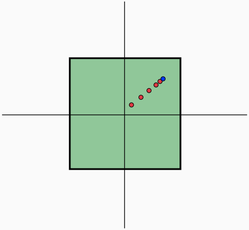
Theorem (Sufficiency of the NPA hierarchy)

Let p be a strategy such that there exists a valid moment matrix Γ^n for all $n \geq 1$. Then p is in the quantum set.

1)

$$\Gamma^n = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \to \hat{\Gamma}^n = \begin{bmatrix} \cdot & \cdot & \cdot & 0 & 0 & \dots \\ \cdot & \cdot & \cdot & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$\to x^n := \mathrm{vec}(\Gamma^n) = [\dots, 0, 0, \dots]^T$$



$$x^n \in B_\infty(0,1)$$

$$\implies x^{n_i} \to x^\infty \iff \hat{\Gamma}^{n_i} \to \Gamma^\infty$$

Banach-Alaoglu

Convergence to the quantum value

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Let p be a strategy such that there exists a valid moment matrix Γ^n for all $n \geq 1$. Then p is in the quantum set.

2)

There exists infinite set of vectors $V = \{|S\rangle : |S| = 1, 2, \dots\}$ such that

$$\Gamma^{\infty}_{S,T} = \langle S|T\rangle \quad (\text{sequential Cholesky decomposition})$$

Then take $\mathcal{H} = \mathrm{span}(V)$, and measurement operators

$$\hat{E}_{a|x} = \mathrm{proj}(\mathrm{span}(\{|M_{a|x}S\rangle : |M_{a|x}S| = 1, 2, \dots\})).$$

One can verify the following:

1. $\hat{E}_{a|x}|\mathbb{1}\rangle = |M_{a|x}\rangle \implies \langle \mathbb{1}|\hat{E}^{\dagger}_{a|x}\hat{E}_{b|y}|\mathbb{1}\rangle = \Gamma^{\infty}_{M_{a|x},M_{b|y}} = p_{abxy}$.
2. $\hat{E}_{a|x}\hat{E}_{a'|x} = \delta_{aa'}\hat{E}_{a|x}$.
3. $[\hat{E}_{a|x}, \hat{E}_{b|y}] = 0$. $\square$

Conclusion

- $\omega_1 \geq \omega_2 \geq \cdots \geq \omega :=$ quanutm value.

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- For tensor product quantum strategies, it was shown in [JNV$^+$20] that $\mathrm{MIP}^* = \mathrm{RE}$. Perhaps $\mathrm{MIP}^{\mathrm{co}} = \mathrm{coRE}$?
- Also interesting for foundations – the existence of a quantum analogue of a joint probability measure implies the conditions of $\mathcal{Q}^1$ [DHW14].
    - How do we rule out alternate versions of quantum theory?

References i

- Nicolas Brunner, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner.
  Bell nonlocality.
  Reviews of Modern Physics, 86(2):419–478, Apr 2014.
- B. S. Cirel'son.
  Quantum generalizations of bell's inequality.
  Letters in Mathematical Physics, 4(2):93–100, March 1980.
- Fay Dowker, Joe Henson, and Petros Wallden.
  A histories perspective on characterizing quantum non-locality.
  New Journal of Physics, 16(3):033033, mar 2014.
- Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen.
  Mip*=re, 2020.

Miguel Navascués, Stefano Pironio, and Antonio Acín.
A convergent hierarchy of semidefinite programs characterizing the set of quantum correlations.
New Journal of Physics, 10(7):073013, jul 2008.