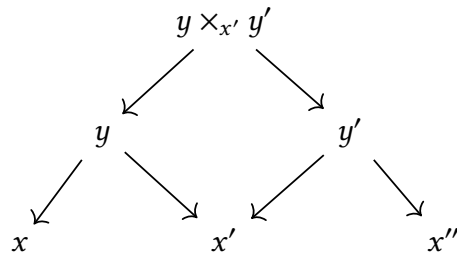


# 1 Introduction

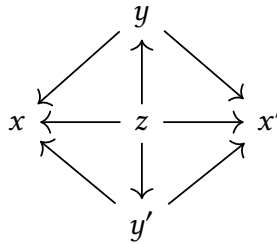
## 1.1 Description

### 1.1.1 Categories of correspondences

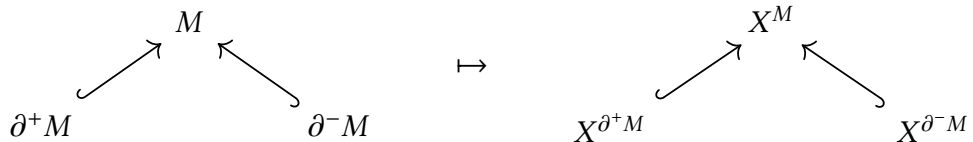
- $\mathcal{C} \rightsquigarrow \mathbf{Corr}(\mathcal{C}) = \mathbf{Span}(\mathcal{C})$



- $\mathcal{C}^{\otimes} \rightsquigarrow \mathbf{Corr}(\mathcal{C})^{\otimes}$  (monoidal structure)
- $\mathcal{C} \rightsquigarrow \mathbf{Corr}_2(\mathcal{C}), \mathbf{Corr}_n(\mathcal{C})^{\otimes}$  (where perhaps  $\otimes$  is a symmetric monoidal  $n$ -category)



$$\text{TFT}: M \mapsto X^M$$

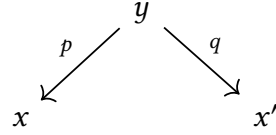


Haugsgeng: Iterated Spans and Classical TFTs

$$\mathbf{Bord}_n^{\otimes} \rightarrow \mathbf{Corr}_n(\mathcal{C})^{\otimes} \overset{?}{\dashrightarrow} \mathbf{Vect}^{\otimes}$$

### 1.1.2 Transfer theories

Let  $F: \mathcal{C} \rightarrow \mathcal{D}$ , and let  $x \in \mathcal{C}$ .



## 1.2 Motivation

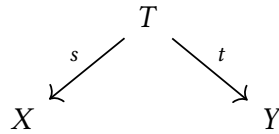
Let  $X$  and  $Y$  be sets, say representing the physical states of some physical system. For example, imagine a photon sent out by some emitter, passing through a screen with holes, and then being detected. The set  $X$  might be

{left hole, middle hole, right hole},

and the set  $Y$  might be

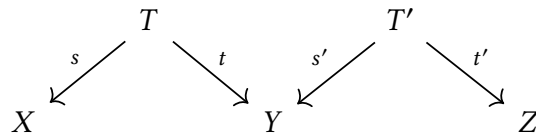
{detected at  $A$ , detected at  $B$ }.

Consider a set  $T$ , equipped with maps  $s: T \rightarrow X$  and  $t: T \rightarrow Y$ .

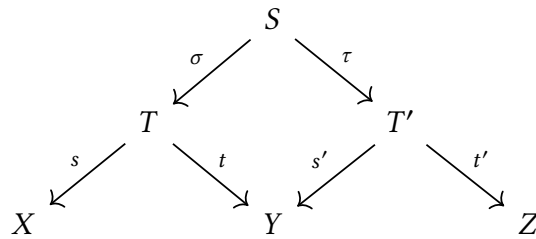


We can think of  $T$  as consisting of a set of ways of going from an element of  $X$  to an element of  $Y$ ; that is, (using the universal property for products) the fiber  $T_y^x := (s, t)^{-1}(x, y)$  can be thought of as the set of ways of going from  $x \in X$  to  $y \in Y$ . In our example, the fiber  $T_y^x$  would correspond to the set of classical trajectories starting at hole  $x$  and ending at detector  $y$ .

Consider concatenating two such processes.



Taking the pullback in **Set** gives us a set of ways of going from  $X$  to  $Z$ .



That is, the fibers  $S_z^x$  consist of pairs  $(t, t')$ , where  $t$  is a process from  $x$  to  $y$  and  $t'$  is a process from  $y$  to  $z$ .

For a set  $X$ , denote by  $L(X)$  the free  $\mathbb{C}$ -vector space on  $X$ . Consider a span  $T: X \rightarrow Y$  as before. We can assign to  $T$  a linear map

$$L(T): L(X) \rightarrow L(Y)$$