

# Tarski Fixed Point Computation and the Arrival Problem

Angus Joshi

University of Edinburgh

*s1712180@ed.ac.uk*

December 2, 2023

## 1 Basic Definitions and Algorithmic Results

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# The integer lattice, monotone functions, Tarski's theorem

## Definition (Bounded Integer Lattice)

The *bounded  $d$ -dimensional integer lattice*  $[N]^d = \{1, \dots, N\}^d$  is equipped with a lattice ordering where for  $(x_1, \dots, x_d), (x'_1, \dots, x'_d) \in [N]^d$ ,  $(x_1, \dots, x_d) \leq (x'_1, \dots, x'_d)$  if  $x_i \leq x'_i$  for each  $i \in [d]$ .

## Definition (Monotone function)

A function  $f : [N]^d \rightarrow [N]^d$  is monotone if whenever  $x, x' \in [N]^d$  with  $x \leq x'$ ,  $f(x) \leq f(x')$ .

## Theorem (Tarski)

If  $f : [N]^d \rightarrow [N]^d$  is monotone, then there is a point  $x \in [N]^d$  with  $f(x) = x$ .

# The problem, Lower Bounds

## Definition ( $\text{TARSKI}$ )

The problem  $\text{TARSKI}(N, d)$  is, given oracle access to a monotone function  $f : [N]^d \rightarrow [N]^d$ , find a point  $x \in [N]^d$  such that  $f(x) = x$ .

## Theorem (Etesami, Papadimitriou, Rubinfeld, Yannakakis)

*The query complexity of  $\text{TARSKI}(N, 2)$  is  $\Theta(\log^2 N)$ .*

## Corollary

*The query complexity of  $\text{TARSKI}(N, d)$  is  $\Omega(\log^2 N)$  for  $d \geq 2$ .*

# Known Upper Bounds

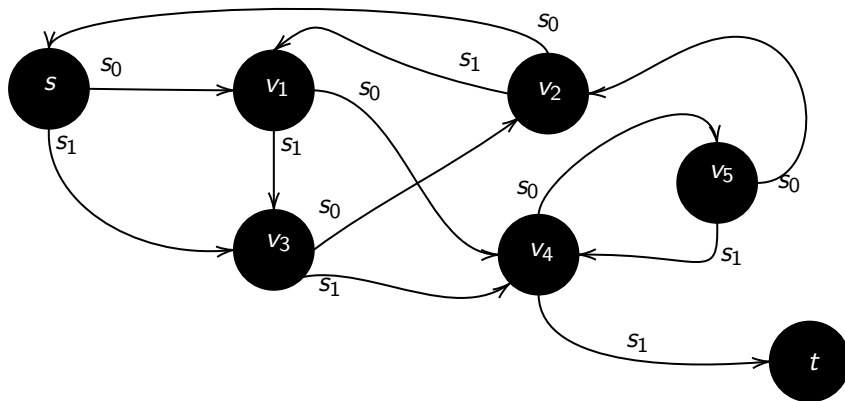
## Theorem (Fearnley, Pálvölgyi, Savani)

*The query complexity of TARSKI  $(N, 3)$  is  $\Theta(\log^2 N)$ .*

## Theorem (Chen, Li)

*The query complexity of TARSKI  $(N, d)$  is  $O(\log^{\lceil (d+1)/2 \rceil} N)$ .*

# The Arrival Problem



Theorem (Gärtner, Haslebacher, Hoang)

*ARRIVAL is polynomial-time reducible to TARSKI.*

# Progress and Next Steps

- Progress
  - Explored solving the 4-dimensional TARSKI problem,
  - Implemented Fearnley, Pálvölgyi, Savani algorithm,
  - Applied the algorithm to randomly generated instances of ARRIVAL.
- Next Steps
  - Explore solving the 4-dimensional TARSKI problem in the special case of monotone functions corresponding to arrival instances.



# Experimental Data

