1 Higher categories

1.1 2-categories

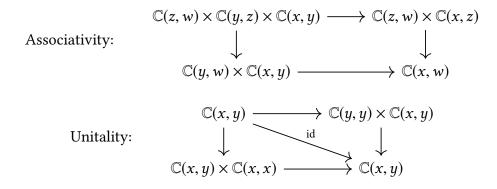
Definition 1 (2-category). A <u>2-category</u> is a category enriched over categories. More explicitly, a 2-category \mathbb{C} consists of the following.

- A set Obj(ℂ) of objects.
- For every two objects $x, y \in \text{Obj}(\mathbb{C})$, a category $\mathbb{C}(x, y)$ of morphisms.
- For every object $x \in \text{Obj}(\mathbb{C})$, an object $\text{id}_x \in \text{Obj}(\mathbb{C}(x, y))$.
- For every three objects x, y, z, a functor

$$\mu \colon \mathbb{C}(y,z) \times \mathbb{C}(x,y) \to \mathbb{C}(x,z)$$

implementing composition.

These must make the following diagrams commute.



Example 2. The category Cat is a 2-category.

- The objects Obj(Cat) are given by small categories.
- The morphisms Cat(C, D) is the category [C, D].
- The identity morphism is the identity functor. We have already seen that this is a functor.
- The composition is given by composition of functors. We must show that this is functorial, i.e. that

These satisfy the conditions:

- 1. Indeed, the identity functor functions as a left and right identity, and whiskering by the identity is the identity.
- 2. Indeed, composition of functors is associative, and vertical composition is associative, although I don't know a nice way to prove this.