1 More on model categories

1.1 Combinatorial model categories

In general, it is difficult to construct model categories. However, in certain special cases we can leverage existing tools. One such tool is the notion of a combinatorial model category.

Definition 1 (presentable category)**.** Let $\mathcal{C}$ be a category. We say that $\mathcal{C}$ is presentable if it satisfies the following conditions:

1. The category $\mathcal{C}$ admits all (small) colimits.
2. There exists some set S of objects of $\mathcal{C}$ which generates $\mathcal{C}$ under small colimits in the sense that any object of $\mathcal{C}$ is a colimit of a diagram taking values in S.
3. Every object in $\mathcal{C}$ is small, i.e. compact in the sense of Definition **??** for some small regular cardinal κ.
4. Each hom-set $\mathrm{Hom}(X, Y)$ in $\mathcal{C}$ is small; that is, $\mathcal{C}$ is locally small.

Note 2. Condition 3. is equivalent to demanding that only the objects of S be compact.

Definition 3 (combinatorial model category)**.** Let $\mathcal{A}$ be a model category. We say that $\mathcal{A}$ is combinatorial if the following conditions are satisfied.

1. The category $\mathcal{A}$ is presentable.
2. There exists a set I of *generating cofibrations* such that $\mathcal{C}\mathrm{of} = \overline{I}$, the saturated hull of I.
3. There exists a set J of *generating trivial cofibrations* such that $\mathcal{C}\mathrm{of} \cap \mathcal{W} = \overline{J}$, the saturated hull of J.

Definition 4 (perfect class of morphisms)**.** Let $\mathcal{C}$ be a presentable category, and W a class of morphisms of $\mathcal{C}$. We say that W is perfect if the following conditions are satisfied.

1. The class W contains all isomorphisms.
2. The class W satisfies the 2/3 law.
3. The class W is stable under filtered colimits, in the sense that for each filtered poset I and functors $F, G \colon I \to \mathcal{C}$ such that each morphism in I

1.2 Homotopy limits and colimits